Optimal Banking with Delegated Monitoring

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Abstract

Risk-neutral firms with risky projects require external funding from lenders. The project's realized return is private information of each firm. We study a financial intermediation with costly-state verification, where agents cannot commit to their verification strategy, there is aggregate uncertainty and lenders may be risk averse. Static bank contracts are Pareto optimal without aggregate uncertainty, but they may not be so with it. However, regulator can implement a Pareto optimal dynamic contract even with aggregate uncertainty, via a resolution mechanism that uses franchise values as threat to discipline the bank undertake costly monitoring in bad states. Bailout policy, subject to ex ante budget-balancedness, can be welfare-improving if the dynamic bank contract is not financially stable.

1 Introduction

The defining events of the Global Financial Crisis of 2007-2008 are the defaults and near defaults of large financial intermediaries. As a result, regulations regarding the supervision and potential resolution of such institutions took center stage in the subsequent reforms. Indeed, both aspects are prominent in the Dodd-Frank Act, which includes a "living will" feature, i.e., a strategy for rapid and orderly resolution in the event of material financial distress or failure of a financial intermediary, as well as the establishment of the Orderly Liquidation Authority to oversee the resolution, potentially with government funding to support the process. These provisions are highly debated,[1] partly because they provide an alternative (to the usual court procedure) in case of bankruptcy of certain financial firms and because the Orderly Liquidation Authority can access public funding. Accordingly, such regulations require the regulator to set expectations about the supervision and resolution of financial intermediaries and how depositors, as well as tax-payers, get repaid in the event of a crisis.

We contribute to this debate by taking seriously the underlying asymmetric information problem that renders financial intermediation necessary, and its implications to banking regulations. We take into account that bankruptcy is costly and that resolution mechanisms have incentive implications which affect the behavior of all relevant parties in financial intermediation. One advantage of this approach is that we are able to derive Pareto optimal financial contracts as simple debt contracts, according to which default means failure to repay the debt level and the creditors receive whatever is left in that event. Moreover, we derive the Pareto optimality of financial firms to act as intermediaries.

However, as seen in the Financial Crisis, default in financial intermediaries is triggered by defaults in the ultimate borrowers, and a novel feature of our model is to introduce aggregate uncertainty to discuss this. By doing so, our model highlights the two-sided nature of financial contracts: as financial intermediaries, defaults from their borrowers have material implications to the lenders; at the same time, the resolution procedure

[1] See, for example, Acharya and Richardson (2012), Section 4.3, and Bernanke (2017) who have argued for their continual existence.

and the fact that what they get from their borrowers would, at least in part, go to their creditors, also affects the incentives for the intermediaries to undergo costly bankruptcy procedures to recover assets from their borrowers.

We formalize these ideas in a costly-state-verification model where financial intermediaries are set up so that investors can delegate costly monitoring activities, a story behind seminal papers such as Diamond (1984) and Williamson (1986), but with two novel features that make it suitable to address the challenges in financial regulations revealed by the recent crisis mentioned above. First, we assume that the lenders cannot commit to their verification strategies, an important feature that endogenously gives rise to debt contracts as the optimal arrangement.[2] This lack of commitment brings in a novel moral hazard problem in financial intermediation. Thus, default in our model is endogenously determined by the contractual arrangement: a minimum return to the depositors is needed to induce voluntary participation, but a higher interest rate to the lender would imply a higher rate of costly monitoring.

Second, we allow for aggregate uncertainty and hence complete diversification may not be feasible. This feature makes the moral hazard issue faced by financial intermediaries prominent due to their two-sided contracts. In particular, in a crisis situation where defaults from entrepreneurs are more prevalent, financial intermediaries may find it unprofitable to engage in costly verification in the interim stage unless they can retain some proceeds from those activities for themselves; in other words, they would need to pay less to their depositors. With risk-averse depositors, these financially unstable contracts are undesirable and could potentially eliminate the benefits of financial intermediation.

We obtain two sets of results. First, even with this moral hazard issue, financial intermediation can make Pareto improvement without any regulations if there is no aggregate uncertainty. We show that the financial intermediary can provide a two-sided contract

[2] The optimality of debt contract under costly-state-verification goes back to Townsend (1979) and Gale and Hellwig (1985). More recently, Krasa and Villamil (2000) have shown the optimality of the debt contract, by assuming that lenders cannot commit to their verification strategy and allowing for renegotiation at the interim stage, which addressed the criticism that the earlier results are not robust to randomization.

that dominates the optimal direct contract. While Williamson (1986) and Krasa and Villamil (1992) obtain similar results when ex-ante commitment to a monitoring strategy is assumed, we extend this to a setting without such commitments.

Our second set of results highlight our main contribution that deals with optimal regulation under aggregate uncertainty and the situation where there can be instability with financial intermediation. We show that a well-designed resolution mechanism in the form of orderly liquidation similar to the one envisioned in the Dodd-Frank Act can make Pareto improvement. The main incentive issue under aggregate uncertainty is the following. Because of lack of commitment, the bank needs incentives to monitor at the interim stage, which requires that the bank's profit, net of the monitoring cost, be nonnegative in every state. This incentive compatibility constraint limits the bank's ability to provide a deposit contract that smooths depositors' return across different states. As a result, if the firms' aggregate returns are worse than the depositors' outside options in some states, then the bank contract cannot be financially stable in the sense that the depositors get the same return in all states.

Under this financial instability, we show that a resolution mechanism regulated by a banking authority can be welfare-improving by providing dynamic incentives to the bank. Under this mechanism, the bank may engage in costly self verification similar to a conservatorship process, and report the results to the banking authority. Depending on the result, the authority may terminate the bank's charter privilege. The authority can then use this threat to the bank's future profits as incentive for the bank to perform costly monitoring of the firms. We show that, under some mild conditions, such a dynamic bank contract can always Pareto dominate any optimal direct contract, and, unless the bank can provide a financially stable static contract, such dynamic contract can always make Pareto improvement.

Both risk aversion and costly monitoring play a role in obtaining our results. If the static contract is not financially stable, the depositors need to monitor the bank occasionally and that is costly. In contrast, in the dynamic setting, because of future profits, the bank has incentive to self-verify even if it is costly, and that is more efficient.

Moreover, since under dynamic contract the bank can make a negative profit at some states, the bank can provide a deposit contract that second order stochastically dominates the static one, and this is welfare-improving for risk averse depositors. These results also show the socially beneficial role of the franchise value of the bank, which are essential for the provision of dynamic incentives.

Finally, we show that a bailout policy can be welfare improving if the above dynamic contract is not financially stable due to feasibility constraints. Under this scheme, the bank has to pay a premium in good states but may request a bail-out in bad sates. However, to request a bail-out, costly self-verification is necessary for the authority to ascertain the state and the amount of funds already available to the bank. Depending on the result, the bank charter may be terminated. In this case, the scheme needs to provide an incentive for the bank to pay the premium in good states, and to monitor firms as well as to self-verify in bad states. We show that, for sufficiently high discount factors and sufficiently low self-verification cost, a financially stable bank contract is always incentive compatible and is welfare-improving under such policy. Again, risk aversion is crucial for these results: the bailout policy allows for the bank to engineer a contract that second order stochastically dominates the original one, and that makes risk averse depositors strictly better off.

Related Literature

We follow the literature that explains financial intermediation based on asymmetry information (e.g., Freixas and Rochet, 1997). Our paper builds on two intertwining literatures that micro-found financial intermediation and debt contracts as the optimal contractual arrangement, building on the intuition that monitoring borrowers is costly. Diamond (1984) formalized the argument that financial intermediaries can save monitoring costs on behalf of the ultimate lenders. Instead of relying on the nonpecuniary cost of monitoring, our approach adopts the costly-state-verification story introduced by Townsend (1979) and Williamson (1986). Like Winton (1995), we allow for risk averse investors and solve for optimal contract with multiple investors as a baseline. Krasa and Villamil (1992) adopt the same approach to justify financial intermediation, but allow for finitely many lenders instead of the continuum. We advance in this literature by considering the lack of commitment in the verification strategy, with the advantage that debt contracts endogenously emerge as the optimal contracts, even if stochastic verification is allowed, using the logic of Krasa and Villamil (2000). Our main contribution, however, lies in the analysis of how this lack of commitment, which is essential to obtain debt contracts, affects the efficiency of financial intermediation and how policies, such as liquidation procedures and bailouts, can be welfare improving.

There is a large literature that considers banks' risk-taking behavior and optimal regulations to mitigate it, including Hellman et al. (2000) and Allen and Gale (2000). Boyd and De Nicoló (2005) emphasize the importance to take both the asset and the liability sides of the bank balance sheet into account in this debate, and argue that both can be endogenously determined by policy. Keeley (1990) demonstrates the role of banks' franchise values, the discounted sum of its future profits, in disciplining banks' risk taking behavior empirically. Our paper focuses on a new moral hazard problem, the incentive for banks to take costly monitoring activities, that would endogenously determine the bank's asset values and focus on the resolution mechanism instead of capital requirements. The optimal regulation in our model also emphasizes the role of the bank's franchise value.

Our paper is also related to the literature that studies optimal resolution of bank failures and the trade-offs of bailout policies, with the emphasis on their ex post benefits and ex ante incentive effects. Acharya and Yorulmazer (2008) study the trade-off to provide bailouts to surviving banks in the banking crisis due to firesale of assets that leads to inefficient closure. More recent papers include Keister (2016) and Bianchi (2016). Keister (2016) adopts the Diamond-Dybvig (1983) model of banking where banking panics occur due to coordination failures and studies bailout policies in such a context. In contrast, in our model bank failures can occur in all equilibria and bailout policies, bundled with a well-designed liquidation procedures, rely on dynamic incentives. Bianchi (2016) follows the tradition of financial frictions in the style of Kiyotaki and Moore (1997), in which bailout policies help smooth consumptions. Our rationale for bailouts share a similar consumption-smoothing motive but we focus on the micro-foundation of the financial intermediations and primitive frictions that give rise to both the need of intermediation as well as the regulation/intervention needed.

2 Model

Consider an economy with a measure one continuum of firms and a continuum of investors of measure M. For simplicity, we assume that M is a natural number. Each investor is endowed with one unit of funds that may be costlessly stored and can generate a gross return $r > 1$. Each firm has an investment project that requires M units of funds from investors and, if funded, the project generates a stochastic return $w \in W := [0, \bar{w}]$ distributed according to cumulative distribution function, $F : W \to [0, 1]$. We assume that F is absolutely continuous with respect to the Lebesgue measure, that F has a full support, and that $\mathbb{E}_F(w) > Mr$.[3] Firms are risk-neutral and short-lived. Investors are long-lived and may be risk averse. The instantaneous utility of an investor is a strictly increasing and concave function, $u(c)$, of the consumption level c, and the investor has discount factor $\beta \in (0, 1)$.

In this section, we consider direct contracts where one firm borrows directly from M investors as the benchmark case. Since firms last for just one period, only static contracts are feasible. The information friction is that the firm's return, w, is its private information. The investors who lend to the firm, however, may verify the return by paying a cost γ, which is additive to their utilities of consumption. The verified return is private information to the verifier that cannot be credibly shared with other lenders. Before the verification, however, the firm can communicate to its lenders via private messages, and the lenders, after seeing the messages, decide whether to verify the firm or not. A crucial assumption is that the investors cannot commit to this verification strategy ex ante. As

[3] The assumptions about absolute continuity and full support are mainly for expositional reasons. None of our results rely on absolute continuity. The full support assumption, however, guarantees that the asymmetric-information problem is substantial.

a result, the revelation principle does not hold and, following Krasa and Villamil (2000), we consider the following contractual forms and the time line of interactions. We restrict our attention to symmetric contracts to be described as follows.

A direct contract is a pair of functions, (b, ρ), with $b : W \to \mathbb{R}$ the payment to each investor without verification and $\rho : W \times \{1, .., M\} \to \mathbb{R}$ the residual payment after verification, specified as follows. As in Krasa and Villamil (2000), b is essentially voluntary repayment but ρ can be costlessly enforced ex post.[4] After the return $w \in W$ is realized, the firm makes a report to each of its investors. Each investor i then decides to verify or not, based on $\tilde{w}_i$, the return reported to that investor. If investor i does not verify, she receives payment $b(\tilde{w}_i)$. If investor i verifies, she pays a cost γ, and receives a payment $\rho(v, m)$, where v is the amount of funds available *after* paying those who did not verify and m is the number of verifiers. Recall that both the message, $\tilde{w}_i$, and verified output, v, are investor i's private information and cannot be (credibly) shared with other investors.

