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Price competition with repeat, loyal buyers

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Abstract Extant theoretical models suggest that greater consumer loyalty increases a firm's market power and leads to higher prices and fewer price promotions (Klemperer, *Quarterly Journal of Economics* 102(2):375–394, 1987a, *Economic Journal* 97(0):99–177, 1987b, *Review of Economic Studies* 62(4):515–539, 1995; Padilla, *Journal of Economic Theory* 67(2):520–530, 1995). However, in some markets large, national brands that are able to generate more consumer loyalty than their rivals offer lower prices and promote more frequently. In this paper, we develop a two-period game-theoretic, asymmetric duopoly model in which firms differ in their ability to retain repeat, loyal buyers. In this market, we demonstrate that it is optimal for a firm that generates more loyalty to offer a lower average price and promote more frequently than a weaker competitor. Numerical analysis of a more general infinite period version of this asymmetric model leads to three additional results. First, we show that there is an inverted-U relationship between a weak firm's ability to attract repeat, loyal consumers and strong firm profits. Second, we show that the relative ability of firms to attract repeat buyers affects whether serial and contemporaneous price correlations are positive or negative. Finally, we highlight the effect of dynamics on firms' expected prices and profits.

Keywords Price promotions · Promotion depth and frequency · Loyalty

JEL Classifications C70 · C72 · D40 · D43 · L00 · L11 · M21 · M31

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1 Introduction

Numerous marketing studies document that consumer loyalty or state dependence varies across consumers, brands, and categories (Guadagni and Little 1983; Seetharaman et al. 1999). Extant theoretical models suggest that greater consumer loyalty increases a firm's market power and leads to higher prices and fewer price promotions (Klemperer 1987a, b, 1995; Padilla 1995). However, in some markets large, national brands that are able to generate more consumer loyalty than their rivals offer lower prices and promote more frequently. For example, using the ERIM data researchers find that peanut butter and stick margarine are two categories in which many consumers exhibit state dependence (Seetharaman et al. 1999) and large share brands have more state dependence or loyalty (Van Oest and Frances 2003). Surprisingly, the largest national brands in these categories, Peter Pan (peanut butter) and Parkay (stick margarine), offer a lower average price and more frequent promotions than rival national brands.

Such pricing strategies are inconsistent with predictions from the extant literature in economics and marketing. The economics literature has emphasized that switching costs or state dependence should lead to higher prices (Klemperer 1987a, b, 1995; Padilla 1995), but in the previous examples we observe the opposite. In marketing, the theoretical literature on price promotions has examined competition between weak brands and strong brands, which are assumed to have more loyal consumers (Narasimhan 1988; Raju et al. 1990; Lal 1990; Rao 1991). However, these models predict that the optimal strategy for a strong brand is to offer infrequent, deep discounts or frequent, shallow discounts. In sum, the existing literature cannot explain why a brand that creates more consumer loyalty than its rivals would compete by offering a lower average price and more frequent promotions.

In this paper, we develop a dynamic, game theoretic model that offers an explanation for this pricing behavior. Similar to past research on state dependence or switching costs, we allow consumers who are initially indifferent to become loyal to a brand. However, the key difference in our model is that we assume firms are asymmetric in their ability to create consumer loyalty or state dependence. In our duopoly model we refer to a firm as strong if it converts a greater fraction of trial purchasers into repeat, loyal consumers; analogously, we refer to the competing firm as weak. In packaged goods, an asymmetry in each firm's ability to create state dependence is consistent with the notion that some brands are able to create more favorable purchase experiences that lead to consumer loyalty. Asymmetries in state dependence may also be present in business markets. For example, a firm that offers a proprietary, closed software system may be able to create lock-in or state dependence while rivals who sell open software systems may not have the same degree of loyalty.

Our assumption that loyalty is in-part created by a firm's pricing strategy endogenizes the size of a firm's loyal base of consumers. This creates a trade-off between investing and harvesting as a firm optimizes its pricing strategy. To invest in creating a loyal base of consumers, a firm may offer a low price that attracts new consumers and some of these buyers may become loyal. In contrast, once a firm establishes a large base of loyal consumers they can harvest the value created by charging a high price.[1] In every period, a firm must assess the market to determine whether it is optimal to invest or harvest.

A key insight from the model is that the incentive to invest and harvest differs for strong and weak firms. If a weak firm attracts few repeat, loyal buyers then the only benefit of a low price is the current increase in market share. A strong firm that attracts more loyal consumers benefits from a low price in both the current period and future periods. In a two period model, we show analytically that the incentive to create future loyalty may lead a strong firm to offer a lower average price and promote more frequently than a weak firm. As shown in Table 1, this prediction is consistent with the pricing strategies used by Peter Pan peanut butter and Parkay margarine in Springfield, MO. Both leading brands offer a lower average price, promote more frequently, and offer a higher percentage discount compared to the second largest brand in the category.

Our analysis of a more general infinite period version of this asymmetric model allows us to obtain three additional results numerically. First, we show that there is an inverted-U relationship between the weak firm's ability to attract repeat, loyal consumers and the strong firm's profits. Second, we show that the relative ability of firms to attract repeat buyers affects whether serial and contemporaneous price correlations are positive or negative. Finally, we highlight the effect of dynamics on firms' expected prices and profits. Next, we briefly discuss each result.

The inverted-U relationship in profits can be explained by two competing effects. When a competitor is able to capture more repeat, loyal consumers there is a direct loss of market share for the rival. But, an increase in the number of loyal customers raises all firms' prices. When a competitor is very weak, the loss in market share is small compared to the benefit of increased prices and both firms' profits increase. However, as the rival firm attracts more loyal buyers, the loss in market share is the dominant effect and own firm profits decrease. This result suggests that when firms are very asymmetric in their ability to attract repeat, loyal buyers, a strong firm may want to accommodate a weaker rival's attempts to increase repeat purchases. As the firms become symmetric, each firm should react aggressively to either firm's attempt to attract repeat, loyal buyers.

Our model also shows that firms' relative ability to attract repeat, loyal consumers determines whether serial and contemporaneous price correlations are positive or negative. When firms are symmetric and attract a similar number of repeat buyers, then both contemporaneous and serial price correlations are negative. In practice, this is analogous to firms engaging in asynchronous, high-low pricing strategies (Lal 1990). When the weak firm attracts few repeat, loyal consumers the weak firm's serial and contemporaneous price correlations are positive. The positive correlation occurs because of the weak firm's incentive to mimic the behavior of the strong firm (i.e., strategic price effect).

Our results on price correlations complement findings from extant promotion models. In static models, price correlations are zero (Shilony 1977; Varian 1980;

[1] We consider myopic consumers who do not anticipate that prices may increase in future periods. This assumption simplifies the analysis but we expect the intuition from our model to extend to a setting with forward-looking consumer behavior. This may require a different model formulation.

Table 1 Average price and promotion frequency

Category	Brand	Market share (%)	Average price	Promotion frequency* (%)	Percentage discount** (%)
Peanut butter	Peter Pan	25	$1.80	10	10
Peanut butter	Jif	9	$1.85	6	6
Margarine	Parkay	32	$0.57	15	17
Margarine	Blue Bonnet	11	$0.61	13	11

* A promotion is defined as a price change of more than 5%.
** Percentage discount is defined as the average percentage discount when there is a promotion.

Narasimhan 1988; Raju et al. 1990; Lal 1990; Rao 1991) and hence cannot speak to these issues. The price cycles literature (Conlisk et al. 1984; Sobel 1984; Villas-Boas 2004a, 2006) predicts that a monopolist will offer periodic promotions, which implies negative serial correlation. In our model, serial correlation of the strong firm is negative, which is consistent with this literature. However, our model shows that strategic price effects can be significant enough for the serial price correlation of a weak firm to be positive. Finally, other competitive promotion models have shown that the contemporaneous price correlation can be either negative (Lal 1990) or positive (Sobel 1984). In a single model, we show how one factor (i.e., the number of repeat, loyal consumers) can determine whether price correlations are positive or negative.

We assume that both firms in our model optimize discounted, long-run profits. An important question is whether firms' concerns for future profits softens or sharpens competition. We find that if the firms were to set prices myopically, the expected profits of the weak firm would increase. In contrast, myopic pricing does not necessarily benefit the strong firm. If the weak firm attracts few repeat, loyal buyers then myopic pricing may decrease profits of the strong firm. We conclude that the effect of myopic pricing depends on a firm's relative market position (i.e., strong or weak) and on the degree of asymmetry in each firm's ability to attract repeat, loyal buyers.

Our model contributes to the marketing and economics literatures on switching costs, which have been shown to affect price levels, market attractiveness to a new entrant, and tacit collusion (Klemperer 1987a, b, 1989; Beggs and Klemperer 1992; Padilla 1995, Anderson et al. 2004). A summary of the switching cost literature is provided in Klemperer (1995). A common force in these models is that firms may have an incentive to offer low prices to attract consumers and then offer higher prices in later periods when consumers face switching costs. Indeed, the strong firm in our model engages in this strategy. Two key features distinguish our model from the switching cost literature. First, we focus on the number of customers who become loyal (i.e., develop switching costs) and second, we analyze firms that differ in their ability to attract repeat, loyal buyers (i.e., asymmetric firms). In contrast, a typical switching cost model analyzes the magnitude of a consumer's switching cost and considers competition between symmetric firms.

