Asset Ownership and Contractibility of Interaction

Andreas Roider∗
University of Bonn†

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Abstract

In a property-rights framework, I study how organizational form and quantity contracts interact in generating investment incentives. The model nests standard property-rights and hold-up models as special cases. I admit general message-dependent contracts, but provide conditions under which non-contingent contracts are optimal. First, I contribute to the foundation of the property-rights theory: I characterize under which circumstances its predictions are correct when trade is contractible. Second, I study how the optimal use of the incentive instruments depends on the environment. Finally, the model offers a new perspective on the classic Fisher Body case, and it produces implications that are empirically testable.

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†University of Bonn, Wirtschaftspolitische Abteilung, Adenauerallee 24-42, 53113 Bonn, Germany; ++49-(0)228-739246; andreas.roider@wiwi.uni-bonn.de.
1 Introduction

The property-rights theory of the firm (henceforth PRT) addresses fundamental questions initially raised by Coase (1937). Namely, why are certain transactions conducted within firms and not in markets, and hence what determines the boundaries of the firm? Grossman and Hart (1986), Hart and Moore (1990), and Hart (1995) approach this question by defining a firm as a collection of non-human assets. They study how the allocation of ownership rights influences the incentives to engage in non-verifiable, relationship-specific investments. They assume that only the allocation of property rights can be specified in a contract, and derive optimal ownership structures endogenously. Their incomplete contracts approach has become a cornerstone of recent discussions about the boundaries of the firm.

However, asset ownership often interacts with other instruments in generating investment incentives. This aspect has been neglected by the PRT, but has been emphasized by Holmstrom and Milgrom (1994), Holmstrom and Roberts (1998), and Holmstrom (1999). These authors argue that a theory of the firm should not ignore explicit and implicit contracts: ownership patterns often cannot be explained by property-rights considerations alone. The literature on the hold-up problem provides support for this criticism. There, simple trade contracts that specify trade quantities and payments often suffice to induce first-best investments.\(^1\) In these models the boundaries of the firm would be irrelevant because the parties would be able to sign the optimal quantity contract independent of the underlying ownership structure. In this light, it is somewhat surprising that some empirical studies find support for the PRT when contractual provisions regarding the level of activity are persuasive (see e.g., Elfenbein and Lerner, 2003).

\(^1\)See e.g., Chung (1991), Hermalin and Katz (1993), Aghion, Dewatripont, and Rey (1994), Nöldeke and Schmidt (1995) and Edlin and Reichelstein (1996). However, Hart and Moore (1999), Segal (1999) and Che and Hausch (1999) provide conditions under which the absence of a trade contract is optimal.
Several questions emerge from this discussion. First, under what conditions are the predictions of the PRT correct when trade or, more generally, the degree of interaction is contractible? Besides its theoretical relevance, an answer to this question would generate new testable hypotheses, and might lead to a better understanding of observed ownership patterns. Second, how does the contractibility of trade change the predictions of the PRT? For example, are we more likely to observe integration when quantity contracts are feasible? And third, how do asset ownership and quantity contracts interact in generating investment incentives? For example, how do optimal quantity contracts between firms differ from quantity contracts within firms?2

The present paper addresses these questions. I study the interaction of asset ownership and quantity contracts in a setting of symmetric information that contains standard property-rights and hold-up models as special cases. In the model, two parties want to interact with each other in order to create a surplus. The degree of interaction (e.g., the trade quantity or the number of joint projects) is ex-ante contractible. The parties may trade simultaneously with each other and with an outside-market. I focus on ex-ante investments embodied in an asset. Such investments might be physical themselves (e.g., a new plant), or they might represent effort to increase the size of the market for the asset’s product (e.g., marketing).3 The parties sign a (possibly message-dependent) contract specifying the organizational form, the degree of interaction, and a transfer payment. The purpose of the contract is to generate investment incentives.

In this setting, I show in a preliminary step that the parties can restrict themselves to message-independent contracts if only one of the parties invests, or if investments are transferable across parties. If investments are transferable, it does not matter which of the parties actually makes the investment. Hart (1995) argues that this is true for many investments in physical capital because

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2 Holmstrom and Roberts (1998) provide examples of such “inside contracting.”

3 If only one of the parties invests, the results qualitatively also hold for investments in human capital.
such investments are frequently not specific to a particular individual. The restriction to such non-contingent contracts allows me to fully characterize optimal contracts. Thereby, it is possible to study how the optimal use of organizational form and degree of interaction depends on the environment.

The main findings are as follows: (1) If only one of the parties invests, the organizational form predicted by the PRT is optimal even when trade is contractible. However, other organizational forms that are suboptimal according to the PRT might be optimal as well. (2) If both parties invest, the organizational form predicted by the PRT may not be optimal. (3) Even when the right choice of organizational form matters, the parties may sign a quantity contract. Such quantity contracts reduce or even eliminate the inefficiency that the property-rights approach suggests. Hence, while in the standard property-rights and hold-up literatures only one incentive instrument (organizational form or quantity contract) matters for generating investment incentives, in the present model both may be important. This is also in contrast to earlier work on the foundation of the property-rights approach by Maskin and Tirole (1999). (4) More generally, I provide conditions under which the right choice of both incentive instruments is important. This is in the spirit of the literature on multi-tasking and organizational form (see e.g., Holmstrom and Milgrom, 1991; Holmstrom and Tirole, 1991; and Baker, Gibbons, and Murphy, 2001, 2002). However, this literature usually employs a moral hazard framework, and restricts attention to linear incentive contracts and non-contingent assignments of ownership. (5) Even when it is irrelevant which organizational form the parties choose (which is often the case), the model imposes restrictions on the optimal combinations of incentive instruments. Hence, the model can provide some guidance for future empirical work. (6) Finally, the model permits interesting comparative statics with respect to the contractibility of interaction and the payoff functions of the parties. To illustrate this, I revisit the

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classic but controversial Fisher Body case, and show that the model lends support to a view held by Klein (2000): after a large increase in demand, integration of Fisher Body by General Motors was necessary in order to restore incentives for investments in physical capital.

What drives these results? Most of the property-rights literature has focussed on *selfish investments*, from which only the investor benefits directly. However, if both parties profit from a more valuable asset, investments are *cooperative* in the sense of Che and Hausch (1999). In this case, quantity contracts might not be very effective in generating investment incentives: a cooperative investment by one party increases the threatpoint payoff of the trading partner from internal trade. Thus, the available renegotiation surplus is reduced. This lowers the ex-post payoff of the investor, and consequently his expected marginal investment return. When trading externally, this effect can be avoided by giving ownership of the asset to the investor. In this case, only he can use the asset for external trade. Hence, the set of optimal contracts crucially depends on the degree to which each of the parties profits from an increase in asset value. For example, I will show that if an increase in asset value has an asymmetric effect on the parties, the PRT may predict an organizational form that is not optimal.

The remainder of the paper is structured as follows. In Section 2 I set up the model. In Section 3 some assumptions are introduced. In Sections 4 and 5 equilibrium outcomes in the one-sided investment case and the transferable investments case respectively are derived. In Section 6 it is shown that the parties cannot gain by considering more complicated, message-dependent contracts. Finally, in Section 7 I discuss implications for future empirical work, followed by concluding remarks in Section 8.

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4Other papers that consider cooperative investments include e.g., MacLeod and Malcomson (1993), Bernheim and Whinston (1998), Rosenkranz and Schmitz (1999) and Guriev (2003).
2 Model

A downstream buyer (B) and an upstream seller (S), both of whom are risk-neutral, want to trade a variable quantity of a good. If one interprets the level of trade more generally as the level of interaction between the parties, the results below equally apply to horizontal relationships. Parties B and S are assumed to have symmetric information, and may simultaneously trade with each other (internal trade) and with a competitive outside-market (external trade). The parties may use an asset A (for example a machine) for production and/or trade, and the organizational form \( O \in \{B, S, X\} \) determines which party has the residual rights of control over A. The asset may either be owned by the buyer \((O = B)\) or by the seller \((O = S)\). Under both \(O = B\) and \(O = S\) the parties are free to trade externally. However, the parties may also sign an exclusive dealing clause \((O = X)\) that forbids external trade. Note that residual rights of control will only matter for external trade. Hence, if the parties sign an exclusive contract, it does not matter which of the parties owns the asset. Joint ownership of the asset would have the same effect on investment incentives as \(O = X\).\(^5\) Similarly, exclusivity agreements that forbid only one of the parties to trade externally would lead to the same outcome as some \(O \in \{B, S\}\). Hence, these organizational forms are not explicitly introduced into the model.

