Positive Political Theory II

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Egregious Errata

Positive Political Theory II (University of Michigan Press, 2005) regrettably contains a variety of obscurities and errors, both typographical and substantive. Most of these are apparent and the appropriate corrections evident. Unfortunately, a few of the mistakes to surface are egregious (and thus correspondingly embarrassing ...). So, with apologies to Jeff and to those using this book, the mistakes within this category identified so far are located and corrected below. (The University of Michigan Press has kindly agreed to publish a corrected version of the ms in 2015.)

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p.4, following discussion of Definition 1.2.

To avoid the occasional ambiguity in some subsequent examples, a sentence has been added that reads, "Hereafter, unless explicitly stated otherwise, assume an aggregation rule f satisfies unrestricted domain, $\mathcal{D} = \mathcal{R}^n$." Corresponding changes are thereafter made where appropriate.

p.5, remarks on simple rules and the Nakamura number.

The examples using majority and plurality rule on this page (from the top to the final paragraph) presume unrestricted domain.

p.22, Definition 2.4.

As written, this is insufficiently tight. Replace the current definition and subsequent paragraph with:

> **Definition 2.4** *Fix a collective choice function $\varphi : R^n \to X$ with range Π_φ. Say that φ respects unanimity if and only if, for all $\rho \in R^n$ and all sets $X^*_\rho \subseteq \Pi_\varphi$ such that $(x, y) \in X^*_\rho \times \Pi_\varphi \backslash X^*_\rho$ implies $P(x, y; \rho) = N$, $\varphi(\rho) \in X^*_\rho$.*
>
> In words, φ respects unanimity if, at any profile for which it is possible to partition the range of φ into two subsets such that every alternative in one subset is strictly preferred by all individuals to every alternative in the other subset, then φ surely selects an alternative from the universally more-preferred-to set. It is easy to see that if there are multiple such sets for any profile, then they must

be ordered by set-inclusion. Although similar in spirit to the Pareto criterion, respecting unanimity and weak Pareto are not equivalent (where a choice function φ is *weakly Paretian* if, for all $x, y \in X$ and all $\rho \in \mathcal{R}^n$, $P(x, y; \rho) = N$ implies $\varphi(\rho) \neq y$).

p.38, line 5.

Should have k and $n - k + 1$ below the equality, *not* i_k and $n - i_k + 1$, respectively.

p.44, Lemma 2.5.

The constructive argument used in the text to prove Lemma 2.5 turns out to have paid insufficient attention to difficulties at the boundaries of the feasible set of alternatives. In particular, the particular choice of profiles for the argument in one case need not be feasible. Below is a complete restatement of the required (slightly extended) Lemma and proof.

Lemma 2.5 *Let $\varphi : S_Q \rightarrow X$ be strategy-proof and satisfy citizen sovereignty. Then φ is weakly Paretian and satisfies peak only.*

Proof (Weakly Paretian) We need to prove $\min\{x_i\}_{i \in N} \leq \varphi(\rho) \leq \max\{x_i\}_{i \in N}$ for all $\rho \in \mathcal{S}_Q$. First show that strategy-proofness and voter sovereignty imply unanimity. To this end, let $\rho \in \mathcal{S}_Q$ be any profile such that, for all $i \in N$, $x_i = x$; we want to show $\varphi(\rho) = x$. By citizen sovereignty there exists a profile $\rho' \in \mathcal{S}_Q$ such that $\varphi(\rho') = x$. By strategy-proofness, for all admissible single-peaked R'_1, $\varphi(R_1, \rho'_{-1}) R_1 \varphi(R'_1, \rho'_{-1})$; in particular, since $\varphi(R'_1, \rho'_{-1}) \equiv \varphi(\rho') = x_1$ by hypothesis and ideal points are uniquely defined, we must have $\varphi(\rho') = \varphi(R_1, \rho'_{-1})$. But $\varphi(R_1, \rho'_{-1}) = x$ and $x = x_2$; hence, by strategy-proofness, it must also be that $\varphi(R_1, \rho'_{-1}) = \varphi(R_1, R_2, \rho'_{-\{1,2\}})$. Repeating the argument for $j = 3, .., n$ yields $\varphi(\rho) = x$. Thus φ satisfies unanimity.