In related papers, Villas-Boas develops two period (2004a) and infinite period (2006) dynamic models in which forward-looking consumers are initially uncertain about how well a product fits their preferences. After a purchase experience consumers learn about the true fit of the product. Our model assumes consumers are myopic but has a similar feature in that some consumers are initially indifferent and a fraction of these consumers become loyal. While we focus on the effect of such behavior on firms' pricing strategies Villas-Boas focuses on "the competitive effects of the potential informational advantages of a product that has been tried by the consumer" (Villas-Boas 2004a, p. 142).

Our work also adds to the empirical consumer choice literature that documents both dynamics and state dependence in consumer preferences (Guadagni and Little 1983; Erdem and Keane 1996; Mela et al. 1997; Foekens et al. 1999; Anderson and Simester 2004). Our model complements these empirical studies as we provide a theory that relates consumer dynamics to competitive behavior. In addition, our results provide a framework to explain empirically observed patterns of competitive price response (Leeflang and Wittink 1992, 1996; Kopalle et al. 1999).

The remainder of this paper is organized as follows. In Section 2 we consider a two period model that allows us to derive our two main results analytically. In Section 3, we extend this model to an infinite horizon, overlapping generations model and derive analytic expressions that fully characterize equilibrium pricing strategies and payoffs. Because the pricing strategies involve complex analytic expressions, many important results can only be shown through numerical simulation. In Section 4, we present results of our numerical analysis of the infinite horizon model. Two of these results replicate findings from the two period model and we also present additional results on price correlations and the effect of forward-looking firm behavior. We conclude with a brief discussion in Section 5.

2 Two-period model

We consider a market with two competing firms that we label s for strong and w for weak. Each firm has zero marginal cost and sells one product over two periods to three types of consumers: static loyal consumers, dynamic loyal consumers and switchers. Both firms have a mass of l static loyal consumers who purchase from their preferred firm as long as price is less than the reservation price (i.e., $p_j \leq r$. where $j \in \{s,w\}$). In period 1, we assume there is a unit mass of switchers who behave myopically and purchase from the firm that offers the lowest price as long as $p \leq r$. We assume that a customer who is initially a switcher may become a loyal consumer in the subsequent period. We refer to consumers who are initially switchers but become loyal as dynamic loyal consumers.

Unlike previous models of state dependence, a key assumption in our model is that each firm differs in their ability to convert switchers into loyal consumers. We assume that the strong brand is able to convert θ_s switchers into dynamic loyal consumers while the weak brand converts θ_w consumers, where $\theta_s > \theta_w$. This assumption is supported by empirical studies that show that large share brands tend to generate more state dependence (Van Oest and Frances 2003). Intuitively, one might expect a leading brand in a category, such as Tide detergent, to generate more state dependence than its rivals.

To simplify our analysis we assume that θ_s=1 and allow θ_w to vary between 0 and 1. This implies that if the weak brand offers the lowest price in period 1 then θ_w of the switchers become loyal to the weak brand in period 2. In this case, the market in period 2 consists of l consumers who are loyal to the strong firm, $l+\theta_w$ consumers who are loyal to the weak firm and $(1-\theta_w)$ switchers. In contrast, if the strong firm offers the lowest price in period 1 then all switching consumers become loyal to that firm. The market in period 2 consists of l+1 consumers who are loyal to the strong firm, l consumers who are loyal to the weak firm and zero switchers. Relaxing our assumptions to allow θ_s<1 does not substantively change any of our main results but adds complexity to the analysis.

We solve the game by backward induction from period 2. Given our assumptions, there are two possible states in period 2 that we label state 0 and state 1. In state 0, the strong firm has θ_s dynamic loyal consumers while in state 1 the weak firm has θ_w dynamic loyal consumers. In state 0 there are no switchers, hence, both firms charge the reservation price and the profit of each firm is: $\pi_{s20} = r(l+1)$ and $\pi_{w20} = rl$ where π_{ijk} represents the profit of firm i in period j in state k. In state 1, there is no equilibrium in pure strategies but there is a unique mixed strategy equilibrium. Let $F_{j21}(p)$ represent the CDF of firm j in period 2 in state 1 and assume each firm mixes over prices in the range $[\underline{p}_{21}, r]$. Given that the weak brand has more loyal consumers it is easily shown that F_{w21} has a mass point at $p = r$ but the strong firm has no mass point in its distribution. The lower bound of the support is $\underline{p}_{21} = r(l+\theta_w)/(l+1)$, which is the lowest price the weak firm would offer. The equilibrium pricing strategies of the firms in state 1 must satisfy the following conditions:

$$p[l + (1-\theta_w)(1 - F_{w21}(p))] = \underline{p}_{21}(l + (1-\theta_w)), \tag{1}$$

$$p[l + \theta_w + (1-\theta_w)(1 - F_{s21}(p))] = r(l+\theta_w). \tag{2}$$

In Eqs. 1 and 2, the left hand side represents the expected second period payoff in state 1 given the rival's mixing distribution and the right hand side represents the reservation profits of the strong and weak firm, respectively. Solving Eqs. 1 and 2 we obtain the equilibrium mixing distributions in period 2 in state 1:

$$F_{w21}(p) = 1 - \frac{l(1+l)(r-p) + r\theta_w(1-\theta_w)}{p(1+l)(1-\theta_w)}, \forall p \in \left[\underline{p}_{21}, r\right) \tag{3}$$

$$F_{s21}(p) = \frac{p(1+l) - r(l+\theta_w)}{p(1-\theta_w)}, \forall p \in \left[\underline{p}_{21}, r\right] \tag{4}$$

We now turn to the first period strategies of both firms. Consistent with our previous analysis, there is no pure strategy equilibrium in period 1 and the unique equilibrium is in mixed strategies. While both firms have the same number of loyal consumers in period 1 the strong firm is able to convert a greater fraction of switchers into static loyal consumers. This creates an incentive for the strong firm to offer lower prices than the weak firm in period 1. In equilibrium, both firms price over the range $[\underline{p}_1, r]$, the strong firm has no mass points and the weak firm has a point mass at $p = r$. The lower bound of the support is $\underline{p}_1 = r(l - \delta\theta_w)/(l+1)$, which is the lowest price the weak firm is willing to offer in period 1. The equilibrium pricing strategies of the firms in period 1 satisfy the following conditions:

$$\left\{ \begin{array}{l} p[l + (1 - F_{w1}(p))] \\ +\delta[(1 - F_{w1}(p))\pi_{s20} + F_{w1}(p)\pi_{s21}] \end{array} \right\} = \underline{p}_1(l+1) + \delta\pi_{s20}, \tag{5}$$

$$\left\{ \begin{array}{l} p[l + (1 - F_{s1}(p))] \\ +\delta[(1 - F_{s1}(p))\pi_{w21} + F_{s1}(p)\pi_{w20}] \end{array} \right\} = rl + \delta rl, \tag{6}$$

The left hand side of Eq. 5 is the strong firm's discounted expected profit when charging a price of p. The right hand side of Eq. 5 is the strong firm's discounted expected profit from charging $p = \underline{p}_1$ in period 1. At this price, the strong firm sells to all the switchers, earns $\underline{p}_1(l+1)$ in period 1 and transitions to state 0. The term $\delta\pi_{s20}$ is the discounted, second period profit of the strong firm in state 0. Equation 6 is an analogous expression for the weak firm. Solving Eqs. 5 and 6 we obtain the equilibrium mixing distributions in period 1:

$$F_{w1}(p) = \frac{(1+l)(p - l(r-p) + \delta r\theta_w)}{((1+l)p + r\delta(1 + l - \theta_w(1-\theta_w)))}, \forall p \in \left[\underline{p}_1, r\right) \tag{7}$$

$$F_{s1}(p) = 1 - \frac{l(r-p)}{p + \delta r\theta_w}, \qquad \forall p \in \left[\underline{p}_1, r\right] \tag{8}$$

The expected profits of the weak and strong firm are:

$$\Pi_w = rl + \delta rl \tag{9}$$

$$\Pi_s = r(l - \delta\theta_w) + \delta r(l+1) \tag{10}$$

Equations 9 and 10 follow from the right hand side of Eqs. 5 and 6. The weak firm's expected profits are equal to charging $p = r$ in both periods and generating no additional loyal consumers. However, inspection of Eq. 10 offers a different interpretation of the strong firm's expected profit. If the strong firm offers a low price in period 1 and sells to the switchers, this leads to discounted future profits of $\delta r(l+1)$. Equation 10 shows that the low first period price results in an opportunity cost for the strong firm in period 1. To see this, note that the strong firm could offer $p = r$ in period 1, sell to only loyal consumers and earn a profit of rl. The expected first period profit of the strong firm equals $r(l - \delta\theta_w)$, which implies an expected opportunity cost of $\delta r\theta_w$ in period 1. The strong firm is willing to make this investment since the discounted period 2 payoff in state 0 exceeds the discounted expected payoff in state 1 by $\delta r(1 - \theta_w(1 - \theta_w))(1+l)$. We conclude that the strong firm's pricing strategy is analogous to loss-leader pricing. The strong firm offers low prices in period 1 and incurs an opportunity cost but recoups this loss in period 2.

Our two period model yields two key results. First, we show that the incentive to create repeat, loyal buyers results in the strong firm offering lower average prices and promoting more frequently. We state this formally as:

2.1 Result 1

(a) The expected price of the strong firm is less than the weak firm.
(b) The strong firm promotes more frequently than the weak firm.

As shown in the Appendix, Result 1a follows from stochastic dominance of the firms' pricing strategies. In period 1, the strong firm has an incentive to offer lower prices to attract switchers. In period 2, both firms charge the same price in state 0 and the weak firm has more loyal consumers and charges a higher price in state 1. Together, this implies that the expected price of the strong firm is lower than the weak firm.