Sequence of events

At date 1, the parties sign a non-contingent contract \(C = (O, q)\) that is registered with the courts. The contract specifies the organizational form \(O\) and the internal trade quantity \(q \in [0, 7]\). Thus I

\(^5\)However, it will become clear below that if not only joint ownership, but also exclusive contracts are feasible, the boundaries of the firm only matter for a smaller range of parameters.
assume that it is not possible to produce or trade an unlimited quantity internally. Besides $O$ and $q$, a transfer payment $t \in \mathbb{R}$ from $B$ to $S$ at date 4, and messages that the parties might exchange between dates 2 and 3 are assumed to be verifiable by a court. Because a fixed transfer payment will have no effect on incentives, $t$ is set equal to zero without loss of generality.

The assumption that the parties can only allow or forbid external trade, but that they are unable to regulate it in more detail deserves some discussion.\(^6\) Suppose that, due to a large number of potential external trading partners, courts are unable to directly observe external trade. First, I argue that courts may nevertheless be able to verify indirectly whether external trade has taken place at all. Suppose that certain verifiable actions have to be taken to initiate external trade. For example, external trade may require modifications of the asset (the external market may demand a different variety of the good); or hiring of additional transport capacity may be an indicator that external trade took place. In such an informational environment, only exclusive contracts but not more complicated arrangements with respect to external trade would be enforceable. Second, if courts cannot even verify whether external trade did or did not take place, the propositions below continue to hold. Only $O = X$ has to be replaced by joint ownership of the asset because under this organizational form each party can block the other party from using the asset for external trade (see e.g., Cai, 2003).

At date 2, $B$ and $S$ make relationship-specific investments $\beta \in [0, \overline{\beta}]$ and $\sigma \in [0, \overline{\sigma}]$ respectively in order to increase the value $a(\beta, \sigma)$ of the asset $A$, where $a$ is continuously differentiable in both arguments. Since the results below do not depend on the presence of ex-ante uncertainty about the ex-post state of nature, for ease of exposition, it is not introduced into the model. Denote the ex-post state of the world by $\theta \equiv (\beta, \sigma) \in \Theta \equiv [0, \overline{\beta}] \times [0, \overline{\sigma}]$. Because the parties have symmetric

\(^6\)Segal and Whinston (2000) impose the same assumption. They do not study asset ownership, but explore the possible irrelevance of exclusive dealing contracts for providing investment incentives.
information, they always renegotiate the initial contract to an ex-post efficient outcome at date 3. Renegotiations are discussed in more detail below. Finally, at date 4, production, trade, and payments take place.

**Threatpoint payoffs**

If renegotiations fail, the threatpoint payoffs \( \hat{b}(O, q, a) \) and \( \hat{s}(O, q, a) \) of B and S are determined by the initial contract \( C = (O, q) \). Both payoff functions are assumed to be continuously differentiable in \( q \) and \( a \), and non-decreasing in \( a \). If \( q > 0 \), the contract obliges parties to trade quantity \( q \) internally. I denote by \( \hat{b}(X, q, a) \) and \( \hat{s}(X, q, a) \) the respective threatpoint payoffs from internal trade, and assume that \( \hat{b}_q(X, q, a), \hat{s}_q(X, q, a), \hat{b}_{aq}(X, q, a), \hat{s}_{aq}(X, q, a) \geq 0 \) for all \( q \) and \( a \). Note that subscripts denote partial derivatives throughout. Moreover, if \( O \in \{B, S\} \), parties are free to trade with the outside-market. Consequently, \( [\hat{b}(O, q, a) - \hat{b}(X, q, a)] > 0 \) and \( [\hat{s}(O, q, a) - \hat{s}(X, q, a)] > 0 \) for \( O \in \{B, S\} \) represent the respective threatpoint payoffs from external trade. The external threatpoint payoffs are assumed to be independent of the internal trade quantity (i.e., \( \hat{s}_q(X, q, a) = \hat{s}_q(X, q, a) = \hat{b}_q(O, q, a) - \hat{b}_q(X, q, a) = 0 \) for all \( O, q \) and \( a \)). This assumption is solely made to simplify the exposition. The model could easily be extended to allow for spillovers between internal and external trade. Finally, I make the following assumptions: (i) internal payoffs are zero if no internal trade takes place (i.e., \( \hat{b}(X, 0, a) = \hat{s}(X, 0, a) = 0 \) for all \( a \)), and (ii) an owner has residual control rights over the asset. Hence, he can block the non-owner from using it for external trade. This implies that the external payoff of the non-owner does not vary in the asset value (i.e., \( \hat{b}(S, q, a) - \hat{b}(X, q, a) = \hat{b}(B, q, 0) - \hat{b}(X, q, 0) \) and \( \hat{s}(B, q, a) - \hat{s}(X, q, a) = \hat{s}(S, q, 0) - \hat{s}(X, q, 0) \) for

\(^7\)Under certain circumstances the parties might be able to commit not to renegotiate. However, this is controversial (see e.g., Hart and Moore, 1999).
all \( q \) and \( a \). (iii) The possibility of external trade raises the marginal value of the asset for the owner (i.e., \( \hat{b}_a (B, q, a) > \hat{b}_a (X, q, a) \) and \( \hat{s}_a (S, q, a) > \hat{s}_a (X, q, a) \) for all \( q \) and \( a \)).

**First-best solution**

I assume that it is ex-post efficient for \( B \) and \( S \) to cooperate. By working together they create an ex-post surplus \( \phi (a) \), possibly through internal and external trade, and I assume that \( \phi_a (a) > 0 \) and \( \phi_{aa} (a) < 0 \) for all \( a \). Hence, the efficient best-response investment functions are given by:

\[
\beta^* (\sigma) = \arg \max_{\beta \leq \beta^F} \{ \phi (a(\beta, \sigma)) - \beta \}, \quad \text{and} \quad (1)
\]

\[
\sigma^* (\beta) = \arg \max_{\sigma \leq \sigma^F} \{ \phi (a(\beta, \sigma)) - \sigma \}. \quad (2)
\]

The ex-ante efficient investment pair \( (\beta^{FB}, \sigma^{FB}) \) satisfies \( \beta^* (\sigma^{FB}) = \beta^{FB} \) and \( \sigma^* (\beta^{FB}) = \sigma^{FB} \). Hence, the efficient asset value is given by \( a^{FB} \equiv a(\beta^{FB}, \sigma^{FB}) \), where \( a^{FB} \) is assumed to be unique and interior.