Now consider any profile $\rho^\circ \in \mathcal{S}_Q$ such that $x_1 \leq x_2 \leq \ldots \leq x_n$ and $x_1 < x_n$ and suppose, contrary to φ weakly Paretian, $\varphi(\rho^\circ) > x_n$. For $k = 1, 2, \ldots, n-1$, fix an admissible profile $\rho^k = (\rho'_{\{1,\ldots,k\}}, \rho^\circ_{-\{1,\ldots,k\}}) \in \mathcal{S}_Q$ such that, for all $j \leq k$, $x'_j = x_n$ and $\varphi(\rho^\circ) P'_j a_t$ for all $a_t < x_n$. By φ strategy-proof, $\varphi(\rho^\circ) R_1 \varphi(\rho^1)$. By R_1 single-peaked, therefore, either $\varphi(\rho^1) \geq \varphi(\rho^\circ) > x_n$ or $\varphi(\rho^1) \leq \iota_1(\varphi(\rho^\circ)) < x_1$. Suppose the latter case holds, so $\varphi(\rho^1) < x_1 < x_n$ and, by definition of P'_1, $\varphi(\rho^\circ) P'_1 \varphi(\rho^1)$. But then $j = 1$ can profitably manipulate φ by reporting R_1 instead of R'_1 at ρ^1. So, $\varphi(\rho^1) \geq \varphi(\rho^\circ)$. Moreover, since φ strategy-proof requires $\varphi(\rho^1) R'_1 \varphi(\rho^\circ)$ which, by R'_1 single-peaked, implies $\varphi(\rho^1) \leq \varphi(\rho^\circ)$, we must have $\varphi(\rho^\circ) = \varphi(\rho^1)$. Now applying this argument iteratively, we find $\varphi(\rho^{n-1}) = \ldots = \varphi(\rho^1) = \varphi(\rho^\circ) > x_n$. But by construction, ρ^{n-1} is such that $x'_j = x_n$ for all $j \in N$, in which case, φ unanimous implies $\varphi(\rho^{n-1}) = x_n$: contradiction. A symmetric argument accounts for the case $\varphi(\rho^\circ) < x_1$. Hence, φ must be weakly Paretian.

(Peak only) Now fix $\rho \in \mathcal{S}_Q$, arbitrary $i \in N$ and any single-peaked preference ordering R_i' with $x_i = x_i'$. To prove the lemma, it suffices to show $\varphi(\rho) = \varphi(R_i', \rho_{-i})$. If $\varphi(\rho) = x_i'$, then the result is trivial. So, without loss of generality, assume $x_i' = x_i > \varphi(\rho)$. There are two cases (in what follows, the alternative $\iota_i'(y)$ is defined in exactly the same way as the alternative $\iota_i(y)$, above, but with respect to the preference ordering R_i' rather than to the ordering R_i).

(1) $\iota_i'(\varphi(\rho)) < \iota_i(\varphi(\rho))$: see Figure 2.3.

Figure 2.3: Lemma 2.5, case (1)

Suppose $\varphi(\rho) \neq \varphi(R_i', \rho_{-i})$. Then, depending on the relative position of the alternative $\varphi(R_i', \rho_{-i})$ in Figure 2.3, there are three possibilities, each of which (given single-peakedness) contradicts strategy-proofness.

(1a):

$$\begin{aligned} \varphi(R_i', \rho_{-i}) &< \varphi(\rho) \Rightarrow \varphi(R_i', \rho_{-i}) < \varphi(\rho) < x_i' \\ &\Rightarrow \varphi(\rho) P_i' \varphi(R_i', \rho_{-i}) \\ &\Rightarrow \varphi \text{ manipulable at } (R_i', \rho_{-i}). \end{aligned}$$

(1b):

$$\begin{aligned} & \varphi(\rho) < \varphi(R_i', \rho_{-i}) \leq \iota_i'(\varphi(\rho)) \\ \Rightarrow\ & \varphi(\rho) < \varphi(R_i', \rho_{-i}) < \iota_i(\varphi(\rho)) \\ \Rightarrow\ & \varphi(R_i', \rho_{-i}) P_i \varphi(\rho) \\ \Rightarrow\ & \varphi \text{ manipulable at } \rho. \end{aligned}$$