Consistent with previous theoretical price promotion models, we interpret prices of $p < r$ as promotions (Narasimhan 1988; Raju et al. 1990). Result 1b shows that the strong firm promotes more frequently than the weak firm. In period 1, the strong firm always promotes but the weak firm may not promote. In period 2, neither firm promotes in state 0 and the strong firm promotes more frequently in state 1. This implies that the strong firm promotes more frequently than the weak firm.

A second key result from our model is how the weak firm's ability to build a base of loyal consumers, as measured by θ_w, affects the strong firm's profits. Our model illustrates that θ_w affects the strong firm's profits in two ways and this is best illustrated by considering payoffs in state 1.

2.2 Result 2

The expected profits in state 1 of the strong firm are increasing for low values of θ_w and decreasing for large values of θ_w.

Using the right hand side of Eq. 1, one can show that the expected profit of the strong brand in state 1 equals $\underline{p}_{21}(l + (1 - \theta_w))$. Substitution and simplification yields $\pi_{s20} = r\left(l^2 + l + \theta_w - \theta_w^2\right)/\left(\overline{l} + 1\right)$. Hence, there is an inverted-U relationship between the strong firm's profits and the weak firm's ability to create loyalty. For $\theta_w <$ 0.5 the strong firm's profits are increasing in θ_w and for $\theta_w > 0.5$ the strong firm's profits are decreasing in θ_w.

In Result 2, two forces are at work and we refer to these as the direct effect and strategic effect. An increase in θ_w results in more repeat buyers for the weak firm and a direct loss in market share for the strong firm. Thus, the direct effect is always negative for the strong firm. But, an increase in the number of loyal consumers raises the expected price of the weak firm. Since prices are strategic complements, an increase in the expected price of the rival allows the strong firm to raise its expected price. The strategic effect is always positive. When θ_w is small, the strategic effect dominates and the strong firm's profits are increasing in θ_w. However, when θ_w is large, the direct effect dominates and the expected profit of the strong firm is decreasing in θ_w.

The extant literature on price promotions and state dependence has focused on the number of static loyal consumers, l, and the degree to which a consumer is loyal to a brand. The latter is often measured as a switching cost, c, which is the price premium a consumer is willing to pay for the preferred brand. Results 1 and 2 contribute to the extant literature by highlighting that changes in l and c have a different impact than changes in a firm's ability to create dynamic loyal consumers, θ_j.

Static promotion models show that a firm with more loyalty offers either deep, infrequent promotions or shallow, frequent promotions (Narasimhan 1988; Raju et al. 1990; Rao 1991). Result 1 shows that a strong brand should promote more often and offer a lower expected price, and this result is not predicted by the extant literature. Result 2 shows that changes in a rival's ability to attract repeat, loyal buyers may either increase or decrease own firm profits. Again, this contrasts with predictions from the extant literature on changes in a rival's l and c. For example, in Narasimhan (1988), if the fraction of static loyal buyers increases for the weak competitor there is no change in the strong firm's profits. Analogously, in Raju et al. (1990) an increase in the degree of loyalty to the weak firm increases the weak firm's expected profits but has no effect on the strong firm's profits.

An advantage of the two period model is that we can analytically derive two key results. However, a limitation of this approach is that we do not fully explore on-going competition between two firms that are both trying to attract new consumers and capture profits from existing consumers. In the next section, we extend this model to an infinite period game and this allows us to address this issue more fully.

3 Infinite-period model

In this section, we extend our two period model to an infinite horizon, overlapping generations model (OLG). Most of our assumptions are identical to the two period model but we also relax several assumptions. We assume that a cohort of consumers enters the market each period, buys at most one unit each period and lives for two periods. Each cohort has a unit mass of switching consumers and a mass of l_s and l_w static loyal consumers, where $l_s \geq l_w$. Since $(1 + l_s + l_w)$ consumers enter the market and exit after two periods the market size is always $2(1 + l_s + l_w)$.[2]

We maintain the same assumptions about each firm's ability to convert switchers into dynamic loyal consumers. An example helps to clarify the market structure and dynamics. Let p_{jt} equal the price offered by firm j in period t. If the firms offer p_{sl}=1/2 and p_{wl}=3/4 in period 1 then all switching consumers buy from the strong firm. In period 2 there are θ_s dynamic loyal consumers for the strong firm and zero dynamic loyal consumers for the weak firm. The strong firm sells to $2l_s + \theta_s$ loyal consumers and the weak firm sells to $2l_w$ loyal consumers and both firms compete for the $(2 - \theta_s)$ switching consumers. If

[2] An alternative interpretation is there are $2(l_s + l_w)$ consumers in all periods and a unit mass of switching consumers, who live 2 periods, enter each period.

the firms offer p_{s2}=1 and p_{w2}=1/4 in period 2 then in period 3 there are θ_w dynamic loyal consumers for the weak firm and zero dynamic loyal consumers for the strong firm. The strong firm sells to $2l_s$ loyal consumers and the weak firm sells to $2l_w + \theta_w$ loyal consumers and both firms compete for the $(2 - \theta_w)$ switching consumers.

There are an infinite number of periods but only two possible states in our model that we label state 0 and state 1. The current state summarizes all payoff relevant information, which allows us to drop the period subscripts in our notation. In state 0, the strong firm has θ_s dynamic loyal consumers while in state 1 the weak firm has θ_w dynamic loyal consumers. Let V_{jk} equal the continuation payoff to firm j in state $k \in \{0,1\}$ and $F_{jk}(p)$ equal the CDF of each firm in each state. In state k each firm mixes over prices in the range $[\underline{p}_k, r)$ and at most one firm offers $p = r$ in each state. In state 0, the continuation payoffs are:

$$V_{s0} = p[2l_s + \theta_s + (2 - \theta_s)(1 - F_{w0}(p))] + \delta[(1 - F_{w0}(p))V_{s0} + F_{w0}(p)V_{s1}], \quad (11)$$

$$V_{w0} = p[2l_w + (2 - \theta_s)(1 - F_{s0}(p))] + \delta[(1 - F_{s0}(p))V_{w1} + F_{s0}(p)V_{w0}]. \quad (12)$$

In Eqs. 11 and 12, the first term represents the current period payoff and the second term represents the discounted expected future payoff. If the strong firm offers a lower price than its rival (probability $1 - F_{w0}(p)$), then the strong firm's current payoff is $p(2l_s + \theta_s + 2 - \theta_s)$ and in the next period firms are again in state 0. In contrast, if the weak firm offers a lower price than its rival (probability $1 - F_{s0}(p)$), then the weak firm's current payoff is $p(2l_w + 2 - \theta_s)$ and in the next period firms transition to state 1. The continuation payoffs in state 1 are:

$$V_{s1} = p[2l_s + (2 - \theta_w)(1 - F_{w1}(p))] + \delta[(1 - F_{w1}(p))V_{s0} + F_{w1}(p)V_{s1}], \quad (13)$$

$$V_{w1} = p[2l_w + \theta_w + (2 - \theta_w)(1 - F_{s1}(p))] + \delta[(1 - F_{s1}(p))V_{w1} + F_{s1}(p)V_{w0}]. \quad (14)$$

The interpretation of these equations is analogous to state 0. We solve Eqs. 11–14 for $F_{jk}(p)$ to obtain the pricing policies of the strong and the weak firm in each state.

$$F_{s0}(p) = 1 - \frac{V_{w0}(1 - \delta) - 2l_w p}{p(2 - \theta_s) - \delta(V_{w0} - V_{w1})}, \qquad \underline{p}_0 \le p < r \quad (15)$$

$$F_{w0}(p) = \frac{2p(1 + l_s) - V_{s0}(1 - \delta)}{p(2 - \theta_s) + \delta(V_{s0} - V_{s1})}, \qquad \underline{p}_0 \le p < r \quad (16)$$

$$F_{s1}(p) = \frac{2p(1 + l_w) - V_{w1}(1 - \delta)}{p(2 - \theta_w) - \delta(V_{w0} - V_{w1})}, \qquad \underline{p}_1 \le p < r \quad (17)$$

$$F_{w1}(p) = 1 - \frac{V_{s1}(1 - \delta) - 2l_s p}{p(2 - \theta_w) + \delta(V_{s0} - V_{s1})}, \qquad \underline{p}_1 \le p < r. \quad (18)$$

We note that Eqs. 15–18 are functions of the continuation payoffs, V_{jk}, and lower bounds of the support, $\underline{p}_k$. To fully characterize the equilibrium we need to identify six unknowns (V_{jk}, $\underline{p}_k$) and this implies that there are six relevant equations. The CDFs evaluated at the lower bound of the support have zero mass for each firm in each state and this leads to four equations:

$$F_{jk}\left(\underline{p}_k\right) = 0, \qquad \forall j = \{s, w\}, k = \{0, 1\}. \tag{19}$$

The final two equations are determined by whether firm j has a mass point on r in state k. In each state, one firm has zero mass on r while the other firm has positive mass on r and this results in four possible cases. For example, in Case 1 the two additional equations that define the equilibrium are $F_{w0}(r) = 1$ and $F_{s1}(r) = 1$. The solutions and conditions for each case are provided in the Appendix.

