Reputation and Conflict

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Abstract

We study reputation in conflict games. The players can use their first round actions to signal their types. With strategic substitutes, a reputation for being hostile is desirable, because it causes the opponent to be more dovish in period two. Dovish behavior in round one is a sign of weakness, which makes the opponent more hawkish. With strategic substitutes, neither player wants to be seen as weak. Therefore, the desire to build a reputation for toughness will lead to increased hawkishness in period one. This can make both players worse off in the stage where they build reputation. With strategic complements, the situation is reversed. A reputation for being hostile causes the opponent to become more hawkish, which is undesirable.*Keywords*: conflict, reputation, strategic complements, strategic substitutes.

1 Introduction

In a repeated relationship, a player observes his opponent's past behavior, and adjusts his own actions accordingly. This allows for rich strategic interactions. The *folk theorem* for infinitely repeated games of complete information shows that with very patient players, the set of subgame perfect equilibrium outcomes is very large (Fudenberg and Maskin [3]). In the infinitely repeated prisoner's dilemma, patient players can support cooperation by using trigger strategies (Friedman [2]). However, real-world decision makers may behave myopically. In democratic countries, political leaders naturally focus on winning the next election. Dictators should, in theory, be less myopic. However, there are many countries where the typical dictator cannot expect to hold on to power for very long (Acemoglu and Robinson).

A model of interaction between players who interact repeatedly but lack a truly long-run perspective is a *finitely* repeated game. In finitely repeated games of complete information, the folk theorem doesn't apply. For example, there cannot be any cooperation in subgame perfect equilibrium of a finitely repeated prisoner's dilemma. However, this strong result is an artefact of the complete information assumption, which makes reputation building impossible. Suppose instead that each player has a privately known type, which doesn't change over time. As the game unfolds, a rational player may learn something about the opponent's true type by observing his actions. Accordingly, it may be rational for the opponent to give up a short run benefit in order to "build a reputation". By choosing his actions carefully, he can *signal* his type. This can make cooperation rational in a finitely repeated prisoner's dilemma (Kreps, Milgrom, Roberts and Wilson [5]).

To study reputation building in the simplest possible way, we suppose the following conflict game is played twice:

	H	D
H	$h_i - c$	h_i
D	$-d$	0

Thus, there are two rounds. The types, h_1 and h_2, are independently drawn before the first round, and remain the same in both rounds. After being privately informed of their own types, the two players make simultaneous choices (D or H) in round 1. Between the two rounds, they learn what the opponent did in round 1, which allows them to update their beliefs about the opponent's type. Finally, they simultaneously choose D or H in round 2. For simplicity, we suppose there is no discounting, so each player strives to maximize the sum of his payoffs in the two rounds.

The players can use their first round actions to signal their types. Specifically, playing hawk in the first round will lead to a reputation for being hostile. This may or may not be desirable, however, depending on whether the game has strategic substitutes or complements. With strategic substitutes, a reputation for being hostile is desirable, because it causes the opponent to be more dovish in period two. Dovish behavior in round one is a sign of weakness, which makes the opponent more hawkish. With strategic substitutes, neither player wants to be seen as weak. Therefore, the desire to build a reputation for toughness will lead to increased hawkishness in period one.

This corresponds to what the Industrial Organization literature calls the *top dog* effect (Fudenberg and Tirole [4]).

With strategic complements, the situation is reversed. A reputation for being hostile causes the opponent to become more hawkish, which is undesirable. With strategic complements, neither player wants a reputation for toughness, which leads to reduced hawkishness in period one. This corresponds to what the Industrial Organization literature calls the *puppy dog* effect (Fudenberg and Tirole [4]).

Thus, the incentive to signal depends on whether the underlying game has strategic complements (like stag-hunt) or strategic substitutes (like chicken). For this reason, we will also consider a "hybrid" model, where each players not only does not know the hostility of the opponent - he does not even know whether the opponent thinks the game is chicken or stag-hunt. In the hybrid game, playing hawk in round one may make the opponent either more or less likely to play hawk in round two, depending on whether the opponent sees actions as strategic complements or strategic substitutes. Therefore, an additional layer of deception is overlaid on the signaling incentives. A hostile player who sees the game as chicken, but thinks the opponent very likely sees the game as a stag-hunt, may play dove in round one. This deception will give the opponent a false sense of security which can be exploited in round two.

We now turn to the formal analysis of the twice-repeated game. To simplify the analysis, we maintain the following assumption.

Assumption 5.1 (i) $k_1 = k_2 = 0$; (ii) η_1 and η_2 are independently drawn from the uniform distribution on $[\underline{h}, \bar{h}]$, where normalize so that $\bar{h} - \underline{h} = 1$; (iii) $\underline{h} < c - d$ and $\bar{h} > 0$.

Notice that part (iii) implies that both dominant strategy doves and dominant strategy hawks exist. Under these assumptions, Theorems 3 and 4 in Baliga and Sjöström [1] imply that the corresponding *one-shot* game has a unique equilibrium. The cut-off point $\tilde{h}$ for the one-shot game satisfies the indifference condition

$$\Psi^i(\tilde{h}, \tilde{h}) \equiv \tilde{h} + (d - c)(\bar{h} - \tilde{h}) = 0$$

so

$$\tilde{h} = \frac{\bar{h}}{1 + c - d}(c - d) \tag{1}$$

where we notice that $1 + c - d > 1 + \underline{h} = \bar{h} > 0$.

2 Strategic Complements

In this section we suppose $c < d$. To avoid dealing with different cases, assume

$$0 < c < \bar{h} < d \tag{2}$$

By assumption, a player's type is the same in both rounds. Therefore, with strategic complements, the players will have an incentive to signal dovishness in round 1 in order to encourage the opponent to be dovish in round 2.

Proposition 1 *With strategic complements, there is a PBE where* in each round *the probability of conflict is strictly lower than in the unique equilibrium of the one-shot game.*

Proof. The proof is by construction. Under our assumptions, the following quadratic polynomial in h has two negative real roots:

$$\Omega(h) \equiv h^2 + \left(2 - \left(\bar{h} + d\right)\right) h + \left(\bar{h} - c\right)(d - c)$$

The biggest root is denoted $\hat{h}$. Thus, $\Omega(\hat{h}) = 0$. It can be shown that $c - d < \hat{h} < 0$.[1] Define

$$h^* \equiv \frac{\hat{h} - (c - d)}{\hat{h} - \underline{h}}(c - d)$$

[1] Write $\Omega(h) = h^2 + Ah + B$. The roots of the polynomial are $-\frac{1}{2}A \pm \frac{1}{2}\sqrt{A^2 - 4B}$. Our assumptions imply $\underline{h} < 0$, $c < d$ and $c < \bar{h} = 1 + \underline{h} < 1$, so

$$\bar{h} + d = 1 + \underline{h} + d < 1 + c < 2.$$

Therefore, $A > 0$ and $B > 0$. Now,

$$A^2 - 4B = \left(2 - \left(\bar{h} + d\right)\right)^2 - 4\left(\bar{h} - c\right)(d - c)$$

The derivative of this expression with respect to $\bar{h}$ is $-2A - 4(d - c) < 0$. Since $\bar{h} < 1 - (d - c)$, we obtain

$$A^2 - 4B > (1 - c)^2 - 4(1 - d)(d - c).$$

For any d, we have $4(1 - d)(d - c) \leq (1 - c)^2$, so $A^2 - 4B > 0$. Therefore, the roots are real. Since $B > 0$, they are both negative. Notice that $\Omega(c - d) = 2(c - d)\left(c - d + 1 - \bar{h}\right) < 0$, since $1 + c > \bar{h} + d$. Since $\hat{h}$ is the biggest root of $\Omega(h) = 0$, we must have $\hat{h} > c - d$.

