This appendix provides details on the model of optimal contracting with subjective performance measures and discusses the existence of optimal relational contracts.

**APPENDIX C: SUBJECTIVE PERFORMANCE MEASURES.**

Consider a full review contract that in its initial period specifies effort $e$, payments $w$ and $b(y)$, and continuation payoffs $u(y), \pi(y)$ contingent on output, with reversion to static no trade equilibrium following a deviation in payments or in proposing or accepting the contract. Define $W(y) \equiv w + b(y)$ and expected payoffs:

$$u \equiv (1 - \delta) \mathbb{E}_y [W - c | e] + \delta \mathbb{E}_y [u(y) | e] ,$$

$$\pi \equiv (1 - \delta) \mathbb{E}_y [y - W | e] + \delta \mathbb{E}_y [\pi(y) | e] ,$$

and $s \equiv u + \pi$. This contract is self-enforcing if and only if (i) both parties are willing to participate, $u \geq \overline{u}$ and $\pi \geq \overline{\pi}$; (ii) the agent will choose $e$, i.e.

$$e \in \arg \max_e \mathbb{E}_y \left[ b(y) + \frac{\delta}{1 - \delta} u(y) | \hat{e} \right] - c(\hat{e}) ;$$

(iii) the principal will truthfully report, i.e. for all $y, y'$

$$b(y) + \frac{\delta}{1 - \delta} \pi(y) = b(y') + \frac{\delta}{1 - \delta} \pi(y') ;$$

(iv) both parties will make payments, i.e. for all $y$,

$$b(y) + \frac{\delta}{1 - \delta} u(y) \geq \frac{\delta}{1 - \delta} \overline{\pi} ,$$

$$\bar{b}(y) + \frac{\delta}{1 - \delta} \pi(y) \geq \frac{\delta}{1 - \delta} \overline{\pi} ,$$

and (v) for all $y$, the continuation payoffs $u(y), \pi(y)$ correspond to a self-enforcing contract.

**The Equilibrium Payoff Set.** Assuming an optimal contract exists that generates surplus $s^*$, an argument identical to in the proof of Theorem 1 implies that the set of payoffs achievable with a self-enforcing full review contract is equal to $\{ (u, \pi) : u \geq \overline{u}, \pi \geq \overline{\pi} \text{ and } u + \pi \leq s^* \}$.

**Termination Contracts are Optimal.** I now argue constructively, along the lines of Theorem 2, that there is a termination contract that achieves the optimal surplus $s^*$. Suppose the contract described above is optimal. Then,
\[ s^* = (1 - \delta) \mathbb{E}_y [y - c \mid e] + \delta \mathbb{E}_y [s(y) \mid e]. \]  

(1)

Note that because \( s^* \) is optimal, \( s(y) \leq s^* \) for all \( y \) and hence \( \mathbb{E}_y [y - c \mid e] \geq s^* \).

Now, let \( u^* \in [\bar{u}, s^* - \bar{\pi}] \) be given and define \( \pi^* \equiv s^* - u^* \). I construct a termination contract with these payoffs. Suppose the contract specifies effort \( e \), payments \( u^* \) and \( b^* (y) \), and probabilities of continuation \( \alpha^* (y) \). Define \( \alpha^* (y) \) so that the expected continuation surplus following any outcome \( y \) is the same as under the original contract:

\[ \bar{s} + \alpha^* (y) (s^* - \bar{s}) \equiv s(y). \]  

(2)

Let \( u^* (y) \equiv \bar{\pi} + \alpha^* (y) (u^* - \bar{\pi}) \) and \( \pi^* (y) \equiv \bar{\pi} + \alpha^* (y) (\pi^* - \bar{\pi}) \) be the expected continuation payoffs following output \( y \). Define \( b^* (y) \) to so that agent’s expected future payoff following outcome \( y \) is the same as under the initial contract:

\[ b^* (y) + \frac{\delta}{1 - \delta} u^* (y) \equiv b(y) + \frac{\delta}{1 - \delta} u(y). \]  

(3)

Combining (2) and (3) implies that for all \( y \),

\[ -b^* (y) + \frac{\delta}{1 - \delta} \pi^* (y) \equiv -b(y) + \frac{\delta}{1 - \delta} \pi(y). \]

Finally, define the fixed payment \( w^* \) so that the agent’s expected payoff is \( u^* \):

\[ u^* = (1 - \delta) \mathbb{E}_y [w^* + b^* (y) \mid c] + \delta \mathbb{E}_y [u^* (y) \mid c]. \]

I claim that this termination contract generates surplus \( s = s^* \) and is self-enforcing. To see this, observe that the surplus generated, \( s \), satisfies:

\[ s \equiv (1 - \delta) \mathbb{E}_y [y - c \mid e] + \delta \left\{ \bar{s} + \mathbb{E} [\alpha^* (m) \mid e] (s - \bar{s}) \right\}. \]

Substituting (1) and (2) into this expression shows that \( s = s^* \). Moreover, this termination contract, with effort \( e \), payments \( w^* \), and contingent continuation payoffs \( u^* (y) \) and \( \pi^* (y) \) satisfies the constraints (i)-(v) by definition. Hence, it is self-enforcing.