3 **Assumptions and preliminary analysis**

The surplus generated through renegotiating the initial contract is given by:

\[
\Delta(O, q, a) \equiv \phi (a) - \hat{b} (O, q, a) - \hat{s} (O, q, a) \geq 0 \quad \text{for all} \quad O, q \text{ and } a. \quad (3)
\]
I assume that $B$ and $S$ divide $\Delta(O,q,a)$ in Nash-bargaining in equal parts.\footnote{The optimal ownership structure might depend on the nature of the bargaining game (see e.g., DeMeza and Lockwood, 1998; and Chiu, 1998).} Hence, the post-renegotiation payoffs of $B$ and $S$ (net of investment costs) are given by

$$\tilde{B}(O,q,a) \equiv \hat{b}(O,q,a) + \frac{1}{2} \Delta(O,q,a) = \frac{1}{2} \left[ \phi(a) + \hat{b}(O,q,a) - \hat{s}(O,q,a) \right], \quad \text{and}$$

$$\tilde{S}(O,q,a) \equiv \hat{s}(O,q,a) + \frac{1}{2} \Delta(O,q,a) = \frac{1}{2} \left[ \phi(a) + \hat{s}(O,q,a) - \hat{b}(O,q,a) \right],$$

respectively. It follows from (4) and (5) that the best-response investment functions are given by

$$\beta^{BR}(\sigma;C) \equiv \arg \max_{\beta} \tilde{B}(O,q,a(\beta,\sigma)) - \beta, \quad \text{and}$$

$$\sigma^{BR}(\beta;C) \equiv \arg \max_{\sigma} \tilde{S}(O,q,a(\beta,\sigma)) - \sigma,$$

respectively. An investment equilibrium $(\tilde{\beta}(C), \tilde{\sigma}(C))$ is implicitly defined by $\tilde{\beta}(C) = \beta^{BR}(\tilde{\sigma}(C); C)$ and $\tilde{\sigma}(C) = \sigma^{BR}(\tilde{\beta}(C); C)$. Define $\tilde{a}(C) \equiv a(\tilde{\beta}(C), \tilde{\sigma}(C))$. Hence, for a given contract, the net equilibrium surplus of the relationship between $B$ and $S$ is given by:

$$\tilde{W}(C) \equiv \phi(\tilde{a}(C)) - \tilde{\beta}(C) - \tilde{\sigma}(C).$$

In order to characterize optimal contracts, I now derive some properties of the post-renegotiation payoffs and introduce some assumptions. It directly follows from (4), (5) and the properties of the
threatpoint payoffs that:

\[ \hat{B}_{aq}(O, q, a) = -\hat{S}_{aq}(O, q, a) \text{ for all } O, q \text{ and } a, \] (9)

\[ \hat{S}_a(S, q, a) > \hat{S}_a(X, q, a) > \hat{S}_a(B, q, a) \text{ for all } q \text{ and } a, \text{ and} \] (10)

\[ \hat{B}_a(B, q, a) > \hat{B}_a(X, q, a) > \hat{B}_a(S, q, a) \text{ for all } q \text{ and } a. \] (11)

If an increase in \( q \) has a positive impact on the marginal investment return of party \( S \), it automatically has a negative impact on the marginal investment return of party \( B \), and vice versa. This arises from the fact that the payoffs of the parties depend on investments only through the asset value \( a \). Moreover, because only the owner can use the asset for external trade, the marginal investment returns can be unambiguously ordered across \( O \).

Define:

\[ n(q, a) = \hat{s}(X, q, a) - \hat{b}(X, q, a). \] (12)

For the remainder of the paper, the following assumptions are maintained.

**Assumption 1.** \( \bar{\alpha}(C) > 0, \bar{\beta}(C) < \bar{\beta}, \text{ and } \bar{\sigma}(C) < \bar{\sigma} \) for all \( C \).

**Assumption 2.** \( \frac{\partial^2}{\partial \beta^2} \hat{B}(O, q, a(\beta, \sigma)), \frac{\partial^2}{\partial \sigma^2} \hat{S}(O, q, a(\beta, \sigma)) < 0 \) for all \( O, q, \beta \) and \( \sigma \).

**Assumption 3.** Either \( n_{aq}(q, a) > 0 \) for all \( q \) and \( a \), or \( n_{aq}(q, a) < 0 \) for all \( q \) and \( a \).
The first two assumptions are of technical nature. Assumption 1 implies that the equilibrium investment of at least one of the parties is strictly positive. Assumption 2 guarantees that the investment equilibrium can be characterized by the appropriate first-order conditions. Unfortunately, even if $\phi(\cdot), \tilde{b}(\cdot)$ and $\tilde{s}(\cdot)$ are well-behaved in $a$, and $a(\cdot)$ is well-behaved in $\beta$ and $\sigma$, this is not guaranteed automatically. Finally, Assumption 3 ensures that the nature of the incentive problem does not vary across $q$ and $a$. The term $n_a(q,a)$ represents the net selfish effect on the seller that arises through internal trade. Analogously, $-n_a(q,a)$ represents the net selfish effect on the buyer. If Assumption 3 holds, it does not dependent on $q$ and $a$ whether the net selfish effect of an investment is positive or negative.

In accordance with the property-rights theory I assume that investments are relationship-specific: investments yield higher returns within the relationship than in the outside-market.

Define:

\[
\begin{align*}
\tilde{s}^e(a) &\equiv \tilde{s}(S,q,a) - \tilde{s}(X,q,a), \text{ and} \\
\hat{b}^e(a) &\equiv \hat{b}(B,q,a) - \hat{b}(X,q,a).
\end{align*}
\]

Formally, this assumption amounts to:

**Assumption 4.** $\tilde{s}^e_a(a), \hat{b}^e_a(a) < \phi_a(a)$ for all $a$.

An optimal contract $\tilde{C} = (\tilde{O}, \tilde{q})$ solves the following problem:

\[
\tilde{C} \in \arg \max_{\tilde{C}} \tilde{W}(\tilde{C}).
\]
In general the optimal contract is not unique. Denote the set of optimal organizational forms by \( \Omega \equiv \{ O \in \{ B, S, X \} \mid O = \tilde{O} \text{ for some } \tilde{C} \} \). Assumption 3 ensures that, for given \( O \in \Omega \), the optimal trade quantity is unique. Define \((\beta^{EQ}, \sigma^{EQ}) \equiv (\beta^{(C)}, \sigma^{(C)})\) and \(a^{EQ} \equiv a(\beta^{EQ}, \sigma^{EQ})\). As discussed above, if \( X \in \Omega \), it does not matter whether the asset is given to party \( B \) or party \( S \). In this case, the optimal exclusive quantity contract can be signed regardless of the underlying ownership structure. Hence, if \( \Omega = \{ B \} \), or if \( \Omega = \{ S \} \), it is said to be relevant who owns \( A \), otherwise it is irrelevant who owns \( A \). Similarly, if \( \tilde{q} > 0 \), quantity contracts are relevant, while they are irrelevant otherwise.\(^9\)

4 One-sided investment case

In this subsection, I focus on the case where only the seller is able to make an investment. That is, it is assumed that \( \beta \equiv 0 \). For simplicity, assume that \( a = \sigma \).

Frequently, \( O = S \) is interpreted as two separate firms, while \( O = B \) is interpreted as an integrated firm in which \( B \) procures the good internally from \( S \) (see e.g., Whinston, 2003). Given this terminology, the property-rights approach predicts that two separate firms are formed, but party \( S \) underinvests. That is, \((S,0)\) is the solution to (15) under the constraint \( q \equiv 0 \) (see Proposition 1 below). This standard property-rights result follows immediately from Assumption 4. Under which circumstances does this sharp prediction uphold when trade is contractible?

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\(^9\)Suppose that exclusive contracts are not feasible. This would imply \( O \in \{ B, J, S \} \), where \( O = J \) denotes joint ownership of the asset. In this case, it would only be irrelevant who owns \( A \) if \( \Omega = \{ B, J, S \} \).
Intuition

As laid out in the introduction, by investing, the seller exerts a positive externality on the buyer if the buyer profits from a more valuable asset. For example, the better the asset, the higher might be the quality of its product. It is obvious from (5) that such an externality has a negative effect on investment incentives. The extent of this externality depends on the contract terms $O$ and $q$.

To illustrate this, note that

$$\tilde{S}_a (S, q, a) = \frac{1}{2} [\phi_a (a) + n_a (q, a) + \tilde{s}_a (a)].$$  \hspace{1cm} (16)

First, suppose that the externality on the buyer is sufficiently strong. That is, suppose that $n_a (q, a) < 0$ for $q > 0$. In this case, Assumption 3 implies that $n_a (q, a)$ is decreasing in $q$. Hence, $\tilde{q} = 0$ is optimal, and it follows from the discussion above that $\Omega = \{S\}$.