(1c):

$$\begin{aligned} \iota_i'(\varphi(\rho)) &< \varphi(R_i', \rho_{-i}) \Rightarrow x_i' < \iota_i'(\varphi(\rho)) < \varphi(R_i', \rho_{-i}) \\ &\Rightarrow \varphi(\rho) P_i' \varphi(R_i', \rho_{-i}) \\ &\Rightarrow \varphi \text{ manipulable at } (R_i', \rho_{-i}). \end{aligned}$$

Therefore, if, contrary to the claim, $\varphi(\rho) \neq \varphi(R_i', \rho_{-i})$, we must have $\varphi(R_i', \rho_{-i}) \geq \iota_i(\varphi(\rho)) > x_i'$ and $\iota_i(\varphi(\rho)) \leq \iota_i'(\varphi(\rho))$, the second case.

(2) $\iota_i(\varphi(\rho)) \leq \iota_i'(\varphi(\rho))$. In this case, we need to take a more indirect approach than in case (1) to proving the claim. To this end, let $L = \{j \in N : x_j > x_i'\}$. If $L = \emptyset$ then $x_i' = x_i \geq x_k$ for all $k \in N$.

But since, $\varphi(\rho) \neq \varphi(R_i', \rho_{-i})$ implies $\varphi(R_i', \rho_{-i}) > x_i'$ from Case (1), this contradicts φ weakly Paretian. So assume $L \neq \emptyset$. By relabeling if necessary, let $L = \{1, ..., \ell\}$ and $x_1 \leq x_2 \leq \ldots \leq x_\ell$. For all $j \in L$, let R_j' be such that $x_j' = x_i'$ and $x_j \leq \iota_j'(\varphi(\rho)) \leq \iota_j(\varphi(\rho))$: see Figure 2.4 for the situation in which all of the inequalities are strict for i and some $j \in L$.

Figure 2.4: Lemma 2.5, case (2)

Consider individual $\ell \in L$ and the choice $\varphi(R_\ell', \rho_{-\ell})$. Suppose $\varphi(R_\ell', \rho_{-\ell}) \neq \varphi(\rho)$. Arguing similarly to Case (1), single-peaked preferences implies that φ is manipulable by ℓ unless $\varphi(R_\ell', \rho_{-\ell}) \geq \iota_\ell(\varphi(\rho)) > x_\ell$. But since, by definition, $x_\ell \geq x_k$ for all $k \in N$, $\varphi(R_\ell', \rho_{-\ell}) > x_\ell$ contradicts φ weakly Paretian. Hence $\varphi(R_\ell', \rho_{-\ell}) = \varphi(\rho)$. Applying the same argument, *mutatis mutandis*, to $\ell - 1$ then yields $\varphi(R_\ell', R_{\ell-1}', \rho_{-\{\ell,\ell-1\}}) = \varphi(R_\ell', \rho_{-\ell}) = \varphi(\rho)$. And continuing in this way iteratively for $\ell - 2, \ell - 3, \ldots, 1$, conclude $\varphi(\rho) = \varphi(\rho_L', \rho_{-L})$. Now consider the profile $\rho' = (\rho_{L\cup\{i\}}', \rho_{-L\cup\{i\}})$. There are five possibilities for the relative location of $\varphi(\rho')$. In four of these, $\varphi(\rho') \neq \varphi(\rho)$ and we show that i has opportunity to manipulate the outcome profitably; thus we must have $\varphi(\rho') = \varphi(\rho)$ and, in this case we show $\varphi(\rho') = \varphi(R_i', \rho_{-i})$ as claimed.

(2a): $\varphi(\rho') < \varphi(\rho)$. Referring to Figure 2.4, observe

$$\begin{aligned} \varphi(\rho') &< \varphi(\rho) \Rightarrow \varphi(\rho') < \varphi(\rho) < x_i' \\ &\Rightarrow \varphi(\rho) P_i' \varphi(\rho') \\ &\Rightarrow \varphi \text{ manipulable at } \rho'. \end{aligned}$$

