Integrating Models of Price Discrimination †

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Abstract

The marketing, economics, and operations management literatures have recognized many ways in which a firm can price discriminate. The question of how to price discriminate (e.g. how to price a product line) has received considerable attention but most of this work has assumed that price discrimination is optimal. However, the question of whether to price discriminate is also important as firms often forgo this option. For example, a firm may offer only a single product or version, not engage in intertemporal pricing, offer a single customer service queue, not offer advance purchase discounts, or not offer coupons.

In this paper, we develop a very general model of monopoly price discrimination that yields two important contributions. First, we derive a single intuitive condition that determines whether price discrimination is profitable. The condition relates to changes in the social surplus, which is the difference between consumer benefits and firm costs, when a consumer upgrades from a low quality product to a high quality product. We show that price discrimination is profitable if and only if the percentage change in social surplus from product upgrades is increasing in consumers' willingness to pay. We refer to this as an *increasing percentage differences condition*. Second, while applications of second-degree price discrimination have much in common, few papers attempt to unify them in a single model. Using our framework, we both recover and generalize many results from applications of price discrimination in the marketing, economics, and operations management literatures. Our paper unifies these seemingly disparate applications of price discrimination by explicitly recognizing their common elements.

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1. Introduction

The marketing, economics, and operations management literatures have recognized many ways in which a firm can price discriminate. Examples include product line pricing (e.g. BMW 3 and 5 series), damaged goods (e.g., Intel 486 SX and DX), intertemporal pricing (e.g. Talbot's semi-annual sale), service queues (e.g. priority customer support), advance purchase discounts (e.g. airline, rail and hotel tickets), and coupons (e.g. FSI inserts). When sellers use these strategies, it is common to offer a menu of choices at different prices and allow consumers to self-select into the offer of their choice (i.e., second-degree price discrimination). For example, a customer who shops at Talbot's has the option of purchasing today at the regular price or waiting for the semi-annual sale. In this instance, waiting may be less attractive due to delayed consumption or lack of future product availability.

Given the numerous means available to price discriminate, a fundamental question a firm faces is whether and how to optimally price discriminate. Within specific applications of price discrimination, the question of how to price discriminate has received considerable attention (e.g. how to price a product line) but much less attention has been paid to the question of whether to price discriminate. Yet, there are numerous instances where a firm does not offer multiple products, does not use intertemporal pricing, offers a single service queue, does not offer advance purchase discounts, or does not offer coupons.

One explanation for why this topic has received less scrutiny is that applications of price discrimination have been largely developed independently. For example, the question of whether to price discriminate has received some attention in the economics literature (Salant 1989, Stokey 1979) but little recognition in other fields or applications. This lack of integration has led to a fragmented view of price discrimination with each application being developed in isolation.

In this paper, we develop a very general model of product line pricing that integrates numerous applications of price discrimination. For a firm the questions of whether and how to price discriminate are both important, but this paper will emphasize whether a firm should price discriminate. Importantly, we will consider situations where it is feasible to price discriminate but a firm chooses to forgo this option because it is not profitable. Surprisingly, we show that a single, elegant and intuitive condition characterizes whether price discrimination is optimal. Specifically, we show that when a continuum of product qualities is feasible, price discrimination is profitable if and only if the percentage change in social surplus from product upgrades is increasing in consumers' willingness to pay. We refer to this as an *increasing percentage differences* condition.

The heart of this paper, however, is our application of this condition to the seemingly unrelated strands of literature on price discrimination in marketing, economics, and operations management. We develop our model around a firm offering multiple product qualities at different prices. However, the application to product line pricing is purely for ease of exposition as the model seamlessly extends to other forms of price discrimination. For example, purchasing with a coupon affords a customer a lower price but the hassle cost of redeeming a coupon may result in disutility for some consumers. In this sense, a coupon is a low quality / low price product but paying the full price is a high quality / high price product. We illustrate how our model relates to applications of price discrimination such as product line pricing, versioning information goods, damaged goods, intertemporal pricing, service queues, advance purchase discounts and coupons. By applying our framework, we are able to both recover and generalize many results from these literatures. In sum, our paper unifies numerous applications of price discrimination by explicitly recognizing their common elements.

Our analysis explicitly recognizes the role that quality constraints play in price discrimination. Upper bounds on quality are often implicit in price discrimination models because they are rather natural in many applications for at least two reasons. First, firms are endowed with a given product technology or service level, which bounds the maximum level of quality. For example, past R&D investments limit the fastest processor that Intel can offer. Second, and perhaps more importantly, the technologies available for lowering product quality (e.g., coupons, travel restrictions, disabling product features, and delaying delivery times) are often much richer and more diverse than the technologies available for raising quality.

Before we proceed, it is important to note that we take only one of many potential views on what constitutes price discrimination. To explain our view, we first ask, "What would a firm offer if it could directly, or *perfectly*, segment its customers?" Our model assumptions guarantee that in this situation a firm would offer all customers the same product or service, but at different prices. The ability to perfectly segment its customers means that there is no leakage – customers are unable to purchase at lower prices intended for other customers. We then ask: "What would a firm offer if it could only indirectly, or *imperfectly,* segment its customers?" In this case, the price the firm can charge to each segment is constrained by its customers' willingness to buy the product that it is offering for its other segments. In this case, if the firm chooses to offer multiple products we interpret this as price discrimination. This definition of price discrimination is appealing because it corresponds to asking whether the solution to the monopolist's problem is separating or pooling. However this definition is not always appropriate. For example, when a firm sells multiple units to consumers, uniform pricing is not considered price discrimination even when it induces separation.