Second, suppose that the externality is relatively weak (i.e., $n_a (q, a) > 0$ for $q > 0$) implying $n_{aq} (q, a) > 0$. It follows from inequality (10) that in this case $S \in \Omega$ still holds, but $O = S$ is not necessarily uniquely optimal. Only if the maximal net selfish effect $\pi_a \equiv n_a (\tilde{q}, a^{FB})$ (evaluated at the efficient asset value) is not too large, $\Omega = \{S\}$ holds. In particular, only if $\pi_a$ is below $\phi_a (a^{FB}) = 1$, an exclusive contract $O = X$ is not optimal as well (see inequality (10)). To summarize, if $\pi_a < 1$, $\Omega = \{S\}$ holds. If $0 < \pi_a < 1$, a strictly positive trade quantity is optimal: in this parameter range, $\tilde{q}$ will be chosen if $\tilde{a} (S, \tilde{q}) \leq a^{FB}$ holds, while an interior quantity will be chosen otherwise.

Third, if more than one organizational form turns out to be optimal (i.e., if $\pi_a \geq 1$), it immediately follows from (10) that the first-best is reached. In any contract that induces the first-best
optimal trade quantities are strictly positive and (generically) interior. For a given organizational form, such quantities \( q^O \) are implicitly defined by \( \tilde{S}_O(O, q^O, a^{FB}) = 1 \), where \( q^B > q^X > q^S \).

Define the following threshold values for \( \pi_a \): \( v^1 \equiv 0 \), \( v^2 \equiv 1 - \tilde{b}^e_a(a^{FB}) \), \( v^3 \equiv 1 \) and \( v^4 \equiv 1 + \tilde{b}^e(a^{FB}) \). Assumption 4 and \( \tilde{b}^e_a(a), \tilde{b}^e_a(a) > 0 \) for all \( a \) imply \( v^1 < v^2 < v^3 < v^4 \). Note that Assumption 3 rules out the case \( \pi_a = 0 \). Finally, for ease of notation define \( a^{S,0} \equiv \sigma BR (0; S, 0) \) and \( a^{S,\pi} \equiv \sigma^{BR} (0; S, \pi) \). It is now possible to state the following result.

**Proposition 1.** Suppose Assumptions 1-4 hold, and only party \( S \) invests.

(i) Suppose trade is not contractible. Then \( \Omega = \{S\} \) and \( a^{EQ} = a^{S,0} < a^{FB} \).

(ii) Suppose trade is contractible. Then the equilibrium outcome \((\Omega, \tilde{q}, a^{EQ})\) is given by Table 1.

**Proof.** See the Appendix.

<table>
<thead>
<tr>
<th>Parameter region</th>
<th>Set of optimal organizational forms ( \Omega )</th>
<th>Optimal trade quantity ( \tilde{q} )</th>
<th>Equilibrium asset value ( a^{EQ} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ( \pi_a &lt; v^1 )</td>
<td>( {S} )</td>
<td>0</td>
<td>( a^{S,0} &lt; a^{FB} )</td>
</tr>
<tr>
<td>(b) ( v^1 &lt; \pi_a &lt; v^2 )</td>
<td>( {S} )</td>
<td>( \pi )</td>
<td>( a^{S,\pi} &lt; a^{FB} )</td>
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<tr>
<td>(c) ( v^2 \leq \pi_a &lt; v^3 )</td>
<td>( {S} )</td>
<td>( q^O )</td>
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<tr>
<td>(d) ( v^3 \leq \pi_a &lt; v^4 )</td>
<td>( {S, X} )</td>
<td>( q^O )</td>
<td>( a^{FB} )</td>
</tr>
<tr>
<td>(e) ( v^4 \leq \pi_a )</td>
<td>( {S, X, B} )</td>
<td>( q^O )</td>
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Foundations of the property-rights theory

Proposition 1 contributes to the foundation of the property-rights theory of the firm. (1) In the one-sided investment case the contractibility of trade has no impact on the optimality of \( O = S \). This is a strength of the property-rights approach because the PRT derives this prediction under the assumption \( q \equiv 0 \), and hence in a simpler framework. (2) Among others, Holmstrom (1999) has argued that the ability of the property-rights approach to rule out suboptimal arrangements is perhaps even more important than identifying optimal once. However, if trade is contractible, the PRT may fail on this account. An exclusive contract or even integration may also be optimal. However, if the buyer profits sufficiently from a more valuable asset, the PRT correctly predicts \( \Omega = \{S\} \). (3) Previous foundations of the PRT have described environments where the lack of a quantity contract is optimal. In the present model, this is the case if and only if the externality on the buyer is so strong that \( \pi_{\alpha} < 0 \). Beyond this irrelevance result, Table 1 provides conditions under which the PRT is well founded (i.e., \( \Omega = \{S\} \)) and quantity contracts are relevant at the same time.

Contractibility and organizational form

Proposition 1 has implications beyond the foundations issue. (1) An integrated firm (i.e., ownership by the buyer) can only be optimal if trade is contractible. If it turns out that \( \Omega = \{S, X\} \), then, in addition to integration, the parties need to sign an exclusive contract. If, however, \( \Omega = \{S, X, B\} \), integration is optimal with and without an exclusivity agreement. (2) Suppose that trade is initially non-contractible, but subsequently becomes contractible (e.g., due to standardization after a development phase). In this case, the model suggest that a change of organizational form is not
necessary because $S \in \Omega$ holds in both cases.

**Interaction between organizational forms and quantity contracts**

Even when the model allows only limited or no predictions regarding $O$, it still imposes restrictions on the optimal combinations of $O$ and $q$. (1) Once more than one organizational form is observed, $q^S < q^X < q^B$ has to hold. That is, if both separate and integrated firms are present in a market, contracted quantities should be higher in the latter than in the former. (2) However, if the parties sign an exclusive contract, the model predicts the same trade quantities independent of who owns the asset. (3) The lack of a quantity contract, which can only be optimal between firms, indicates that the optimal organizational form is unique. In this case, separate firms should be observed in all relationships identical to the one under consideration.

**Efficiency**

By observing certain contract terms one can draw inferences about the efficiency of the relationship. (1) Whenever $q = 0$ or $q = \overline{q}$, generically the first-best is not achieved. Moreover, in this case $O = S$ is uniquely optimal, and hence no other organizational form should be observed empirically. (2) Conversely, if one observes (i) more than one organizational form, (ii) that the parties sign an exclusive contract, or (iii) an interior trade quantity, the model implies that the first-best is achieved.

Finally, the results above could be extended to the case of a *one-sided investment in human capital*: given that the parties interact, $B$ may profit from a human capital investment by $S$ if this
enables $S$ to produce a higher quality product. This would cause an externality on $B$. Independent of the ownership structure, such an externality does not arise when the parties trade externally: when the buyer obtains the good on the outside-market, he does not profit from the higher quality product of the seller. Two-sided investments in human capital are discussed in Section 6.

5 Transferable investments

When both parties may invest, the model allows additional insights. For the remainder of this section, I restrict attention to two-sided investments that are transferable across parties. Transferable investments are interesting from an applied point of view, and possess some useful analytical properties. Formally:

Assumption 5. $a(\beta, \sigma) \equiv \beta + \sigma$.

In Section 6 I show that such investments allow an optimal restriction to non-contingent contracts. If investments are non-transferable, parties can in general gain by randomizing over organizational forms. This complicates the analysis considerably, and, from a theoretical point of view, there is no obvious reason why randomization in the initial contract should be ruled out.

Investment equilibrium

If investments are transferable then, for a given investment $\sigma$ by the seller, the buyer will invest up to the point where $\widehat{B}_a(O, q, \beta + \sigma) = 1$. Hence, the investment best-response functions are given by $\beta^{BR}(\sigma; C) = \max\{a^B(C) - \sigma, 0\}$ and $\sigma^{BR}(\beta; C) = \max\{a^S(C) - \beta, 0\}$, where
\[ a^B(C) \equiv \arg \max_a \{ \bar{B}(O, q, a) - a \} \quad \text{and} \quad a^S(C) \equiv \arg \max_a \{ \bar{S}(O, q, a) - a \}. \]

Hence, the investment equilibrium \((\beta(C), \sigma(C))\) is given by (i) \((a^B(C), 0)\) if \(a^B(C) > a^S(C)\), (ii) \((0, a^S(C))\) if \(a^B(C) < a^S(C)\), and (iii) \((\beta, \sigma)\) such that \(\beta + \sigma = a^B(C)\) otherwise. Total equilibrium investment is given by \(\bar{a}(C) = \max\{a^B(C), a^S(C)\}\). Note that since the \(a^j(C)\)'s are continuous in \(q\), \(\bar{a}(C)\) is continuous in \(q\) as well.