(2b): $\varphi(\rho') = \varphi(\rho)$. In this subcase, we need to show: $\varphi(\rho') = \varphi(R_i', \rho_{-i})$. We first show $\varphi(\rho') = \varphi(R_1, \rho_{-1}')$; to do this, suppose the contrary, $\varphi(\rho') \neq \varphi(R_1, \rho_{-1}')$. Then arguing as for Case (1), φ strategy-proof implies $\iota_1'(\varphi(\rho') \leq \varphi(R_1, \rho_{-1}') \leq \iota_1(\varphi(\rho'))$ with $x_1 < \varphi(R_1, \rho_{-1}')$. Now, for all $j \neq 1$, let R_j'' be an admissible single-peaked ordering such that $x_j'' = x_1$ and

$$\begin{aligned} j &\in L\backslash\{1\} \Rightarrow \iota_j'(\varphi(R_1, \rho_{-1}')) < \iota_j''(\varphi(R_1, \rho_{-1}')), \\ j &\notin L \Rightarrow \iota_j(\varphi(R_1, \rho_{-1}')) < \iota_j''(\varphi(R_1, \rho_{-1}')). \end{aligned}$$

Figure 2.5 illustrates the construction.

Figure 2.5: Lemma 2.5, case (2b)

Using a similar argument to that above for $\varphi(\rho) = \varphi(\rho_L', \rho_{-L})$, conclude $\varphi(R_1, \rho_{-1}') = \varphi(R_1, \rho_{-1}'')$. But then $x_1 = x_2'' = ... =$

$x_n'' < \varphi(R_1, \rho'_{-1})$, contradicting φ unanimous; therefore, $\varphi(\rho') = \varphi(R_1, \rho'_{-1})$ necessarily. Repeating the preceding argument for each $j \in L$ yields

$$\varphi(\rho') = \varphi(R_1, \rho'_{-1}) = \varphi(R_1, R_2, \rho'_{-\{1,2\}}) = ... = \varphi(\rho_L, \rho'_{-L}) = \varphi(R'_i, \rho_{-i})$$

as required.

(2c): $\varphi(\rho) < \varphi(\rho') < \iota_i(\varphi(\rho)))$. But then $\varphi(\rho) = \varphi(\rho'_L, \rho_{-L})$ implies

$$\begin{aligned} \varphi(\rho'_L, \rho_{-L}) &< \varphi(\rho'_{L\cup\{i\}}, \rho_{-L\cup\{i\}}) < \iota_i(\varphi(\rho)) \\ &\Rightarrow \varphi(\rho') P_i \varphi(\rho) \\ &\Rightarrow \varphi \text{ manipulable at } \rho. \end{aligned}$$

(2d): $\iota_i(\varphi(\rho)) \leq \varphi(\rho') \leq \iota'_i(\varphi(\rho))$. In this case, a similar argument as that for (2b) yields a contradiction of φ unanimous [Exercise].

(2e): $\varphi(\rho') > \iota'_i(\varphi(\rho))$. Referring to Figure 2.4, observe

$$\begin{aligned} \varphi(\rho') &> \iota'_i(\varphi(\rho)) \Rightarrow x'_i < \iota'_i(\varphi(\rho)) = \iota'_i(\varphi(\rho'_L, \rho_{-L})) < \varphi(\rho') \\ &\Rightarrow \varphi(\rho'_L, \rho_{-L}) P'_i \varphi(\rho') \\ &\Rightarrow \varphi \text{ manipulable at } \rho'. \end{aligned}$$

Because the situation where $\varphi(\rho) > x'_i$ is completely symmetric, the lemma is proved. $\square$

p.67, Section 2.10: Exercises.

Exercises 2.1 and 2.9 of the original text have been eliminated.

p.79, four lines up from Example 3.3.

The sentence beginning, "Formally, therefore, ..." should read:

"Formally, therefore, Maskin monotonicity implies monotonicity which in turn implies weak monotonicity; ... "

p.82, Definition 3.7.

The definition should have "... $|\cap_{y \in X} R(x, y; \rho)| \geq n - 1$ implies $x \in \varphi(\rho)$", rather than $R(x, y; \rho) = n - 1$ for all $y \in X$ implies ...".

p.92, Second sentence of paragraph immediately preceding Corollary 3.2.

The vote profiles $m \in \mathcal{M}$ should be understood as rationalizable preference relations (see PPT I, ch.1, on rationalizable preferences).

p.139, from the end of line 5 to the middle of line 12.