It can be shown that $h^* < \hat{h}$.[2]

Consider the following strategies. In the first round, player i chooses H iff $h_i \geq \hat{h}$. Second round behavior is as follows. If both players chose H in round 1, then in round 2, player i chooses H iff $h_i \geq c - d$. If both chose D in round 1, then in round 2, player i chooses H iff $h_i \geq 0$. If player i chose H and player j chose D in round 1, then in round 2, player i chooses H iff $h_i \geq h^*$, and player j chooses H iff $h_j \geq c - d$.

We claim this is a PBE. First, consider round 2. Any player who chose H in round 1 is expected to choose H again in round 2. Therefore, if the opponent chose H in round 1, only dominant strategy doves choose D in round 2. If both players chose D in round 1, then each is expected to choose D again in round 2. Therefore, only dominant strategy hawks choose H in round 2. Finally, suppose player i chose H and player j chose D. Then, player j is expected to choose D only if he is a dominant strategy dove. Conditional on player j's type being less that $\hat{h}$ (which is indicated by the fact that he chose D in round 1), he is a dominant strategy dove with probability $\frac{(c-d)-\underline{h}}{\hat{h}-\underline{h}}$. Therefore, leader i prefers to choose H if and only if

$$h_i - \frac{\hat{h} - (c-d)}{\hat{h} - \underline{h}} c \geq -\frac{\hat{h} - (c-d)}{\hat{h} - \underline{h}} d$$

This is equivalent to $h_i \geq h^*$. Therefore, the strategies are sequentially rational in round 2.

Next, consider round 1. It suffices to show that the cut-off type $\hat{h}$ is indifferent between H and D in round 1. Notice that if type $\hat{h}$ chooses H

[2] We claim

$$\frac{\hat{h} - (c-d)}{\hat{h} - \underline{h}} (c-d) < \hat{h}$$

which can be rewritten as

$$\hat{h}^2 + \hat{h}(1 - \bar{h} + d - c) + (d-c)^2 > 0 \tag{3}$$

Now $\Omega(\hat{h}) = 0$. Using the definition of Ω, some straightforward calculation yields

$$\begin{aligned} \hat{h}^2 + \hat{h}(1 - \bar{h} + d - c) + (d-c)^2 &= \hat{h}^2 + \hat{h}(1 - \bar{h} + d - c) + (d-c)^2 - \Omega(\hat{h}) \\ &= (2d - c - 1)\hat{h} + (d - \bar{h})(d - c) \end{aligned}$$

The last expression is strictly positive because $\hat{h} < 0 < d - c$ and our assumptions imply $2d - c - 1 < d - \bar{h}$. Therefore, (3) holds.

in round 1, then in round 2, his best choice is surely H. So suppose type $\hat{h}$ chooses H in both rounds. With probability $\bar{h} - \hat{h}$, the opponent also chooses H in both rounds. With probability $\hat{h} - (c - d)$, the opponent chooses D in round 1 but switches to H in round 2. With probability $(c - d) - \underline{h}$, the opponent is a dominant strategy dove who chooses D in both rounds. Therefore, type $\hat{h}$ gets expected payoff

$$2\hat{h} - \left(\bar{h} - \hat{h}\right) 2c - \left(\hat{h} - (c - d)\right) c \tag{4}$$

from choosing H in both rounds.

Now, if type $\hat{h}$ instead chooses D in round 1, then in round 2, his best choice is D if player 1 chose D in round 1, and H otherwise. This is so because $h^* < \hat{h} < 0$.[3] If he plays like this, his payoff is zero if the opponent's type is less than $\hat{h}$. If his opponent's type is greater than $\hat{h}$, however, he gets $-d$ in round 1, and $\hat{h} - c$ in round 2. Thus,his expected payoff is

$$\left(\bar{h} - \hat{h}\right) (-d + \hat{h} - c) \tag{5}$$

The expressions (4) and (5) are equal if $\Omega(\hat{h}) = 0$, which holds by construction. Therefore, in round 1, type $\hat{h}$ is indifferent between H and D. Thus, we have constructed a PBE. It remains only to show that the probability of conflict is lower in this PBE than it would be in the one shot game.

Notice that they coexist peacefully iff both choose D. In the twice repeated game, this happens *in both rounds* if both types are below $\hat{h}$. In contrast, in the one-shot game, they coexist peacefully iff both types are below $\tilde{h}$, defined by (1). So it suffices to show $\hat{h} > \tilde{h}$. Some calculation yields

$$\Omega(\tilde{h}) = \left(1 - \frac{\bar{h}}{1 + c - d}\right) \left(\frac{\bar{h}}{1 + c - d} + c\right) (c - d)$$

This expression is strictly negative, because $0 < \bar{h} = 1 + \underline{h} < 1 + c - d$. Since $\hat{h}$ is the biggest root of $\Omega(h) = 0$, we must have $\hat{h} > \tilde{h}$. ■

[3] If $h^* > \hat{h}$, the cut off type would actually prefer to choose D in period 2, if he chose H and the opponent D in period 1. Also types slightly above $\hat{h}$, who in equilibrium choose H in period 1, would prefer to switch to D in period 2 if the opponent chooses D in period 1. Thus, in equilibrium, players with type above h^* would always choose H in both periods. Players with types between $\hat{h}$ and h^* choose H in period 1 but switch to D in period 2 if the opponent chooses D in period 1. This complication is ruled out by the assumption $\bar{h} > c$, which guarantees that $h^* < \hat{h}$ (see previous footnote). Thus, we could relax the assumption $\bar{h} > c$, and get a slightly more complex two-period equilibrium.

Now suppose the game is repeated for $T > 2$ periods. Let $h^* = \left(\bar{h} - (c-d)\right) c$ and consider the following strategies: in period 1, player i chooses H iff $h_i \geq h^*$. If both players chose H in round 1, then, in subsequent rounds, player i chooses H iff $h_i \geq c - d$. If player i chose H and player j chose D in round 1 or the reverse, then player i chooses H iff $h_i \geq c - d$. If both players chose D in round 1, in periods 2 to $T - 2$, behavior is as follows: player i chooses D iff $h_i \leq h^*$.