Given a contract and the above timeline, a strategy for the firm is a measurable function $s_f : W \to \Delta(W^M)$, which maps realized returns to reported returns to each of the M investors. Firms can randomize over messages to investors and need not be truthful. However, each investor may verify this return by paying a cost γ, which is additive to the investor's utility of consumption. A strategy for an investor is a measurable function $s_i : W \to \{0, 1\}$, where 0 indicates no verification and 1 indicates verification. Here we only allow pure strategies on verifications for the investors. Krasa and Villamil (2000) show that this is with no loss of generality when investors cannot commit to the verification strategies ex ante. The intuition is the following. Without commitment, the investor will adopt a randomized verification strategy only if she is indifferent between verifying or not, and this would not survive renegotiation where it is optimal for the borrower to "bribe" the investor not to do it.[5] Finally, the contract between the firm and investors define

[4] Note that, since the firm can choose any message as it wishes, essentially this amounts to a voluntary payment to the lenders, as we will discuss below.

[5] Strictly speaking, we need also to assume that the firm also incur a small cost when being verified for this logic to work. Adding that would complicate the notations without adding any insights here, and one can think of our setting as the limiting case where that cost goes to zero.

payments as a function of the reported returns if there is no verification and the realized returns if lenders decide to verify.

We have the following feasibility constraints on the contract (and strategies):

(F1) Feasibility of ρ: $\rho(v, m)m \leq v$ for all $v \in W$.

(F2) Feasibility of s_f under b: $\sum_{i=1}^{M} b(\tilde{w}_i) \leq w$ for all $(\tilde{w}_1, ...\tilde{w}_M)$ in the support of $s_f(w)$ and for all $w \in W$.

We focus on PBE that are symmetric in the following sense: first, all investors use the same strategy, s_i, and, for the firm, s_f has finite support,[6] and, if a profile of message $(\tilde{w}_1, ..., \tilde{w}_M)$ is in the support of the equilibrium strategy s_f, then all permutations of that message occur with equal probabilities.

We say that a direct contract (b, ρ) is **implementable** if investors are willing to participate. We let W_m be the subset of W in which m investors verify the firm with positive probability. Note that if, in equilibrium, upon receiving a message $\tilde{w}$ the investor does not verify the firm's return, the firm simply pays $b(\tilde{w})$, it must be the case that the payment is the same under b across all such messages that are sent with positive probability, which we will denote by $\bar{b}$. Hence, we may assume that there is only one message corresponding to the subset of W where no lender verifies the firm, W_0. Furthermore, because investors cannot commit to their verification strategies upon signing the contract, they need to be incentivized to verify after the report from the firm comes in. This requires that γ is smaller than the expected payment when receiving a message that implies verification in equilibrium. Formally, let $\tilde{w}$ be a message such that in equilibrium the investor verifies, it must be the case that

$$\mathbb{E}_{w,m} \left\{ u \left[\rho(w - (M - m)\bar{b}, m) \right] \mid \tilde{w} \right\} \geq \gamma.$$

Since this condition has to hold for all messages $\tilde{w}$ which are sent with a positive probability and for which the investor verifies in equilibrium, it implies that

$$\mathbb{E} \left\{ u \left[\rho(w - (M - m)\bar{b}, m) \right] \mid \text{ verification} \right\} \geq \gamma.$$

[6]This assumption is mainly for technical convenience.

As a result, it is then without loss of generality to assume that in equilibrium the firm only sends two messages, $\tilde{w}_0$ or $\tilde{w}_1$, where $\tilde{w}_0$ indicates no verification while $\tilde{w}_1$ indicates verification.

We make one assumption regarding the cost γ:

(A0) $\gamma F(Mr) < \int_0^{Mr} u\left(\frac{w}{M}\right) \mathrm{d}F(w)$.

When $M = 1$, assumption (A0) is essentially identical to assumption (A.2) in Krasa and Villamil (2000), and hence it generalizes that assumption for general M and it plays a similar role here. In particular, note that we only consider equilibria where the investors use a pure strategy, which, as mentioned earlier, can be justified along the renegotiation-proof requirement introduced by Krasa and Villamil (2000) under (A0).[7] The following theorem characterizes (Pareto) optimal contracts.

Theorem 2.1. *Assume (A0). Any optimal contract, $(\bar{b}, \rho)$, takes the form $\rho(v, m) = v/m$. Moreover, under an optimal contract $W_0 = [M\bar{b}, \bar{w}]$.*

According to Theorem 2.1, the (Pareto) optimal contract is a simple debt contract from the firm's perspective: it is characterized by a debt level, $M\bar{b}$, and if the return is above that level the firm repays the debt to all investors equally outright; otherwise, at least some of the investors verify the firm. The fact that the firm pays outright $\bar{b}$ to all investors when it can is a direct implication that the firm must pay the same amount to all who do not verify in equilibrium. To show that a debt contract is optimal, however, one needs to demonstrate that the firm pays out *all* its funds whenever it cannot pay *all* investors the debt level.

For the case $M = 1$, the result is based on the following well-known intuition (e.g., Townsend, 1979; Gale and Hellwig, 1985): any contract deviating from the debt contract necessarily requires the firm to keep *some* of the funds when verified for *some* returns that

[7] Krasa and Villamil (2000) assume risk neutrality and that the firm also occurs a small cost when it is verified, and show that equilibria with random verification is not sustainable under (A0) with $M = 1$. Our result will not be affected by the introduction of a small cost to the firm and the logic to exclude random verification in Krasa and Villamil (2000) does not seem to depend on M nor risk neutrality.

occur with positive probability. It then follows that, while keeping the *expected* payment from the firm constant, one can adjust the contract by lowering the debt level but requiring the firm to pay all its funds whenever it cannot pay the debt level and is verified. This is Pareto improvement because it strictly decreases the probability of verification while holding the expected return the same for the investor. While most papers assume that the investor is risk neutral, the argument is even stronger for risk-averse investors: the original contract is a mean-preserving spread of the new contract, and hence a risk-averse investor would strictly prefer the return distribution of the latter over the former, even without taking the lower verification cost into account.

Our analysis sheds some more light on the interaction between verification cost and risk aversion in addition to this established result. As mentioned earlier, for $M = 1$ there is no conflict between verification cost and risk aversion: whenever the firm cannot pay the debt level the investor has to verify. It is more subtle when there are multiple investors. Indeed, when the firm cannot pay its debt level to all investors but can do so to some, the optimal payment schedule faces a trade-off between verification cost and risk aversion. On the one hand, to save the verification cost, it is better to ask as few lenders as possible to verify. On the other hand, risk-averse investors prefer a payment schedule with more equal payments to *all* investors, but this would require more investors than the minimum to verify the firm.

As a result, under risk neutrality, optimal contract only minimizes verification cost and hence it calls the firm to pay $\bar{b}$ to the maximum number of investors. The resulting contract is the random seniority as in Winton (1995). In contrast, for strictly risk averse investors, the optimal contract calls for *all* investors to verify whenever the firm cannot pay *all* investors the debt level for γ sufficiently small. In general, under the optimal contract some investors are chosen to be paid outright while others are called to verify the firm to divide whatever is left. We fully characterize the optimal number of investors asked to verify, with details given in the Supplemental Appendix, A.1. Finally, although (A0) is sufficient for Theorem 2.1, a weaker condition may also suffice, depending on the fundamentals. See also the Appendix for more discussion.

Theorem 2.1 will be useful for our later analysis as it gives welfare bounds to both the firms and to the investors under direct contracting. Indeed, under optimal direct contracts with debt level $B = M\bar{b}$, the firm's payoff is

$$\int_{w \geq B} (w - B)\ \mathrm{d}F(w).$$

Each investor's payoff is bounded from above by

$$\int_{w<B} u(w/M)\ \mathrm{d}F(w) + u(B/M)[1 - F(B)] - (1/M) \sum_{m=1}^{M} F(mB/M)\gamma. \tag{1}$$

The first two terms reflect the expected payoff to each investor, assuming that they share the returns whenever the firm fails to pay all investors the debt level, which is the upper bound for that payoff, the third term reflects the cost of verification, assuming the fewest number of investors verifying and all have equal chance to do so, which is a lower bound of the verification cost. We will use this bound to compare welfare with bank contracts, which we consider next.

3 Banking: static contracts

We now consider how banks may emerge in such an environment. Here we consider the static case where there are no aggregate shocks, and returns to firms are iid. In this case we show that a static bank contract is sufficient to make Pareto improvement against direct contracting.

As in Section 2, there is a continuum of investors of measure M and a continuum of firms of measure 1 with i.i.d. returns. We assume that the LLN holds exactly when we aggregate the firms' returns and hence there is no aggregate shock.[8] We envision the bank as a risk-neutral agent capable of taking deposits from investors and then using the proceeds to lend to firms. When the bank decides to verify a firm, it has to pay a cost

[8] The literature has recognized issues related to i.i.d. returns in a continuum economy; see, e.g. Sun (2006). Our use of this assumption, however, is mainly for expositional convenience. All our results can be generalized to a setting with a finite but large number of depositors and firms, as will be discussed in concluding remarks.

c_E. This monitoring cost is interpreted as the effort of the banker and does not affect the funds available for distribution to investors.[9] Again, the verified returns are private information to the bank and cannot be credibly shared with investors.

Since debt contracts are optimal without financial intermediation, to show that the bank can make Pareto improvement, we only need to consider such contracts. We call a two-sided contract between the bank and investors on the one side and between the bank and firms on the other side, a *bank contract*. Assuming that it is a debt contract for both sides, we may denote it by (B, d), where B is the debt level each firm owes the bank and d is the promised repayment to each investor. Later on we will establish that such contracts are in fact optimal. One of the insights from Section 2 is that it is optimal to restrict the message space to $\{0, 1\}$, where 0 indicates repayment without verification and 1 indicates verification, and, if the lender, which can be either the bank or the investor, fails to verify, he or she gets nothing.

The two-sided nature of bank contracts introduces new incentive issues that are not present in direct contracting. To formalize those issues we give a precise description of the game as follows. The game is played among firms, the bank, and investors. We assume that all agents have agreed to the contract (B, d) and discuss agents' participation decisions later.

1. Firms' returns are realized and firms simultaneously report (either 0 or 1) to the bank.
2. The bank decides, based on the reports, whether or not to verify each firm.
3. After the bank receives all the payments from the firms, the bank sends a massage (again, either 0 or 1) to each investor simultaneously.
4. After seeing the individual message, each investor decides whether or not to verify the bank.

[9] Note that the cost is c_E per firm. The bank corresponds to a large entity in reality with potential monitoring issues within itself; here we abstract away from those complications.

Given a bank contract (B, d), we formulate the agents' strategies as follows. The strategies for firms and for investors are straightforward extensions from the ones under direct contracting. Each firm decides, as a function of its realized profit, to either repay the debt in full or request verification. The bank's strategy comes in two stages. The first is a verification strategy for each firm, as a function of all the information it has available, i.e., the bank knows which firms have repaid the debt and which requested verification. Second, the bank makes a recommendation to each of its investors, asking them to verify it or repaying the investors in full. Each investor's strategy is then a function which takes the bank's recommendation and maps it to a binary decision (to verify or not). The following gives the formal definitions.

Definition 3.1. (a) For a firm, its strategy is a measurable function $s_f : W \to \{0, 1\}$ that maps its return to its report to the bank.

(b) The bank's strategy has two components. The first is a measurable function $s_B^1 : \mathcal{M}([0,1]) \times [0,1] \to \Delta(\{0,1\})$. The first argument is the subset of firms sending 1, and the second argument is the identity of the firm.[10] Thus, for any given received reports, s_B^1 specifies a (randomized) verification decision for each firm. Then, after the verifications, the bank's strategy has a second component, a measurable function $s_B^2 : W \times [0, M] \to \Delta(\{0, 1\})$, in which the first argument is the amount of available funds to the bank (in per depositor terms) and the second is the identity of the investor, and hence it maps the available funds to the bank's (randomized) report to each investor.[11]

(c) An investor's strategy is a function $s_i : \{0,1\} \to \{0,1\}$, which maps the report from the bank to the verification strategy.

As before, symmetric PBE is our solution concept, i.e., all firms use the same strategy s_f and all investors use the same strategy s_i in equilibrium, and, for the bank, although we allow randomization, we only focus on equilibrium strategies in which s_B^1 verifies all firms that cannot pay B.[12]

[10] $\mathcal{M}([0,1])$ is the set of all measurable subsets of $[0,1]$ (endowed with the sup norm).

[11] In general, s_2^B may also depend on the history that leads up to the available funds.

[12] This assumption is in line with the observation from Krasa and Villamil (2000). In particular, if we

We now discuss the participation decisions of agents. We say that a bank contract (B, d) is **implementable** if there is a symmetric PBE in which the investors are willing to participate, the bank is willing to verify the firms' returns whenever they cannot meet their debt level B, and the bank's expected payoff (net of monitoring costs) is nonnegative.