4 Numerical results

In the Appendix, we characterize the equilibrium payoffs and pricing strategies of the infinite horizon game with analytic closed form solutions. But, analytic expressions for many properties of the model, such as the expected price, promotion frequency, and price correlations are not analytically tractable. To derive the properties of the infinite horizon model we use numerical analysis. In models with many parameters, general insights from numerical analysis are difficult to obtain. However, in our model the main parameter of interest is the relative ability of firms to attract repeat, loyal customers. Thus, we set $\theta_s = k$ and allow θ_w to vary from 0 to θ_s; with no loss in generality we report results for k=1. The level of r in our model is arbitrary and we normalize to r=1. We assume a discount factor of δ=0.9 and note that all our results hold provided firms are sufficiently patient (δ>0). To simplify the exposition and highlight the role of repeat loyal consumers we assume both firms have the same number of static loyals ($l_w = l_s = l$). To allow variation in the relative importance of dynamic loyalty we simulated numerous levels of l>0. However, to simplify exposition we present figures for $l \in \{0.01, 0.50\}$.

Given our assumptions, the unique equilibrium for these parameter values is Case 1 (see Appendix). We focus our results on Case 1 because this is the only solution where the strong firm has a mass point on $p = r$ in state 0 and the weak firm has a mass point on $p = r$ in state 1. This equilibrium arises when the firms are relatively symmetric, which leads to more strategic interaction. Case 1 is also a general solution to the model considered by Padilla (1995) and Anderson et al. (2004).

We present four numerical results from the infinite period model. The first two results replicate Results 1 and 2 from the two period model. Result 3 shows how changes in θ_w affects both serial and contemporaneous price correlations. Finally, Result 4 considers how the pricing strategies and expected profits of the firms change when they are myopic rather than forward looking.

A plot of the expected price of both firms illustrates Result 1a for the infinite period model (see Fig. 1a). For all levels of θ_w and both levels of l the expected price of the strong firm exceeds the expected price of the weak firm. Figure 1b is a plot of the promotion frequency of both firms. If there are enough static loyal consumers

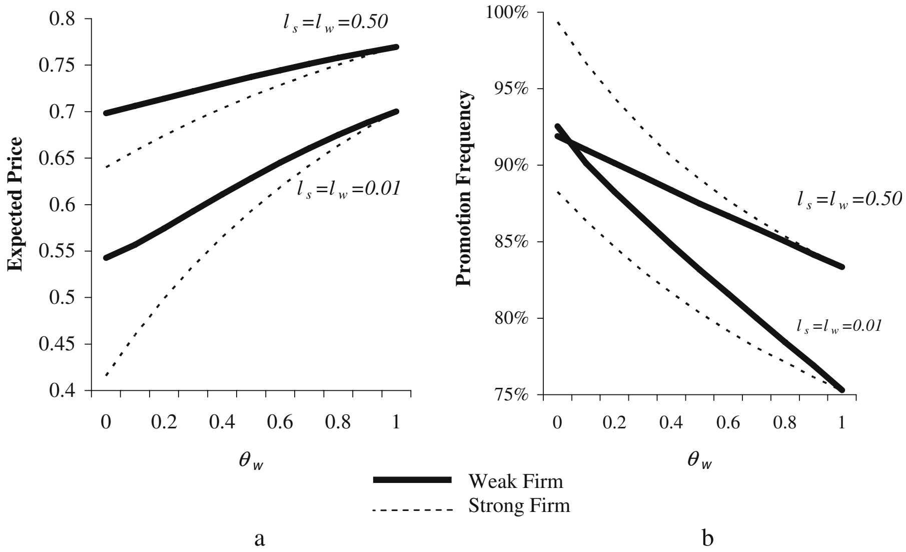

Fig. 1 Expected price and promotion frequency

(e.g. l=0.5) then the strong firm promotes more frequently than the weak firm. But, when there are few static loyal consumers this result may not hold. Extensive numerical simulations verify that Result 1b holds provided $l > l^*$.

To offer some intuition for why Result 1b depends on the level of l, we first recognize that the difference in expected price of the weak and strong firms is decreasing in θ_w (see Fig. 1a). The relative magnitude of the strategic and direct effects in state 0 explains this result. In state 0, the weak firm has zero dynamic loyal consumers and there is little incentive to offer a low price to build future loyalty because θ_w is small (i.e., the direct effect is small). At the same time, the strong firm has many dynamic loyal consumers and charges high prices to capture margin on its large, loyal base of consumers. In reaction, the weak firm raises its price in state 0 due to the strategic effect. Surprisingly, for low values of θ_w this effect can be large enough that the weak firm earns a higher expected margin in state 0 (no dynamic loyal consumers) than state 1 (see Appendix, Fig. 6).

Now consider how the strong firm's incentive to promote is affected by l. The strong firm's strategy must incorporate the impact of current promotion frequency on future profits. When the strong firm does not promote ($p_s = r$) there is a short-run gain in margin and a loss in volume. But there is also a future cost because the strong firm immediately transitions to state 1, which is a less profitable state with zero dynamic loyal consumers. As l increases the transition probability from state 1 to state 0 decreases because the gap between the strong firm's price and weak firm's price narrows (see Fig. 1a). Spending more time in state 1 is undesirable and the strong firm reacts to this expected future cost by increasing its promotion frequency in state 0. In turn, this decreases the likelihood of transitioning to state 1. These future costs (i.e. dynamic effects) are less significant for a weak firm that generates few dynamic loyal consumers. Thus, a weak firm's promotion frequency is affected primarily by the short-run trade-off of volume versus margin. Dynamics play a greater role for the strong firm and this explains why it promotes more frequently than a weak rival.

Result 2 showed that an increase in the ability of the weak firm to attract repeat, loyal buyers may increase profits of the strong brand for low values of θ_w. However, for larger values of θ_w, profits of the strong brand are decreasing in θ_w. Figure 2 illustrates that Result 2 holds for the expected profits in the infinite period model. Thus, while the analysis of state 1 in the two-period model offered intuition for Result 2, the finding holds more generally for the infinite period game. An increase in θ_w decreases market share of the strong firm and this has a negative impact on profit. However, an increase in θ_w raises the price of both firms (Result 1a). The relative magnitude of these effects dictates whether the profits of the strong firm increase or decrease. If θ_w is small then the strategic price effect dominates and while the market share of the strong brand declines its profits increase. If θ_w is large then the loss in market share dominates and the strong brand loses both share and profits when its rival attracts more repeat buyers.

With the infinite period model we can now assess the effects of θ_w on serial and contemporaneous price correlations. Result 3 characterizes how firms' relative ability to attract repeat, loyal buyers affects whether these correlations are positive or negative.

4.1 Result 3

(a) The contemporaneous price correlation is positive for low values of θ_w and negative for high values of θ_w.

(b) The serial correlation of the weak firm is positive for low values of θ_w and negative for high values of θ_w. Serial correlation of the strong firm is always negative.

The contemporaneous correlation in prices is depicted in Fig. 3a and illustrates Result 3a. When θ_w is sufficiently small the weak firm's incentive to build loyalty is not strong and the strategic effect is dominant. Thus, when the strong firm offers a high price the weak firm tends to offer a high price and this induces a positive

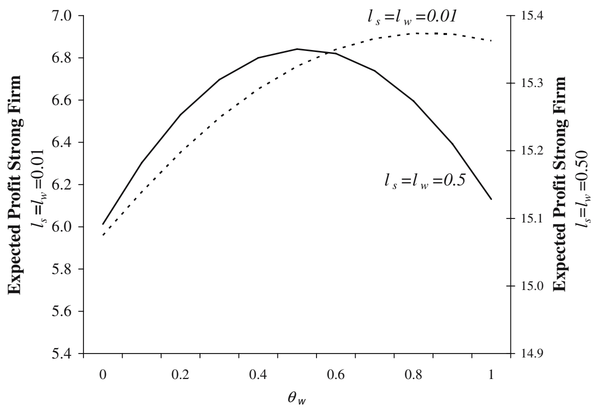

Fig. 2 Expected profit of strong firm

correlation. As θ_w increases, the direct effect becomes important for both firms and each uses high-low pricing. In state 0, the strong firm offers high prices to capture margin on loyal consumers and the weak firm offers low prices to build loyalty. In state 1, the weak firm offers high prices to capture margin on loyal consumers and the strong firm offers low prices to build loyalty (see Appendix, Fig. 6). This results in a negative contemporaneous price correlation for large values of θ_w.

The use of high-low pricing also explains Result 3b, which is shown graphically in Fig. 3b. The strong firm always uses high-low pricing to build loyalty and then capture profits, and this leads to negative serial correlation. Unlike the strong firm, the weak firm does not engage in high-low pricing for low values of θ_w. The main reason is that in state 0 there is no incentive to offer a low price when θ_w is low. Instead, the weak firm raises its price in state 0 and this leads to positive serial correlation.

A surprising insight from Fig. 3 is that the weak firm may have both positive serial and contemporaneous price correlations for low values of θ_w. Positive contemporaneous correlation implies that the weak and strong firm follow similar pricing strategies. Since the strong firm uses high-low pricing, one might predict that the weak firm would also use high-low pricing. However, the model shows that the weak firm's serial price correlation is positive for low values of θ_w. Thus, in a qualitative sense the weak firm probabilistically mimics the strong firm (positive contemporaneous correlation). But, the mimicry is not extreme since the serial price correlation is positive.