2.1 Why democracy interferes with reputation-building

We now show that the happy outcome of the previous section is not necessarily true with democracies. Consider two democracies who play the twice repeated game. The time line is the same as in the previous section, except that between the two rounds, the median voter in each country can decide to replace his current leader with a new leader, drawn again from the uniform distribution on $[\underline{h}, \bar{h}]$. Consider now an equilibrium with a round one cut-off $\hat{h}$. That is, in round 1, the leader chooses H iff $h_i \geq \hat{h}$. In the equilibrium exhibited in the previous section, if leader i chooses H in round 1 and leader j chooses D, then leader j is known to have a low hostility. Nevertheless, if hawkish leader i stays in power, leader j will be very likely to choose H in round 2 in self-defense. To avoid this bad outcome, the median voter of country i can instead replace his hawkish leader with some one drawn from the population. In this case, the second round will be equivalent to a one-shot game where leader i is drawn from $[\underline{h}, \bar{h}]$ and leader j from a truncated distribution $[\underline{h}, \hat{h}]$. The one-shot game has a unique equilibrium (see the lemma below) where leader j is more likely to choose D than he would have been, had the hawkish leader i remained in power. Therefore, the median voter gains from replacing his hawkish leader. Therefore, we conclude that in the game between two democracies, a leader who early on reveals that he is more hawkish than the opponent will be replaced. Now, this has two countervailing effects. First, in the equilibrium of the previous section, one reason not to choose H in round 1 is that, since it reveals a hawkish type, this makes the opponent very likely to choose H in the next round. With democracies, this is no longer true, since the hawkish type is replaced. This increases the incentive to choose H in round 1. On the other hand, the leader who chooses H in round 1 risks losing the rents from office R in round 2. This reduces the incentive to choose H in round 1. Thus, a priori, it is not clear if democracies will have a higher or lower round one cut-off. However, the second effect is weak if R is small. Thus, it will be true that in long run interactions, democracies are less peaceful than dictatorships if rents from office are small (but more peaceful otherwise).

We need the following result to show that a one-shot game, where one player comes from a truncated distribution, has a unique equilibrium. .

Lemma 2 *In the one-shot game, suppose h_1 in drawn from a uniform distribution on $[\underline{h}, h']$ and h_2 is drawn from a uniform distribution on $[\underline{h}, \bar{h}]$. Then, the unique equilibrium has cut-off points $(\hat{h}_1, \hat{h}_2)$ as follows. If $h' > -\bar{h}(d-c)$ then*

$$\hat{h}_1 = -\frac{\bar{h}\left(h' - \bar{h} + 1\right)(d-c) + h'(d-c)^2}{h' - \bar{h} + 1 - (d-c)^2} \tag{6}$$

$$\hat{h}_2 = -\frac{h'(d-c) + \bar{h}(d-c)^2}{h' - \bar{h} + 1 - (d-c)^2} \tag{7}$$

If $h' \leq -\bar{h}(d-c)$ then $\hat{h}_1 = h'$ and $\hat{h}_2 = 0$.

Proof. Uniqueness follows from uniformity. For an interior cut-off equilibrium, the cut-off points $(\hat{h}_1, \hat{h}_2)$ must satisfy

$$\hat{h}_1 - (\bar{h} - \hat{h}_2)c = -(\bar{h} - \hat{h}_2)d$$

and

$$\hat{h}_2 - \frac{h' - \hat{h}_1}{h' - \underline{h}}c = -\frac{h' - \hat{h}_1}{h' - \underline{h}}d$$

The solutions are given by (6) and (7). Moreover, an interior equilibrium requires $\hat{h}_1 < h'$ which holds if $h' > -\bar{h}(d-c)$. Otherwise, player 1 always chooses D. ∎

2.2 The more general case

The previous sections may have given the impression that dominant strategy doves are important in order to allow repetition a chance to reduce conflict. We now indicate that the result that repetition can reduce conflict is true more generally. Let the distribution of types be F, which is not necessarily uniform. Consider the function

$$\Omega(h) = h - c + (1 - F(h))d + F(h)h$$

Notice that

$$\Omega(\underline{h}) = \underline{h} - c + d$$

and

$$\Omega(0) = -c + (1 - F(0))d$$

If there are no dominant-strategy doves, then $\underline{h} - c > -d$, so $\Omega(\underline{h}) > 0$. Moreover, $\Omega(0) < 0$ if $1 - F(0) < \frac{c}{d}$, i.e., if there are not too many dominant strategy hawks. In this case, since Ω is continuous, there is $\hat{h} \in (\underline{h}, \hat{h})$ such that $\Omega(\hat{h}) = 0$.

Notice that if there are no dominant strategy doves, then in the one-shot game, there is an equilibrium where each chooses H with probability one. Moreover, from Theorem 3 in Baliga and Sjöström [1], this is the unique equilibrium if $F' < 1/(d - c)$. We now show that repetition allows them to choose D with some probability.

Proposition 3 *Suppose $\underline{h} - c > -d$ and $1 - F(0) < \frac{c}{d}$. Let $\hat{h} \in (\underline{h}, \hat{h})$ satisfy $\Omega(\hat{h}) = 0$. In the twice repeated game, the following is an equilibrium. In round 1, player i plays H iff $h_i \geq \hat{h}$. If at least one player played H in round 1, then both play H in round 2. If both played D in round 1, then in round 2, player i plays H iff $h_i \geq 0$.*

Proof. It is clear that the strategies are sequentially rational in round 2. Indeed, if both played D in round 1, then each thinks the other player will play D in round 2 as well. Therefore, it is sequentially rational to play H iff $h_i \geq 0$.

Consider round 1. All we need to check is that the cut-off type $\hat{h}$ is indifferent between D and H. Suppose he chooses H in round 1. With probability $1 - F(\hat{h})$ the opponent also chooses H. In round 2, each will surely choose H. Therefore, type $\hat{h}$'s expected payoff is

$$\left(\hat{h} - (1 - F(\hat{h}))c\right) + \left(\hat{h} - c\right) \tag{8}$$

where the first parenthesis is his expectation in round 1, and the second his certain payoff in round 2. Suppose instead he chooses D in round 1. The best thing to do in round 2 is to choose D iff the opponent also chose D in round 1 (since $\hat{h} < 0$). Now in round 1, the opponent chooses H with probability $1 - F(\hat{h})$. Therefore, if type $\hat{h}$ chooses D in round 1, his expected total payoff is

$$\left(1 - F(\hat{h})\right)\left(-d + \hat{h} - c\right) + F(\hat{h}) \times 0 \tag{9}$$

Now, for type $\hat{h}$ to be indifferent between D and H in round 1, (8) and (9) must be equal. But it is easy to check that this is true if and only if $\Omega(\hat{h}) = 0$. ■

Thus, consider a case where dominant strategy hawks exist but no dominant strategy doves. If the game is played once, the unique equilibrium involves both choosing H with probability one. But, if the game is repeated, then if there are not too many dominant strategy hawks, there is an equilibrium where D is chosen with positive probability.