The remainder of the paper is organized as follows. In Section 2, we offer a simple example with two types of consumers and two exogenously given product qualities to illustrate the intuition of our *increasing percentage differences* condition. We extend this example to a monopolist selecting two product qualities subject to an upper bound on quality. Section 3 considers the more general problem in which a monopolist sells to a continuum of consumers. Section 4 analyzes the welfare properties of price discrimination. Section 5, which is the heart of our paper, relates our *increasing percentage differences* condition to the literatures on product line pricing, versioning information goods, damaged goods, intertemporal price discrimination, service queues, advance purchase discounts and coupons. A brief discussion concludes the paper.

2. Two Consumer Types

Consider a monopolist who can sell either or both of two products, one with exogenously given quality $\underline{q}$ and another with exogenously given quality $\overline{q}$, to two distinct groups of consumers, n_H high types, denoted θ_H, and n_L low types, denoted θ_L. The monopolist cannot directly distinguish between consumer types, but can sell a different product to each type as long as the purchase decision is individually rational and incentive compatible. Consumers have unit demands and maximize their consumer surplus, $V(q,\theta_L)-p(q)$ and $V(q,\theta_H)-p(q)$ respectively. The firm has unit costs of production, $c(q)$, that vary with product quality.

We assume $V(\overline{q},\theta_H)>V(\overline{q},\theta_L)>c(\overline{q})$ and $V(\underline{q},\theta_H)>V(\underline{q},\theta_L)>c(\underline{q})$. We also assume $V(\overline{q},\theta_H)-V(\overline{q},\theta_L)>V(\underline{q},\theta_H)-V(\underline{q},\theta_L)$, or equivalently that the consumers who are willing to pay the most for a low quality product are also the consumers that are willing to pay the most to increase the quality from low to high. This is the well-known single-crossing property, which guarantees that price discrimination is feasible. We assume that $V(\overline{q},\theta_L)-V(\underline{q},\theta_L)>c(\overline{q})-c(\underline{q})$. This implies that if the firm were serving *only* the low types it would find it more profitable to sell them high quality. So the high-quality product is efficient for both consumer types.

Figure 1 depicts an example of this problem in which $V(q,\theta)$ and $c(q)$ are continuous in q and $V_q(\overline{q},\theta_L)>c_q(\overline{q})$. In addition, we will assume for this example that $V(0,\theta_H)=V(0,\theta_L)=c(0)=0$. Our results do not depend on these assumptions; their purpose is to make it easier to use a graph to illustrate the solution to our problem.

If the firm served all consumers with a single quality $\overline{q}$ at a single price, it would earn $A + C$ on each sale. The high-type consumers would capture surplus $D + B$ while the low-type consumers would capture 0 surplus. If, instead, the firm chose to offer both a high and a low quality product, it would loose A (earn C) on each sale to a low type but gain B (earn $C + A + B$) on each sale to a high type. So the firm's profits would increase as long as $Bn_H > An_L$, or $A/(A+B)<n_H/(n_L+n_H)$.

If the firm served only the high-type consumers it would be able to earn $A + B + C + D$ on each sale. If instead the firm chose to offer both high and low quality, its profits would increase only if the profit earned on the new low-type consumers covered the lower margin on high-type customers. That is $Cn_L > Dn_H$, or $C/(C+D) > n_H/(n_L+n_H)$. Hence the firm is willing to offer both product qualities if and only if $C/(C+D) > n_H/(n_L+n_H) > A/(A+B)$, or $(A+B)/(C+D) > A/C$, or

$$\frac{V(\bar{q},\theta_H)-c(\bar{q})}{V(\underline{q},\theta_H)-c(\underline{q})} > \frac{V(\bar{q},\theta_L)-c(\bar{q})}{V(\underline{q},\theta_L)-c(\underline{q})}. \tag{1}$$

This condition implies that both products are offered only if the ratio of the high type's total surplus to the low type's total surplus is increasing in quality. Equivalently both products are offered only if the marginal surplus from an increase in quality as a percentage of the total surplus is increasing in the consumer type. We call this condition *increasing percentage differences.* There exist values of n_L and n_H such that offering both products is optimal only if this condition is met.

A function $f(x,y)$ satisfies *increasing percentage differences* if $(f(x_1,y)-f(x_2,y))/f(x_2,y)$ is increasing in *y* for all $x_1 > x_2$, or equivalently either $f(x_1,y)/f(x_2,y)$ or $\ln f(x_1,y)-\ln f(x_2,y)$ is increasing in *y* for all $x_1 > x_2$. The latter is typically referred to as log supermodularity, and we will use this term extensively throughout the paper. Also note that if $f(x,y)$ is continuously differentiable then it is log supermodular if and only if $f_{xy}f - f_x f_y > 0$. Some additional properties of log supermodular functions are included in the Appendix.

This example is easily generalized to allow the firm to choose its product quality optimally. Suppose there are n_L buyers of type θ_L and n_H buyers of type θ_H. Buyer θ's consumer surplus from purchasing a product of quality *q* at price *t* is $V(q,\theta)-t$, and buyers purchase the product that gives them the greatest consumer surplus. Assume the firm's cost for selling *n* units of quality *q* is $nc(q)$. We assume that *V* and *c* are continuously differentiable with respect to *q*, that $V_q(q,\theta) > 0$, and that $V(q,\theta)$ and $V_q(q,\theta)$ are increasing in θ. We define

$S(q,\theta)=V(q,\theta)-c(q)$, so $S(q,\theta)$ is the total surplus from selling a product of quality q to a single consumer of type θ. Quality is constrained to be less than or equal to one. We assume $V(0,\theta)-c(0)\le 0,\forall\theta$ and $V(q,\theta)-c(q)>0$ for all θ and some $q\in(0,1)$, which implies that whether the firm sells one or two products, it will always choose quality to be strictly positive. Finally, we assume that $V_q(1,\theta)-c_q(1)\ge 0$.