Result 3b offers an explanation for the variation in competitive price response shown by Kopalle et al. (1999) in the dishwashing detergent category. The authors estimate the price response function proposed by Leeflang and Wittink (1992, 1996) for six brands of dishwashing detergent. For the 30 brand pairs, they find eight significant negative coefficients and eight significant positive coefficients. Closer inspection reveals that the variation in these competitive response coefficients is

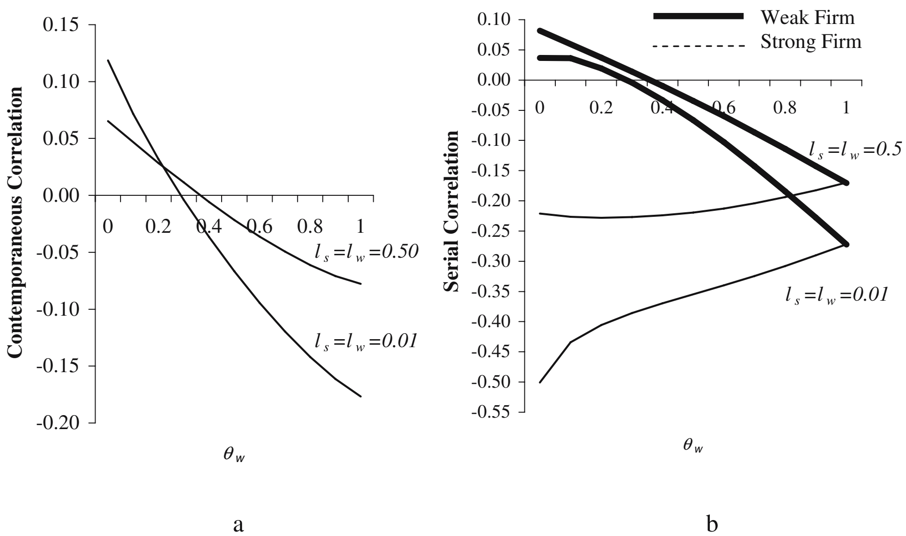

Fig. 3 Contemporaneous and serial correlation

systematic. To uncover the underlying pattern, we categorize the three largest share brands (Dawn, Palmolive, and Sunlight) as strong brands and the three lowest share brands (Ivory, C.W. Octagon, and Dove) as weak brands in Table 2.

Negative (positive) coefficients in Table 2 indicate that the temporal correlation of price promotions is negative (positive). In other words, a brand is more (less) likely to promote in the current period if a competing brand promoted last period. Table 2 shows that the effect of strong brands on other strong brands is either negative or not significant implying alternating retail price promotions.[3] For competition between weak brands, the effect is mostly insignificant suggesting the timing of promotion to be independent of competitive action. In contrast, the effect of strong brands on weak brands tends to be positive (4 of 5 cases) or not significant (4 cases). A similar pattern appears for the effect of weak brands on strong brands (4 of 6 positive, 3 not significant). This pattern of response coefficients is broadly consistent with predictions from Result 3b.

Finally, in our model, firms' pricing strategies incorporate discounted future profits. This contrasts with static promotion models (Narasimhan 1988; Raju et al. 1990) in which firms optimize current profits. To benchmark our dynamic model against static promotion models, we consider the case where firms ignore future profits (δ=0) and the equilibrium pricing strategy for both firms is identical to Narasimhan (1988). To illustrate our results we will focus on the case where firms are perfectly myopic, δ=0, and compare against our assumed discount factor, δ=0.9. In the dynamic game the continuation payoff is $V = \pi + \delta V$ and to compare profits with the static game we focus on π, which is the expected current period profit. The average prices in the myopic and dynamic cases are plotted in Fig. 4 and expected current period profits are plotted in Fig. 5. Our results are summarized as follows.

4.2 Result 4

If firms are perfectly myopic (δ=0), then relative to the dynamic model (δ>0)

(a) The average price of each firm increases if the number of dynamic loyal consumers is sufficiently large,
(b) Expected current period profits of the weak firm always increases and expected current period profits of the strong firm increases if the number of dynamic loyal consumers is sufficiently large.

When firms are myopic they ignore the effect of their current pricing strategy on future profits. The effect of myopia differs in each state and varies with θ_w because the number of loyal consumers changes. In state 0, a myopic strong firm faces many loyal consumers, focuses on short-run profits and ignores the possibility of building future loyal consumers. Lack of concern for future profits raises the average price of a myopic strong firm in state 0, which in-turn raises the price of a myopic weak firm in state 0. In state 1, two related effects lead to a lower average price in the myopic game for low values of θ_w. First, competition for switching consumers lowers both firms' average price and second, the intense price competition increases the occurrence of state 1. Larger values of θ_w increase the number of loyal consumers

[3] Such an alternating pattern is also noted in Lal (1990) and Kopalle et al. (1999).

Table 2 Price reaction coefficients from Kopalle et al (1999)

Effect on:	Effect of:	
	Strong brands(high share brands)	Weak brands(low share brands)
Strong brands (high share brands)	4 negative 0 positive 2 not significant	2 negative 4 positive 3 not significant
Weak brands (low share brands)	1 negative 4 positive 4 not significant	1 negative 0 positive 5 not significant

in state 1 and this softens price competition in the myopic game. In addition, prices increase at a faster rate compared to the dynamic game because firms ignore the possibility of building future loyal consumers. Eventually myopia leads to higher prices in state 1 compared to the dynamic game.

When θ_w is low the effects in state 1 dominate and the average price decreases in the myopic game. That is, the increase in average price in state 0 is outweighed by both the lower average price in state 1 and the increased occurrence of state 1. As θ_w increases there is less price competition in state 1 and a lack of concern for future profits leads to higher prices versus the dynamic game.

Our analysis of myopia highlights two marginal effects in the dynamic game that arise due to a concern for future profits. First, when the market is composed of primarily switching consumers there is less price competition, which increases the average price. Second, even when firms sell to many loyal consumers there is a concern for building a future base of loyal consumers. This has a marginal effect of lowering prices in the dynamic game.

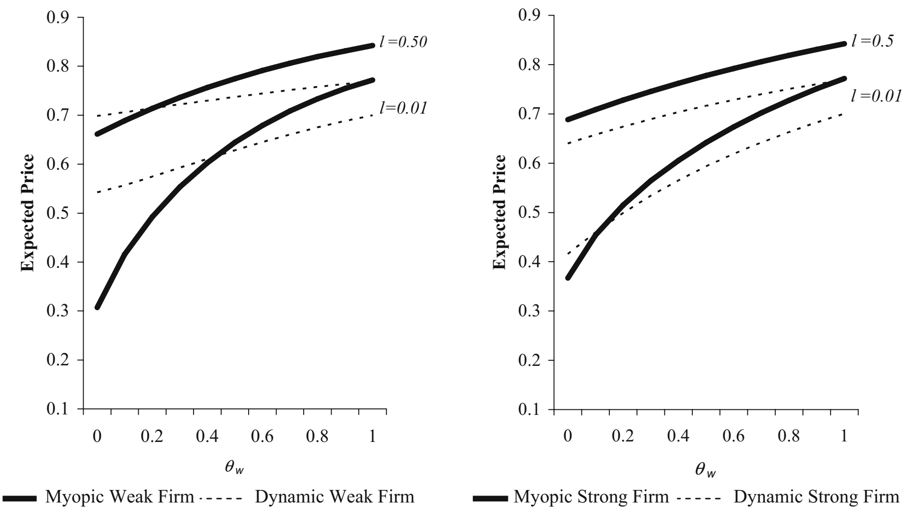

Fig. 4 Impact of dynamics on expected price

The impact of myopia on each firm's profits is shown in Fig. 5 and differs for each firm. Surprisingly, the weak firm always benefits from myopia. While low values of θ_w lower the average price of the weak firm, a gain in market share compensates for lost margin. In contrast, myopia decreases profits of the strong firm for low values of θ_w. As θ_w increases price competition becomes less intense and the strong firm benefits from the higher prices induced by myopia.

It is useful to contrast this result with Chintagunta and Rao (1996), who also analyze pricing strategies in a dynamic duopoly. Similar to our model, they find that myopic pricing leads to higher expected prices when there is dynamic, consumer loyalty. Chintagunta and Rao (1996) also claim that myopic pricing leads to lower profits, which differs from Result 4. This difference can be explained by the relative importance of the direct and strategic effects in each model. In both models, myopic firms fail to recognize the future benefits of building a loyal base of consumers. This results in higher prices and a loss in market share, which is the direct effect. But, in our model the strategic effect outweighs the direct loss in market share when both firms attract sufficient numbers of repeat, loyal buyers. The reason for this difference is that Chintagunta and Rao (1996) focus on long-run, steady state pricing strategies. Consumer dynamics plays a role in the evolution of pricing strategies but does not play a role in steady-state. In contrast, consumer dynamics always plays a role in our model since consumers continually enter and exit the market.

5 Discussion and conclusions

Many empirical studies have established that state dependence varies across consumers, brands, and categories. Previous analytic models have assumed that state dependence is a customer characteristic that does not vary across firms. In contrast, we take the view that state dependence varies by firm. That is, a well-

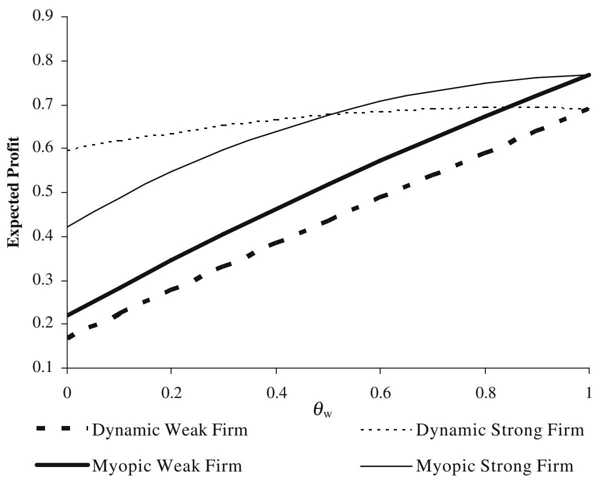

Fig. 5 Impact of dynamics on expected profit

known national brand, such as Tide detergent, may be able to attract more repeat, loyal buyers than a lesser-known rival brand. While this view of state dependence is supported by empirical research (Van Oest and Frances 2003), it has not been previously considered in analytic models.