**Self-Enforcing Termination Contracts.** Next I identify simple conditions under which a termination contract is self-enforcing (analogous to Theorem 3). Consider a termination contract with effort \( e \), payments \( w \) and \( b (y) \), and continuation probabilities \( \alpha (y) \). The expected payoffs \( u, \pi \) satisfy the recursive equations:

\[ u \equiv (1 - \delta) \mathbb{E}_y [W(y) - c \mid e] + \delta \bar{\pi} + \delta \mathbb{E}_y [\alpha (y) \mid e] (u - \bar{\pi}) \]

\[ \pi \equiv (1 - \delta) \mathbb{E}_y [y - W(y) \mid c] + \delta \bar{\pi} + \delta \mathbb{E}_y [\alpha (y) \mid e] (\pi - \bar{\pi}), \]
while the surplus $s \equiv u + \pi$ depends only on the effort and continuation probabilities:

$$s \equiv (1 - \delta) \mathbb{E}_y [y - c \mid e] + \delta \bar{s} + \delta \mathbb{E}_y [\alpha (y) \mid e] (s - \bar{s}) \quad (4)$$

This contract is self-enforcing if and only if it satisfies: (i) the participation constraints $u \geq \bar{u}, \pi \geq \bar{\pi}$; (ii) incentive compatibility for the agent

$$e \in \arg \max \mathbb{E}_y \left[ b (y) + \frac{\delta}{1 - \delta} \alpha (y) (u - \bar{\pi}) \mid \tilde{e} \right] - c (\tilde{e}),$$

and (iii) for the principal,

$$- b (y) + \frac{\delta}{1 - \delta} \alpha (y) (\pi - \bar{\pi}) \text{ is constant in } y;$$

(iv) willingness to make payments,

$$\frac{\delta}{1 - \delta} \alpha (y) (\pi - \bar{\pi}) \geq b (y)$$

$$\frac{\delta}{1 - \delta} \alpha (y) (u - \bar{\pi}) \geq - b (y)$$

and finally (v) for all $y$, $0 \leq \alpha (y) \leq 1$.

The next result reduces these constraints in a manner analogous to Theorem 3.

**Lemma 1** A self-enforcing termination contract can implement effort $e$ with continuation probabilities $\alpha : Y \to [0, 1]$ if and only if:

$$e \in \arg \max \mathbb{E}_y \left[ b (y) + \frac{\delta}{1 - \delta} \alpha (y) (s - \bar{s}) \mid \tilde{e} \right] - c (\tilde{e}) \quad (5)$$

where $s$ is defined by (4) and is greater than or equal to $\bar{s}$.

**Proof.** ($\Rightarrow$) If a termination contract defined by $e, w, b (y), \alpha (y)$ is self-enforcing, it must satisfy (i)-(v) above. Adding the constraints (ii) and (iii) implies it must satisfy the stated constraint, and from (i) it must satisfy $s \geq \bar{s}$.

($\Leftarrow$) Given $e, \alpha : Y \to [0, 1]$ satisfying the stated constraint, with $s \geq \bar{s}$, complete the termination contract by adding $b (y) \equiv 0$ for all $y$ and $w \equiv \mathbb{E} [y - \bar{\pi} \mid e]$. This completed contract gives expected payoffs $u \equiv s - \bar{\pi}$ and $\pi = \bar{\pi}$. To see that it is self-enforcing, observe that it satisfies (i), (iii) and (iv) by definition, (v) because $\alpha (y) \in [0, 1]$, and (ii) because $b (y) \equiv 0$ and $u - \bar{\pi} \equiv s - \bar{s}$, so the assumption (5) implies incentive compatibility. Q.E.D.

**Optimal Incentive Structure.** Given the above, an optimal contract solves:

$$\max_{e \in [0, \bar{s}], \alpha : Y \to \mathbb{R}} s \quad \text{subject to } (4),(5),$$

and $0 \leq \alpha (y) \leq 1$ for all $y \in Y$. 