In contrast to Krasa and Villamil (1992), because of the lack of commitment, we need to discuss the bank's incentive to verify firms after returns realize and requests come in. In particular, two deviations are crucial for incentive compatibility. First, the bank can choose to verify none of the firms and, regardless of investors' reactions, the bank gets at least zero profit. Second, the bank has to decide whether to verify each firm which requests it, and the bank will do so as long as the expected gain (net of what it expects to pay to investors at the margin) from doing so exceeds the cost. Under the contract B, $F(B)$ fraction of the firms request verification. We have the following lemma.

Lemma 3.1. *Let (B, d) be a bank contract. Bank verification of the firms is incentive compatible if and only if*

$$\left\{[1 - F(B)]B + \int_0^B w \mathrm{d}F(w)\right\} \geq Md + F(B)c_E, \tag{2}$$

$$\int_0^B w \mathrm{d}F(w) \geq F(B)c_E. \tag{3}$$

To understand the two conditions above, first note that under the bank contract, (B, d), the bank's expected profit (per investor per firm) is given by

$$\frac{1}{M}\left[[1 - F(B)]B + \int_0^B w \ \mathrm{d}F(w) - F(B)c_E\right] - d. \tag{4}$$

Hence, condition (2) simply says that the bank is making a nonnegative profit and this deters the first deviation mentioned above. Indeed, the left-side of (2) represents the available funds to the bank (in per-firm terms), and the right-side represents two expenditures to be covered, the repayment to investors and the labour cost of verification. The repayment to investors is specified by the contract, but the verification cost requires proper incentives as it is decided in the interim stage. Conditional on the firm sending a message

introduce renegotiation then randomized verification is unlikely to be sustained in equilibrium.

that recommends verification under contract (B, d), the bank's expected payment to be received is given by

$$\frac{\int_0^B w \, \mathrm{d}F(w)}{F(B)},$$

and hence condition (3) requires that conditional expected return to at least cover the cost of verification, c_E, and this would deter the second deviation. The proof essentially shows that these two are the only relevant deviations.

For bank contract to be more efficient, we need the following assumptions.

(A1) Efficient bank verification: $c_E < u^{-1}(\gamma) \sum_{m=1}^{M} \frac{F\left(\frac{mB}{M}\right)}{F(B)}$.

(A2) $\int_{w \in [0,B]} w \mathrm{d}F(w) + (1 - F(B))B - F(B)c_E > Mr$.

Assumption (A1) is necessary: if c_E is large relative to γ, then the bank contract cannot save on monitoring costs. However, the precise comparison is related to the curvature of u, since γ enters the investors' payoff in comparison with u. M also plays a role, as the bank monitors a firm for M investors. Without (A2), the bank is better off by storing cash and generating return r for each investor. The following theorem shows that these two assumptions are also sufficient to ensure that the bank contract can make Pareto improvement for all parties involved.

Theorem 3.1. *Assume (A0). For any implementable direct contract with debt level B that satisfies (A1) and (A2), there is an implementable (static) bank contract that Pareto dominates the direct contract.*

Theorem 3.1 is proved by constructing a bank contract that Pareto dominates the direct contract with debt level B. The bank contract is given by (B, d), where the debt level is left unchanged, and d is given by

$$d = \frac{1}{M} \left\{ \int_0^B w \mathrm{d}F(w) + [1 - F(B)]B - F(B)c_E - \varepsilon \right\}. \tag{5}$$

The proof shows that for $\varepsilon > 0$ but small, $u(d)$ is strictly higher than the payoff upper bound for lenders given by (1). Moreover, by (4), the bank has profit at least ε. This implies that the bank contract is Pareto superior to the direct contract. The crucial observation for $u(d)$ to be higher than the payoff from direct contract is the following.

Notice that, by construction of d, $d+\frac{1}{M}F(B)c_E+\varepsilon$ is the expected payment from the firm to each investor under the direct contract. Moreover, assumption (A1) ensures that the monitoring cost is lower under the bank contract. As a result, even risk aversion implies that having a sure payment of d, which is guaranteed under the bank contract, is strictly better than the risky repayment from the direct contract minus the monitoring cost. This then illustrates the advantage of the bank contract through diversification: first, it can reduce risk by pooling the funds, and, secondly, it can reduce monitoring, as the bank can offer a deposit contract that the bank can almost surely pays to the depositors and hence only monitoring of the bank on behalf of M investors per firm is required.

Assumption (A0) ensures that the optimal direct contracts are simple debt contracts and the investors' payoffs are bounded by equation (1). The assumptions (A1) and (A2) are tight for Theorem 3.1. We have seen the necessity of (A2), which is in fact implied by implementability of the direct contract under risk neutrality. When u is linear, (A1) is also necessary for the bank to be able to provide a better contract. When u is strictly concave, however, (A1) can be relaxed, since the constructed bank contract is strictly better because it is able to provide the certainty equivalence of what the direct contract can provide to the investors.

4 Aggregate shock and financial stability

Here we consider aggregate shocks, so that firms' returns are correlated. We begin by extending the static bank contracts to this environment, and then consider liquidation mechanisms under a regulator who uses the franchise value as an important dynamic motivation to incentivze the bank to perform costly monitoring even when it is not profitable to do so in an economic downturn.

4.1 Environment and static bank contracts

Suppose that the aggregate returns depend on an aggregate state, s, which can be either h (high) or ℓ (low). We assume that the state s is i.i.d. across time according to distribution π (π_s denotes the probability of state s for $s = h, \ell$), and that the firms' returns are i.i.d. according to $F_s(w)$ conditional on s. Suppose that, F_h first-order stochastically dominates F_ℓ, so that for any $B \in [0, \bar{w}]$, $F_h(B) < F_\ell(B)$. We assume that the aggregate state s, once realized, is observable to the bank and firms, but not to investors. Investors learn s only if they verify the bank.[13] Note that this assumption implies that results regarding direct contracting are not affected by this aggregate shock at all: for each specific firm, its return is characterized by the distribution function $F = \pi_h F_h + \pi_\ell F_\ell$.

For bank contracts, the timing is as follows. In the beginning of each period, before the aggregate state is realized, the bank agrees a contract with firms and investors. The bank contract with a firm takes the same form as in direct contracting, (ρ, b). Since the realization of the state is observable to the bank and to the firms and hence is common knowledge among them, the payment without verification can be dependent on the aggregate state, and we use B_s to denote that payment under state $s = h, \ell$.

The bank contract with depositors is slightly more complicated, but can be specified as follows. First, as in Section 2, the contract specifies a repayment amount without verification, d. Note that d has to be state independent as investors do not know the aggregate state without verification. The deposit contract also describes what happens if the bank does not pay back d to all and some investors verify. This is specified by the function $\xi : \{h, \ell\} \times \mathbb{R}_+ \times [0, M] \to \mathbb{R}_+$, where $\xi(s, y, m)$ is the amount paid to each verifying investor when the aggregate state is s, the available funds (in per investor terms) are y and m is the measure of depositors verifying. The bank contract with depositors will thus be denoted by (d, ξ).

The optimal bank contract can be quite complicated in general. However, under an

[13] Even if the bank does not observe s, it can learn it based on reports made from the firms. This assumption therefore eliminates the potential asymmetric information between the bank and firms about the state.

analogous assumption to (A0) in direct contracting, we can show that the debt contract with the firms is in general optimal. Formally, the assumption is given by (A3) below, and the proof of the optimality of debt contract under (A3) can be found in the Supplemental Appendix A.2 (see, in particular, Lemma A.4).

(A3) $\int_0^{Mr} w dF_s(w) > c_E F_s(Mr)$ for both $s = h, \ell$.

Intuitively, since both the bank and firms are risk neutral, (Pareto) optimal contracts only aim at saving the verification cost and the simple debt contract would do the job, regardless of the contract with the investors, (d, ξ).

Besides ensuring the optimality of debt contracts with firms, assumption (A3) also implies that, in equilibrium, each firm who request verification by the bank will indeed be verified, if the bank can keep the verified return. Such a contract may be denoted by $\mathbf{B} = (B_h, B_\ell)$, where B_s is the debt level under state s, $s = h, \ell$. However, depending on the deposit contracts, banks may not be able to keep all the returns from verifying a firm, and additional considerations about the bank's incentive to verify firms are necessary.

The deposit contract is more complicated and, significantly, the bank's incentive to verify firms is linked to its repayment to investors. To see this, for a given $\mathbf{B}$, the contract between the bank and firms, and a contract with investors, (d, ξ), the corresponding equilibrium outcome may be characterized by (d_h, d_ℓ), where d_s is the *average* payment to investors under state s. From this information one can infer the fraction of verifying investors at each state. Thus, any equilibrium outcome can be represented by $(\mathbf{B}, \mathbf{d})$, where $\mathbf{d} = (d, d_h, d_\ell)$. We restrict our attention to contracts such that $d_\ell \leq d_h$.

We highlight a key incentive problem for implementing bank contracts: it requires the bank to make a nonnegative *ex post* profit in both states. This is due to the fact that the bank cannot commit to its verification strategy. So, if the bank were to make a negative profit at some state, it would deviate by not verifying any firms and guarantee at least a zero profit. To respect this incentive constraint, it is necessary that, for both $s = h, \ell$,

$$\frac{1}{M}\left\{\int_0^{B_s} w \mathrm{d}F_s(w) + [1 - F_s(B_s)]B_s\right\} - \frac{1}{M}F_s(B_s)c_E - d_s \geq 0. \tag{6}$$

The first term in (6) is the amount of funds (per investor) the bank collects in equilibrium, which is the maximum amount that the bank can repay to investors to satisfy feasibility. Incentive compatibility, however, requires the payout to investors to be significantly lower than that due to the cost of verification, the second term in (6), as the bank has to keep some of the available funds to compensate for effort costs. A welfare-improving scheme, where the bank repays all its available funds at state ℓ (and makes a loss) but makes a higher profit at state h, may not be incentive compatible.

To study to what extent the incentive problem (6) prevents a better arrangement, we consider the benchmark case where the bank contracts can provide full risk sharing for the depositors.

Definition 4.1. A bank contract, $[\mathbf{B}, (d, \xi)]$, is **financially stable** if there is a PBE in which the bank's expected payoff is nonnegative, and all investors obtain $u(d) \geq u(r)$ in both states $s = h, \ell$.

Our main result here shows that static bank contracts, due to the incentive compatibility condition, (6), has limited scope in achieving financial stability. We have the following theorem.

Theorem 4.1. *Let* $[\mathbf{B}, (d, \xi)]$ *be an implementable bank contract. If*

$$[1 - F_\ell(B_\ell)]B_\ell + \int_0^{B_\ell} w \mathrm{d}F_\ell(w) \leq Mr, \tag{7}$$

then it is not financially stable.

Theorem 4.1 states that if the amount of available funds to the bank in state ℓ is lower than Mr, and if this contract is implementable, then it is not financially stable. Condition (7) essentially states that the amount of available fund to the bank can only cover the repayment to investors at most $d = r$, but, as we have seen earlier, to do that would imply that the bank cannot cover its verification cost. As a result, by (6), it must be the case that $d_\ell < r$. However, implementability requires the investor's expected utility must be at least $u(r)$ and hence d_h must be more than compensating the low d_ℓ relative to r. This implies financial instability.

Theorem 4.1 also highlights the possibility that aggregate shocks may hinder the beneficial role of financial intermediation, when no regulator is present. In particular, since investors do not observe the aggregate state when it is realized, results in Section 2 go through with $F = \pi_h F_h + \pi_\ell F_\ell$. Thus, when F_ℓ is much lower than F_h but the average gives a good return, direct contracting can dominate any static bank contract due to financial instability. In the next section, we introduce a liquidation mechanism and show that banking regulations and dynamic incentives can restore the beneficial role of financial intermediation, even under aggregate shocks.

4.2 Liquidation mechanisms and dynamic contracts

In this section we introduce a regulator to enforce a liquidation mechanism with a dynamic bank contracts, which can be implemented by a bank charter system. A banking charter is a privilege in the sense that once revoked, the bank can no longer operate and can no longer receive future profits. Under this system the regulator can implement dynamic contracts with the bank through threats of revoking the charter and by requiring self-verification from the bank. Importantly, even when the bank cannot repay investors the debt level, the regulator can allow the bank to continue to operate if the self-verification result is satisfactory. As seen in the previous section, under a static contract incentive compatibility requires the bank to make a nonnegative profit in every state, and this requirement limits the bank's ability to offer a financially stable contract. In contrast, a dynamic contract can overcome this incentive issue and improve social welfare.