Importantly, we show a firm's relative ability to attract repeat, loyal buyers has surprising implications for firms' pricing strategies. We show that this single effect can explain why leading brands such Parkay and Peter Pan may offer both the lowest average price and the most frequent promotions. For managers, this result shows that frequent, deep promotions may be optimal to maintaining a dominant market position. In this sense, we identify an alternative pricing strategy that a manager should consider to build and maintain a leading brand. While a strategy of low prices and frequent promotions is not always profitable, our model identifies conditions under which this strategy is optimal.

We also find that profits of a firm may increase when a weak competitor is able to attract more repeat, loyal buyers. This shows that if a firm becomes weaker it may lower the price of all firms and erode industry profits. An implication for managers is that a leading brand must be cautious using tactics that weaken a rival as this may lead to the unexpected consequence of lower industry prices and profits for both firms.

Our findings on price correlations complement extant theoretical research on price promotions and price cycles. Consistent with this literature, we show that strong firms exhibit high-low pricing or negative serial price correlation. However, strategic price effects may lead a weak competitor to mimic the behavior of the strong firm, which may lead to both positive serial and contemporaneous price correlations. We note that these findings are entirely due to competitive effects; in the absence of competition, a weak firm would exhibit negative serial price correlation. For practitioners, these results show when it is optimal for a weaker brand to mimic a stronger rival's pricing strategy. If the firms are sufficiently asymmetric in their ability to attract repeat, loyal buyers then mimicry of the competitor's pricing strategy is optimal.

A limitation of the infinite period model is that we allow consumers to live for only two periods. We maintain this assumption for tractability but this assumption does not drive our results. To illustrate this point, we extend the model to a three period overlapping generations model (see Appendix). All of our results hold in the three period OLG model, which illustrates that the assumption is not restrictive.

An additional limitation of our model is that we do not allow for strategic consumer behavior. That is, a strategic consumer may forgo a low price today if the consumer anticipates becoming loyal and paying a high price tomorrow. Incorporating strategic behavior is clearly important (Villas-Boas 2004a, b, 2006), but is not feasible in this model structure. However, we can show that for low values of θ_w and l consumer behavior is dynamically consistent in our model. This suggests that our results will extend to a game with strategic consumer behavior, which is an area of future research.

This paper incorporates consumer dynamics that are well-established in empirical marketing studies. The model allows us to highlight how firms' ability to attract repeat, loyal buyers affects pricing strategies and profits. Reassuringly, there is consistency between model predictions and extant empirical findings, but additional empirical research is required to formally test these predictions.

Appendix

Two-period model

Proof of Result 1

Proof:

(a) In period 2 state 0 both firms charge r. In state 1, the weak brand has a point mass at r and both firms randomize in the interval $[\underline{p}_{21}, r]$.

Note that: $F_{s21}(p) - F_{w21}(p) = \frac{\theta_w(p(1+l) - r(l+\theta_w))}{p(1+l)(1-\theta_w)} > 0$. This implies that $F_{w21}(p)$ first-order stochastically dominates $F_{s21}(p)$ so that the expected price charged by the weak brand in state 1 is greater than that charged by the strong brand. In period 1, once again the weak brand has a point mass at r and both firms randomize in the interval $[\underline{p}_1, r]$.

$$F_{s1}(p) - F_{w1}(p) = \frac{r\delta(1 + l - \theta_w)(1 - \theta_w)(p - l(r - p) + r\delta\theta_w)}{(p + r\delta\theta_w)(p(1 + l) + r\delta(1 + l - (1 - \theta_w)\theta_w))} > 0$$

Given that $F_{w1}(p)$ first-order stochastically dominates $F_{s1}(p)$ the expected price charged by the weak brand in period 1 is greater than that charged by the strong brand. Together this implies that the expected price of the strong brand is less than the weak brand.

(b) In period 2 state 0 neither firm promotes. In state 1, the weak brand has a point mass at r and hence the strong brand promotes more frequently in state 1. In period 1 the weak brand has a point mass at r so once again the strong brand promotes more frequently.

Result 2

The expected profits in state 1 of the strong firm are increasing for low values of θ_w and decreasing for large values of θ_w.

Proof: $\pi_{s21} = \underline{p}_{21}(l + (1 - \theta_w)) = \frac{r(l+\theta_w)}{(l+1)}(l + (1 - \theta_w))$

$$\frac{\partial \pi_{s21}}{\partial \theta_w} = \frac{r}{(l + 1)}(1 - 2\theta_w)\left.\begin{matrix} > 0, \theta_w < 1/2 \\ \leq 0, \theta_w \geq 1/2 \end{matrix}\right\}$$

Solutions for infinite horizon model

Let V_{jk}^{c*} and $\underline{p}_k^{c*}$ be the equilibrium solution for firm j in state k for case c. Define $\underline{p}_{w0}$, $\underline{p}_{s0}$, $\underline{p}_{w1}$ and $\underline{p}_{s1}$ as the solution to:

$$r2l_w + \delta V_{w0} = \underline{p}_{w0}(2l_w + 2 - \theta_s) + \delta V_{w1} \quad (20)$$

$$r(2l_s+\theta_s)+\delta V_{s1}=\underline{p}_{s0}(2l_s+2)+\delta V_{s0} \tag{21}$$

$$r(2l_w+\theta_w)+\delta V_{w0}=\underline{p}_{w1}(2l_w+2)+\delta V_{w1} \tag{22}$$

$$r2l_s+\delta V_{s1}=\underline{p}_{s1}(2l_s+2-\theta_w)+\delta V_{s0} \tag{23}$$

Case 1

In this case $F_{w0}(r)=1$ and $F_{s1}(r)=1$. Case 1 is feasible if $\underline{p}_{s1}<\underline{p}_1^{1*}$ and $\underline{p}_{w0}<\underline{p}_0^{1*}$, which is equivalent to conditions $C1$ and $C2$, respectively.

$$V_{w0}^{1*}=\left(\frac{r(2l_w+\theta_w)}{\delta(1-\delta^2)}+\frac{\left(r(2l_w+2-\theta_s)\left(\begin{array}{l}4\delta l_w+4l_s(1+l_w+\delta(1+2l_w))\\+2(1+l_w)(1+\delta)\theta_s\\+2\delta\theta_w(1+l_s-l_w)-\delta\theta_w^2\end{array}\right)\right)}{\left(\left(1-\delta^2\right)\left(\begin{array}{l}2(1+l_s)(2+l_w(2+4\delta)+\delta(4+\delta\theta_s))\\+\delta^2(2l_w+2-\theta_s)\theta_w\end{array}\right)\right)}\right) \tag{24}$$

$$V_{w1}^{1*}=\frac{\left(2(1+l_w)r\left(\begin{array}{l}\delta(2-\theta_s)(2l_s+\theta_s)+2l_w(2+l_s(2+4\delta)+\delta(2+\theta_s))\\+2(1+l_s)(1+\delta)\theta_w\end{array}\right)\right)}{\left((1-\delta)\left(2(1+l_s)(2+2l_w(1+2\delta)+\delta(4+\delta\theta_s))+\delta^2(2l_w+2-\theta_s)\theta_w\right)\right)} \tag{25}$$

$$V_{s0}^{1*}=\frac{\left(2(1+l_s)\left(\begin{array}{l}2r(1+l_w)\left(1-\delta^2\right)(2l_s+\theta_s)\\+r\delta(1-\delta)(2l_s+2-\theta_w)(2l_w+\theta_w)\end{array}\right)\right)}{\left(4(1+l_s)(1+l_w)\left(1-\delta^2\right)^2-\delta^2(1-\delta)^2(2l_w+2-\theta_s)(2l_s+2-\theta_w)\right)} \tag{26}$$

$$V_{s1}^{1*}=\frac{1}{\delta}\left(\frac{2(1+l_s)\left(\begin{array}{l}2r(1+l_w)(1+\delta)(2l_s+\theta_s)\\+r\delta(2l_s+2-\theta_w)(2l_w+\theta_w)\end{array}\right)}{\left(\begin{array}{l}4(1+l_s)(1+l_w)(1+\delta)^2(1-\delta)\\-\delta^2(1+\delta)(2l_w+2-\theta_s)(2l_s+2-\theta_w)\end{array}\right)}-r(2l_s+\theta_s)\right) \tag{27}$$

$$\underline{p}_0^{1*}=\frac{\left(r\left(4l_w\delta+4l_s(1+l_w+\delta+2l_w\delta)+2\theta_s(1+l_w)(1+\delta)+2\delta\theta_w(1+l_s-l_w)-\delta\theta_w^2\right)\right)}{\left(2(1+l_s)(2+2l_w(1+2\delta)+\delta(4+\delta\theta_s))+\delta^2\theta_w(2l_w+2-\theta_s)\right)} \tag{28}$$

$$\underline{p}_1^{1*}=\frac{\left(r(4l_w(1+l_s+\delta+2l_s\delta)+2\delta l_w\theta_s+\delta(2-\theta_s)(2l_s+\theta_s)+2\theta_w(1+l_s)(1+\delta))\right)}{\left(2(1+l_s)(2+2l_w(1+2\delta)+\delta(4+\delta\theta_s))+\delta^2\theta_w(2l_w+2-\theta_s)\right)} \tag{29}$$

$$C1:\frac{\left(r\left(\begin{array}{c}2(1+l_s)(\delta\theta_s(4-\theta_s(1-\delta))-4l_w(1+\delta\theta_s)+2l_s(2+\delta\theta_s))\\+\left(\begin{array}{l}4+4l_s^2-4l_w+4l_s(2-l_w-\delta(1+l_w))\\+2l_s\delta\theta_s-(1-\delta)\delta(2+2l_w-\theta_s)\theta_s\end{array}\right)\theta_w-2(1+l_s)\theta_w^2\end{array}\right)\right)}{\left((2+2l_s-\theta_w)\big(2(1+l_s)(2+l_w(2+4\delta)+\delta(4+\delta\theta_s))+\delta^2(2+2l_w-\theta_s)\theta_w\big)\right)}>0$$

$$C2:\frac{\left(r\left(\begin{array}{l}8(1+l_w)(l_s-l_w)+4((1-l_s+l_w)(1+l_w)-(1+l_s)l_w\delta)\theta_s-2(1+l_w)\theta_s^2\\+2\delta(2(2+2l_s-l_w)(1+l_w)-(1-l_w+l_s(1-\delta)-\delta)\theta_s)\theta_w\\-\delta(1-\delta)(2+2l_w-\theta_s)\theta_w^2\end{array}\right)\right)}{\left((2+2l_w-\theta_s)\big(2(1+l_s)(2+l_w(2+4\delta)+\delta(4+\delta\theta_s))+\delta^2(2+2l_w-\theta_s)\theta_w\big)\right)}>0$$