Proposition 1: *Let N^* denote the open interval* $\left(\frac{V_q(1,\theta_L)-c_q(1)}{V_q(1,\theta_H)-c_q(1)},\frac{V(\tilde{q},\theta_L)-c(\tilde{q})}{V(\tilde{q},\theta_H)-c(\tilde{q})}\right)$ *where*

$$\tilde{q}=\arg\max_{q\le 1} n_L S(q,\theta_L)+n_H S(1,\theta_H)-n_H\left(V(q,\theta_H)-V(q,\theta_L)\right). \tag{2}$$

a) N^* is non-empty and the firm will offer multiple qualities only if $V(q,\theta)-c(q)$ is log supermodular.

b) If $V(q,\theta)-c(q)$ is log supermodular, then N^* is non-empty and the firm will offer multiple qualities if and only if $n_H/(n_H+n_L)\in N^*$.

Proof: See Appendix

Proposition 1 shows a monopolist will price discriminate only if the ratio of the marginal social value of quality to the total social value of quality is increasing in the consumer's type, θ. Proposition 1 also characterizes the distributional conditions that, along with log supermodularity of the surplus function, are sufficient for price discrimination to be profitable. Note that the expression $\tilde{q}$ characterizes the firm's optimal product quality for the low-type consumers conditional on selling to both low and high-type consumers. So establishing that price discrimination is optimal is equivalent to establishing that the firm sells a product $\tilde{q}<1$ to low-type consumers.

This result is perfectly consistent with a two-type version of Mussa and Rosen (1978). In particular, if the quality constraint does not strictly bind, then $q^*(\theta_H)=1$ implies $V_q(1,\theta_H)-c_q(1)=0$. It follows that $V(q,\theta)-c(q)$ is log supermodular at $q=1$. In the absence of a quality constraint, Proposition 1 predicts that there always exists a distribution of consumer types such that the firm will price discriminate. Also, $V_q(1,\theta_H)-c_q(1)=0$ implies

$V_q(1,\theta_L)-c_q(1)<0$, so the minimum value in N^* is negative or equivalently $[0,\hat{n}]\subset N^*$ for some $\hat{n}\in(0,1)$. So price discrimination is always profitable if n_L is sufficiently large and price discrimination is unprofitable if and only if n_H is sufficiently large.

Proposition 1 implies that the firm never sells the same product to both types when quality is unconstrained. The firm may choose to sell only to the high types, but if it chooses to sell to both the high and the low types, then it offers each type a different product. Whether or not the firm sells to both types of consumers depends on the distribution of types, but whether or not it price discriminates conditional on selling to both types does not. However, as can be seen in the two-product example we first examined, whether or not the firm price discriminates does depend on the distribution of types when the firm is choosing between a finite set of exogenous products.

Proposition 1's value is most evident when $V_q(1,\theta)-c_q(1)>0$, i.e., the quality constraint strictly binds. In this case, price discrimination is feasible, but it is optimal only if the surplus function is log supermodular *and* if the distribution of types satisfies $n_H/(n_H+n_L)\in N^*$.

We will return to this model in Section 4, in which we discuss the welfare impact of price discrimination, but first we generalize the model to a continuum of consumer types.

3. The Continuous-Consumer-Type Model

In this section we analyze a general model in which there are a continuum of heterogeneous buyers. Buyers' types, θ, are distributed with probability distribution $f(\theta)$ and cumulative distribution $F(\theta)$ on the interval $[\underline{\theta},\bar{\theta}]$. We assume that $J(\theta)=(1-F(\theta))/f(\theta)$ is monotonically decreasing, that is, the distribution F has a monotone hazard rate.

Consumers maximize their consumer surplus, equal to their strictly positive utility, $V(q,\theta)$, less the price, $p(q)$. We assume that V satisfies $V_q>0$, $V_\theta>0$, and the Spence-Mirlees condition, $V_{q\theta}>0$ for all $q\in[0,1]$ and $\theta\in[\underline{\theta},\bar{\theta}]$. The firm can produce any number of products of any quality, q, subject to the constraint that $q\le 1$. The firm's unit cost of production is $c(q)$.

Let $S(q,\theta)=V(q,\theta)-c(q)$ denote the total surplus function. Throughout, we assume quality is constrained to less than 1 for all θ. We assume that the solution, $q^*(\theta)$, to

$$\max_{\theta_L, q(\theta)} \int_{\theta_L}^{\bar{\theta}} S\left(q(\theta),\theta\right) dF(\theta), \text{ subject to } q(\theta) \leq 1, \tag{3}$$

is a nondecreasing function and that $\theta_L^* > \underline{\theta}$. The later implies that the firm will never chose to sell to all of the consumers. We also assume that $S(0,\theta) - c(0) < 0$ for all θ, so that the quality the firm chooses is always strictly positive.