We first describe the liquidation mechanism. Under this mechanism, the bank has to be chartered by a regulator. We assume that the bank is long-lived and has discount factor β. The fact that this discount factor coincides with investors' discount factor plays no role in our analysis, but simplifies notation.

The regulator has the power to terminate the bank's charter and hence uses future profits as an incentive.[14] However, the regulator does not observe the funds collected by

[14] The key differences between our work and the large literature on dynamic principal-agent problems (e.g., Thomas and Worrall, 1990) is that we have to attend to the agent's (the bank, in our case) incentive

the bank, nor the realized aggregate state, s. We assume a technology with which the bank can self-verify and make both the state and available funds in the bank credibly known to the regulator at a cost of c_B per investor.[15] This self-verification can be done by hiring an external auditor, for example.

The liquidation mechanism implemented by the regulator operates as follows. If the bank fails to pay the investors in full and fails to self-verify, its charter is automatically terminated: in this case, the investors may verify the bank and receive whatever funds are left. Otherwise, termination of the charter is determined by the following dynamic contract. First the contract specifies a promised payment to investors, d, which corresponds to the repayment amount without any verification. If the bank does not pay d to all investors and engages in self-verification, the contract then specifies a termination policy, $\tau(s, y) \in \{0, 1\}$ (here 0 indicates termination), and a payment to each investor, $\chi(s, y) \in \mathbb{R}_+$, where s is the aggregate state and y is the available funds per investor revealed through verification. Since τ governs the fate of the bank's future operation given the self-verification report, we regard this as a liquidation mechanism in the event of a default, i.e., failure to repay d. This contract also has a deposit side that governs the repayments to investors in such an event, given by (d, τ, χ). Note that, in contrast to the last subsection, here χ is a contract between the regulator (on behalf of the investors) and the bank.

After termination, the bank's continuation payoff is zero and hence, if designed appropriately, the termination policy can provide the bank with an incentive to verify the firms. Moreover, these dynamic considerations can sometimes make financially stable contracts feasible in equilibrium, even though a static contract cannot.

to pay third-parties (investors), which the principal (regulator) also cares about. So, while we use future utility promises to incentivise truthful reporting of private information, our construction is quite different to those papers; in particular, the agent (bank) will necessarily need to make a profit (in order to ensure it has an incentive to pay depositors) and thus the agent cannot be held down to his participation constraint, as in Thomas and Worrall (1990).

[15]While we assume a fixed cost per investor, our analysis goes through with a cost which varies with the size of the bank's balance sheet.

Given the dynamic contract, the sequence of actions needs some modification. The timing in each period is as follows (after all parties agree to participate):

1. After the firms' returns realize, all firms simultaneously make reports (either 0 or 1) to the bank.
2. The bank decides, based on the reports from the firms, whether or not to verify each firm.
3. After the bank receives all the payments from the firms it can take one of the following actions:
   (a) pay all investors d (which may not be feasible);
   (b) engage in self-verification—and is thus subject to termination policy, $\tau(s, y)$, and repayment schedule, $\chi(s, y)$;
   (c) do nothing—in which case the bank's charter is automatically terminated.
4. If the bank fails to pay all investors d and fails to sell-verify, each investor decides whether or not to verify the bank.

Given the dynamic nature and the modified timing of the game, the agents' strategies can be defined in an analogous manner to those in Definition 3.1. We emphasize a few key differences. First, strategies have to be indexed by time and may depend on observed histories from previous periods as well as the state s. Furthermore, s_B^2, the bank's reporting strategy has to be amended to allow for the possibility in item 3 above (we'll denote paying d by 0, self-verification by 1, and doing nothing by 2). Finally, s_i, the strategy of the investor, chooses whether to verify the bank or not only when the bank decided to do nothing. While this seems complicated, we will show that when discount rates are high and the cost of self-verification is low, investors have a relatively simple optimal strategy—they will decide to verify if given the opportunity.

Note that the dynamic contract is more efficient than static contracts only if the cost c_B is small, and that will be the case we focus on. Moreover, when c_B is small, the optimal dynamic contract will use investors' verification only as an off-equilibrium threat. Because of that the decisions of investors are straightforward, so our focus is on the interaction between the bank and the regulator, as well as the bank and the firms.

As before, we focus on symmetric equilibria in which the interactions between the bank and firms form a PBE with the property that the bank verifies all firms that request it. We say that a dynamic contract is **implementable** if there is such a PBE in which the investors are willing to participate and the bank is making a nonnegative (expected) profit at the beginning of each period.[16] The following theorem shows that the dynamic bank contracts can make Pareto improvement.

Theorem 4.2. *Assume that u is strictly concave and that c_E satisfies (A3).*

(a) Assume (A0). For any implementable direct contract with debt level B that satisfies (A1) and (A2) for $F = \sum_{s \in S} \pi(s) F_s$, there is an implementable dynamic bank contract that Pareto dominates the direct contract for sufficiently high β and sufficiently low c_B.

(b) Let $[\mathbf{B}, (d, \xi)]$ be an implementable static bank contract that is not financially stable. Then, for sufficiently high β and sufficiently low c_B, there is an implementable dynamic bank contract that Pareto dominates it.

The main ingredient of the proof, which is also the main contribution of the above theorem, is in the design of a dynamic contract which allows for the usual repeated-games arguments and, thus, a Pareto improvement on the original contract. While the details differ for parts (a) and (b), the main ideas can be outlined as follows. We use expected future profits to incentivise the bank to suffer a short-term loss, which allow the contract to increase the depositors' returns at state ℓ at the expense of returns at state h; when designed properly, this is welfare-improving given that investors are risk averse. Once we obtain a contract that both improves investors' welfare as well as gives the bank a positive profit, we can apply the usual repeated-game argument that a sufficiently high β and a sufficiently low c_B ensures that the dynamic incentives are powerful enough to discipline the bank. These requirements are indispensable; for low β's, the dynamic incentives have

[16] Hence, implicitly we allow the bank to leave the charter system at any point.

no bite, and for high c_B's, the self-monitoring technology of the bank is not sufficiently efficient to be useful.

According to Theorem 4.2 (a), one can always devise a dynamic contract to dominate the direct contract, even under aggregate uncertainty, as long as (A1), (A2), and (A3) are satisfied. This result does not hold for static contracts; indeed, for F_ℓ sufficiently concentrated on returns that are close to zero, the depositors have no incentive to monitor the bank at state ℓ. Instead, in the dynamic bank contract, the bank is motivated to self-verify because of concerns about future profits.

Part (b) of Theorem 4.2 shows that a dynamic bank contract can improve upon any implementable static bank contract that is not financially stable. Recall that under the static contract, the bank's incentive to verify defaulting firms requires a nonnegative profit for the bank state by state, condition (6). Given this condition, we construct a dynamic contract in which the bank takes a short-term loss at state ℓ and pays all its available fund to each depositor, a loss because of the monitoring cost. This dynamic contract, although violating (6), is incentive compatible because of future profits: we can decrease d_h, the payment to investors in state h, so that the bank is making a positive expected profit. This change improves the welfare of the investors as they are strictly risk averse and prefer the higher payment in the low state.

We remark here that while bank profit is crucial to use the dynamic incentives, it also limits the benefits to the investors. In fact, for any given static contract, there is a maximal (average) profit that can give to the bank to ensure that the investor is better off, which depends the investors' risk aversion. Given the profit, there is then a cut-off discount factor that makes the dynamic contract incentive feasible.

Finally, although the designed liquidation mechanism can provide better contracts, it may not guarantee financial stability as defined earlier. In particular, the ability to increase d_ℓ depends on both the available funds in state ℓ and the incentive compatibility condition for the bank to suffer losses. In the next section we show that the first issue can be solved by the introduction of a bailout policy.

4.3 Orderly Liquidation and Bailouts

In this section, we introduce a bailout policy under our liquidation mechanism. We assume that there is a monopoly bank. The policy is essentially an insurance scheme, which sets an amount x of transfers from the regulator to the investors (in terms of per investor) and an amount z of premium paid by the bank to the regulator (in terms of per investor). Consistent with the previous section, to claim for the transfer or a "bailout," the bank has to pay a per depositor cost c_B to self-verify to make its available funds and the state known to the regulator. The transfer x may depend on the funds y and the state s. Alternatively, the bank may simply pay the premium. If the bank fails to pay the premium and fails to self-verify at the same time, the bank is declared bankrupt (with depositors rushing in to claim their d) and its charter is terminated, resulting in a zero continuation payoff. Thus, under deposit insurance, the dynamic contract has three components: the promised payment d, the termination and payment rules, $\tau(s, y)$ and $\chi(s, y)$, and the bailout policy with the premium z and the transfer $x(s, y)$.

Given the deposit insurance policy, the sequence of events is as described in Section 4.2, except that one needed to modify item 3, describing the bank's options after it receives payments from the firms, as follows:

(a) pay all depositors d and the premium z, if feasible;

(b) engage in self-verification (subject to payment and termination policy, $\chi(s, y)$ and $\tau(s, y)$, and bailout policy pays $x(s, y)$ in addition to investors);

(c) do nothing—in which case the bank's charter is automatically terminated.

We say that a liquidation mechanism with a bailout policy is **implementable** if the investors are willing to participate, the bank has a nonnegative (expected) profit every period, and that the scheme is *ex ante* budget balanced. More precisely, let $\bar{y}_s$ be the expected funds available at the bank for state s in equilibrium. Then,

$$-\sum_{s\in L} \pi_s x(s, \bar{y}_s) + \sum_{s\notin L} \pi_s z \geq 0, \tag{8}$$

where L is the set of states at which the bank self-verifies. We have the following theorem.

Theorem 4.3. *Assume (A3) and assume that u is strictly concave. For any implementable dynamic bank contract that is not financially stable, for β sufficiently high, there exists an implementable bailout policy and a bank contract that Pareto dominates it and is financially stable.*

Theorem 4.3 shows that, with a well-designed bailout policy in the dynamic bank contract, one can always achieve financial stability, and the scheme is Pareto optimal. We emphasize that, although the bailout policy may resemble deposit insurance, the rationale in our model is drastically different from that in Diamond and Dybvig (1983). In particular, the bailout policy has to pay out its funds *in equilibrium*, while in Diamond and Dybvig (1983) the deposit insurance scheme is used to restore good equilibria, but does not pay out in equilibrium.

Theorem 4.3 assumes ex ante budget balancedness. This amounts to assuming that the external lender to the bailout policy can commit to future lending (such as US Treasury). In the Supplemental Appendix A3, we show that this commitment is not necessary. Instead, the external lender can walk away from this lending relationship with the regulator or the government at any point in time. As shown there, what we need is that the regulator can commit not to borrow from a lender who has walked away, and we can device a similar scheme that, provided that the lender is sufficiently patient, gives the lender some profits from the lending relationship to keep the lender in the game.

5 Concluding remarks

We showed how financial intermediaries can improve welfare in the presence of asymmetric information about returns between firms who require external funds and potential lenders. A large bank, which can take deposits from a large number of depositors and then lend to the firms, can reduce the cost of monitoring relative to direct contracts and hence improve welfare. In our setup where the lender cannot commit to ex post monitoring, a new incentive issue emerges: with bank contracts, the incentive to induce the bank to monitor the firms is a nontrivial issue. That issue is even more severe under aggregate uncertainty, and we showed that a resolution mechanism can be welfare improving. If such a dynamic contract cannot reach financial stability, a bailout policy embedded into this resolution mechanism is welfare improving.

While we mainly focused on whether financial intermediaries and regulators can make Pareto improvements, one can use our framework to discuss limits on rents for the financial intermediary or to discuss other welfare criteria. In particular, since our framework also provides results regarding welfare bounds on direct contracting, these bounds set a natural limit to the rents the financial intermediary can extract. Moreover, the framework can be used to discuss how regulators may want to set limits on bank profits. However, the friction we identify also shows that the franchise value can play an important role for incentive compatibility. Finally, while we assumed a single financial intermediary, our model can be extended to allow for multiple industries where each industry has a specialized bank (which has a comparative advantage in monitoring that industry). In such an extension, we can get a similarly functioning resolution mechanism and bailout policy.

A Appendix: Proofs

Proof of Theorem 2.1

Before proving the theorem, we first give the following useful lemma.