The claim made in this section is that condition (*) is a *necessary* condition for an alternative y to be agenda independent. This is false and the comments immediately following (*) are thereby nonsense. The text beginning on line 5 with the sentence "In other words ..." to the end of the sentence concluding on line 12 with "... $S(\Gamma_\alpha, P) = y$", should be deleted and replaced with the following:

> "On the other hand, a sufficient condition for an alternative $y \in X\backslash\{x_0\}$ to be the sophisticated outcome irrespective of the agenda $\Gamma_\alpha \in \mathcal{A}(x_0)$, is that
>
> $$y \in P(x_0) \text{ and, } \forall z \in P(x_0)\backslash\{y\},\ y \in P(z). \tag{*}$$
>
> That is, if y satisfies (*) then, for all $\Gamma_\alpha \in \mathcal{A}(x_0)$, $S(\Gamma_\alpha, P) = y$. To see this, recall that every terminal node of an amendment agenda pairs the status quo x_0 against an alternative from the agenda, with every such alternative appearing on at least one terminal node. By the first property of (*), that $y \in P(x_0)$, and the earlier logic for solving binary voting games, y must be the sophisticated equivalent of every terminal node at which y is compared with x_0. Now consider any alternative $z \neq y$. If z is the sophisticated equivalent of some terminal node, then either $z = x_0$ or $z \in P(x_0)$. In either case, (*) implies that y must be the sophisticated equivalent of any pairwise comparison between y and z at the next stage; and so on back up the voting tree, thus establishing the claim. In other words ... "

p.172, Figure 5.12.

The payoffs identified in this diagram for Example 5.8 are incorrect. A corrected example is as follows.

> **Example 5.8** Consider the extensive form game G summarized in Figure 5.12, where the outcome associated with each terminal node of the game tree is defined in terms of the payoffs (u_1, u_2) to individuals 1 and 2 respectively.

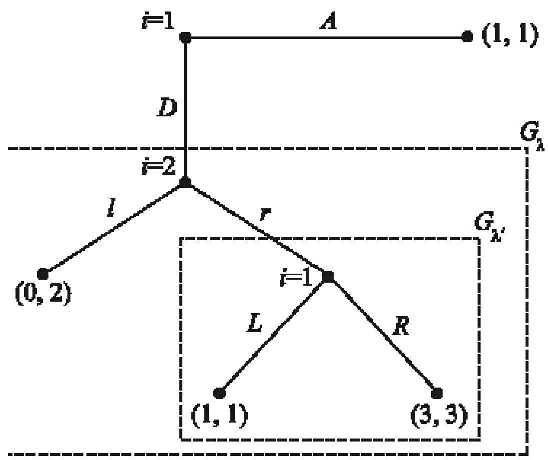

Figure 5.12: The game G

The feasible strategy sets for the two individuals are

$$\begin{aligned}\mathcal{M}_1 &= \{(D,L),(D,R),(A,L),(A,R)\},\\ \mathcal{M}_2 &= \{l,r\}.\end{aligned}$$

There are three Nash equilibria $\sigma = (\sigma_1, \sigma_2) \in \mathcal{M}_1 \times \mathcal{M}_2$ to G:

$$\sigma^* = ((A,L),l);\ \sigma^{**} = ((A,R),l);\ \text{and}\ \sigma^{***} = ((D,R),r).$$

To check σ^* and σ^{**} are both Nash equilibria note that, given $\sigma_2^* = l$, the best 1 can do is play A at his first decision node to obtain a payoff of one rather than zero and, given 1 chooses A, no subsequent decision nodes are reached, so specifying 2 plays l is as good as anything else since 2 receives payoff one in any case; similarly, given 2 plays l, 1 is indifferent between L or R at the final decision node. That σ^{***} is also a Nash equilibrium follows easily from the observation that it yields the unique best payoff for each individual. Moreover, we claim σ^{***} is the only subgame perfect Nash equilibrium to G.

To see that σ^* and σ^{**} are not subgame perfect, consider the two proper subgames, G_λ and $G_{\lambda'}$, in turn. The restriction of σ^* to G_λ is the strategy pair (L,l) which is easily checked to be a Nash equilibrium for G_λ. On the other hand, the restriction of σ^* to $G_{\lambda'}$ is the decision "L" for individual 1, which is clearly not a best response at this decision node. Thus σ^* is not a subgame perfect equilibrium strategy profile for G. Similar reasoning shows σ^{**} is not subgame perfect: here, the restriction of σ^{**} to the (trivial) subgame $G_{\lambda'}$ has individual 1 choosing a best response, but then the restriction of σ^{**} to the subgame G_λ is not a Nash equilibrium for G_λ. This is because individual 2 choosing l is not a best response in G_λ to 1's choice of R, despite the fact that $\sigma_2^{**} = l$ is a best response in G to $\sigma_1^{**} = (A, R)$. Finally, it is easy to check that σ^{***} is a subgame perfect Nash equilibrium to G as claimed. $\square$

p.178, line 4.
The definition of $R(x)$ should read, $R(x) = \{y \in X : yRx\}$.