**Intuition**

In a first step, consider optimal organizational forms. If trade is non-contractible, the optimal organizational form is (generically) unique. This is analogous to the one-sided case. However, it depends on the parameters of the model whether \(\Omega = \{S\}\) or \(\Omega = \{B\}\).\(^{10}\) In the following, I restrict attention to the case that \(\Omega = \{B\}\) when trade is non-contractible.\(^{11}\) First, as laid out in the introduction, if both parties may invest and trade is contractible, the property-rights approach may fail to identify the optimal organizational form. That is, the maximization of (15) subject to \(q \equiv 0\) may deliver misleading results. To illustrate this, suppose that when trading internally, party \(S\) profits substantially more from a better asset than party \(B\). If trade is contractible, this implies that the incentives of \(S\) (\(B\)) are increasing (decreasing) in \(q\). As a consequence, it may be optimal to raise the incentives of \(S\) further by giving him ownership of the asset. Second, note that generically only one of the parties invests in equilibrium. Hence, the choice of organizational form is only relevant if specifying a large trade quantity generates relatively low incentives for either

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\(^{10}\)In general, an exclusive contract may be optimal when investments are in physical capital. However, this is not possible when investments are transferable. This has previously been shown by Rosenkranz and Schmitz (2001), who study a dynamic property-rights framework.

\(^{11}\)This case is chosen because it fits the Fisher-Body example, which is discussed below. For the alternative case \(\Omega = \{S\}\), the results are completely analogous.
party. That is, $|\pi_a|$ must not be too large. This is the case if the (internal) gains from a better asset are relatively similar for both parties.

In a second step, consider optimal trade quantities. First, generically $\bar{q}$ can only be optimal if the first-best cannot be achieved. This is analogous to the one-sided case. Define $a^{B, \bar{q}} \equiv \beta^{BR}(0; B, \bar{q})$. Second, given $\Omega = \{B\}$ when trade is non-contractible, quantity contracts can only be irrelevant within an integrated firm. Moreover, for this to be the case the investment incentives of the buyer need to be decreasing in $q$. However, in contrast to the one-sided case, this is not sufficient for the lack of a quantity contract. The contract $(B, 0)$ is only strictly optimal if contract $(S, \bar{q})$ leads to a lower investment by the seller. That is, if $a^{B, 0} \equiv \beta^{SR}(0; B, 0) > a^{S, \bar{q}}$. Third, suppose that more than one organizational form is optimal. In this case, inequalities $(10)$ and $(11)$ imply that $\bar{q}$ is lower if the party that invests in equilibrium owns the asset. For all $O \in \Omega$, quantities $q^O (q^O)$ are implicitly defined by $\hat{B}_a(O, \bar{q}^O, a^{FB}) = 1 (\hat{S}_a(O, q^O, a^{FB}) = 1)$, where $\bar{q}^O (q^O)$ is strictly positive and generically interior, and $\bar{q}^S > \bar{q}^X > \bar{q}^B (q^S < q^X < q^B)$.

Finally, define the following threshold values: $\eta^1 \equiv -1 - \hat{s}_a^e(a^{FB})$, $\eta^2 \equiv -1$, $\eta^3 \equiv -1 + \hat{b}_a^e(a^{FB})$, $\eta^4 \equiv 0$, $\eta^5 \equiv 1 - \hat{s}_a^e(a^{FB})$, $\eta^6 \equiv 1$, $\eta^7 \equiv 1 + \hat{b}_a^e(a^{FB})$ and $\eta \equiv \phi_a(a^{S, \bar{q}}) + \hat{b}_a^e(a^{S, \bar{q}}) - 2$. Note that $\eta^k < \eta^{k+1}$ for $k = 1, \ldots, 6$ holds. It is now possible to state the following result.

**Proposition 2.** Suppose Assumptions 1-5 hold.

(i) Suppose trade is not contractible. Then

- if $\hat{b}_a^e(a^{B, 0}) > \hat{s}_a^e(a^{B, 0})$, it holds that $\Omega = \{B\}$, $(\beta^{EQ}, \sigma^{EQ}) = (a^{B, 0}, 0)$, and $a^{B, 0} < a^{FB}$, and
- if $\hat{b}_a^e(a^{B, 0}) < \hat{s}_a^e(a^{B, 0})$, it holds that $\Omega = \{S\}$, $(\beta^{EQ}, \sigma^{EQ}) = (0, a^{S, 0})$, and $a^{S, 0} < a^{FB}$.

(ii) Suppose trade is contractible, and $\hat{b}_a^e(a^{B, 0}) > \hat{s}_a^e(a^{B, 0})$ holds. Then the equilibrium outcome $(\Omega, \hat{q}, \beta^{EQ}, \sigma^{EQ})$ is given by Table 2.
Proof. See the Appendix.

Table 2: Equilibrium outcome in the transferable investment case with contractible trade

<table>
<thead>
<tr>
<th>Parameter region</th>
<th>Set of optimal organizational forms $\Omega$</th>
<th>Optimal trade quantity $\tilde{q}$</th>
<th>Equilibrium asset value $a^{EQ} = \beta^{EQ} + \sigma^{EQ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $\pi_a \leq \eta^1$</td>
<td>${B, X, S}$</td>
<td>$\tilde{q}^O$</td>
<td>$a^{FB} + 0$</td>
</tr>
<tr>
<td>(b) $\eta^1 &lt; \pi_a \leq \eta^2$</td>
<td>${B, X}$</td>
<td>$\tilde{q}^O$</td>
<td>$a^{FB} + 0$</td>
</tr>
<tr>
<td>(c) $\eta^2 &lt; \pi_a \leq \eta^3$</td>
<td>${B}$</td>
<td>$\tilde{q}$</td>
<td>$a^{FB} + 0$</td>
</tr>
<tr>
<td>(d) $\eta^3 &lt; \pi_a &lt; \eta^4$</td>
<td>${B}$</td>
<td>$\tilde{q}$</td>
<td>$a^{B,\pi} + 0 &lt; a^{FB}$</td>
</tr>
<tr>
<td>(e) $\eta^4 &lt; \pi_a &lt; \eta^5$ and $\eta &gt; 0$</td>
<td>${B}$</td>
<td>$0$</td>
<td>$a^{B,0} + 0 &lt; a^{FB}$</td>
</tr>
<tr>
<td>(f) $\eta^4 &lt; \pi_a &lt; \eta^5$ and $\eta &lt; 0$</td>
<td>${S}$</td>
<td>$\tilde{q}$</td>
<td>$0 + a^{S,\pi} &lt; a^{FB}$</td>
</tr>
<tr>
<td>(g) $\eta^5 \leq \pi_a &lt; \eta^6$</td>
<td>${S}$</td>
<td>$\tilde{q}^O$</td>
<td>$0 + a^{FB}$</td>
</tr>
<tr>
<td>(h) $\eta^6 \leq \pi_a &lt; \eta^7$</td>
<td>${S, X}$</td>
<td>$\tilde{q}^O$</td>
<td>$0 + a^{FB}$</td>
</tr>
<tr>
<td>(j) $\eta^7 \leq \pi_a$</td>
<td>${S, X, B}$</td>
<td>$\tilde{q}^O$</td>
<td>$0 + a^{FB}$</td>
</tr>
</tbody>
</table>

Note that in the knife-edge case $\eta = 0$, both $(S, \tilde{q})$ and $(B, 0)$ are optimal. Under the former (latter) contract, only party $S$ ($B$) invests $a^{S,\tilde{q}}$ ($a^{B,0}$), where $a^{S,\pi} = a^{B,0} < a^{FB}$. In the following, I highlight which differences and additional insights arise relative to the one-sided investment case.