Lemma A.1. *An equilibrium outcome is characterized by a tuple, $(W_0, W_1, ..., W_M, \rho, \bar{b})$, where $W_m \subset [0, \bar{w}]$ is the set of returns under which m lenders verify with a positive probability. For all $k = 0, 1, ..., M-1$, we have $[k\bar{b}, (k+1)\,\bar{b}) \subset W_{M-k} \cup ... \cup W_M$, where $\bar{b}$ is the repayment to each lender without verification. Moreover, for all $m > 0$ and for all $w \in W_0 \cap W_m$, $\rho(w - (M-m)\bar{b}, m) = \bar{b}$.*

Proof. Let (ρ, b) be a given contract and let (s_l, s_f) be a PBE. Let $A = \{\tilde{w} \in W : s_l(\tilde{w}) =$

$1\}$ be the set of messages resulting in verification. Now, for each $m = 0, ..., M$, define

$$A_m = \{(\tilde{w}_1, ..., \tilde{w}_M) \in W^M : \tilde{w}_{i_1}, ..., \tilde{w}_{i_m} \in A \text{ and } \tilde{w}_{i_{m+1}}, ..., \tilde{w}_{i_M} \notin A\}, \tag{9}$$

the set of messages for which m lenders verify in equilibrium, and let

$$W_m = \{w \in W : s_f(w) \text{ assigns a positive probability to } A_m\},$$

that is the realized returns which induce messages resulting in m lenders verifying in equilibrium.

For each m and each $(\tilde{w}_1, ..., \tilde{w}_M) \in A_m$ that is in the support of s_f, if $\tilde{w}_i \notin A$, then

$$\bar{b} \equiv \min\{b(\tilde{w}) : \tilde{w} \notin A \text{ and } \tilde{w} \text{ assigned a positive probability under } s_f\} = b(\tilde{w}_i). \tag{10}$$

To see this, if this inequality does not hold, or if the minimum does not exist, then the firm always have a positive deviation to send another message to replace the message $\tilde{w}_i$ so that a lower payment without verification is feasible.

By (10) and feasibility, it is easy to see that $[k\bar{b}, (k+1)\,\bar{b}) \subset W_{M-k} \cup ... \cup W_M$. Suppose that $w \in W_0 \cap W_m$, so that at the realized w, the firm has the option of paying everybody $\bar{b}$ and asking them not to verify. If both messages are sent by the firm, it must mean that the firm is indifferent, which can only happen when $\rho(w - (M-m)\bar{b}, m) = \bar{b}$. $\square$

We are now ready to prove theorem 2.1.

Let (ρ, b) be a given contract and let (s_l, s_f) be a PBE in which the lender's expected payoff is at least $u(r)$. For each $w \in W$ and each $m = 1, ...M$, we use $\kappa_m(w)$ to denote the probability that the firm sends a message asking m lenders to verify. By symmetry of s_f, conditional on having m lenders verify, s_f induces a uniform distribution over the identities of the lenders. Obviously, $\kappa_m(w) > 0$ only if $w \in W_m$.

By Lemma A.1, for all $w \in W_0$, the firm's payment is a constant, $M\bar{b}$. Now, let $\bar{b}'$ solve:

$$\begin{aligned} &\int_0^{M\bar{b}'} w \, \mathrm{d}F(w) + [1 - F(M\bar{b}')]M\bar{b}' \qquad (11) \\ = \; &\int_{W_0} M\bar{b} \, \mathrm{d}F(w) + \sum_{m=1}^{M} \int_{W_m} \left[m\rho\left(w - (M-m)\bar{b}, m\right) + (M-m)\bar{b}\right] \kappa_m(w) \, \mathrm{d}F(w). \end{aligned}$$

The above equates the expected payments to lenders of an arbitrary contract (ρ, b) and a debt-like contract. By feasibility, for each $w \in W_m$ with $m \geq 1$,

$$m\rho\left(w - (M-m)\bar{b}, m\right) + (M-m)\bar{b} \leq w, \tag{12}$$

it follows that $\bar{b}' < \bar{b}$, unless the above holds as an equality for almost every w below $M\bar{b}'$ (i.e., the firm pays out all its income, w), in which case (ρ, b) is already a debt contract.

Construct a new contract, (ρ', b') and a new PBE (s'_f, s'_l) as follows. The strategy s'_f only sends two possible messages, $\tilde{w}_0$ and $\tilde{w}_1$, to each lender, and $s'_l(\tilde{w}_0) = 0$ and $s'_l(\tilde{w}_1) = 1$. We set $b'(\tilde{w}_0) = \bar{b}'$ and $b'(\tilde{w}) > \bar{b}'$ for any $\tilde{w} \neq \tilde{w}_0, \tilde{w}_1$, and $\rho'(v, m) = v/m$ if $v \leq M\bar{b}'$ and $\rho'(v, m) = \bar{b}$ otherwise.

Now, for each $w \geq M\bar{b}'$, $s'_f(w)$ sends $\tilde{w}_0$ to all lenders with probability 1. Since $\bar{b}' \leq \bar{b}$, for any $w < M\bar{b}'$, $w \notin W_0$ by feasibility. For each $w < M\bar{b}'$, note that $\sum_{m=1}^{M} \kappa_m(w) = 1$ by feasibility. Thus, for each m, $s'_f(w)$ is a two-stage lottery: first it chooses m with probability $\kappa_m(w)$; second, conditional on choosing m, it sends exactly $\binom{M}{m}$ messages in the support of this second lottery, each of which is sent with equal probability and designates the m lenders verifying (and $M - m$ lenders getting fully repaid).

Now consider the two lotteries, X and Y, that describe each lender's payment to be received from the borrower, denoted p, induced by contracts $\left(\rho, \bar{b}\right)$ and $\left(\rho', \bar{b}'\right)$ and the above PBE's, respectively. By (11) the two lotteries have the same expectation, i.e., $E_X[p] = E_Y[p]$. Let F_X and F_Y denote the distribution function for the two lotteries. Since the two lotteries have the same expectation, we have that $\int_0^{\bar{b}} F_X(x)\ \mathrm{d}x = \int_0^{\bar{b}} F_Y(x)\ \mathrm{d}x$. We claim that for each $p < \bar{b}'$,

$$F_X[X < p] \geq F_Y[Y < p]. \tag{13}$$

To see this, fix some $p < \bar{b}'$. The realization of X and Y depend on the realization of w and the resolution of the randomization in s_f and s'_f. Given w, in both s_f and s'_f, there are two stages of randomization: the first involves the number of lenders to verify and the second involves the identities of lenders to verify.

Now, since $Y < p$ implies that $w < M\bar{b}'$, by construction of how s'_f assigns number of verifiers using κ_m's and by symmetry of the two strategies, the two-stage lotteries in the two strategies have exactly the same distributions, conditional on such w. As a result, for any such w that leads to $Y < p$, it corresponds to the event in which the lender receives $\tilde{w}_1$ at that w under $s'_f(w)$. As argued above, conditional on w, that probability is exactly the same as under $s_f(w)$, and by (12), at that event $X \leq Y < p$. This implies that F_X and F_Y cross exactly once at the point $p = \bar{b}'$. Immediately above this point we have $F_X\left[X \in \left(\bar{b}', \bar{b}\right)\right] < 1 = F_Y\left[Y \in \left(\bar{b}', \bar{b}\right)\right]$. The fact that we have single crossing CDFs, with the same expectation then implies that $\int_0^t F_X(x)\ \mathrm{d}x \geq \int_0^t F_Y(x)\ \mathrm{d}x$ for all t, with the inequality holding strictly for $t \in \left(\bar{b}', \bar{b}\right)$. That is, $\left(\rho', \bar{b}'\right)$ second order stochastically dominates $\left(\rho, \bar{b}\right)$. Hence all lenders prefer $\left(\rho', \bar{b}'\right)$ to $\left(\rho, \bar{b}\right)$, since their utility function u is increasing and concave.

We can make a strict pareto improvement for both the firm and lenders, by slightly adjusting the above. The overall probability of verification is strictly smaller under $\left(\rho', \bar{b}'\right)$ than under $\left(\rho, \bar{b}\right)$, since under $\left(\rho', \bar{b}'\right)$, the corresponding verification regions, $W'_m = W_m \cap [0, M\bar{b}')$ for all $m \geq 1$, $\kappa_m(w)$ is the same under s'_f for all $w < M\bar{b}'$, and since $\bar{b}' < \bar{b}$. Thus the contract $\left(\rho', \bar{b}'\right)$ strictly improves the lenders' expected utilities, while leaving the borrower's expected utility unchanged. This also implies $\bar{b}' > r$ as the original contract is implementable.

Note that under the debt contract $\left(\rho', \bar{b}'\right)$, the payoffs of the two parties are continuous in $\bar{b}'$ (decrease the full repayment level, keep all W_m regions the same). This follows since the contract and utility functions of all agents are continuous. Since $\bar{b}' > r$, we can find $\bar{b}'' < \bar{b}'$ such that the contract $\left(\rho', \bar{b}''\right)$ still strictly improves the lenders' expected utilities, while the borrower now makes a strictly smaller expected payment and thus we have a strict Pareto improvement.

Finally, we need to show that (s'_f, s'_l) does constitute a PBE. It is straightforward to see that s'_f is optimal. Now, to show that s'_l is optimal, we need to show that the lender has the incentive to verify whenever seeing message $\tilde{w}_1$ and has incentive to take $\bar{b}'$ whenever seeing message $\tilde{w}_0$. The latter follows since the lender can never get more than $\bar{b}'$ by verifying. For the former, we note that under contract $\left(\rho', \bar{b}'\right)$ lenders are asked to verify (weakly) less often than under contract $\left(\rho, \bar{b}\right)$ and because $\rho'(v, m) = v/m$ lenders get a (weakly) higher payoff if the do verify. Thus, given that s_l was a PBE, s_l' must be optimal for contract $\left(\rho', \bar{b}'\right)$. ■

Proof of Lemma 3.1

We first show necessity. Suppose that (2) does not hold. Then, the bank is making a negative profit in equilibrium. But by not verifying any firm at all, the bank can make a nonnegative profit and hence verification is not optimal. Similarly, suppose that (3) does not hold. We have shown that (2) is necessary, and that, together with the assumption that (3) does not hold, implies

$$[1 - F(B)]B > d.$$

Now, this implies that the bank would have sufficient funds to pay to each depositor without verifying any firm that sent message 1. Since verifying each individual firm results in a net loss for the bank, it is better off not to do it.

Here we show sufficiency. We show that in equilibrium the bank sends message 0 to all depositors and no depositor verifies the bank, and if the bank sends message 1, the verifying depositors share all the remaining funds. We separate two cases.

(a) Suppose that $[1 - F(B)]B \geq Md$. Then, the bank can pay off d to each depositor without verifying any firm that sent message 1. However, by (3), the bank is making a positive expected profit by verifying such a firm, and hence the bank is willing to do that.

(b) Suppose that $[1 - F(B)]B < Md$. Thus, the bank cannot meet its obligations to all depositors unless it verifies a positive fraction of firms that sent message 1. Let f^* be the minimum fraction that the bank can meet that, and let f be the fraction of such firms that the bank actually verifies. Since the bank will pay out all its available funds when being verified, if $0 < f < f^*$, then the bank's profit is negative; if $f^* \leq f < 1$, then it is optimal for the bank to pay off d to all depositors, and its payoff (in per depositor terms) is given by

$$\frac{1}{M}\left[f\int_0^B [w - c_E]\mathrm{d}F(w) + [1 - F(B)]B\right] - d,$$

which is maximized at $f = 1$ by (3). Finally, having $f = 1$ is better than having $f = 0$

because of (2).

Proof of Theorem 3.1

First we claim that (A0) and (A1) imply that

$$u'(r)c_E < \sum_{m=1}^{M} \frac{F\left(\frac{mB}{M}\right)}{F(B)}\gamma. \tag{14}$$

To see this, note that (A0) implies that

$$\gamma < \frac{\int_0^{Mr} u(w/M)\mathrm{d}F(w)}{F(Mr)} < \frac{\int_0^{Mr} u(r)\mathrm{d}F(w)}{F(Mr)} = u(r).$$

Hence, by concavity of u, we have

$$c_E < u^{-1}(\gamma)\sum_{m=1}^{M} \frac{F\left(\frac{mB}{M}\right)}{F(B)} \leq \frac{\sum_{m=1}^{M} \frac{F\left(\frac{mB}{M}\right)}{F(B)}\gamma}{u'\left[u^{-1}(\gamma)\right]} < \frac{\sum_{m=1}^{M} \frac{F\left(\frac{mB}{M}\right)}{F(B)}\gamma}{u'(r)}.$$

This shows (14). Moreover, we show that

$$c_E F(B) < \int_0^B w\mathrm{d}F(w). \tag{15}$$

Note that implementability of the direct contract implies that

$$\gamma < \frac{\sum_{m=0}^{M-1}\int_{mB/M}^{(m+1)B/M} u((w-mB)/(M-m))\mathrm{d}F(w)}{\sum_{m=0}^{M-1} F((m+1)B/M)} < \frac{\int_0^B u(w/M)}{F(B)} < u\left(\frac{\int_0^B \frac{w}{M}\mathrm{d}F(w)}{F(B)}\right).$$

Therefore,

$$c_E < u^{-1}(\gamma)\sum_{m=1}^{M} \frac{F\left(\frac{mB}{M}\right)}{F(B)} < \frac{\int_0^B \frac{w}{M}\mathrm{d}F(w)}{F(B)}\sum_{m=1}^{M} \frac{F\left(\frac{mB}{M}\right)}{F(B)} < \frac{\int_0^B w\mathrm{d}F(w)}{F(B)}.$$

Consider the following bank contract, (B, d). The bank has a simple debt contract with each firm with debt level B. For the deposit side, the depositor receives d that will be specified below without verification, and, in case the bank refuses to pay d, all depositors verify and receive equal payments from the remaining funds.