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It is useful to make the following change of variable. Given a termination contract with effort \(e\) and continuation probabilities \(\alpha : Y \rightarrow [0, 1]\), define the “per-period” loss following outcome \(y\) to be:

\[
\tau (y) \equiv (1 - \alpha (y)) (s - \bar{s}).
\]

Making the change of variable, the optimal contract solves:

\[
\max_{e \in [0, e^FB]} s = \mathbb{E}_y [y - c \mid e] - \frac{\delta}{1 - \delta} \mathbb{E}_y [\tau (y) \mid e]
\]

s.t.

\[
\frac{d}{de} \left[ -\mathbb{E}_y [\tau (y) \mid e] - c (e) \right] = 0,
\]

\[
s - \bar{s} \geq \tau (y) \geq 0 \text{ for all } y.
\]

Note that I have used a first-order condition in place of the agent’s effort constraint, a valid substitution under the Mirrlees-Rogerson conditions.

This optimization problem is linear in \(\tau (y)\). Given the monotone likelihood ratio property, the solution has \(e \leq e^FB\) and

\[
\tau (y) = \begin{cases} 
  s - \bar{s} & \text{if } y < \hat{y} \\
  0 & \text{if } y \geq \hat{y}
\end{cases}
\]

for some \(\hat{y} \in Y\). Reversing the change of variables shows that under an optimal termination contract \(\alpha (y) = 0\) for all \(y < \hat{y}\) and \(\alpha (y) = 1\) for all \(y > \hat{y}\). This proves Theorem 7.

**Limit Inefficiency.** Consider a solution to the optimal contract defined by a pair \(e, \hat{y}\) and a per-period loss \(\tau (y)\) as described above. Given this, and after some algebra, the agent’s first-order condition can be written as:

\[
-\mathbb{E}_y [\tau (y) \mid e] \frac{F_e (\hat{y} | e)}{F (\hat{y} | e)} - c' (e) = 0.
\]

Now, because \(f > 0\) and \(f_e / f\) is continuous on \([\underline{y}, \bar{y}]\), it follows that \(f_e / f \in [-l, l]\) for some finite \(l > 0\). It follows that \(F_e / F \in [-l, l]\). Consequently,

\[
\mathbb{E}_y [\tau (y) \mid e] = \frac{c'(e)}{|F_e / F (\hat{y} | e)|} \geq \frac{c'(e)}{l}.
\]

This provides a bound on expected surplus independent of the discount factor:

\[
s \leq \mathbb{E} [y - c \mid e] - \frac{c'(e)}{l} < s^{FB}.
\]

So even as \(\delta \to 1\), an optimal contract cannot approximate the first-best.
I now consider the existence of optimal relational contracts in the model of Section I. I show that for the relevant cases studied in the paper, an optimal contract will exist. The argument is closely fashioned on the self-generation construction of Abreu et al. (1990). They prove that in a broad class of games with a finite number of hidden actions and a continuous noisy performance measure, the equilibrium payoff set is compact — this implies the existence of optimal equilibria. I cannot directly adopt their result or proof, but instead use a slightly different argument.

Define $s_{FB} \equiv \mathbb{E}_{\theta,y} [y - c | e_{FB}(\theta)]$ to be the first-best joint surplus. Let $E \subseteq [s, s_{FB}]$ be the set of joint surpluses that are consistent with a self-enforcing contract. To characterize $E$, I first define an associated contracting problem (Problem C) for each $s \in [\overline{s}, s_{FB}]$.

$$\max_{e(\cdot), W(\cdot)} (1 - \delta) \mathbb{E}_{\theta,y} [y - c | e(\theta)] + \delta s$$

subject to $e(\theta) \in \arg \max_{e} \mathbb{E}_{y} [W(\varphi) | e] - c(e, \theta)$ for all $\theta$,

$$\frac{\delta}{1 - \delta} (s - \overline{s}) \geq \sup_{\varphi} W(\varphi) - \inf_{\varphi} W(\varphi).$$

Problem C looks for the optimal effort schedule among those that could be enforced given a fixed continuation surplus $s$. To account for possibility that the parties could forego trade, allow $e(\cdot)$ to be chosen either as a function $e : \Theta \rightarrow [0, \overline{s}]$ or as $e(\theta) = 0$, where $c(0) \equiv 0$ and $\mathbb{E}[y - c | e = 0] \equiv \overline{s}$.

**Assumption** Problem C admits a solution for all $s \in [\overline{s}, s_{FB}]$.

Define $m(s)$ to be the maximized value of Problem C for each $s \in [\overline{s}, s_{FB}]$.