Case 2

In this case $F_{w0}(r)=1$ and $F_{w1}(r)=1$. Case 2 is feasible if $\underline{p}_{w1}<\underline{p}_1^{2*}$ and $\underline{p}_{w0}<\underline{p}_0^{2*}$ or equivalently if conditions $C3$ and $C4$ hold.

$$V_{w0}^{2*}=\frac{r(2l_w+(2-\theta_s))(2l_s+\theta_s(1-\delta))}{2(1+l_s)}+\frac{2r\delta(1+l_w)(2l_s-\delta\theta_s)}{(1-\delta)(2l_w+(2-\theta_w))} \tag{30}$$

$$V_{w1}^{2*}=\frac{2r(1+l_w)(2l_s-\delta\theta_s)}{(1-\delta)(2l_w+(2-\theta_w))} \tag{31}$$

$$V_{s0}^{2*}=r\left(\frac{2l_s}{1-\delta}+\theta_s\right) \tag{32}$$

$$V_{s1}^{2*}=r\left(\frac{2l_s}{1-\delta}\right) \tag{33}$$

$$\underline{p}_0^{2*}=\frac{r(2l_s+\theta_s(1-\delta))}{2(1+l_s)} \tag{34}$$

$$\underline{p}_1^{2*}=\frac{r(2l_s-\delta\theta_s)}{(2l_s+2-\theta_w)} \tag{35}$$

Substituting Eqs. 30–35 in the conditions $\underline{p}_{w1}<\underline{p}_1^{2*}$ and $\underline{p}_{w0}<\underline{p}_0^{2*}$ we obtain the following two conditions:

$$C3:\frac{\left(\begin{array}{c}2r(1+l_s)\Big(4(l_s-l_w)+2(1+l_w-l_s(1-\delta)-2(1+l_w)\delta)\theta_s-(1-\delta)^2\theta_s^2\Big)+\\r\Big(4(l_w-l_s(1-\delta-l_w\delta))+2(l_s-(1+l_w)(1-\delta))(1-\delta)\theta_s+(1-\delta)^2\theta_s^2\Big)\theta_w\end{array}\right)}{(2(1+l_s)(2+2l_w-\theta_s)(2+2l_s-\theta_w))}>0 \tag{36}$$

$$C4:\frac{\left(r\left(\begin{array}{l}2(1+l_s)(2l_s(2+\delta\theta_s)-\delta\theta_s(4-\theta_s+\delta\theta_s)-4l_w(1+\delta\theta_s))-\\ \left(\begin{array}{l}4+4l_s^2-4l_w+4l_s(2-l_w-\delta(1+l_w))\\ +2l_s\delta\theta_s-\delta(1-\delta)(2+2l_w-\theta_s)\theta_s\end{array}\right)\theta_w\\ +2(1+l_s)\theta_w^2\end{array}\right)\right)}{(4(1+l_s)(1+l_w)(2+2l_s-\theta_w))}>0 \quad (37)$$

Case 3

In this case $F_{s0}(r)=1$ and $F_{w1}(r)=1$. Case 3 is feasible if $\underline{p}_{w1}<\underline{p}_1^{3^*}$ and $\underline{p}_{s0}<\underline{p}_0^{3^*}$ or equivalently if conditions $C5$ and $C6$ hold.

$$V_{w0}^{3^*}=\frac{2l_wr}{1-\delta} \quad (38)$$

$$V_{w1}^{3^*}=\frac{(4(1+l_w)r(1-\delta)(2(1+l_s)l_w\delta-l_s(1-\delta)(2+2l_w-\theta_s)))}{\left((1-\delta)^2\left(4(1+l_s)(1+l_w)\delta^2-(1-\delta)^2(2l_w+2-\theta_s)(2l_s+2-\theta_w)\right)\right)} \quad (39)$$

$$V_{s0}^{3^*}=\frac{(4(1+l_s)r(2(1+l_s)l_w-2(l_s+l_w+2l_sl_w)\delta-l_w(1-\delta)\theta_w))}{\left(2(1+l_s)\left(2(1+l_w)(1-\delta)(1-2\delta)-(1-\delta)^3\theta_s\right)-(1-\delta)^3(2+2l_w-\theta_s)\theta_w\right)} \quad (40)$$

$$V_{s1}^{3^*}=\frac{2l_sr}{1-\delta} \quad (41)$$

$$\underline{p}_0^{3^*}=\frac{(2r(2l_s\delta-l_w(l_s(2-4\delta)+(1-\delta)(2-\theta_w))))}{\left(2(1+l_s)\left(2(1+l_w)(2\delta-1)+(1-\delta)^2\theta_s\right)+(1-\delta)^2(2l_w+2-\theta_s)\theta_w\right)} \quad (42)$$

$$\underline{p}_1^{3^*}=\frac{(2r(2l_w\delta+l_s(l_w(2-4\delta)+(1-\delta)(2-\theta_s))))}{\left(2(1+l_s)\left(2(1+l_w)(2\delta-1)+(1-\delta)^2\theta_s\right)+(1-\delta)^2(2l_w+2-\theta_s)\theta_w\right)} \quad (43)$$

Substituting Eqs. 38–43 in the conditions $\underline{p}_{w1}<\underline{p}_1^{3^*}$ and $\underline{p}_{s0}<\underline{p}_0^{3^*}$ we obtain the following conditions:

$$C5:\frac{\left(\begin{array}{l}2(1+l_s)r\left(4(l_s-l_w)+2(1+l_w-l_s(1-\delta)-2(1+l_w)\delta)\theta_s-(1-\delta)^2\theta_s^2\right)\\ +r\left(4(l_w-l_s(1-\delta(1+l_w)))+2(l_s-(1-\delta)(1+l_w))(1-\delta)\theta_s-(1-\delta)^2\theta_s^2\right)\theta_w\end{array}\right)}{\left(2(1+l_s)\left(2(1+l_s)\left(2(1+l_w)(2\delta-1)+(1-\delta)^2\theta_s\right)+(1-\delta)^2(2+2l_w-\theta_s)\theta_w\right)\right)}>0 \quad (44)$$

$$C6:\frac{\left(r\begin{pmatrix}8(l_w-l_s)(1+l_w)+4(l_s-l_w(1-\delta)+l_sl_w\delta)\theta_s\\+2\begin{pmatrix}2(1+l_w)(1+l_s-l_w(1-\delta)-2\delta(1+l_s))\\+(1-\delta)(1-l_w-l_s(1-\delta)+\delta)\theta_s\end{pmatrix}\theta_w\\-(1-\delta)^2(2+2l_w-\theta_s)\theta_w^2\end{pmatrix}\right)}{\left(2(1+l_w)\left(2(1+l_s)\left(2(1+l_w)(2\delta-1)+(1-\delta)^2\theta_s\right)+(1-\delta)^2(2+2l_w-\theta_s)\theta_w\right)\right)}>0 \tag{45}$$

Case 4

In this case $F_{s0}(r)=1$ and $F_{s1}(r)=1$. Case 4 is feasible if $\underline{p}_{s1}<\underline{p}_1^{4*}$ and $\underline{p}_{s0}<$ or equivalently if conditions $C7$ and $C8$ hold. V_{jk}^{4*} and $\underline{p}_k^{4*}$ are symmetric to Case 2 with labels of the weak and strong firm reversed and the labels of state 0 and state 1 reversed.

$$C7:\frac{\left(r\begin{pmatrix}8(1+l_w)(l_w-l_s)+4((1+l_s)l_w\delta-(1-l_s+l_w)(1+l_w))\theta_s\\+2(1+l_w)\theta_s^2-2\delta(2(2+2l_s-l_w)(1+l_w)-(1-l_w-l_s(1-\delta)-\delta)\theta_s)\theta_w\\+\delta(1-\delta)(2+2l_w-\theta_s)\theta_w^2\end{pmatrix}\right)}{(4(1+l_s)(1+l_w)(2+2l_w-\theta_s))}>0 \tag{46}$$

$$C8:\frac{\left(r\begin{pmatrix}8(1+l_w)(l_w-l_s)+4(l_s-l_w(1-\delta)+l_sl_w\delta)\theta_s\\+2\begin{pmatrix}2(1+l_w)(1+l_s-l_w(1-\delta)-2\delta-2l_s\delta)\\+(1-\delta)(1-l_w+l_s(1-\delta)-\delta)\theta_s\end{pmatrix}\theta_w\\-(1-\delta)^2(2+2l_w-\theta_s)\theta_w^2\end{pmatrix}\right)}{(2(1+l_w)(2+2l_w-\theta_s)(2+2l_s-\theta_w))}>0 \tag{47}$$

Numerical simulation

To initialize the model in t=1, we assume the state is s=0. We then simulate a game with one million periods for each vector of parameters. We used the same seed for the random number generator for each new vector of parameters; our results are not sensitive to this given the large number of draws. In the symmetric case ($\theta_s=\theta_w$ and $l_w=l_s$) the expected profits, prices, and promotion frequency are symmetric in each state and we use this to verify the accuracy of our numerical results. Our numerical results are accurate to at least four decimal places (i.e., >10^(−4)).