Let

$$d = \frac{1}{M}\left\{\int_{w\in[0,B]} w\mathrm{d}F(w) + (1-F(B))B - F(B)c_E - \varepsilon\right\}. \tag{16}$$

For any $\varepsilon > 0$, (2) is satisfied with a strict inequality and, recalling that (15) implies (3), the bank has strict incentive to verify firms' returns. It also implies that the bank has a strictly positive payoff.

Now, let U be the expected payoff for each lender from the direct contract. Then,

$$U \leq \int_0^B u\left(\frac{w}{M}\right) \mathrm{d}F(w) + (1 - F(B))u\left(\frac{B}{M}\right) - \frac{1}{M}\sum_{m=1}^{M} F\left(\frac{mB}{M}\right)\gamma. \tag{17}$$

We claim that $u(d + \varepsilon/M) > U$ for any $\varepsilon > 0$ (note that d is defined by (16)) and hence $u(d) > U$ for ε small. To see this, since u is concave, we have:

$$u\left[\frac{1}{M}\left\{\int_0^B w\mathrm{d}F(w) + (1 - F(B))B\right\}\right] \geq \int_{w\in[0,B)} u\left(\frac{w}{M}\right) \mathrm{d}F(w) + (1 - F(B))u\left(\frac{B}{M}\right),$$

that is,

$$u\left[d + \varepsilon/M + \frac{1}{M}F(B)c_E\right] \geq \int_{w\in[0,B)} u\left(\frac{w}{M}\right) \mathrm{d}F(w) + (1 - F(B))u\left(\frac{B}{M}\right). \tag{18}$$

Now, (A1) implies that $d + \varepsilon/M > r$ and hence, by concavity of u,

$$\begin{aligned} u(d + \varepsilon/M) &\geq u\left[d + \frac{\varepsilon}{M} + \frac{1}{M}F(B)c_E\right] - u'\left[d + \frac{\varepsilon}{M}\right]\frac{1}{M}F(B)c_E \\ &> u\left[d + \frac{\varepsilon}{M} + \frac{1}{M}F(B)c_E\right] - \frac{1}{M}\sum_{m=1}^{M} F\left(\frac{mB}{M}\right)\gamma \geq U, \end{aligned}$$

where the last inequality follows from (17) and (18), and the second inequality follows from (14) and $d + \varepsilon/M > r$, which ipmply

$$c_E < \sum_{m=1}^{M} \frac{F\left(\frac{mB}{M}\right)}{F(B)}\gamma/u'(r) < \sum_{m=1}^{M} \frac{F\left(\frac{mB}{M}\right)}{F(B)}\gamma/u'(d + \varepsilon/M).$$

Finally, we need to show that the depositors have incentive to verify the bank's return when the bank does not pay off d. For the continuum model this is off-equilibrium path and we assume that, when receiving the message 1, the depositors believe that the bank has d. For finite but large number of depositors, this follows from the CLT, and one can show that

$$\lim_{N\to\infty} \mathbb{E}[u(y_N)|y_N < \mathbb{E}(y_N) - F(B)c_E] = u(d) > U \geq u(r) > \gamma,$$

where y_N is the random variable that represents the average funds available in the bank to each depositor when there are N firms. □

Proof of Theorem 4.1

Let d_s denote the average amount paid to the depositors at state s in equilibrium under the bank contract, $[\mathbf{B}, (d, \xi)]$. Theorem 4.1 follows from the lemma below.

Lemma A.2. *Let $[\mathbf{B}, (d, \xi)]$ be an implementable bank contract. Then, it satisfies (6) for both $s = h, \ell$, and it has to satisfy the following conditions.*

(a) Suppose that $d = d_h = d_\ell$. Then, $u(d) \geq u(r)$, and

$$\int_0^{B_s} w \mathrm{d}F_s(w) > F_s(B_s)c_E \text{ for both } s = h, \ell. \tag{19}$$

(b) If $d_\ell < d_h$, there are two subcases:

(b.1) $d_h = d$ and letting m_ℓ be the fraction of depositors monitoring at state ℓ,

$$\int_0^{B_h} w \mathrm{d}F_h(w) > F_h(B_h)c_E; \tag{20}$$

$$u\left(\frac{Md_\ell - (M - m_\ell)d}{m_\ell}\right) \geq \gamma; \tag{21}$$

$$\pi_h u(d) + \pi_\ell \left[\frac{M - m_\ell}{M} u(d) + \frac{m_\ell}{M} u\left(\frac{Md_\ell - (M - m_\ell)d_h}{m}\right) - \frac{m_\ell}{M}\gamma\right] \geq u(r). \tag{22}$$

(b.2) $d_h < d$ and letting m_s be the fraction of depositors monitoring at state $s = h, \ell$,

$$\sum_{s=h,\ell} \frac{\pi_s m_s}{\pi_h m_h + \pi_\ell m_\ell} u\left(\frac{Md_s - (M - m_s)d}{m_s}\right) \geq \gamma; \tag{23}$$

$$\sum_{s=h,\ell} \pi_s \left[\frac{M - m_s}{M} u(d) + \frac{m_s}{M} u\left(\frac{Md_s - (M - m_s)d}{m_s}\right) - \frac{m_s}{M}\gamma\right] \geq u(r). \tag{24}$$

Proof. The necessity in (a) uses the same arguments as in Lemma 3.1. Consider (b). Conditions (22) and (24) are individual rationality conditions for depositors. (21) and (23) are necessary for depositors to verify the bank. For (b.1), since (6) holds, for the bank to verify in full at state h, (20) is necessary. □

Now, suppose, by contradiction, that the bank can offer a financially stable contract with debt level with depositors d. By Lemma A.2, (7) implies that

$$d = d_\ell \leq \frac{1}{M}\left\{\int_0^{B_\ell} w \mathrm{d}F_\ell(w) - F_\ell(B_\ell)c_E + [1 - F_\ell(B_\ell)]B_\ell\right\} < r,$$

which leads to a contradiction with implementability. □

Proof of Theorem 4.2

(a) Let B be a direct contract between the firm and the lenders that satisfies (A1) and (A2) for $F = \pi_h F_h + \pi_\ell F_\ell$. Note that implementability requires $B \geq Mr$. Concavity of u implies that, together with (A1),

$$u'(r)c_E < \frac{\sum_{s=h,\ell} \pi_s \left[\sum_{m=1}^{M} F_s\left(\frac{mB}{M}\right)\right]}{\sum_{s=h,\ell} \pi_s F_s(B)} \gamma. \tag{25}$$

To see this, note that (A0) implies that $\gamma < u(r)$. Now,

$$\begin{aligned} c_E &< u^{-1}(\gamma)\frac{\sum_{s=h,\ell} \pi_s \left[\sum_{m=1}^{M} F_s\left(\frac{mB}{M}\right)\right]}{\sum_{s=h,\ell} \pi_s F_s(B)} < \frac{\sum_{s=h,\ell} \pi_s \left[\sum_{m=1}^{M} F_s\left(\frac{mB}{M}\right)\right]}{\sum_{s=h,\ell} \pi_s F_s(B)} \frac{\gamma}{u'\left[u^{-1}(\gamma)\right]} \\ &< \frac{\sum_{s=h,\ell} \pi_s \left[\sum_{m=1}^{M} F_s\left(\frac{mB}{M}\right)\right]}{\sum_{s=h,\ell} \pi_s F_s(B)} \frac{\gamma}{u'(r)}, \end{aligned}$$

where the last inequality follows from the fact that $u^{-1}(\gamma) < r$ and that u is strictly concave.

Now we construct the following bank contract, (B, d), and show that it dominates the original direct contract, B. The bank has a simple debt contract with each firm with debt level B (and hence the firms are indifferent between the original contract and the constructed contract), and let $Y_s(B)$ denote the funds available to the bank in equilibrium under state s, that is,

$$Y_s(B) = \frac{1}{M}\left\{\int_0^B w \mathrm{d}F_s(w) + [1 - F_s(B)]B\right\}. \tag{26}$$

The bank equilibrium revenue, the available funds deducted from the cost of verification, is given by

$$R_s(B) = Y_s(B) - F_s(B_s)c_E = \frac{1}{M}\left\{\int_0^{B_s} w \mathrm{d}F_s(w) + [1 - F_s(B_s)]B_s\right\} - F_s(B_s)c_E. \tag{27}$$

Now, let U be the expected payoff for each lender from the direct contract. Then,

$$U \leq \sum_{s=h,\ell} \pi_s \left\{\int_0^B u\left(\frac{w}{M}\right) \mathrm{d}F_s(w) + (1 - F_s(B))u\left(\frac{B}{M}\right) - \frac{1}{M}\sum_{m=1}^{M} F_s\left(\frac{mB}{M}\right)\gamma\right\}. \tag{28}$$

We consider two cases.

(a.1) Suppose that $Y_\ell(B) \geq \sum_{s=h,\ell} \pi_s R_s(B)$, that is, the available fund at state ℓ is higher than the average revenue. Then, set $\hat{d} = \frac{1}{M} \sum_{s=h,\ell} \pi_s R_s(B)$, that is, $\hat{d}$ is set to be the average revenue. Obviously, if we set d to be slightly below $\hat{d}$, then the bank will make a positive average profit.

We claim that $u(\hat{d}) > U$ and hence $u(\hat{d}-\varepsilon) > U$ for ε small. This then implies that the depositors strictly prefer the constructed contract. To see this, since u is strictly concave,

$$u\left[\hat{d} + \sum_{s=h,\ell} \pi_s \frac{1}{M} F_s(B) c_E\right] > \sum_{s=h,\ell} \pi_s \left\{ \int_{w\in[0,B]} u\left(\frac{w}{M}\right) \mathrm{d}F_s(w) + (1 - F_s(B)) u\left(\frac{B}{M}\right) \right\}. \tag{29}$$

Now, (A2) implies that $\hat{d} > r$ and hence, by strict concavity of u,

$$\begin{aligned} u(\hat{d}) &> u\left[\hat{d} + \sum_{s=h,\ell} \pi_s \frac{1}{M} F_s(B) c_E\right] - u'(\hat{d}) \left[\sum_{s=h,\ell} \pi_s \frac{1}{M} F_s(B) c_E\right] \\ &> u\left[\hat{d} + \sum_{s=h,\ell} \pi_s \frac{1}{M} F_s(B) c_E\right] - \frac{1}{M} \sum_{s=h,\ell} \pi_s \left[\sum_{m=1}^{M} F_s\left(\frac{mB}{M}\right)\right] \gamma \geq U, \end{aligned}$$

where the second inequality follows from (25) and $\hat{d} > r$, and the last inequality follows from (28) and (29). Thus, for $\varepsilon > 0$ small, $u(\hat{d} - \varepsilon) > U$. Now, set the contract with the depositors as $d = \hat{d} - \varepsilon$. This then implies that depositors are better off.

Moreover, there is no self-verification on the equilibrium path. On the off-equilibrium path, we set $\tau(s, y) = 0$ and $\chi(s, y) = y$ for all y and for both s. The depositors are then better off against the direct contract. The bank makes a strictly positive expected profit, which (in per depositor terms) equals

$$\frac{1}{M} \sum_{s=h,\ell} \pi_s R_s(B) - d = \hat{d} - (\hat{d} - \varepsilon) = \varepsilon > 0. \tag{30}$$

Now we consider the bank's incentive compatibility conditions. There are three such conditions: the participation condition for the bank, the incentive to verify *all* firms that request verification firstly against no firms and secondly against only a fraction of firms in both aggregate states.

By (30) the bank makes a strictly positive profit on average and hence is willing to participate. Since $B \geq Mr$ (as the direct contract is implementable), by (A3) we have

$$\int_0^B w dF_s(w) > c_E F_s(B) \text{ for both } s = h, \ell. \tag{31}$$

This also implies that, under both aggregate state, if the bank is willing to verify all firms that request verification against none, it is not profitable for the bank to verify only a fraction of them.