3 Playing Repeated Chicken

> "[W]hat is in dispute is ..everyone's expectations about how a participant will behave in the future. To yield may be to signal that one can be expected to yield." Schelling [7], p.118
>
> "Think about what would have happened if we had sat down to negotiate [in the Paris Summit Conference of 1960] without having an apology from the United States. The aggressors would have wanted to bend us. But if we had bent our back, they would immediately have thrown a saddle on us, and then they would have sat themselves on top of us and begun to drive on us." Khrushchev quoted in Leites [6], pp. 11-12

Consider again the twice-repeated game model of Section 2, but now $c > d$. Again, the distribution of types is uniform on $[\underline{h}, \bar{h}]$, where $\bar{h} - \underline{h} = 1$, and $\underline{h} < 0 < d < c < \bar{h}$. Thus, both dominant strategy hawks and doves exist. The unique one-shot equilibrium cut-off point $\tilde{h}$ is again given by (1),

$$\tilde{h} = \frac{\bar{h}}{1 + c - d}(c - d). \tag{10}$$

Notice that $0 < \tilde{h} < c - d$ as $c - d > 0$ and $\bar{h} = 1 + \underline{h} < 1$

We now show that repeating the game may cause an *increase* in the level of conflict in round 1 which of course makes both players worse off.

Proposition 4 *In twice repeated chicken, there is a PBE where* in round 1 *the probability of conflict is strictly higher than it would be in the unique equilibrium of the one-shot game.*

Proof. The proof is by construction. Let

$$h^{**} - \left(\frac{\bar{h} - h^{**}}{\bar{h} - h}\right) c = -\left(\frac{\bar{h} - h^{**}}{\bar{h} - h}\right) d \text{ or}$$

$$h^{**}(h) \equiv \frac{\bar{h}\,(c-d)}{\left(\bar{h} - h\right) + (c-d)}. \tag{11}$$

Also, let

$$h^{*} - \left(\frac{h - h^{*}}{h - \underline{h}}\right) c = -\left(\frac{h - h^{*}}{h - \underline{h}}\right) d \text{ or}$$

$$h^{*}(h) \equiv \frac{h\,(c-d)}{(h - \underline{h}) + (c-d)}. \tag{12}$$

Finally, let

$$\Omega(h) = h - \left(\bar{h} - h\right)(c-d) + \left(h^{**}(h) - h\right) d + \left(h - h^{*}(h)\right) c \tag{13}$$

Note that

$$\begin{aligned} \Omega(\tilde{h}) &= \tilde{h} - \left(\bar{h} - \tilde{h}\right)(c-d) + \left(h^{**}(\tilde{h}) - \tilde{h}\right) d + \left(\tilde{h} - h^{*}(\tilde{h})\right) c \\ &= \left(h^{**}(\tilde{h}) - \tilde{h}\right) d + \left(\tilde{h} - h^{*}(\tilde{h})\right) c. \end{aligned}$$

As $\frac{\bar{h}}{1+c-d}(c-d) = \tilde{h} > 0 > \underline{h}$, we have $h^{*}(\tilde{h}) < \tilde{h} < h^{**}(\tilde{h})$. This implies that $\Omega(\tilde{h}) > 0$ under our assumptions. Also, $\Omega(0) = \left(\frac{d}{\bar{h}+(c-d)} - 1\right)\bar{h}\,(c-d) < 0$ as $d < c < \bar{h}$. Hence, by continuity of Ω, there is a $\hat{h} \in (0, \tilde{h})$ such that $\Omega(\hat{h}) = 0$. Let $\hat{h}$ be the highest solution to $\Omega(h) = 0$ where $h \in [0, \tilde{h}]$. Also, notice that for $\hat{h} \in (0, \tilde{h})$, $0 < h^{*}(\hat{h}) < \hat{h} < h^{**}(\hat{h}) < c - d$.

Consider the following strategies. In round 1, player i chooses H iff $h_i \geq \hat{h}$. If both choose H in round 1, then in round 2, player i chooses H iff $h_i \geq h^{**}(\hat{h})$. If both choose D in round 1, then in round 2, player i chooses H iff $h_i \geq h^{*}(\hat{h})$. If player i chooses H and player j chooses D in round 1, then in round 2, player i chooses H iff $h_i \geq 0$, and player j chooses H iff $h_j \geq c - d$.

We claim this is a PBE. First, consider round 2. Any player who chose H in round 1 must have a hostility level greater than $\hat{h}$ and is expected to choose H again if and only if $h \geq h^{**}(\hat{h})$. But then from (11), type $h^{**}(\hat{h})$ is indifferent between H and D and it is sequentially rational to play H iff

$h \geq h^{**}(\hat{h})$. Similarly, if both players play D in the first round, from (12), type $h^*(\hat{h})$ is indifferent between playing H and D and it is sequentially rational to play H iff $h \geq h^*(\hat{h})$. Finally, suppose player i chose H and player j chose D. Then, as $\hat{h} < c - d$,player j is known not be a dominant strategy hawk and player i is known not to be a dominant strategy dove. Hence it is an equilibrium for player j to play D unless he is a dominant strategy hawk and for player i to play H unless he is a dominant strategy dove. This leads to the profile (H, D) on the equilibrium path.

Next, consider round 1. It suffices to show that the cut-off type $\hat{h}$ is indifferent between H and D in round 1. Notice that if type $\hat{h}$ chooses H in round 1, then in round 2, as $0 < h^*(\hat{h}) < \hat{h} < h^{**}(\hat{h}) < c - d$, he chooses D if his opponent chose H in round 1 and H if his opponent chose D. The first event occurs with probability $\bar{h} - \hat{h}$ and the second with probability $\hat{h} - \underline{h}$. Therefore, type $\hat{h}$ gets expected payoff

$$\hat{h} - \left(\bar{h} - \hat{h}\right) c - \left(\bar{h} - \hat{h}\right) \frac{\left(\bar{h} - h^{**}(\hat{h})\right) d}{\bar{h} - \hat{h}} + \left(\hat{h} - \underline{h}\right) \hat{h}. \tag{14}$$

Now, if type $\hat{h}$ instead chooses D in round 1, as $0 < h^*(\hat{h}) < \hat{h} < c - d$, he chooses D if his opponent chose H in round 1 and H if his opponent chose D. The first event occurs with probability $\bar{h} - \hat{h}$ and the second with probability $\hat{h} - \underline{h}$. Therefore, type $\hat{h}$ gets expected payoff

$$-\left(\bar{h} - \hat{h}\right) d - \left(\bar{h} - \hat{h}\right) d + \left(\hat{h} - \underline{h}\right) \left(\hat{h} - \frac{\left(\hat{h} - h^*(\hat{h})\right) c}{\hat{h} - \underline{h}}\right) \tag{15}$$