The firm chooses $p(q)$ to maximize its profits. Without loss of generality we assume that the firm uses a direct revelation mechanism $\{q(\theta), p(\theta)\}$. Note that because of the incentive constraints, the firm can only implement mechanisms that serve every consumer in some interval $\left[\theta_L, \bar{\theta}\right]$, so we can write the firm's problem as:

$$\max_{\theta_L, q(\theta), p(\theta)} \int_{\theta_L}^{\bar{\theta}} \left[p(\theta) - c\left(q(\theta)\right)\right] dF(\theta) \tag{4}$$

subject to incentive compatibility constraints, $q(\theta) \in \arg\max_{q \leq 1} V(q,\theta) - p(q), \forall\theta$, participation constraints, $v(\theta) \equiv \max_q V(q,\theta) - p(q) \geq 0, \forall\theta \geq \theta_L$, and the technology constraint, $q(\theta) \leq 1, \forall\theta$.

We simplify the firm's problem as follows:

Lemma 1: *The firm's problem can be written as*

$$\max_{\theta_L, q(\theta)} \int_{\theta_L}^{\bar{\theta}} \left[S\left(q(\theta),\theta\right) - J(\theta) S_\theta\left(q(\theta),\theta\right)\right] dF(\theta) \tag{5}$$

subject to the constraints that $q(\theta)$ *is nondecreasing and* $q(\theta) \leq 1$ *for all* θ.

Proof: *The proof is standard and is included in the Technical Appendix.*[1]

Let $X(\theta) = S\left(q(\theta),\theta\right) - J(\theta) S_\theta\left(q(\theta),\theta\right)$ denote the integrand of the firm's objective function. This is often referred to as the firm's virtual profit function. The second term, $J(\theta) S_\theta\left(q(\theta),\theta\right)$, is often referred to as information rent. Ignoring the constraint that $q(\theta)$ is nondecreasing, the firm's problem can be solved by pointwise constrained maximization of *X*. In this case, $q(\theta)$ is implicitly defined by

$$X_q(\theta) = S_q\left(q(\theta),\theta\right) - J(\theta) S_{q\theta}\left(q(\theta),\theta\right) - \lambda(\theta) = 0, \forall\theta \tag{6}$$

[1] See Fudenberg and Tirole (1991), Myerson (1991), and especially Hermalin (2006).

where the Lagrangian multiplier, $\lambda(\theta)$, is strictly positive only if $q(\theta)=1$ and $\lambda(\theta)=0$ otherwise; and θ_L is implicitly defined by

$$X(\theta_L)=S(q(\theta_L),\theta_L)-J(\theta_L)S_\theta(q(\theta_L),\theta_L)=0 \tag{7}$$

Using the implicit function theorem, (6) implies that $q(\theta)$ is either strictly increasing or equal to 1 for all θ if $X_{qq}<0$ and $X_{q\theta}>0$. Conditions $X_{qq}<0$ and $X_{q\theta}>0$ both hold given our stated assumptions on *S* and *J* if in addition we assume that $S_{qq}<0$ and $S_{qq\theta}>0$. Henceforth we assume that $X_{qq}<0$ and $X_{q\theta}>0$. It follows that ignoring the constraint that $q(\theta)$ is non-decreasing is without loss of generality. In other words, $q(\theta)$ is defined by (6) and (7).

Since $q(\theta)\geq q(\theta_L)$ and $X_{q\theta}>0$, (7) implies

$$S(q(\theta),\theta)-J(\theta)S_\theta(q(\theta),\theta)>0,\forall\theta\in(\theta_L,\bar{\theta}]. \tag{8}$$

Equations (6) and (8) imply that a necessary condition for $q(\theta)<1$ for some $\theta\in(\theta_L,\bar{\theta})$ is

$$\frac{S_{q\theta}(q(\theta),\theta)}{S_q(q(\theta),\theta)}=\frac{1}{J(\theta)}>\frac{S_\theta(q(\theta),\theta)}{S(q(\theta),\theta)}, \tag{9}$$

or equivalently, that $S(q,\theta)$ is log supermodularity at $(q(\theta),\theta)$.

We formally state this result as follows:

Proposition 2: *If $S(q,\theta)$ is log submodular for all $q<1$ and all θ the firm's optimal strategy is to produce a single product, i.e., $q(\theta)=1$ for all θ.*

It also follows that $q(\theta)<1$ for some θ if $S(q,\theta)$ is log supermodular in a neighborhood of $(1,\theta)$. Equations (6) and (7) imply

$$\frac{S_{q\theta}(q(\theta_L),\theta_L)}{S_q(q(\theta_L),\theta_L)-\lambda(\theta_L)}=\frac{1}{J(\theta_L)}=\frac{S_\theta(q(\theta_L),\theta_L)}{S(q(\theta_L),\theta_L)},\text{ or} \tag{10}$$

$$S_{q\theta}(q(\theta_L),\theta_L)S(q(\theta_L),\theta_L)-S_q(q(\theta_L),\theta_L)S_\theta(q(\theta_L),\theta_L)=-\lambda(\theta_L)S_\theta(q(\theta_L),\theta_L). \tag{11}$$

The right hand side of (11) is nonpositive, so if $S(q,\theta)$ is log supermodular at $(1,\theta_L)$, (11) implies $q(\theta_L)<1$. That is, the firm produces multiple products. We formally state this result as follows:

Proposition 3: *If $S(q,\theta)$ is log supermodular at $(1,\theta)$ for all θ then the firm's optimal strategy is to produce multiple products, i.e., $q(\theta)$ is strictly increasing for some θ.*

Propositions 2 and 3 are perfectly consistent with Mussa and Rosen (1978), Moorthy (1984), and related papers on firms' product line decision without a quality constraint. First, suppose that the quality constraint $q \leq 1$ is never binding, that is that $S_q(1,\theta) \leq 0$ for all θ. In this case $S_q(1,\theta)S_\theta(1,\theta) \leq 0$, so S is guaranteed to be log supermodular at $(1,\theta)$ for all θ. That is, the firm always produces multiple products. The interesting case is when $q \leq 1$ is strictly binding. Suppose that $S_q(1,\theta) > 0$ for all θ. Then $S_q(1,\theta)S_\theta(1,\theta) > 0$ and S is log supermodular only if $S_{q\theta}$ is sufficiently large.