Foundations of the property-rights theory

(1) If both parties invest, the property-rights approach may predict the wrong ownership structure: in cases (f) and (g) of Table 2 integration is strictly optimal when trade is non-contractible, while
separate firms are strictly optimal when quantity contracts are feasible. This result is in stark contrast to the one-sided investment case.\textsuperscript{12} This problem can, however, not arise if the buyer profits more from a better asset than the seller (cases (a)-(d)). Note that in case (h) integration is still optimal as long as it is accompanied by an exclusive contract. (2) Besides providing the exact conditions under which the property-rights approach is well founded (i.e., when $\Omega = \{ B \}$), Proposition 2 shows that the choice of ownership structure is relevant over a larger parameter range: if $-1 < \pi_a < 1$, the choice of ownership structure is relevant, but $\Omega = \{ B \}$ is only possible in the smaller parameter range $-1 < \pi_a < 1 - \xi_a^c(a^{FB})$.

**Efficiency and investing party**

Proposition 2 predicts that for all $O \in \Omega$ it is the same party that invests in equilibrium. Whenever $|\pi_a|$ is sufficiently large, the first-best outcome can be induced. Moreover, whenever the non-owner invests, the ownership structure is generically irrelevant, and the first-best outcome is achieved.

**Interaction between organizational forms and quantity contracts**

In contrast to the one-sided case, it depends on the model parameters whether contracted quantities are larger in integrated or in separate firms. However, Proposition 2 predicts that if only the seller (buyer) invests in equilibrium, contracted quantities should be lower (higher) between separate firms than within an integrated firm.

\textsuperscript{12}This problem exists in general, and does not depend on Assumption 5.
Comparative statics: Fisher Body revisited

The model allows interesting comparative static exercises. For example, one can explore how the optimal organizational form, quantity contracts, the efficiency of the relationship, and the identity of the investing party evolve due to exogenous shocks to the payoff functions of the parties. To illustrate this point, the classic Fisher Body (henceforth FB) case that has attracted considerable attention in the transaction cost and property-rights literatures is revisited. While there are rivaling interpretations of the case, the present model provides support to a view held by Klein (2000): following a substantial increase in demand for closed automobile bodies, FB’s refusal to invest in a new plant necessitated full vertical integration of FB by General Motors (henceforth GM) in 1926. Afterwards GM built the plant itself. Since 1919 the two had operated under a supply contract that obliged GM to buy all of its automobile bodies from FB while allowing FB to trade externally. The fact that the contract included explicit price terms gives some evidence that trade was contractible. In the period prior to 1926, FB had built several plants to meet GM’s needs.\(^{13}\)

In terms of the present model, the Fisher Body case could be expressed in the following way. Prior to 1926, the contract \((S, \overline{7})\) was in place, where \(O = S\) is again interpreted as two separate firms. In 1926 the arrangement switched to \((B, 0)\) because no explicit quantity contract was in place after that date. The model can rationalize this switch. Note that these two contracts correspond to neighboring parameter regions (see cases (e) and (f) in Table 2). The boundary between these

\(^{13}\)See Klein (2000) and the references cited therein for more detailed descriptions and discussions of the Fisher Body case.
parameter regions is defined by:

$$\phi_a (a^{S, \vartheta}) + \hat{b}_a (a^{S, \vartheta}) - 2 = 0. \quad (17)$$

As long as the left-hand side of (17) is negative, \((S, \vartheta)\) is optimal. Arguably, an exogenous increase in demand would have made a new plant more valuable for GM causing an upward shift in its marginal payoff function \(\hat{b}_a (\cdot)\). Consequently, the left-hand side of (17) turned positive, and the contract \((B, 0)\) became optimal. Note that the observed investment behavior is consistent with the theoretical prediction.\(^\text{14}\) Klein (2000) argues informally that, due to insufficient reputational capital, the rise in demand shifted the initial supply contract outside its “self-enforcing range”. However, he remained unsure what exactly it was “about the large, unexpected demand increase by GM that caused Fisher to take advantage of the imperfect body supply regime” (Klein, 2000, p. 129). The present model suggests that the rise in demand increased the contractual externality of the investment, and thereby led to the suboptimality of the supply contract regime.

6 Non-contingent contracts suffice

In this section, it is shown that the parties cannot gain by considering more complicated, message-dependent contracts.\(^\text{15}\) I denote by \(\theta^B\) and \(\theta^S\) messages that \(B\) and \(S\) respectively might send between dates 2 and 3. To prove the claim I employ the mechanism design with renegotiation approach as advanced by Maskin and Moore (1999). The revelation principle allows to restrict

\(^{14}\)Proposition 2 builds on the assumption that \(\Omega = \{B\}\) when trade is non-contractible. In the Fisher Body case, this is consistent with the observed contracts. Suppose to the contrary that \(\Omega = \{S\}\) when trade is non-contractible. In this case, contract \((B, 0)\) could not be optimal when trade is contractible.

\(^{15}\)If parties invest sequentially, option contracts may improve upon non-contingent contracts (see e.g., Nöldeke and Schmidt, 1998; and Edlin and Hermalin, 2000).
attention to direct revelation mechanism under which both parties have an incentive to report
the ex-post state of the world truthfully. In the more general setting considered in this section,
a contract $C$ is defined as a mapping $(O, q, t) : \Theta^2 \rightarrow \{B, S, X\} \times [0, \bar{q}] \times \mathbb{R}$. The best-response
investment functions, given this more general class of contracts, are defined in the proof of the
proposition below. I build on the finding of Segal and Whinston (2002), that in the present setting
every sustainable interior investment equilibrium can be sustained by a non-contingent contract that
randomizes over organizational forms.\footnote{This result holds even if Assumption 5 is not satisfied. However, non-randomizing contracts are in general not optimal in this case. In the absence of a randomizing device the results of Segal and Whinston (2002) are not directly applicable because $O$ represents a discrete choice. Whether in this case a restriction to non-contingent contracts is possible awaits future research.} This result is extended in two ways. First, by focussing
on the set of optimal contracts, the result is extended to non-randomizing contracts. Second, if in-
vestments are transferable, investment equilibria are in general not interior. Segal and Whinston’s
(2002) result is extended to accommodate this case.\footnote{With two-sided investments in human capital, the results of Segal and Whinston (2002) are not applicable. In this case, internal payoffs might depend on the investments of both parties. However, when trading externally, a party only has access to its own human capital. Hence, in general the investments cannot be aggregated into one dimension in the decision-dependent parts of the post-renegotiation payoffs as required by Condition A in Segal and Whinston (2002).}

**Proposition 3.** Take Assumptions 2-4 as given, and suppose that Assumption 1 holds for any
message-dependent contract $C$ as defined above. If only one party can invest, or if investments are
transferable across parties, the set of optimal contracts always contains a non-contingent contract
of the form $(O, q, t)$ for some $O \in \{B, X, S\}$, $q \in [0, \bar{q}]$ and $t \in \mathbb{R}$.

**Proof.** See the Appendix.
anisms, the equilibrium pre-renegotiation decisions $\tilde{O}$ and $\tilde{q}$ are in general indeterminate. Starting from any incentive-compatible mechanism one can always change the equilibrium decisions and modify the equilibrium transfers in a way such that the equilibrium payoffs remain unchanged. In this case, the incentive-compatibility conditions still hold. Hence, predictions regarding equilibrium decisions are only possible if one imposes restrictions on the set of feasible contracts. In the setting of Proposition 3 the restriction of attention to non-contingent contracts seems to be justified if one assumes that there is an (arbitrarily) small cost of writing more complex agreements.\(^{18}\)

7 Implications for empirical testing

The model has implications for the growing empirical literature on the PRT (for a review, see e.g., Whinston, 2003). First, the model may shed some light on the puzzle why some empirical studies, like Elfenbein and Lerner’s (2003) recent study of internet portal alliances, support the PRT even though such alliance agreements contain on average two quantity provisions relating to project output. Elfenbein and Lerner (2003) find that in promotional agreements in general one of the parties provides most of the effort, and they report a significant relationship between the identity of this party and the allocation of ownership rights. They interpret this as support for the PRT. However, they are puzzled by a varying degree of completeness of the contracts with respect to quantity provisions. The present model suggests that, given transferability of investments, the optimal organizational form, quantity provisions and the identity of the investing party are all determined endogenously. Moreover, the choice of ownership structure is only relevant under

\(^{18}\)Non-contingent contracts are a special case of continuous contracts in which decisions are continuous in messages. Segal and Whinston (2002) have shown that under some assumptions the expected decision in any optimal continuous contract is equal to the decision in the optimal non-contingent contract. Unfortunately, the assumptions underlying their result, e.g., a one-dimensional decision space, are not satisfied in the present model (see Condition S of Segal and Whinston, 2002).
certain circumstances. Second, in contrast to the PRT, the model suggests that frequently more than one organizational form will be optimal. This poses the problem that the theoretical prediction might not be unique. Hence, it seems to be promising to look at industries that experience a shift in underlying parameter values (e.g., in the demand or costs structure) such that certain organizational forms are only optimal before the shift. Third, Whinston (2003) emphasizes that the informational requirements of empirical test of the PRT are high. In principle, marginal contributions have to be observed. The present model might ease such problems as it imposes joint restrictions on potentially observable variables, like the organizational form, the contracted level of interaction, and the identity of the investing party.