The expressions (14) and (15) are equal if $\Omega(\hat{h}) = 0$, which holds by construction. Therefore, in round 1, type $\hat{h}$ is indifferent between H and D. Thus, we have constructed a PBE. It remains only to show that the probability of conflict is higher in round 1 in this PBE than it would be in the one shot game. But this is obviously the case because $\hat{h} < \tilde{h}$. ■

Remark 5 *Conflict in round 2 can be higher or lower than in the unique equilibrium of the one-shot game. The two players coexist peacefully in round 2 if they both played H but both have types below $h^{**}(\hat{h})$. They coexist peacefully in round 2 if they both played D but both have types below $h^*(\hat{h})$. Hence,* *the total probability of peaceful coexistence for an arbitrary round 1 cutoff h is*

$$h^2 + \left(\bar{h} - h\right)\left(h^{**}(h)\right)^2 + \left(h - \underline{h}\right)\left(h^*(h)\right)^2 \text{ or}$$
$$h^2 + \left(\bar{h} - h\right)\left(\frac{\bar{h}\,(c-d)}{\left(\bar{h} - h\right) + (c-d)}\right)^2 + \left(h - \underline{h}\right)\left(\frac{h\,(c-d)}{\left(h - \underline{h}\right) + (c-d)}\right)^2.$$

The derivative of this (according to Maple calculations reported at bottom of document) with respect to h is

$$2h + \frac{\bar{h}^2\,(c-d)^2}{\left(c-d+\bar{h}-h\right)^3}\left(\left(\bar{h}-h\right)-(c-d)\right) + h\frac{(c-d)^2}{\left(c-d+h-\underline{h}\right)^3}\left((3h-2\underline{h})\left(c-d-\underline{h}\right)+h^2\right).$$

If $\bar{h} > (1+(c-d))\,(c-d)$, when $h \in [0,\tilde{h}]$, this expression is positive. Hence, as $0 < \hat{h} < \tilde{h}$, the total level of conflict increases when the game is repeated.

Proposition 6 *The equilibrium constructed in proposition 4 is the unique cut-off equilibrium, provided that $c - d < \min\{\bar{h}, -\underline{h}\}$.*

Proof. We now provide conditions which guarantee there are no other cutoff equilibria: $\bar{h} - (c-d) > c-d$ and $-\underline{h} > c-d$. Notice that because $1 > \bar{h} = 1 + \underline{h} > c-d$, $\bar{h} > 2(c-d) > (1+c-d)\,(c-d)$ and hence, the total probability of conflict increases with repetition under these assumptions.

Consider a round 1 cut-off equilibrium with cutoff $\hat{h}$. As types below $\hat{h}$ play D and types above play H, we must have $0 \leq \hat{h} \leq c-d$. Notice that given our assumption $\bar{h} - \hat{h} \geq \bar{h} - (c-d) > c-d$ and $\hat{h} - \underline{h} \geq -\underline{h} > c-d$. This implies the flatness condition is satisfied on conditional distributions created at the cutoff $\hat{h}$: $\frac{1}{\bar{h}-\hat{h}} < \frac{1}{c-d}$ and $\frac{1}{\hat{h}-\underline{h}} < \frac{1}{c-d}$. In round 2, after (H,H) or (D,D) are played, there is unique cutoff equilibrium. The only question is what action profile is played in round 2 when player i plays H and player j plays D in round 1?

We claim that the unique equilibrium is for this profile to be played again. This can be seen by examining the reaction functions for the two players. Player i plays H for certain if the probability that that player j plays H, p_j, is less than $\frac{\hat{h}}{c-d}$. In that case, $h - p_j c \geq -p_j d$ for $h \geq \hat{h}$. If $p_j > \frac{\hat{h}}{c-d}$, player i plays H if $h \geq p_j\,(c-d)$. Hence, the probability that player i plays H is

$\frac{1-F(p_j(c-d))}{1-F(\hat{h})} = \frac{\bar{h}-p_j(c-d)}{(\bar{h}-\hat{h})}$. That is, the probability that player i plays H is

$$\begin{aligned} p_i &= 1 \text{ if } 0 \leq p_j \leq \frac{\hat{h}}{c-d} \\ &= \frac{\bar{h} - p_j (c-d)}{\left(\bar{h} - \hat{h}\right)} \text{ if } 1 \geq p_j > \frac{\hat{h}}{c-d}. \end{aligned}$$

Player j plays D for certain if the probability that that player i plays $H,\ p_i$, is greater than$\frac{\hat{h}}{c-d}$. In that case, $h - p_i c \leq -p_i d$ for $h \leq \hat{h}$. If $p_i < \frac{\hat{h}}{c-d}$, player j plays H if $h \geq p_i (c-d)$. Hence, the probability that player i plays H is $1 - \frac{F(p_i(c-d))}{F(\hat{h})} = \frac{\hat{h}-p_i(c-d)}{\hat{h}-\underline{h}}$. That is, the probability that player i plays H is

$$\begin{aligned} p_j &= 0 \text{ if } p_i \geq \frac{\hat{h}}{c-d} \\ &= \frac{\hat{h} - p_i (c-d)}{\hat{h} - \underline{h}} \text{ if } 0 \leq p_i \leq \frac{\hat{h}}{c-d}. \end{aligned}$$

Note that p_j ranges from 0 to $\frac{\hat{h}}{\hat{h}-\underline{h}}$. Also, the last equality can be written as

$$p_i = \frac{\hat{h}}{c-d} - \frac{p_j(\hat{h} - \underline{h})}{c-d}. \tag{16}$$

It can be verified that for all $p_j \in [0, \frac{\hat{h}}{\hat{h}-\underline{h}}]$ (16) lies below player i's reaction function. Hence, the only point where they intersect is at the corner of their flat and vertical portions and the unique equilibrium is for player i to play H and player j to play D. ∎

3.1 The more general case

Now assume that there is a distribution F on $[\underline{h}, \bar{h}]$ and continue to normalize so that $\bar{h} - \underline{h} = 1$. Moreover, to avoid dealing with different "cases", suppose $0 < d < c < \bar{h}$ and $\underline{h} < 0$. Thus, both dominant strategy hawks and doves exist. Also, assume $F' < \frac{1}{c-d}$. Accordingly, if the game is played only once, we know there is a unique equilibrium. The one-shot equilibrium cut-off point $\tilde{h}$ satisfies the indifference condition $\tilde{h} - (1 - F(\tilde{h}))c = -(1 - F(\tilde{h}))d$ or

$$\tilde{h} + \left(1 - F(\tilde{h})\right)(d - c) = 0. \tag{17}$$

As $(1 - F(0))(d - c) < 0 < c - d + (1 - F(c - d))(d - c) = F(c - d)(c - d)$, we must have $0 < \tilde{h} < c - d$.