It is also worth emphasizing that for the continuous type model, whether or not price discrimination is profitable is independent of the distribution of consumer types. Of course, we made assumptions on the distribution (e.g., monotone hazard rate) to guarantee that price discrimination was feasible. But, this is still noteworthy because distributional assumptions *were* an important determinant of profitability in the finite type model.

4. Welfare

In this section, we focus on a specific aspect of welfare: does price discrimination lead to a Pareto improvement? Both Deneckere and McAfee (1996) and Anderson and Song (2004) showed that indirect, or second-degree, price discrimination could lead to a Pareto improvement. A necessary condition for indirect price discrimination to be Pareto improving is that the firm serves more buyers than it would have otherwise. In serving more buyers, the firm will need to sell at a lower price, but by distorting quality, the firm can lower its price without passing on the same price increase to the buyers it would have otherwise served. Nevertheless, incentive compatibility constraints may force the firm to lower its price to all of its buyers.

We first characterize necessary and sufficient conditions for a Pareto improvement to occur when there are two types of buyers. A Pareto improvement occurs when both types of buyers are served when price discrimination is allowed, but only the high type is served when price discrimination is banned. These conditions are summarized in Proposition 4 and illustrated graphically in Figure 2.

Proposition 4: *If there are two types of consumers,* $V(q,\theta)-c(q)$ *is log supermodular in a neighborhood of* $q=1$*,* $n_H/(n_H+n_L)\in N^*$*, and* $n_H/(n_H+n_L)>(V(1,\theta_L)-c(1))/(V(1,\theta_H)-c(1))$*, then offering multiple qualities results in a Pareto improvement.*

If the fraction of high-type consumers is sufficiently high then in the absence of the ability to price discriminate the firm will sell to only the high types. But when the firm is able to price discriminate, the high-type consumers are better off because they capture some surplus due to the incentive compatibility constraint. In contrast, if the fraction of high-type consumers is small, then in the absence of the ability to price discriminate the firm will sell to both consumer types. In this case, when the firm is able to price discriminate, the high-type consumers are worse off because they face a higher price for the same quality.

In the continuous type case, if a firm does not price discriminate it offers a single quality ($q^* \leq 1$) to a segment of consumers ($\theta \geq \theta^*$) at the same price, p^*. If fewer consumers are served under price discrimination, then there cannot be a Pareto improvement. But, if the market expands and more customers are served, then there may be a Pareto improvement. Thus, analogous to the two-type case, market expansion is a necessary condition for a Pareto improvement but it is not sufficient. It is clear that all consumer with types $\theta \leq \theta^*$ are weakly better off under price discrimination. But, when are consumers with types $\theta > \theta^*$ better off?

For these consumers, there are two competing effects. To illustrate, suppose all consumers with types $\theta > \theta^*$ continue to receive quality q^* when the firm is able to price discriminate. Since the price discriminating firm is selling to some consumers with types $\theta < \theta^*$, the firm must be offering all the consumers with types above θ^* a lower price to satisfy their incentive compatibility constraints. This implies that all consumers have strictly higher surplus. However, selling quality q^* to these consumers may no longer be optimal. The consumer with type θ^* may now be sold lower quality $q < q^*$. While the consumer with type θ^* is always weakly better off (because he receives zero surplus under no price discrimination), offering lower quality to this consumer enables the firm to charge higher prices to types $\theta > \theta^*$, which may make them worse off. While this price-quality trade-off applies to all $\theta > \theta^*$, as long as the highest type consumer receives more surplus, then every consumer is strictly better off.

For continuous distributions that approximate a two-type distribution, we prove that there is a Pareto improvement. However these distribution functions do not satisfy the monotone hazard rate property (MHR). We have also analyzed a number of tractable distributions functions that do satisfy MHR assumption, but none of the examples we considered had the property that allowing price discrimination lead to a Pareto improvement. We conclude that while a Pareto improvement is feasible in the continuous type case it is also less likely.

5. Applications

While our results are developed in the context of product line pricing, they readily generalize to other types of indirect, or second-degree, price discrimination. In this section, we link our model to previous research on product line pricing, versioning information goods, damaged goods, intertemporal pricing, priority queuing systems, advance purchase discounts, and coupons (rebates). We show that specific results from these applications are both replicated and generalized by our results. We also use these applications to emphasize several key intuitions from our increasing percentage differences condition.

A. Product Line and Versioning

Our model is directly applicable to an extensive literature in marketing on product line design. Much of this literature has focused on the question of how to optimally price a product line (Reibstein and Gatignon, 1984, Dobson and Kalish, 1988, Moorthy, 1984, and Zenor, 1994) or how to develop an optimal quantity discount schedule (Oren, Smith and Wilson, 1984). These papers begin with the premise that offering a product line or quantity discounts are optimal and then tackle the question of pricing. In contrast, our paper seeks to address the question of whether to offer a single product or a product line.