8 Conclusion

In a property-rights framework, I study how two incentive instruments (organizational form and quantity contracts) are used to generate investment incentives. In a preliminary step, I provide conditions under which the parties can optimally restrict themselves to non-contingent contracts. This simplification allows me to fully characterize optimal contracts. First, the article contributes to the foundation of the property-rights theory of the firm: it is characterized when the PRT does and does not make correct predictions when trade is contractible. I delineate the circumstances under which only the correct choice of a quantity contract, only the correct choice of organizational form, or both are important to reduce the hold-up problem. Second, while maintaining the symmetric information assumption of the PRT the model shows how the two incentive instruments interact. This is in the spirit of the multi-tasking literature, which, however, focuses on settings of asymmetric information. Finally, I illustrate how the model may shed light on the classic Fisher Body case.
Proof of Proposition 1. Ad (i): Inequality (10), Assumptions 1, 2 and 4, and the fact that $a^{FB}$ is assumed to be interior imply that $\tilde{\sigma}(B,0) < \tilde{\sigma}(X,0) < \tilde{\sigma}(S,0) < \sigma^{FB} = a^{FB}$. Ad (ii): I now prove that cases (a)-(e) of Table 1 hold true. Ad (a): Assumption 3 and $n_a(\overline{q},a^{FB}) < 0$ imply that $\tilde{S}_{aq}(S,q,a) < 0 \forall q,a$. Hence, inequality (10) implies that contract $(S,0)$ is optimal. Ad (b): $n_a(\overline{q},a^{FB}) > 0$ and the reasoning in part (a) above imply that the contract $(S,\overline{q})$ maximizes investment incentives. Moreover, $n_a(\overline{q},a^{FB}) < 1 - \tilde{\sigma}_a(a^{FB}) \Leftrightarrow \tilde{S}_a(S,\overline{q},a^{FB}) < 1$. That is, $(S,\overline{q})$ is optimal because the first-best cannot be achieved. Finally, Assumptions 1 and 2 imply that the equilibrium investment is given by $a^{S,\overline{q}} = \sigma^{BR}(0;S,\overline{q})$. Ad (c): $n_a(\overline{q},a^{FB}) \geq 1 - \tilde{\sigma}_a(a^{FB})$ and the reasoning above imply that, generically, the contract $(S,\overline{q})$ leads to overinvestment. However, the fact that $(S,0)$ leads to underinvestment and the Intermediate-Value Theorem imply that there exists a $q^S \in [0,\overline{q}]$ such that $\tilde{S}_a\left(S,q^S,a^{FB}\right) = 1$. Assumptions 1 and 2 imply that $\tilde{\sigma}(S,q^S) = a^{FB}$. Finally, $0 < n_a(\overline{q},a^{FB}) < 1 \Leftrightarrow \tilde{S}_a\left(X,0,a^{FB}\right) < \tilde{S}_a\left(X,\overline{q},a^{FB}\right) < 1$ together with inequality (10) implies $\Omega = \{S\}$. Ad (d): $1 \leq n_a(\overline{q},a^{FB}) < 1 + \tilde{b}_0(a^{FB})$ and the reasoning in part (c) above imply $\Omega = \{S,X\}$ and $\tilde{q} = \overline{q}^O$ for $O \in \Omega$, where $\overline{q}^O$ is implicitly defined by $\tilde{S}_a\left(O,\overline{q}^O,a^{FB}\right) = 1$. Hence, both under $(S,q^S)$ and $(X,q^X)$, the first-best is achieved. Ad (e): Completely analogous to part (d), and therefore omitted. Q.E.D.
Proof of Proposition 2. First, Assumptions 1, 2 and 4 imply:

\[ a^j(S, q) \geq (\leq) a^j(X, q) \geq (\leq) a^j(B, q) \quad \forall q \text{ and } j = S(B), \text{ and} \]

\[ a^j(j, 0) > a^i(j, 0) \text{ for } i, j \in \{B, S\} \text{ and } i \neq j. \tag{A2} \]

The inequalities in (A1) are strict whenever the respective left-hand side (right-hand side) is strictly positive.

Second, Assumption 4 and the Intermediate-Value Theorem imply that, for a given \( O \), the first-best can be achieved if and only if \( \tilde{a}(O, \eta) \geq a^{FB} \). Otherwise, the contract that maximizes investment incentives is optimal because in general only one of the parties invests in equilibrium.

Ad (i): it follows from (A1), and Assumptions 1 and 4 that it depends on \( a^B(B, 0) \) and \( a^S(S, 0) \) whether \( \Omega = \{B\} \) or \( \Omega = \{S\} \). Note, that \( \tilde{S}_a(S, 0, a^B(B, 0)) - 1 = \tilde{S}_a^c(a^B(B, 0)) - \tilde{b}_a^c(a^B(B, 0)) \).

Ad (ii): Define \( \tilde{a}^O \equiv \max_q \tilde{a}(O, q) \). The discussion above and \( \tilde{b}_a^c(a^B(B, 0)) > \tilde{S}_a^c(a^B(B, 0)) \iff a^B(B, 0) > a^S(S, 0) \) imply:

\[ \tilde{a}^B = \max \{ a^B(B, \eta), a^B(B, 0), a^S(B, \eta) \}, \tag{A3} \]

\[ \tilde{a}^S = \max \{ a^S(S, \eta), a^S(S, 0), a^B(S, \eta) \}, \text{ and} \]

\[ \tilde{a}^X = \max \{ a^S(X, \eta), a^B(X, \eta) \}. \tag{A5} \]

Consider \( \tilde{a}^B \): Assumption 3 implies that if \( a^B(B, q) \) is increasing in \( q \), then \( \tilde{a}^B = a^B(B, \eta) \). If \( a^B(B, q) \) is decreasing in \( q \), then \( \tilde{a}^B = \max \{ a^B(B, 0), a^S(B, \eta) \} \). Hence, \( \tilde{a}^B \) is given by the expression above. The proof for \( \tilde{a}^S \) is completely analogous. Assumption 3 and \( a^S(X, 0) = a^B(X, 0) \) imply the solution for \( \tilde{a}^X \). Ad (a): Assumption 2 implies \( a^B(S, \eta) \geq a^{FB} \). Moreover, \( \tilde{B}_a(S, \eta, a^{FB}) \geq 1 \)}
together with Assumption 3 implies \( \hat{B}_{aq}(0,q,a) > 0 \) \( \forall O,q,a \). This observation and the reasoning in part (i) imply \( a^{FB} > a^{S}(S,0) \geq a^{S}(S,\overline{q}) \) from which the result follows immediately. *Ad (b), (c), (g), (h), (j)*: the proofs for these cases are completely analogous to the proof of part (a), and therefore omitted. *Ad (d)*: Assumptions 1, 2 and 3, \(-1 + \hat{b}_a(a^{FB}) < n_a(\overline{q},a^{FB}) < 0\), and \( \hat{b}_a(a^B(B,0)) > \hat{s}_a(a^B(B,0)) \) imply \( a^{FB} > a^B(B,\overline{q}) > a^B(B,0) > a^{S}(S,0) \geq a^{S}(S,\overline{q}) \). Given this observation, the result immediately follows from inequality (A1) and Assumption 3.