We now show that repeating the game may cause an *increase* in the level of conflict. We state some further definitions which are useful for the statement and proof of our result. Let

$$h^{**} - \left(\frac{1 - F(h^{**})}{1 - F(h)}\right) c = -\left(\frac{1 - F(h^{**})}{1 - F(h)}\right) d \text{ or}$$

$$h^{**}(h) \equiv \left(\frac{1 - F(h^{**})}{1 - F(h)}\right)(c - d). \tag{18}$$

As $\frac{\tilde{h}}{(c-d)} = \left(1 - F(\tilde{h})\right)$, we have

$$h^{**}(\tilde{h})\tilde{h} = \left(1 - F(h^{**}(\tilde{h}))\right)(c - d)^2).$$

This implies that $h^{**}(\tilde{h}) > \tilde{h}$. Otherwise we have

$$\tilde{h}^2 \geq h^{**}(\tilde{h})\tilde{h} = \left(1 - F(h^{**}(\tilde{h}))\right)(c - d)^2 \geq \left(1 - F(\tilde{h})\right)(c - d)^2 = \tilde{h}\,(c - d)$$

which contradicts the fact that $\tilde{h} < c - d$.

Also, let

$$h^{*} - \left(\frac{F(h) - F(h^{*})}{F(h)}\right) c = -\left(\frac{F(h) - F(h^{*})}{F(h)}\right) d \text{ or}$$

$$h^{*}(h) \equiv \left(\frac{F(h) - F(h^{*})}{F(h)}\right)(c - d). \tag{19}$$

Notice that

$$h^{*}(\tilde{h}) - \tilde{h} = \left(1 - \frac{F(h^{*}\left(\tilde{h}\right))}{F(\tilde{h})}\right)(c - d).$$

This implies that $h^{*}(\tilde{h}) < \tilde{h}$. Otherwise, we have

$$0 < h^{*}(\tilde{h}) - \tilde{h} = \left(1 - \frac{F(h^{*}(\tilde{h}))}{F(\tilde{h})}\right)(c - d) < 0.$$

Proposition 7 *Assume* $-(1-2F(0))(c-d)+F(h^{**}(0))d-F(h^*(0))c<0$. *In twice repeated chicken, there is a PBE where* in round 1 *the probability of conflict is strictly higher than it would be in the unique equilibrium of the one-shot game.*

Proof. The proof is by construction. Let

$$\Omega(h)=h-(1-F(h))(c-d)+(F(h^{**}(h))-F(h))d+(F(h)-F(h^*(h)))c. \tag{20}$$

At $h=\tilde{h}$, this becomes

$$\begin{aligned}\Omega(\tilde{h}) &= \tilde{h}-\left(1-F(\tilde{h})\right)(c-d)+\left(F(h^{**}(h))-F(\tilde{h})\right)d+\left(F(\tilde{h})-F(h^*(h))\right)c \\ &= \left(F(h^{**}(h))-F(\tilde{h})\right)d+\left(F(\tilde{h})-F(h^*(h))\right)c>0\end{aligned}$$

as $h^*(\tilde{h})<\tilde{h}<h^{**}(\tilde{h})$. Also,

$$\begin{aligned}\Omega(0) &= -(1-F(0))(c-d)+(F(h^{**}(0))-F(0))d+(F(0)-F(h^*(0)))c \\ &= -(1-2F(0))(c-d)+F(h^{**}(0))d-F(h^*(0))c<0\end{aligned}$$

by assumption. Hence, by continuity of Ω, there is a $\hat{h}\in(0,\tilde{h})$ such that $\Omega(\hat{h})=0$. Let $\hat{h}$ be the highest solution to $\Omega(h)=0$ where $h\in[0,\tilde{h}]$. Also, notice that for $\hat{h}\in(0,\tilde{h})$, $0<h^*(\hat{h})<\hat{h}<h^{**}(\hat{h})<c-d$.

Consider the following strategies. In round 1, player i chooses H iff $h_i\geq\hat{h}$. If both choose H in round 1, then in round 2, player i chooses H iff $h_i\geq h^{**}(\hat{h})$. If both choose D in round 1, then in round 2, player i chooses H iff $h_i\geq h^*(\hat{h})$. If player i chooses H and player j chooses D in round 1, then in round 2, player i chooses H iff $h_i\geq 0$, and player j chooses H iff $h_j\geq c-d$.

We claim this is a PBE. First, consider round 2. Any player who chose H in round 1 must have a hostility level greater than $\hat{h}$ and is expected to choose H again if and only if $h\geq h^{**}(\hat{h})$. But then from (11), type $h^{**}(\hat{h})$ is indifferent between H and D and it is sequentially rational to play H iff $h\geq h^{**}(\hat{h})$. Similarly, if both players play D in the first round, from (12), type $h^*(\hat{h})$ is indifferent between playing H and D and it is sequentially rational to play H iff $h\geq h^*(\hat{h})$. Finally, suppose player i chose H and player j chose D. Then, as $\hat{h}<c-d$,player j is known not be a dominant strategy hawk and player i is known not to be a dominant strategy dove. Hence it is an equilibrium for player j to play D unless he is a dominant strategy hawk

and for player i to play H unless he is a dominant strategy dove. This leads to the profile (H, D) on the equilibrium path.

Next, consider round 1. It suffices to show that the cut-off type $\hat{h}$ is indifferent between H and D in round 1. Notice that if type $\hat{h}$ chooses H in round 1, then in round 2, as $0 < h^*(\hat{h}) < \hat{h} < h^{**}(\hat{h}) < c - d$, he chooses D if his opponent chose H in round 1 and H if his opponent chose D. The first event occurs with probability $\bar{h} - \hat{h}$ and the second with probability $\hat{h} - \underline{h}$. Therefore, type $\hat{h}$ gets expected payoff

$$\hat{h} - \left(1 - F(\hat{h})\right) c - \left(1 - F(\hat{h})\right) \left(\frac{1 - F(h^{**})}{1 - F(\hat{h})}\right) d + F(\hat{h})\hat{h}. \qquad (21)$$

Now, if type $\hat{h}$ instead chooses D in round 1, as $0 < h^*(\hat{h}) < \hat{h} < c - d$, he chooses D if his opponent chose H in round 1 and H if his opponent chose D. The first event occurs with probability $\bar{h} - \hat{h}$ and the second with probability $\hat{h} - \underline{h}$. Therefore, type $\hat{h}$ gets expected payoff

$$-\left(1 - F(\hat{h})\right) d - \left(1 - F(\hat{h})\right) d + F(\hat{h}) \left(\hat{h} - \left(\frac{F(\hat{h}) - F(h^*)}{F(\hat{h})}\right) c\right) \qquad (22)$$

The expressions (21) and (22) are equal if $\Omega(\hat{h}) = 0$, which holds by construction. Therefore, in round 1, type $\hat{h}$ is indifferent between H and D. Thus, we have constructed a PBE. It remains only to show that the probability of conflict is higher in round 1 in this PBE than it would be in the one shot game. But this is obviously the case because $\hat{h} < \tilde{h}$. ■

4 Hybrid Game

We have assumed that while a player does not know how hostile his opponent is, he does know if his opponent sees the game as Chicken or Stag Hunt. But we may not know an opponent's true objectives and whether he backs down or not if we become aggressive. To capture this scenario we now add another dimension of asymmetric information. We now assume that d can be low d_L with probability q or high d_H with probability $1 - q$. We assume

$$\underline{h} < 0 < d_L < c < \bar{h} < d_H.$$

For simplicity we assume there are no doves in the stag hunt game: $\underline{h} > c - d_H$. Other details are as above: each player has a hostility level h drawn uniformly and independently from $[\underline{h}, \bar{h}]$. Each player i privately observes his hostility level h_i and his cost of defense d_i. We focus on the case of strategic complements.