While our model is formulated around a firm offering products of varying qualities it can readily be interpreted as a firm offering products with varying quantities. Thus, our model addresses whether a firm should offer all units at a constant price p or offer a menu of q units that are priced at $p(q)$. Our model does not address the question of whether a firm should offer a menu of non-linear contracts (e.g. a menu of two-part tariffs).

We can apply our results to interpret work by Villas-Boas (1998) on product line design in a channel. Villas-Boas considers a vertical channel with a single manufacturer and a single retailer and investigates distortions in the product line offering. He shows that when two product qualities are offered, a manufacturer decreases the quality of the low-end product, even more than a vertically integrated firm would do, in order to induce the retailer to offer both products.

Note that for a given set of manufacturer product qualities, $\bar{q}$ and $\underline{q}$, and manufacturer prices, $\bar{\omega}$ and $\underline{\omega}$, the problem faced by the retailer is identical to the problem faced by the firm in our model. The retailer faces consumers with valuations $V\left(q,\theta\right)$ where $q \in (\bar{q}, \underline{q})$ and $\theta \in (\bar{\theta}, \underline{\theta})$. There are γ consumers of type $\underline{\theta}$ and $1-\gamma$ consumers of type $\bar{\theta}$.

In Proposition 1, Villas-Boas shows that if a naïve manufacturer offers a product line that is optimal for the vertically integrated channel then the retailer does not adopt both qualities. In this case, the manufacturer offers product $\bar{q}$ at price $\bar{\omega} = V\left(\bar{q},\theta_H\right) - V\left(\underline{q},\theta_H\right) + V\left(\underline{q},\theta_L\right)$ and product $\underline{q}$ at price $\underline{\omega} = V\left(\underline{q},\theta_L\right)$. This implies that $V(q,\theta) - \omega$ is log submodular, which is readily seen from equation (1). The denominator on the right hand side of equation (1) is zero and therefore the surplus function is always log submodular.

In Proposition 2, Villas-Boas characterizes conditions for a retailer to adopt both manufacturer products. He correctly claims that a retailer will adopt both products if the difference in manufacturer prices for the low and high quality products is not too large and if the manufacturer prices are not too high. It is straightforward to show that conditions (3) and (4) in

Villas-Boas are equivalent to the conditions $Bn_H > An_L$ and $Cn_L > Dn_H$ in our two consumer-type example. While the result is the same, we believe that our increasing percentage differences interpretation is more intuitive and more general.

When a manufacturer optimizes its product line to encourage a retailer to price discriminate, then the manufacturer's problem is to choose products and prices subject to the constraint that both will be adopted, or equivalently subject to the constraint that they lead to a log supermodular surplus function for the retailer. Villas-Boas shows that the optimal prices are:

$$\underline{\omega} = V\left(\bar{q},\theta_H\right) - \frac{V\left(\underline{q},\theta_H\right) - V\left(\underline{q},\theta_L\right)}{\gamma}, \text{ and}$$

$$\bar{\omega} = \frac{V\left(\underline{q},\theta_L\right)}{\gamma} - \frac{1-\gamma}{\gamma} V\left(\underline{q},\theta_H\right).$$

Substitution of these values into equation (1) and simplification verifies that the surplus function is log supermodular at these qualities and wholesale prices. This leads to the following corollary:

Corrollary A: *In a vertical channel with one manufacturer and one retailer selling two products, the manufacturer's qualities,* $\bar{q}$ *and* $\underline{q}$*, and wholesale prices,* $\bar{\omega}$ *and* $\underline{\omega}$*, must satisfy*

$$\frac{V\left(\bar{q},\theta_H\right) - \bar{\omega}}{V\left(\underline{q},\theta_H\right) - \underline{\omega}} > \frac{V\left(\bar{q},\theta_L\right) - \bar{\omega}}{V\left(\underline{q},\theta_L\right) - \underline{\omega}}$$

for a retailer to offer the full product line.

Our model also sheds light on why firms that sell information goods typically offer a product line. Information goods is a term used to describe goods like software, books, music, newspapers and magazines, which have high fixed costs of production and small or negligible variable costs. The practice of selling multiple versions of information goods (i.e., a product line) has been described informally by Shapiro and Varian (1998) and more formally by Varian (1995 & 2001) and Bhargava and Choudhary (2001b, 2004).

One key intuition from our model is that a decrease in marginal costs increases the likelihood of satisfying the increasing percentage differences condition. Thus, a firm that faces lower marginal costs is more likely to price discriminate. This may in part explain why price discrimination with information goods has received considerable attention.

However, it is not always optimal for a information good seller to price discriminate. For example, if $V(\theta,q)=\theta q$ and costs are zero then it is never optimal to price discriminate. It should be clear that such a utility function does not satisfy our increasing percentage differences condition. For sellers facing zero marginal costs, our model specifies the exact properties of the utility function that determine whether it is optimal to offer multiple versions of a good or service. This extends work by Bhargava and Choudhary (2001b), who provide a necessary condition for versioning when a firm faces zero marginal cost.

Finally, our analysis extends work by Johnson and Myatt (2003) who examine the issue of fighting brands in a product line. The authors show that an *increasing percentage differences* condition is necessary for multiple products to be optimal. Our analysis extends their work in two ways. First, Johnson and Myatt consider an exogenous and finite product space while we endogenize quality subject to a constraint. Second, we prove that the increasing percentage differences condition is both necessary and sufficient.