Uniqueness

To establish uniqueness of the equilibrium we need to show that $\forall n=\{1,2,3,4\}$ when conditions C_{2n-1} and C_{2n} hold, $C_{2n'-1}$ and $C_{2n'}$ cannot simultaneously hold for

$\forall n' \neq n = \{1, 2, 3, 4\}$. Because of the complexity of the expressions we are unable to analytically establish uniqueness but we are able to confirm uniqueness with extensive numerical simulations (available from authors). As an example, consider the following parameters values: $\{r=1,\ \delta=0.9,\ \theta_s=1,\ l_w=0.01,\ l_s=0.01\}$. Evaluating conditions C1–C8 at these parameter values we obtain expressions for these conditions which are a function only of θ_w. Evaluated at these parameter values we find the following:

$$C1 > 0, \forall\theta_w \in [0, 2.02) \cup (3.09, \infty] \text{ and } C1 < 0, \forall\theta_w \in (2.02, 3.09)$$

$$C2 > 0, \forall\theta_w \in [0, 78.10) \text{ and } C2 < 0, \forall\theta_w \in (78.10, \infty]$$

$$C3 < 0, \forall\theta_w \in [0, 2.02) \text{ and } C3 > 0, \forall\theta_w > 2.02$$

$$C4 < 0, \forall\theta_w \in [0, 2.02) \cup (3.09, \infty] \text{ and } C4 > 0, \theta_w \in (2.02, 3.09)$$

$$C5 < 0, \forall\theta_w < 116.78 \text{ and } C5 > 0, \forall\theta_w > 116.78$$

$$C6 > 0, \forall\theta_w \in [0, 0.011) \text{ and } C6 < 0, \forall\theta_w > 0.011$$

$$C7 < 0, \forall\theta_w \in [0, 78.10) \text{ and } C7 > 0, \forall\theta_w > 78.10$$

$$C8 < 0, \forall\theta_w < 2.02 \text{ and } C8 > 0, \forall\theta_w > 2.02$$

Table 3 specifies the unique equilibrium for different values of θ_w. For each range both conditions for a single case are satisfied and at least one condition for the remaining cases is not satisfied. This proves uniqueness. As an example, for $\theta_w \in [0,2.02)$ conditions C1>0 and C2>0, which proves that Case 1 is an equilibrium. In this range the other cases are not feasible (C3<0, C5<0, and C7<0), which proves that Case 1 is a unique equilibrium.

Additional results

Figure 6 further illustrates the average price offered by each firm in each state. In particular, the figure shows that for low values of θ_w the weak firm offers a higher price in state 0 than in state 1.

Model extension

To demonstrate the robustness of our model we analyze a game in which consumers live for three periods rather than two periods. This results in four possible states that

Table 3 Uniqueness: an illustration

Parameter region	Equilibrium
$\theta_w \in [0, 2.02)$	Case 1
$\theta_w \in (2.02, 3.09)$	Case 2
$\theta_w \in (3.09, 78.10)$	Case 1
$\theta_w \in (78.10, \infty)$	Case 4

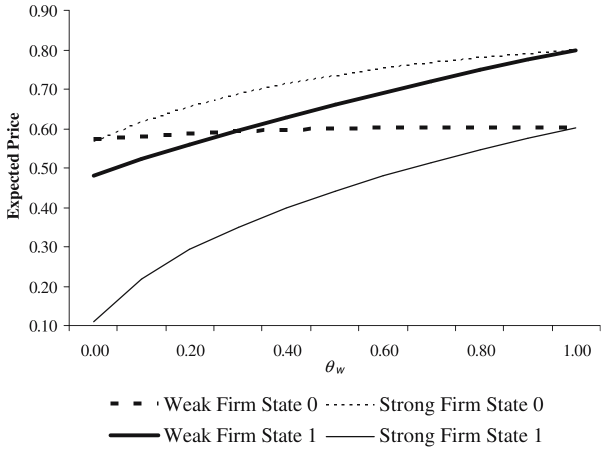

Fig. 6 Expected price in each state for l=0.01

we label k=0, 1, 2, and 3. In state 0 there are $2\theta_s$ dynamic loyal consumers for the strong firm and zero for the weak firm. In state 3 there are $2\theta_w$ dynamic loyal consumers for the weak firm and zero for the strong firm. In states 1 and 2 there are $\theta_s + \theta_w$ dynamic loyal consumers who live for one or two additional periods. We refer to a consumer as "old" if they have one period remaining and "young" if they have two periods remaining. In state 1, there are θ_s old consumers and θ_w young consumers. In state 2, there are θ_s young consumers and θ_w old consumers.

The analysis of the model is analogous to Section 2 and details are available from the authors. We specify eight continuation payoffs and then solve for eight CDFs (2

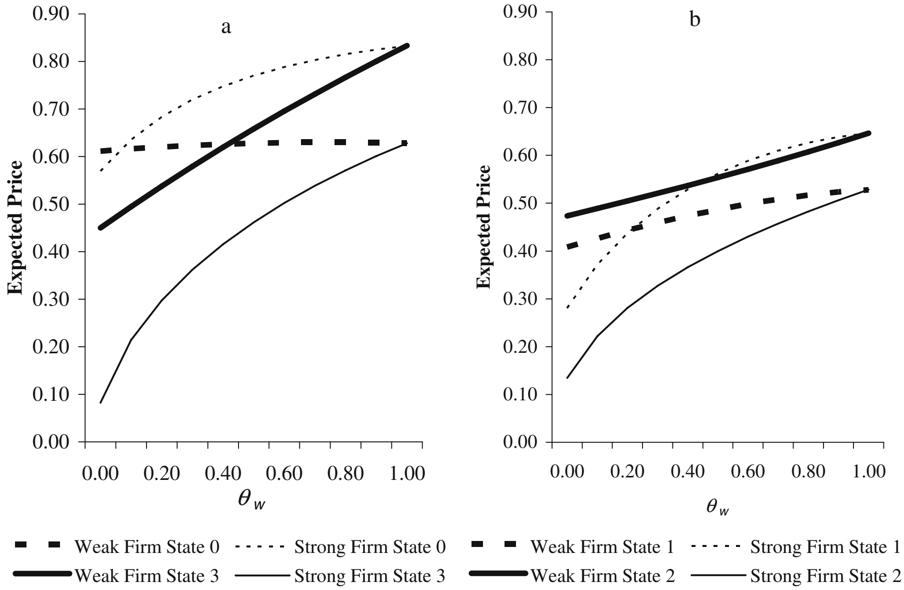

Fig. 7 Expected price in each state for 3 period OLG game

firms × 4 states) as a function of V_{jk} and $\underline{p}_k$. The 12 equilibrium values for V_{jk}^* and $\underline{p}_k^*$ require 12 equations and eight of these are given by $F_{jk}(\underline{p}_k) = 0$. There are four possible states and in each state exactly one firm has a mass point, which leads to 16 (4^2) possible solutions. In our simulations, we focus on a solution analogous to Case 1 where the strong firm has a mass point in states 0 and 2 and the weak firm has a mass point in states 1 and 3. In each of these states, the firm with the mass point offered the lowest price in the previous period and hence acquired a new group of dynamic loyal consumers.

Analysis of this model, which is available from the authors, demonstrates that our results hold in a richer, more complex model. The assumption that consumers live for two periods yields a parsimonious model and our results continue to hold when consumers live for more than two periods. To illustrate the similarity of the models, we plot the expected price in Fig. 7. In Fig. 7a we consider the two extreme states (state 0 and 3) where one firm has $2\theta_j$ dynamic loyal consumers and the other firm has zero. Given the similarity of Figs. 6 and 7a, it is not surprising that our results hold in the three period OLG model. In Fig. 7b, we analyze the intermediate states where each firm has θ_j dynamic loyal consumers. We find that firms offer low prices when they have θ_j young loyal consumers and zero old loyal consumers to build their loyal base of consumers. If successful at acquiring switching consumers, firms then transition to the state where they have $2\theta_j$ dynamic loyal consumers. At this point, they “harvest” and offer high prices to their loyal consumers.

In contrast, when a firm has zero young loyal consumers and θ_j old loyal consumers should it offer high prices (“harvest”) or offer low prices (“invest”)? We find that the strong firm’s expected price is higher in state 1 (θ_s old loyal consumers) compared to state 2 (zero old loyal consumers). Similarly, the weak firm offers higher prices in state 2 (θ_w old loyal consumers) compared to state 1 (zero old loyal consumers). This indicates that the incentive to harvest old loyal consumers dominates the incentive to build a loyal base of consumers.

Overall, the extension demonstrates the robustness of our results. It also suggests that firms will offer high prices in multiple periods to extract profits on their loyal base of consumers. When the loyal base of consumers is sufficiently low, the firm will offer a series of deep promotions to establish a loyal base of consumers. Once the loyal base of consumers is established the cycle repeats.

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