Now we consider the incentive to verify all requesting firms. Since the bank makes a strictly ex post positive profit at state h following the equilibrium behavior but will make more profit by verifying than not according to (31), we only need to consider the incentive under state ℓ. If the bank verifies none of the firms and fails to repay d to all depositors, the bank charter will be terminated according to τ. Note that, since $R_\ell < d$, by (31), $[1 - F_\ell(B)]B < d$, that is, verification is needed to be able to pay back d. Now, verifying is incentive compatible if and only if

$$R_\ell(B) - d + \frac{\beta}{1-\beta}\varepsilon \geq 0, \tag{32}$$

where the left-side of (32) is the continuation payoff following the equilibrium behavior and the right-side is the continuation payoff by verifying none of the requesting firms and having the charter terminated. Since $\varepsilon > 0$, for sufficiently high β, (32) is satisfied.

(a.2) Suppose that $Y_\ell(B) < \sum_{s=h,\ell} \pi_s R_s(B)$. Then, set

$$d_\ell = \frac{1}{M} Y_\ell(B) \text{ and } \hat{d}_h = R_h(B) - \frac{1}{M}\frac{\pi_\ell}{\pi_h} F_\ell(B) c_E.$$

Here d_ℓ will be the average repayment to the depositors at state ℓ and $\hat{d}_h - \varepsilon$ will be the average repayment to the depositors at state h with $\varepsilon > 0$ to be determined below. We claim that

$$\pi_h u(\hat{d}_h) + \pi_\ell u(d_\ell) > U. \tag{33}$$

To see this, since u is strictly concave, we have

$$\begin{aligned} &\pi_h u\left[\hat{d}_h + \frac{1}{M}\left(F_h(B)c_E + \frac{\pi_\ell}{\pi_h}F_\ell(B)c_E\right)\right] + \pi_\ell u(d_\ell) \\ > &\sum_{s=h,\ell} \pi_s \left\{\int_{w\in[0,B]} u\left(\frac{w}{M}\right) \mathrm{d}F_s(w) + (1 - F_s(B))u\left(\frac{B}{M}\right)\right\}. \end{aligned} \tag{34}$$

Now, (A2) and $Y_\ell(B) < \sum_{s=h,\ell} \pi_s R_s(B)$ imply that $\hat{d}_h > r$ and hence, by strict concavity of u,

$$\begin{aligned} u(\hat{d}_h) &> u\left[\hat{d} + \frac{c_E}{M}\left(F_h(B)c_E + \frac{\pi_\ell}{\pi_h}F_\ell(B)\right)\right] - u'(\hat{d}_h)\left[\frac{c_E}{M}\left(F_h(B)c_E + \frac{\pi_\ell}{\pi_h}F_\ell(B)\right)\right] \\ &\geq u\left[\hat{d} + \frac{c_E}{M}\left(F_h(B)c_E + \frac{\pi_\ell}{\pi_h}F_\ell(B)\right)\right] - \frac{1}{M\pi_h}\sum_{s=h,\ell}\pi_s\left[\sum_{m=1}^{M} F_s\left(\frac{mB}{M}\right)\right], \end{aligned}$$

where the second inequality follows from (25) and $\hat{d}_h > r$, and hence,

$$\begin{aligned} &\pi_h u(\hat{d}_h) + \pi_\ell u(d_\ell) \\ > &\sum_{s=h,\ell}\pi_s\left\{\int_{w\in[0,B]} u\left(\frac{w}{M}\right)\mathrm{d}F_s(w) + (1-F_s(B))u\left(\frac{B}{M}\right)\right\} - \frac{1}{M}\sum_{s=h,\ell}\pi_s\left[\sum_{m=1}^{M} F_s\left(\frac{mB}{M}\right)\right] \\ \geq &\ U. \end{aligned}$$

Thus, for $\varepsilon > 0$ small, $\pi_h u(\hat{d}_h - \varepsilon) + \pi_\ell u(d_\ell) > U$. Now, set the contract with the depositors as $d = \hat{d}_h - \varepsilon$, which is paid to the depositors when $s = h$ without self-verification, and, at $s = \ell$, there is self-verification and the bank pays d_ℓ equally to all depositors; i.e., $\chi[\ell, Y_\ell(B)] = d_\ell$. Thus, the depositors are strictly better off against the direct contract. We set $\tau(h, y) = 0$ and $\chi(h, y) = y$ for all y, and $\tau(\ell, y) = 0$ iff $y < Y_\ell(B)$ and $\tau(\ell, y) = y$ for all y.

The bank has a positive expected profit provided that $c_B < \pi_h\varepsilon/\pi_\ell$: indeed, in this case, the expected profit is given by

$$\sum_{s=h,\ell}\pi_s\left\{R_s(B) - d_s\right\} - \pi_\ell c_B = \pi_h\varepsilon - \pi_\ell c_B > 0.$$

As in (a.1) we need to verify the bank incentives, and, as there, participation is taken care of by positive expected profit and conditional on having to the incentive to verify all requesting firms against none of them, the bank has incentive not to verify only a fraction of such firms instead of all because of (A3). Moreover, at state h all incentives are respected as well.

Note that at state ℓ, the bank can maintain its charter only if it verifies *all* requesting firms and engages in self-verification. Thus, the only deviation to worry about is that it might deviate to verify *none* of the requesting firms and have its charter terminated. Now, for the bank to have incentive to verify all requesting firms and to self-verify at state ℓ, we need

$$R_\ell(B) - d_\ell - c_B + \frac{\beta}{1-\beta}[\pi_h \varepsilon - \pi_\ell c_B] \geq 0. \tag{35}$$

Note that while $R_\ell(B) - d_\ell - c_B < 0$, (35) is satisfied for β sufficiently close to one because $\pi_h \varepsilon - \pi_\ell c_B > 0$.

(b) Let $[\mathbf{B}, (d, \xi)]$ be an implementable static contract that is not financially stable. With no loss of generality we assume $d_\ell < d_h$ and $Y_\ell(B_\ell) \leq Y_h(B_h)$ (the fact that the order of the d's align with the order of the Y_s's does not matter either), where Y_s is defined in (26). Incentive compatibility requires $d_\ell \leq R_\ell(B_\ell)$, where R_s is defined in (27).

We consider two cases.

(b.1) Suppose that $\hat{d} = \pi_h d_h + \pi_\ell d_\ell \leq Y_\ell(B_\ell)$. We can choose $\varepsilon > 0$ such that

$$\pi_h u(d_h) + \pi_\ell u(d_\ell) - \frac{d - d_\ell}{d}\gamma < u(\hat{d} - \varepsilon)$$

because u is strictly concave. We now design a dynamic contract, with $\mathbf{B} = (B_h, B_\ell)$ kept the same, but with promise to depositors equal to $d' = \hat{d} = \varepsilon$. Moreover, we will design the regulator's contract (τ, χ) such that in equilibrium $d'_h = d' = d'_\ell$, i.e., the depositors receive d' in both aggregate states.

Since the static contract is implementable, (6) must hold, and hence $\hat{d} \leq \pi_h R_h(B_h) + \pi_\ell R_\ell(B_\ell)$, and the bank's profit is at least $\varepsilon > 0$ under the new contract. Moreover, the depositors are strictly better off against the static contract.

Now we turn to the regulator contract with the bank and the bank's incentive problems. The dynamic contract is given as follows. $\tau(s, y) = 0$ and $\chi(s, y) = y$ for all s, y. By implementability of the static contract we know that $d' > r$ and hence the assumption that $Y(B_\ell) > d'$ implies that $B_\ell > Mr$. Thus, by (A3) and applying the same logic as in (a1), (31) holds, and hence the only relevant deviation for the bank in terms of verification is to choose between verifying all the requesting firms or none of them, but there is no concern for deviating for verifying only a fraction.

Clearly $d' < B_h(B_h)$ and hence the bank has incentive to verify all requesting firms in state h. In state ℓ, the bank has incentive to verify all requesting firms if and only if (32) holds, using the same logic as in (a.1). So it is respected for β sufficiently high.

(b.2) Suppose that $\pi_h d_h + \pi_\ell d_\ell > Y_\ell(B_\ell)$. As before, we keep the bank contract with the firms the same, $\mathbf{B} = (B_h, B_\ell)$ as in the original static contract. We first design the corresponding equilibrium payments under the new dynamic contract, d'_h and d'_ℓ, and then spell out the contractual arrangements to achieve them. Let $\hat{d}_h$ be such that

$$\pi_h \hat{d}_h + \pi_\ell Y_\ell(B_\ell) = \pi_h d_h + \pi_\ell d_\ell.$$

Since the static contract is implementable, (6) holds and hence $d_\ell \leq R_\ell(B_\ell) < Y_\ell(B_\ell)$, this implies that $\hat{d}_h < d_h$. Let $d'_\ell = Y_\ell(B_\ell)$. Since u is strictly concave,

$$\pi_h u(\hat{d}_h) + \pi_\ell u(d'_\ell) > \pi_h u(d_h) + \pi_\ell u(d_\ell) - \frac{d - d_\ell}{d}\gamma,$$

and hence for $\varepsilon > 0$ small,

$$\pi_h u(\hat{d}_h - \varepsilon) + \pi_\ell u(d'_\ell) > \pi_h u(d_h) + \pi_\ell u(d_\ell) - \frac{d - d_\ell}{d}\gamma.$$

Then set $d'_h = \hat{d}_h - \varepsilon$. Thus, if these are the equilibrium payments to the depositors, the depositors are better off relative to the original static contract.

Now we design the new dynamic contract to achieve the depositor payments, and show that the bank also makes a positive profit. First we give the termination policy and depositor payment rules under self-verification:

$$\tau(\ell, y) = 1 \text{ iff } y \geq Y_\ell(B_\ell) = d'_\ell,\ \chi(\ell, y) = y \text{ for all } y;\ \tau(h, y) = 0,\ \chi(h, y) = y \text{ for all } y.$$

The bank contract with depositors has debt level $d' = d'_h$. On the equilibrium path, the bank pays $d' = \hat{d}_h - \varepsilon$ to all depositors at $s = h$ without self-verification, and self-verifies when $s = \ell$. Following the same logic as in (a.2), if $c_B < \frac{\pi_h}{\pi_\ell}\varepsilon$, the bank is making a strictly positive expected profit. The incentive problems also are the same as in (a.2) and they are respected for sufficiently high β. □

Proof of Theorem 4.3

Let d_s be the payment to the depositors under the original dynamic contract at state s. We may assume that $d_h = d$, the promised payment. Indeed, if $d_h < d$, we can devise an alternative dynamic contract where $d' = d_h$ and it would keep all bank's incentive compatibility conditions in tact but save on self-verification cost. Since it is not financially stable, $d_\ell < d$ and the bank has to engage self-verification at $s = \ell$.

As before, we devise a new dynamic contract that uses the bailout scheme to Pareto dominate the original dynamic contract. We do so by keeping the firm contracts unchanged, $\mathbf{B} = (B_h, B_\ell)$. We first design the equilibrium payments to depositors. Let $\hat{d} = \pi_h d_h + \pi_\ell d_\ell$. If $\hat{d} \leq Y_\ell(B_\ell)$, then, using the same arguments as in Theorem 4.2, we can have a financially stable contract that dominates the original one without using bailouts. So suppose that $\hat{d} > Y_\ell(\mathbf{B})$. Since u is strictly concave, for some $\varepsilon > 0$,

$$u(\hat{d} - \varepsilon) > \sum_{s=h,\ell} \pi_s u(d_s). \tag{36}$$

Then, take $d' = \hat{d} - \varepsilon$ as the promised return to depositors in the new contract, and we will design a regulator contract so that this is also the equilibrium payment in both aggregate states.

Now, for the regulator contract, we have

$$\tau(\ell, y) = 1 \text{ iff } y \geq Y_\ell(B_\ell),\ \chi(\ell, y) = y \text{ for all } y;\ \tau(h, y) = 0 \text{ and } \chi(h, y) = y \text{ for all } y.$$

For the bailout scheme, we have

$$\begin{aligned} x(\ell, y) &= \max\{d' - y, 0\} \text{ if } y \geq Y_\ell(B_\ell),\ x(\ell, y) = 0 \text{ otherwise;} \\ z &= \frac{\pi_\ell\,[d' - Y_\ell(B_\ell)]}{\pi_h}. \end{aligned} \tag{37}$$

On the proposed equilibrium path, the bank self-verifies only in state ℓ, and, in that state, the bank pays off all its available funds, $Y_\ell(B_\ell)$, and the bailout scheme pays out additional funds so that all depositors receive d'; at state h, the bank pays d' to all depositors and pays z to the bailout scheme. Note that the equilibrium x is given by $d' - Y_\ell(B_\ell)$ and that on the proposed equilibrium path the budget constraint (8) is satisfied.