B. Intertemporal Price Discrimination

Another common type of price discrimination is intertemporal discounts. Firms can charge higher prices to their less patient customers and lower prices to their more patient customers simply by lowering their price over time. A seminal paper in this literature is Stokey (1979) who considers a monopolist with unit cost of production $k(t)=\kappa\delta^t$ selling to consumers with utility functions $U(\theta,t)=\theta\delta^t$. These assumptions imply that firm cost is independent of time except for the time value of money. A well-known result from this model is that intertemporal price discrimination is never optimal. We now show that this important result follows immediately from Proposition 2.

A monopolist chooses a menu of prices paid at time 0 and delivery times, subject to the constraint that $t \geq 0$, to maximize profits. Similar to Salant (1989), we use a change of variables, $q = \delta^t$, so that $V(\theta, q) = \theta q$ and costs $c(q) = cq$. With this transformation, the firm's problem is to choose the profit-maximizing menu of prices and qualities subject to the constraint that $q \leq 1$. Clearly $V(\theta, q) - c(q) = \theta q - cq$ is not log supermodular and $q = 1$ is the optimal quality for all θ. So by Proposition 2, intertemporal price discrimination is never optimal, even though it is clearly feasible.

Salant (1989) sought to explain the apparently contradictory findings that product line price discrimination is always optimal (Mussa and Rosen, 1978) and that intertemporal price discrimination is not optimal (Stokey, 1979). Salant's was the first paper to emphasize that upper bounds on quality may cause firms to forgo price discrimination.

Salant (1989) made it easier to see that intertemporal price discrimination is optimal with more general cost functions, such as $k(t) = \kappa(t)\delta^t$. After a change of variables, this implies $c(q) = \kappa(\log q / \log \delta) q$. The surplus function, $V(\theta, q) - c(q) = \theta q - c(q)$, is log supermodular if and onl if $c_q(q) > c(q)/q$. So if the marginal cost of quality is positive and greater than the average cost of quality then intertemporal price discrimination is profitable.[2]

Salant showed that $c'(q) > c(q)/q$ was necessary and that

$$\frac{\theta_L - c_q(0)}{\theta_H - c_q(0)} > \frac{n_H}{n_H + n_L} > \frac{\theta_L - c_q(1)}{\theta_H - c_q(1)} \tag{12}$$

was sufficient for intertemporal price discrimination. However, Proposition 1 implies the following corollary, which is more general:

Corollary B: *If $V(\theta, q) = \theta q$ and if $c_q(q) > c(q)/q$ for all $q \in (0,1]$, then offering multiple products is optimal if and only if n_L and n_H satisfy*

[2] Johnson and Myatt (2003) have a related result about the product range of a multiproduct monopolist.

$$\frac{\theta_L\tilde{q} - c(\tilde{q})}{\theta_H\tilde{q} - c(\tilde{q})} > \frac{n_H}{n_H + n_L} > \frac{\theta_L - c_q(1)}{\theta_H - c_q(1)}, \tag{13}$$

where $\tilde{q}$ is given by (2).

Examining (12) and (13), the lower bounds are the same, but the upper bound in (13) is strictly greater than the upper bound in (12). Thus, we both replicate and generalize Salant's previous findings. Further, while (12) is implied by Salant's analysis, he does not formally state his sufficient condition in terms of the fraction of high types in the market.

Using Propositions 2 and 3, we can also generalize Salant's results for discrete types to a market with a continuum of consumer types[3]. First, the following is clearly a corollary of Propositions 2 and 3:

Corollary C: *If $V(q,\theta) = \theta q$ and $q^*(\theta) = 1$ for all θ, then offering multiple products is optimal if $c_q(q) > c(q)/q$ for all $q \in [\hat{q},1]$, $0 \le \hat{q} < 1$, and offering a single product is optimal if $c_q(q) \le c(q)/q$ for all $q \in [\hat{q},1]$, $0 \le \hat{q} < 1$.*

By Corollary C, $c_q(q) > c(q)/q$ is necessary and sufficient condition for price discrimination, and since $c(q) = \kappa(\log q/\log\delta)q$, it immediately follows that $c_q(q) > c(q)/q$ if and only if

$$\kappa\left(\frac{\log q}{\log\delta}\right) + \kappa'\left(\frac{\log q}{\log\delta}\right)\frac{1}{\log\delta} > \kappa\left(\frac{\log q}{\log\delta}\right). \tag{14}$$

Assuming $\delta < 1$ and $\log\delta < 0$, this means that intertemporal price discrimination is profitable if and only if $\kappa'(t) < 0$. Thus, if the firm's production costs are declining over time the firm will offer declining prices and induce some consumers to delay their purchases, while if the firm's production costs are rising over time the firm will offer a constant price over time and all consumers will purchase immediately.[4]

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[3] Salant claims that his results generalize to the *n*-type case, but does not consider a continuum of types.

[4] As Stokey points out, when $\kappa'(t) < r = -\log\delta$ for some *t*, competitive markets will also exhibit this pattern of

C. Damaged Goods

A damaged good is one for which $c_q(q) \leq 0$, that is, it is weakly more expensive to produce lower quality goods. Deneckere and McAfee (1996) derive conditions for optimal price discrimination with damaged goods. They demonstrate that it can be both profitable and Pareto improving to offer a damaged good. They assume a continuum of types with unit demands, and restrict attention to two product qualities, q_L and q_H. Consumers have quasi-linear utilities $V(q_H,\theta) = \theta$ and $V(q_L,\theta) = \lambda(\theta)$.