We assume that

$$c < qd_L + (1-q)\, d_H.$$

It is possible to show that the players have increasing reaction functions. If the game is played only once, we know there is a unique equilibrium and this equilibrium is symmetric. In that equilibrium, the probability that each player plays H is

$$\begin{aligned} p &= 1 - qF(p(c-d_L)) - (1-q)\, F(p(c-d_H)) \\ &= 1 + \underline{h} - pc + p\, (qd_L + (1-q)\, d_H) \end{aligned}$$

or, as $1 + \underline{h} = \bar{h}$,

$$p = \frac{\bar{h}}{1 + c - (qd_L + (1-q)\, d_H)}.$$

Player i with cost of defense d_i is hawkish if and only if $h_i \geq p(c-d_i) \equiv \frac{\bar{h}(c-d_i)}{1+c-(qd_L+(1-q)d_H)}$. Notice that when $q = 0$ and the game is known to be Stag Hunt, the unique equilibrium is for both players to play H.

We now show that in this game, whatever the cost of defense, there is an equilibrium where each player reduces his aggressiveness in the first round. If the cost of defense is high d_H, the logic for this is the same as in Stag Hunt: each player wants to increase the probability that an opponent is cooperative in the second round. But if the cost of defense is low d_L, the logic is different. In that case, even a dominant strategy hawk may play dovishly in the first round to "trick" the opponent into being cooperative in the second round and then take advantage of him.

Proposition 8 *Suppose* $\bar{h}d_H - c < 0$, $1+c > \bar{h}+d_H$, *and* $c < qd_L + (1-q)\, d_H$. *In the twice repeated hybrid game, if the probability of low cost of defense q is sufficiently low, there is a PBE where* in the first round *the probability of conflict is strictly less than it would be in the unique equilibrium of the one-shot game.*

Proof. The proof is by construction. Define

$$\tilde{h} = \frac{q\bar{h}\, (c - d_L)}{1 + q(c - d_L)}. \tag{23}$$

Also, let $\hat{h}_H(q)$ and $\hat{h}_L(q)$ be defined by the following two equations:

$$\begin{aligned}
&-\left(q\left(\hat{h}_H-\left(\bar{h}-\hat{h}_L\right)c+\bar{h}d_H+\left(\hat{h}_L-(c-d_L)\right)\left(\hat{h}_H-c\right)+\left(\hat{h}_L-\tilde{h}\right)d_H\right)\right)= \\
&\qquad (1-q)\left(\hat{h}_H-\left(\bar{h}-\hat{h}_H\right)(c-d_H)+\left(\hat{h}_H-\underline{h}\right)\left(\hat{h}_H-c\right)\right)
\end{aligned} \tag{24}$$

and

$$\begin{aligned}
&q\left(-\left(\bar{h}-\hat{h}_L\right)d_L\right)+(1-q)\left(-\left(\bar{h}-\hat{h}_H\right)d_L\right)-q\left(\bar{h}-\tilde{h}\right)c-(1-q)\,c\left(\bar{h}-\hat{h}_H\right)= \\
&q\left(\hat{h}_L-\left(\bar{h}-\hat{h}_L\right)c\right)+(1-q)\left(\hat{h}_L-\left(\bar{h}-\hat{h}_H\right)c\right)-q\left(\bar{h}-(c-d_L)\right)c-c\,(1-q)\,.
\end{aligned} \tag{25}$$

Let $h^*(q)$ and $h^{**}(q)$ be defined by the following two equations:

$$\begin{aligned}
&\frac{q}{\hat{h}_L-\underline{h}}\left(\left(\hat{h}_L-\underline{h}\right)h^*-\left(\hat{h}_L-(c-d_L)\right)c\right)+(1-q)\,(h^*-c)= \\
&\qquad -\frac{q}{\hat{h}_L-\underline{h}}\left(\left(\hat{h}_L-\underline{h}\right)\hat{h}_L-(c-d_L)\right)d_H-(1-q)\,d_H,\ \text{and}
\end{aligned} \tag{26}$$

$$\frac{q}{\hat{h}_L-\underline{h}}\left(\left(\hat{h}_L-\underline{h}\right)h^{**}-\left(\hat{h}_L-\tilde{h}\right)c\right)+(1-q)\,h^{**}=-\frac{q}{\hat{h}_L-\underline{h}}\left(\hat{h}_L-\tilde{h}\right)c. \tag{27}$$

Finally, define

$$\begin{aligned}
\Omega(\hat{h}_H,q) &\equiv q\left(d_H+\hat{h}_H\right)A-\left(1+q\,(c-d_L)\right)B \text{ where} \\
A &\equiv q\left(\bar{h}\,(c-d_L)-((c-d_L)-\tilde{h})c\right)+(1-q)\left(\left(\bar{h}-\hat{h}_H\right)(c-d_L)+c\left(\hat{h}_H-\underline{h}\right)\right) \text{ and} \\
B &\equiv q\left(\hat{h}_H-\bar{h}c+\bar{h}d_H-(c-d_L)\left(\hat{h}_H-c\right)-\tilde{h}d_H\right)+(1-q)\left(\hat{h}_H-\left(\bar{h}-\hat{h}_H\right)(c-d_H)+\right.
\end{aligned}$$

Solving for $\hat{h}_L$ in (24) and (25), subtracting the two terms and re-arranging gives $\Omega(\hat{h}_H,q)$.