**Lemma 2** The set $\{s : m(s) \geq s\}$ has a largest element $s^*$ and $m(s^*) = s^*$.

**Proof.** First, when $s = \overline{s}$, the only feasible solution to Problem C is to set $e(\theta) = 0$ or $e(\theta) = 0$, so consequently $m(\overline{s}) = \overline{s}$. Second, $m(s)$ is weakly increasing because an increase in $s$ both increases the objective and relaxes the constraints. Third, for all $s \in [\overline{s}, s_{FB}]$, $m(s) \leq s_{FB}$ by the definition of $s_{FB}$. Thus $m : [\overline{s}, s_{FB}] \rightarrow [\overline{s}, s_{FB}]$ is nondecreasing with $m(\overline{s}) = \overline{s}$ and $m(s_{FB}) \leq s_{FB}$. Tarski’s fixed point theorem applied to the function $m$ implies the existence of a largest fixed point. $Q.E.D.$

**Lemma 3** $E \supseteq [\overline{s}, s^*]$. 

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Proof. I first argue that $s^* \in E$. Suppose $e(\theta)$ solves Problem C for $s = s^*$. Because $e(\theta)$ is feasible for Problem C, Theorem 3 implies that it can be implemented by a stationary self-enforcing contract. Moreover, as $m(s^*) = s^*$, this stationary contract will generate surplus $s^*$. Thus $s^* \in E$ and the result follows from convexity. Q.E.D.

Lemma 4 $E \subseteq [\overline{s}, s^*]$. 

Proof. Suppose to the contrary that there is some $s^1 > s^*$ with $s^1 \in E$. Then the contract that supports $s^1$ has some initial effort $e(\theta)$ and continuation payoffs $s^1(\varphi)$ such that:

$$s^1 = (1 - \delta)\mathbb{E}_{\theta,y}[y - c(\theta)] + \delta \mathbb{E}_{\theta,y}[s^1(\varphi)|\theta(\theta)]$$

Self-enforcement implies that for each $\varphi$, $s^1(\varphi) \in E$.

Suppose that for all $\varphi$, $s^1(\varphi) \leq s$. Then $e(\theta)$ must be feasible for Problem C given the parameter $s$ and hence $m(s) \geq s$. But because $s > s^*$ this contradicts the definition of $s^*$. Thus, it must be that $s^1(\varphi) > s$ for some $\varphi$, or in other words that $\sup_{\varphi} s^1(\varphi) > s$. Consequently since $s > s^* > m(s^*) = m(\sup_{\varphi} s^1(\varphi))$. I now argue to a contradiction by constructing a sequence $s^1, s^2, s^3, \ldots$ converging to some $\hat{s}$ with the property that $\hat{s} > s^*$ and $m(\hat{s}) = \hat{s}$.

Define $s^2 \equiv \sup_{\varphi} s^1(\varphi) - \varepsilon^2$, where $\varepsilon^2 > 0$ is chosen small enough so that $s^2 > m(\sup_{\varphi} s^1(\varphi))$. Since $e(\theta)$ must be admissible for Problem C with parameter $\sup_{\varphi} s^1(\varphi)$, the definition of $s^1$ above implies that $m(\sup_{\varphi} s^1(\varphi)) \geq s^1$. Because $m(\sup_{\varphi} s^1(\varphi)) \equiv m(s^2 + \varepsilon^2)$, it follows that $s^2 > m(s^2 + \varepsilon^2) \geq s^1$.

Now, because $s^1(\varphi) \in E$ for all $\varphi$, and $s^2 < \sup_{\varphi} s^1(\varphi)$, it must be the case that $s^2 \in E$. So this same construction can be repeated to find some $\varepsilon^3$ and corresponding $s^3$ with $s^3 > m(s^3 + \varepsilon^3) \geq s^2$. Moreover, it is possible to take $0 < \varepsilon^3 < \varepsilon^2$. Iterating this process yields an increasing sequence $s^n \to \hat{s}$ and a decreasing sequence $\varepsilon^n \to 0$ such that for all $n$,

$$s^{n+1} > m(s^{n+1} + \varepsilon^{n+1}) \geq s^n.$$ 

Taking limits implies that $m(\hat{s}) = \hat{s}$. Since $\hat{s} > s^* > s^*$ this yields a contradiction. Q.E.D.

This establishes that if Problem C admits a solution for any $s$, then $E = [\overline{s}, s^*]$ so an optimal contract exists. Problem C is a relatively straightforward problem. It is easy to check that if $\Theta$ and $Y$ are finite, it will admit a solution. Given that $\Theta$ and $Y$ are continuous, the problem needs to be checked separately for different informational conditions. If there is symmetric information, i.e. $\varphi \equiv \{\theta, e, y\}$, then an assumption that $S$ is concave and $c$ is convex ensures a solution. With hidden information, the approach to combining the
constraints taken in Section III, combined with concavity, ensures a solution. The moral hazard existence problem is more complicated and is discussed by Holmström (1979). Under the Mirrlees-Rogerson conditions, however, the first-order approach is valid, and a solution certainly exists. Thus, for the cases considered in the paper, existence of an optimal contract is not a problem.