The bank's expected profit (in per depositor term) on the proposed equilibrium path is then given by

$$\begin{aligned}
&\pi_h\left[R_h(B_h) - d' - z\right] + \pi_\ell\left[R_\ell(B_\ell) - d_\ell - c_B\right] \\
= &\left[\sum_{s=h,\ell} \pi_s R_s(B_s)\right] - \pi_h d' - \pi_h \frac{\pi_\ell[d' - Y_\ell(B_\ell)]}{\pi_h} - \pi_\ell Y_\ell(B_\ell) - \pi_\ell c_B \\
= &\left[\sum_{s=h,\ell} \pi_s \eta_s(\mathbf{B})\right] - d' - \pi_\ell c_B = \left\{\sum_{s=h,\ell} \pi_s[\eta_s(\mathbf{B}) - d_s]\right\} + \varepsilon - \pi_\ell c_B,
\end{aligned}$$

where the first equality follows from the fact that z is given by (37) and that $d_\ell = Y_\ell(B_\ell)$, and the last from the construction of d'. Note that this profit is then ε above to the original dynamic contract in each period, and hence is strictly positive. Moreover, the depositors are strictly better off under bailout policy than the original unstable contract by (36).

Finally, using exactly the same arguments as in the proof of Theorem 4.2, part (b.2), high β ensures that the bank wants to pay $d' + z$ at state h, self verifies at state ℓ, and (A3) ensures that the bank has incentive to verify all requesting firms. □

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Supplemental Appendix: for online publication only

A.1 Optimal verification regions

Theorem 2.1 proved the Pareto optimality of debt contracts, but didn't characterize their specific form. We now extend this result by finding the optimal verification regions (the W_m) for the debt contract. Let (ρ, b) be a debt contract, where we denote by b the level of debt for each lender. For each $w \in [0, Mb)$, the *ex ante* expected payoff to a lender when m lenders verify is

$$\Upsilon(w, m) \equiv \frac{m}{M}\left[u\left(\frac{w-(M-m)b}{m}\right)-\gamma\right]+\frac{M-m}{M}u(b). \tag{38}$$

The optimal verification strategy maximizes this expected payoff for each w. Note that under a debt contract the payment for the firm is the same for each realization of w (pay out everything up to Mb and just Mb thereafter). So this verification strategy will indeed be Pareto optimal.

We shall define cutoffs $x_M < x_{M-1} ... < x_1$ so that under the contract (ρ, b) is is optimal for m lenders to verify if $w \in [x_{m+1}, x_m)$. We have that $x_1 = Mb$ because for realizations of w sufficiently close to Mb the firm give the verifying lenders a payout arbitrarily close to b and so whatever the level of risk-aversion there is no reason to ask more than one lender to verify. For each $m = M, M-1, ..., 2$, define x_m as follows:

$$x_m = \inf\{w \geq (M-m+1)\, b : \Upsilon(w, m-1) \geq \Upsilon(w, m)\}.$$

For all $w \in [x_m, x_{m-1})$, we have $m-1$ lenders verify, so the firm needs sufficient funds to pay the $M-(m-1)$ lenders who do not verify, i.e., at the cutoff x_m we need to have $w \geq (M-m+1)\, b$. The inequality involving $\Upsilon(w, m)$ in this definition show that lenders cannot be strictly better off at w if fewer than m lenders verify. The following lemma shows that these cutoffs are well defined.

Lemma A.3. *For each $m = 2, ..., M$, x_m is the unique w such that either $\Upsilon(w, m) = \Upsilon(w, m-1)$ or $x_m = (M-m+1)\, b$. Moreover, $x_m > x_n$ for all $m < n$.*

Proof. Fix some m. Suppose that $\Upsilon((M-m+1)b, m) > \Upsilon((M-m+1)b, m-1)$. Observe that

$$\frac{\partial}{\partial w}\left[\Upsilon(w,m) - \Upsilon(w,m-1)\right] = \frac{1}{M}\left[u'\left(\frac{w-(M-m)b}{m}\right) - u'\left(\frac{w-(M-m+1)b}{m-1}\right)\right] < 0$$

for all $w < Mb$, because u is concave and

$$\frac{w-(M-m)b}{m} - \frac{w-(M-m+1)b}{m-1} = \frac{Mb-w}{m(m-1)}.$$

When $w = Mb$, we have $\Upsilon(Mb,m) - \Upsilon(Mb,m-1) = \frac{1}{M}\gamma < 0$, and when $w < (M-m+1)\,b$ we cannot have only $m-1$ lenders verifying by feasibility. Hence there is a unique x_m for each m such that either $\Upsilon(x_m, m) = \Upsilon(x_m, m-1)$ or $x_m = (M-m+1)\,b$.

Next we will show that $x_m > x_{m-1}$ and the result follows by induction. First consider the case where $x_m > (M-m+1)\,b$. Thus $\Upsilon(x_m, m) = \Upsilon(x_m, m-1)$ and $\Upsilon(x_{m-1}, m-1) \leq \Upsilon(x_{m-1}, m-2)$ implies that:

$$\begin{aligned}
0 &= mu\left(\frac{x_m-(M-m)b}{m}\right) - (m-1)u\left(\frac{x_m-(M-m+1)b}{m-1}\right) - u(b) - \gamma \\
&\geq (m-1)u\left(\frac{x_{m-1}-(M-m+1)b}{m-1}\right) - (m-2)u\left(\frac{x_{m-1}-(M-m+2)b}{m-2}\right) - u(b) - \gamma.
\end{aligned}$$

Suppose, by contradiction, that $x_{m-1} \geq x_m$. Then,

$$\begin{aligned}
& mu\left(\frac{x_{m-1}-(M-m)b}{m}\right) - (m-1)u\left(\frac{x_{m-1}-(M-m+1)b}{m-1}\right) \\
\geq\ & mu\left(\frac{x_m-(M-m)b}{m}\right) - (m-1)u\left(\frac{x_m-(M-m+1)b}{m-1}\right) \\
\geq\ & (m-1)u\left(\frac{x_{m-1}-(M-m+1)b}{m-1}\right) - (m-2)u\left(\frac{x_{m-1}-(M-m+2)b}{m-2}\right),
\end{aligned}$$

which implies that

$$\begin{aligned}
& \frac{m}{2m-2}u\left(\frac{x_{m-1}-(M-m)b}{m}\right) + \frac{(m-2)}{2m-2}u\left(\frac{x_{m-1}-(M-m+2)b}{m-2}\right) \\
\geq\ & u\left(\frac{x_{m-1}-(M-m+1)b}{m-1}\right).
\end{aligned}$$

But notice that

$$\frac{x_{m-1}-(M-m+1)b}{m-1} = \frac{m}{2m-2}\left(\frac{x_{m-1}-(M-m)b}{m}\right) + \frac{(m-2)}{2m-2}\left(\frac{x_{m-1}-(M-m+2)b}{m-2}\right),$$

and this leads to a contradiction to the concavity of u.

Now consider the case where $x_m = (M - m + 1)\, b$. Because of the definition of x_{m-1}, we have that $x_{m-1} \geq (M - m + 2)\, b > x_m$. □

A.2 Optimality of debt contract under aggregate uncertainty

The bank contract with the depositors is given by the debt level d and a function $\xi(s, y, m)$ that specifies payment to each verifying depositor, where y is the available funds to each verifying depositor, m is the fraction who verify, and s is the state. Our earlier results regarding the optimality of debt contracts with firms, however, do not directly extend to the bank contract, since the bank's funds are (at least partially) paid to the depositors. In general, a bank contract with the firm may be denoted $(\bar{b}_s, \rho_s)_{s=h,\ell}$, where $\rho_s(w)$ denotes the firm's repayment to the bank when verified return is w and $\bar{b}_s$ is the repayment without verification, both of which may depend on the state s.

Given the contract, the strategies can be defined in an analogous manner to those in Definition 3.1, but with one modification: now both s_f and (s_B^1, s_B^2) may depend on the state s. Moreover, as before, we focus on symmetric equilibria in which the interactions between the bank and the firms in a PBE with the property that the bank verifies all firms that send message 1. The following lemma shows that it is optimal to have debt contracts between the bank and the firms.

Lemma A.4. *Let $\{(\bar{b}_s, \rho_s)_{s=h,\ell}, (d, \xi)\}$ be an implementable bank contract. Under (A3), there is another bank contract in which the contract with the firm is a debt contract with verification occurring iff $w < B_s$ and B_s the debt level at state s and which Pareto dominates the original contract.*

Proof. Suppose that the contracts with the firms are given by $(\bar{b}_s, \rho_s)_{s=h,\ell}$, and suppose that $W_{0,s}$ is the set where no verification occurs from the bank, $s = h, \ell$.

Let

$$R_s = \frac{1}{M}\left\{\int_{W_{1,s}} \rho_s(w)\mathrm{d}F_s(w) - F_s(W_{1,s})c_E + F_s(W_{0,s})\bar{b}_s\right\},$$

and let

$$Y_s = \frac{1}{M}\left\{\int_{W_{1,s}} \rho_s(w)\mathrm{d}F_s(w) + F_s(W_{0,s})\bar{b}_s\right\}.$$

Here R_s is the revenue (net of monitoring cost) from the firms by the bank, and Y_s is the available funds at the bank at state s, both in per investor's term.

Consider the alternative contracts $(B_s)_{s\in S}$ and ξ' as follows. B_s is such that

$$\int_0^{B_s} w\mathrm{d}F_s(w) + [1 - F_s(B_s)]B_s = MY_s,$$

and hence $B_s \leq \bar{b}_s$. Let $W'_{1,s} = [0, B_s) \subset W_{1,s}$, and let

$$R'_s = \frac{1}{M}\left\{\int_0^{B_s} w\mathrm{d}F_s(w) - F_s(B_s)c_E + [1 - F_s(B_s)]B_s\right\} \geq R_s.$$

The above inequality is strict unless the contract with the firm is a debt contract. Moreover, the firm's expected payment is exactly the same conditional on each state under the alternative contract as in the original contract.

Now we consider the bank contract with the investors. Given the bank contract with the investors, (d, ξ), let d_s be the average amount paid to the depositors at state s. Incentive compatibility for the bank to monitor the firms requires $d_s \leq R_s$ for both $s = h, \ell$. We then design the new contract with the investors as follows. Keep d as in the original contract and keep the fraction of verifying investors the same as before for each states. If at state s no investors verify the bank, then it must be the case that $Y_s \geq Mr$ and hence $B_s \geq Mr$. Then, (A3) implies that the bank has incentive to verify the firms at state s. Instead, if the bank is verified at state s and if the bank's available fund and the fraction of verifiers are consistent with bank revenue of Y_s, give the same payments as the original contract. Otherwise, require the bank to pay off all its available funds. This ensures the bank has the same incentive to monitor the firms.

Finally, since the investors receive the same amount of payments when verifying in both states and the probability of verifying remains the same, their incentive to verify remains. □

A.3 Relaxing lander commitment for the bailout scheme

In Section 4.2 we consider a bailout scheme that only requires budget-balancedness from the *ex ante* perspective. This would require an external lender who can commit to a contract with the regulator, who will receive the premium at good states and will pay for the insurance at bad states. Here we relax this assumption and consider the possibility that the external lender cannot commit to his future actions. The regulator, however, can commit and offer a contract as follows. We assume that there is a single lender to simplify the analysis but, to be more realistic, we could have a continuum of identical lenders and each is offered the same contract. We assume that the lender is risk-neutral and has discount factor β.

If the bank is in state h, then the regulator pays ϕ (in per depositor term). If the bank is in state ℓ, then the lenders pays τ. Lack of commitment implies that the lender can walk away from the contract at any point of time. Obviously, the lender would not do so in state h. In state ℓ, however, the lender may choose to leave unless the future benefits from staying in the contract is better than leaving the contract. We assume that the regulator only pays the lender who has always stayed in the contract.

Thus, the incentive compatibility constraint for the lender to remain in the contract is given by

$$-\tau + \frac{\beta}{1-\beta}\left[\pi_h\phi - \pi_\ell\tau\right] \geq 0. \tag{39}$$

Here we show that even with this additional constraint, Theorem 4.3 still holds. The only modification needed in the argument is the construction of the bailout scheme, and, instead of having $\pi_h\phi = \pi_\ell\tau$, we have:

$$\pi_h\phi = \left(\pi_\ell + \frac{1-\beta}{\beta}\right)\tau,$$

This implies that the bank in state h pays $d'' < d'$ (the amount constructed in the proof of Theorem 4.3, but the difference converges to zero as β goes to one. Since we obtained strict Pareto improvement in that theorem, this implies that we can still obtain improvement with β sufficiently high. The incentives to monitor are not altered.