We must have $\Omega(\hat{h}_H,q)=0$ for (24) and (25) to be satisfied simultaneously. We first examine the case where $q=0$ and the game is known to be Stag Hunt. In that case, from (26) and (27), $h^*(0)=c-d_h<h^{**}(0)=0$. Also,

$$\Omega(\hat{h}_H,0)\equiv-\left(\hat{h}_H-\left(\bar{h}-\hat{h}_H\right)(c-d_H)+\left(\hat{h}_H-\underline{h}\right)\left(\hat{h}_H-c\right)\right).$$

Notice that

$$\Omega(0,0) = -\left(-\bar{h}\left(c - d_H\right) + \underline{h}c\right) = c - \bar{h}d_H > 0$$

and

$$\begin{aligned}\Omega(\underline{h},0) &= -\left(\underline{h} - \left(\bar{h} - \underline{h}\right)\left(c - d_H\right)\right) \\ &= \left(c - d_H\right) - \underline{h} < 0.\end{aligned}$$

As Ω is continuous in h, let $\hat{h}_H(0) \in (\underline{h}, 0)$ be the largest root of $\Omega(\hat{h}_H, 0) = 0$. When $q = 0$, in the one-shot game the unique equilibrium is (H, H) and hence there is more peaceful coexistence in the repeated game. Also, from (25),

$$\hat{h}_L(0) = c - \bar{h}d_L + \hat{h}_H d_L.$$

This is increasing in $\hat{h}_H$ and $\hat{h}_L(0) > c - d_L$ at $\hat{h}_H = c - d_H$ by our assumption $1 + c > h + d_H$.

To summarize, when $q = 0$, we have

$$\begin{aligned}\hat{h}_H(0) &\in (c - d_H, 0), \\ h^*(0) &= c - d_h < h^{**}(0) = 0, \\ h^*(0) &< \hat{h}_H(0) < h^{**}(0) \text{ and} \\ \hat{h}_L(0) &> c - d_L.\end{aligned}$$

As (24)-(27) and Ω are continuous in q, these properties are retained if q is small. Henceforth, we assume that q is in fact small and drop the dependence of these variables on q in the notation.

Consider the following strategies. Suppose the player cost of defense is d_H. In round 1, player i chooses H iff $h_i \geq \hat{h}_H$. If he chooses H in round 1 and so does player j, he chooses H in round 2. If he chooses H in round 1 and player j chooses D he chooses H in round 2 if $h > h^*$ where $h^* < \hat{h}_H$. If he chooses D in round 1 and so does his opponent, he chooses D if $h_i \leq h^{**}$ where $h^{**} > \hat{h}_H$. If he chooses D in round 1 and his opponent chooses H, he chooses H. Suppose the player cost of defense is d_L. In round 1, player i chooses H iff $h_i \geq \hat{h}_L > c - d_L$. Note that type $\hat{h}_L$ is a dominant strategy hawk. If player i chose H in round 1 and the opponent chose H, he plays H iff $h \geq c - d_L$. If he chose H in round 1 and the opponent chose D, he plays

H iff $h \geq h'$where h' solves

$$\frac{q}{\hat{h}_L - \underline{h}} \left(\left(\hat{h}_L - \underline{h} \right) h' - \left(\hat{h}_L - (c - d_L) \right) c \right) + (1 - q)(h' - c) = \quad (28)$$
$$- \frac{q}{\hat{h}_L - \underline{h}} \left(\left(\hat{h}_L - \underline{h} \right) h' - (c - d_L) \right) d_L - (1 - q) d_L$$

If he chose D in round 1 and the opponent chose H, he chooses D if and only if $h_i \leq c - d_L$. If he chose D in round 1 and the opponent chose D, he chooses H if $h_i \geq \tilde{h}$.

We claim this is a PBE. First, consider round 2.

Any player who chose H in round 1 is expected to choose H again in round 2. Therefore, if the opponent chose H in round 1, all types with cost d_H choose H in round 2 and only dominant strategy hawks choose H if cost is c_L.

If both players chose D in round 1, then player j is expected to choose H in round 2 if and only if his cost is d_L and $h \geq \tilde{h}$. In this case, in round 2, from (27), when his cost is d_H, type h^{**} is indifferent between playing H and D. Hence, he plays D if $h \leq h^{**}$. In round 2, from (23), when his cost is d_L, type $\tilde{h}$ is indifferent between playing H and D when both players choose D in round 1. Hence, he plays D iff $h \leq \tilde{h}$.

Finally, suppose player i chose H and player j chose D. Then, player j is expected to choose D only if his cost type is d_L and his hostility type $h \leq c - d_L$. Player i is certain to choose H. Then, player j if his cost type is d_H certainly plays H and, if his cost type is d_L, he plays H only if he is a dominant strategy hawk. If player i's cost is d_H, from (26), he is indifferent between playing H and D if his hostility type is h^*. Hence, he plays H if his hostility type $h \geq h^*$. If player i's cost is d_L, from (28), he is indifferent between playing H and D if his hostility type is h'. Hence, he plays H if his hostility type $h \geq h'$. Therefore, the strategies are sequentially rational in round 2.

Next, consider round 1. It suffices to show that the cut-off types $\hat{h}_H$ and $\hat{h}_L$ are indifferent between H and D in round 1. This follows from (24) and 25). as $\hat{h}_i > \frac{\bar{h}(c-d_i)}{1+c-(qd_L+(1-q)d_H)}$, both cost types are less aggressive in round 1 than in the one shot game. ■

A similar phenomenon can occur if the game is likely to have strategic substitutes. Each player plays more aggressively in the first round. If the cost of defense is low d_L, the logic for this is the same as in Chicken: each player wants to increase the probability that an opponent backs off in the second round. But if the cost of defense is low d_H, the logic is different. In that case, a player may play dovishly in the first round to "trick" the opponent into being cooperative in the second round and then play cooperatively with him.

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5 Appendix

$$h^2 + \left(\bar{h} - h\right)\left(h^{**}(h)\right)^2 + \left(h - \underline{h}\right)\left(h^*(h)\right)^2 \text{ or}$$

$$h^2 + \left(\bar{h} - h\right)\left(\frac{\bar{h}\,(c-d)}{\left(\bar{h} - h\right) + (c-d)}\right)^2 + \left(h - \underline{h}\right)\left(\frac{h\,(c-d)}{\left(h - \underline{h}\right) + (c-d)}\right)^2 \quad (29)$$

Rename variables to use maple so $\bar{h} = a$, $\underline{h} = b$:

$$h^2 + (a-h)\left(\frac{a\,(c-d)}{(a-h)+(c-d)}\right)^2 + (h-b)\left(\frac{h\,(c-d)}{(h-b)+(c-d)}\right)^2$$

Then,

$$\frac{d\left((a-h)\left(\frac{a(c-d)}{(a-h)+(c-d)}\right)^2\right)}{dh} = a^2\frac{(c-d)^2}{(a+c-d-h)^3}\,(a-c+d-h)$$

and

$$\frac{d\left((h-b)\left(\frac{h(c-d)}{(h-b)+(c-d)}\right)^2\right)}{dh} = h\frac{(c-d)^2}{(c-d+h-b)^3}\left((3h-2b)\,(c-d-b)+h^2\right)$$

Then returning to original notation, the derivative of (29) with respect to h is

$$2h+\frac{\bar{h}^2\,(c-d)^2}{\left(c-d+\bar{h}-h\right)^3}\left(\left(\bar{h}-h\right)-(c-d)\right)+h\frac{(c-d)^2}{(c-d+h-\underline{h})^3}\left((3h-2\underline{h})\,(c-d-\underline{h})+h^2\right)$$