p.288, last expression in the proof of Theorem 7:10.
By definition of $-\Pi(a^*, y)$, there should be no minus sign on the terms $\omega_i^\beta(\cdot)$ under the summation. The same is true in the argument for Corollary 7.3.

p.288, line 2 of Corollary 7.3.
The assumption that "$p_i(\cdot) = p(\cdot)$" should be replaced by "$\omega_i^c(\cdot)$ is independent of i"; the first line of the proof is then redundant.

p.336, Example 8.2 (5).
Proportional representation, even as described here, is not a scoring rule.

p.383, Exercise 8.3(b).
The last conditional, "... if m is odd" should read "... if $m = 3$". Furthermore, the exercise is intended to concern only pure strategy equilibria.

p.213, Example 6.4

Example 6.4 contains both a typo. and a mystery matrix (pointed out to me by Diwen Si). The corrected version is as follows.

Example 6.4 Let $N = \{1, 2, 3\}$, $\delta_i = \delta$ all i and $q = 2$. By Theorem 6.2, any individual $i \in N$ recognized in the first period proposes an allocation giving a strictly positive amount δV_j to exactly one other committee member $j \neq i$ and nothing to the remaining individual. Consequently, the following three equations must hold in any stationary equilibrium:

$$\begin{aligned} V_1 &= p_1[1 - r_{12}\delta V_2 - r_{13}\delta V_3] + \delta V_1[p_2 r_{21} + p_3 r_{31}]; \\ V_2 &= p_2[1 - r_{21}\delta V_1 - r_{23}\delta V_3] + \delta V_2[p_1 r_{12} + p_3 r_{32}]; \\ V_3 &= p_3[1 - r_{31}\delta V_1 - r_{32}\delta V_2] + \delta V_3[p_1 r_{13} + p_2 r_{23}]. \end{aligned}$$

Suppose two individuals use non-degenerate mixed strategies, say $r_{12}, r_{23} \in (0, 1)$. Then both individuals 1 and 2 must be indifferent over to whom to make an offer, implying that, for all j, $V_j = V$ in equilibrium. In particular, assuming there is a dollar to be distributed, $V = 1/3$. Substituting V for V_j, noting $r_{12} = 1 - r_{13}$ etc, and collecting terms, the preceding system of three equations can be written

$$\delta V \begin{bmatrix} 0 & -p_2 & p_3 \\ p_1 & 0 & -p_3 \\ -p_1 & p_2 & 0 \end{bmatrix} \begin{bmatrix} r_{12} \\ r_{23} \\ r_{31} \end{bmatrix} = [\mathbf{v}] \qquad (*)$$

where $[\mathbf{v}]$ is a 3×1 vector of terms independent of (r_{12}, r_{23}, r_{31}). It is easy to check that the 3×3 matrix is singular. Therefore, there can be a continuum of randomization choices r satisfying $(*)$, that is, there can be a continuum of equilibria differing only in the relative likelihoods of any particular minimal winning majority forming. If $\delta = 0.8$ and $p = (0.45, 0.35, 0.2)$, for instance, any r such that

$$r_{12} = 0.19 + 0.44 r_{31};\ r_{23} = 0.96 + 0.57 r_{31};\ r_{31} \in [0, 0.0625]$$

constitutes stationary equilibrium behavior. $\Box$

p.217, lines 4-7.

The italicized statement is incorrect. Lemma 6.1(3) only shows that if $p_i = p_k$ and $\delta_i < \delta_j$ then $\delta_i V_i \leq \delta_j V_j$, not that $V_i \leq V_j$. Thus, more patient players do not necessarily receive greater equilibrium payoffs than less patient players. See Tomohiko Kawamori (2005), "Players' Patience and Equilibrium Payoffs in the Baron-Ferejohn Model", *Economic Bulletin* 3, 1-5).