*Ad (e) and (f)*: analogous to the reasoning in part (d), \( 0 < n_a(\overline{q},a^{FB}) < 1 - \hat{s}_a(a^{FB}) \), and \( \hat{b}_a(a^B(B,0)) > \hat{s}_a(a^B(B,0)) \) imply \( a^{FB} > a^B(B,0) > a^B(B,\overline{q}) \) and \( a^{FB} > a^{S}(S,\overline{q}) \geq a^{S}(S,0) \).

Hence, it is optimal to maximize investment incentives. If \( \eta > 0 \), then \( a^B(B,0) > a^{S}(S,\overline{q}) \). In this case, (A1) and Assumption 3 imply that \( \Omega = \{ B \} \) and \( \overline{q} = 0 \). Hence, \( (\beta^{EQ},\sigma^{EQ}) = (a^B(B,0),0) \), where \( a^B(B,0) < a^{FB} \). The proof for the case \( \eta < 0 \) is completely analogous, and therefore omitted. *Q.E.D.*

**Proof of Proposition 3. Step 1.** For the purpose of the proof, consider the more general class of contracts \( C''=\{\hat{p}^B,\hat{p}^S,\hat{p}^X,\hat{q},\hat{t}\} : \Theta^2 \rightarrow [0,1]^3 \times [0,\overline{q}] \times \Re \). The functions \( \hat{p}^O(\theta^B,\theta^S) \leq 1 \) \( \forall O \) denote the respective probabilities with which an ownership structure \( O \) is selected ex-post. Hence, \( \sum \hat{p}^O(\theta^B,\theta^S) = 1 \) \( \forall \theta^B,\theta^S \) holds. Renegotiations are assumed to occur after the realization of the randomly determined organizational form. Without loss of generality one can restrict attention to direct mechanisms that induce truth-telling on and off the equilibrium path. Hence, given this more general class of contracts, the best-response investment function of \( B \) is defined by

\[
\beta^{BR}(\sigma;C'') \equiv \arg \max_{\beta} \sum \hat{p}^O(\theta,\theta) \cdot \hat{B}(O,\hat{q}(\theta,\theta),a(\beta,\sigma)) - \beta - \hat{t}(\theta,\theta), \quad (A6)
\]
where $\theta = (\beta, \sigma)$. The best-response function of $S$ is defined analogously.

**Step 2.** Suppose that in the two-sided case the equilibrium investments of both parties are interior, i.e., $0 < \beta(C^\eta) < \beta$ and $0 < \sigma(C^\eta) < \sigma \forall C^\eta$. In the one-sided case, $0 < \beta(C^\eta) < \beta \forall C^\eta$ holds by assumption. For the moment, consider message-independent contract terms $p^O \equiv \tilde{p}^O (\theta^B, \theta^S)$, $q \equiv \tilde{q}(\theta^B, \theta^S)$ and $\tilde{t}(\theta^B, \theta^S) \equiv 0$ for all $\theta^B, \theta^S$, and $O$. Define $q^{\text{max}} \equiv \arg \max_q \{ \tilde{S}_a(S, q, a) \}$ and $q^{\text{min}} \equiv \arg \min_q \{ \tilde{S}_a(B, q, a) \}$. Assumption 3 implies that $q^{\text{max}}, q^{\text{min}} \in \{0, \tilde{q}\}$, and that both $q^{\text{max}}$ and $q^{\text{min}}$ are independent of $a$. These observations and (10) imply that:

$$\tilde{S}_a(B, q^{\text{min}}, a) \leq \sum_O p^O \cdot \tilde{S}_a(O, q, a) \leq \tilde{S}_a(S, q^{\text{max}}, a) \forall p^O, q, a. \quad (A7)$$

Hence, Segal and Whinston’s (2002) Condition H is satisfied in the present framework. Moreover, the decision space $[0, 1]^3 \times [0, \tilde{q}]$ is compact and connected, and the functions $\phi_a$, $\tilde{S}_a$ and $\tilde{B}_a$ are continuously differentiable in all arguments except $O$. Note that $O$ is not a decision variable given the larger class of contracts presently considered. Hence, Condition A of Segal and Whinston (2002) is also satisfied. Finally, Assumption 2 corresponds to their Condition C. This implies that Segal and Whinston’s (2002) Proposition 4 applies in the present framework. That is, if there is a message-dependent contract $C^\eta$ that sustains certain interior investments, then there exists a non-contingent contract $[p^B, p^S, p^X, q]$ that sustains the same investments. With slight abuse of notation, define $a^S(p^B, p^S, p^X, q) \equiv \arg \max_a \{ \sum p^O \cdot \tilde{S}(O, q, a) - 1 \}$, and define $a^B(p^B, p^S, p^X, q)$ analogously. Note, that the $a^i(\cdot)$ and $\tilde{a}(\cdot)$ are continuous in $q$, $p^B, p^S$ and $p^X$.

**Step 3.** Now, the result of Step 2 is extended to situations in the transferable case where one of the parties does not invest in equilibrium. Suppose that an arbitrary contract $C^\eta$ sustains an investment vector $(0, \sigma^\eta)$, where $0 < \sigma^\eta < \sigma$. The case $(\beta^\eta, 0)$, where $0 < \beta^\eta < \beta$, is completely
analogous, and therefore omitted. Since $C''$ sustains $(0, \sigma'')$, the investment $\sigma'' > 0$ is a best-
response to $\beta = 0$. Now, consider the exactly same situation, but assume that only the seller has
the possibility to invests. That is, suppose that $\beta \equiv 0$ is exogenously given. In this corresponding
two-sided investment problem it is still a best-response to choose $\sigma''$ given the original contract $C''$.
Because $\sigma''$ is interior, Step 2 implies that for this corresponding one-sided investment problem
there exists a non-contingent contract $C^{NC} = [p^B, p^S, p^X, q]$, for some $p^B, p^S, p^X$ and $q$, that also
sustains $\sigma''$. Hence, $\sigma'' = a^S(C^{NC})$. Now consider the original two-sided problem. There, it is still
a best-response to choose $\sigma''$ given $\beta = 0$ and $C^{NC}$. Given $C^{NC}$, either $(0, \sigma'')$ or $(\beta, \sigma)$, where
$\beta + \sigma \geq \sigma''$, emerges as investment equilibrium. If $\beta + \sigma \leq a^{FB}$, contract $C^{NC}$ leads to weakly
higher welfare than the initial contract $C''$. If $\beta + \sigma > a^{FB}$, Assumption 4, the Intermediate-Value
Theorem and the observation that $\bar{a}(\cdot)$ is continuous in $p^B, p^S$ and $q$ imply that there exists a
non-contingent contract $C^{FB}$ that induces the first-best.

Step 4. Building on the observations in Steps 2 and 3, it is finally shown that the set of
optimal contracts always contains a contract that assigns probability one to some $O$. Consider an
arbitrary non-contingent (but possibly randomizing) contract $C^{NC}$. For all $O \in \{B, S, X\}$
and an arbitrary $q$, define $C^O \equiv [p^B, p^S, p^X, q]$ where $p^O = 1$. Without loss of generality suppose
that $\bar{a}(C^S) \geq \bar{a}(C^O) \ \forall O$. This implies that $\bar{a}(C^S) \geq \bar{a}(C^{NC})$. If $\bar{a}(C^S) \leq a^{FB}$, then $C^S$ leads to
weakly higher welfare than $C^{NC}$. If $\bar{a}(C^S) > a^{FB}$, then the fact that $\bar{a}(\cdot)$ is continuous in $q$, the
Intermediate-Value Theorem and $\bar{a}(S, 0) < a^{FB}$ imply that there exists a $\tilde{q}$ such that the contract
$[p^B = 0, p^S = 1, p^X = 0, \tilde{q}]$ induces the first-best. Q.E.D.
References