The necessary and sufficient condition derived by Deneckere and McAfee is a special case of our more general condition. Specifically, in Deneckere and McAfee's model, $V(q,\theta) - c(q)$ is log supermodular if and only if

$$\frac{1}{\theta - c_H} > \frac{\lambda'(\theta)}{\lambda(\theta) - c_L},$$

or $\lambda(\theta) - c_L - (\theta - c_H)\lambda'(\theta) > 0$. To see how this is related to the condition derived by Deneckere and McAfee, note that the price a single product firm would charge is $p = V(q_H, \underline{\theta}) = \underline{\theta}$ where $\underline{\theta}$ is defined by $\underline{\theta} - c_H - (1 - F(\underline{\theta}))/f(\underline{\theta}) = 0$. So $V(q,\underline{\theta}) - c(q)$ is log supermodular if and only if $\lambda(\underline{\theta}) - c_L - (1 - F(\underline{\theta}))/f(\underline{\theta})\lambda'(\underline{\theta}) > 0$, which is the necessary and sufficient condition for the provision of damaged goods derived by Deneckere and McAfee.[5]

Note that it follows from our increasing percentage surplus condition that price discrimination is less likely to be profitable for damaged goods. In particular, when consumers' utility is $V(q,\theta) = \theta q$, by Corollary C price discrimination is never optimal if $c_q(q) \leq 0$ but is optimal for all cost functions satisfying $c_q(q) > c(q)/q$.

D. Queuing Systems

Priority service queues are a common device that firms use to more efficiently serve their

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prices and purchases. But by Proposition 4, when competitive market exhibit such delay, the monopoly market exhibit weakly greater delay for all consumers and strictly greater delay for any consumers who don't purchase immediately.

[5] McAfee (2006) extends Deneckere and McAfee (1996) and explicitly identifies the increasing percentage differences condition as a necessary condition for a damaged goods strategy to be profitable.

customers when delays in delivery are unavoidable. Customers who are less time sensitive will accept a lower priority queue in exchange for a lower price. For example, the postal service puts first-class mail in a higher priority queue than bulk mail. Queuing systems are efficient because delays are inevitable when demand is high and resources are scarce. While queuing systems can be used for efficient allocation of scarce resources, they can also be used as a discriminatory mechanism to extract more revenue out of customers. For example, major package delivery firms typically use the same resources to delivery overnight and 2-day service packages. But, the deliveries of 2nd-day service packages are deliberately delayed for an additional day to insure that they do not arrive overnight. Reducing the quality of 2-day service allows the firm to charge more for its overnight service.

While most of the research on queuing systems has focused on using them to increase efficiency, more recent work has emphasized the ability of queuing systems to price discriminate. Afeche (forthcoming) shows that a profit-maximizing firm may be able to benefit from a strategy of deliberately delaying delivery to some consumers in order to increase the price it can charge to others. He considers a model in which type $x \in \{x_1, x_2\}$ consumers derive value $U(x,t) = v - xt$ from consuming the good delivered with delay *t* (Afeche allows v to be stochastic, but independently distributed). Doing a change of variables, $\theta = -x$ and $q = \delta^t$, yields $V(\theta,q) = v + \theta \ln q / \ln \delta$, which is always log supermodular (and satisfies $V_\theta > 0$, $V_q > 0$, and $V_{q\theta} > 0$). It follows from Proposition 1 that as long as there are enough consumers of each type, *and no other source of delay*, the firm would always find it profitable to offer the low θ customer a product that is deliberately delayed as this increases the price it can charge the high value customer. However, in Afeche's paper demand uncertainty and congestion are major reasons for delays, so the conditions under which deliberate delay is profitable are more restrictive.

While Afeche (forthcoming) shows that there exist conditions under which deliberate delay is profitable, he does not offer a full characterization of when this strategy is profitable. Indeed the queuing system literature has considered many models in which this price discrimination strategy would not be profitable.[6] For example, Afeche and Mendelsen (2004) consider utility functions of the form $U(\theta,t)=\theta d(t)-w(t)$. A change of variables, $q=d(t)$, yields $V(\theta,q)=\theta q-w\left(d^{-1}(q)\right)$, which by Corollary C is log supermodular only if $dw\left(d^{-1}(q)\right)/dq$ is increasing, or w is increasing faster than d is decreasing (note that the constraint is $q\le d(0)$ as opposed to $q\le 1$). Another paper, van Meighem (2000), considers utility functions of the form $U(\theta,t)=\theta-w(\theta,t)$, where θ is the consumer's delay-free valuation and w is their delay cost. Using a change of variables, $q=g(t)$, where g is any decreasing function, the resulting utility function $V(\theta,q)$ is log supermodular only if $U(\theta,t)$ is log submodular (see properties of log supermodular functions in the appendix). Note that $U(\theta,t)$ is neither log submodular nor log supermodular when $w(\theta,t)=\theta t$. This suggests that deliberate delay is less likely to be profitable when consumers delay costs are proportional to their delay-free valuations.

More generally, while deliberate delay is potentially profitable, whether or not it is profitable depends on the precise model specification. Our results contribute to the literature on using queuing systems as a mechanism for price discrimination by showing that log supermodularity of the surplus function is a necessary condition (and in the absence of congestion delays, a sufficient condition) for deliberate delay to be profitable.

E. Advance Purchase Discounts

Several recent papers have examined the use of advance purchase requirements for price discrimination, including Shugan and Xie (2001), Courty and Li (2000), and Gale and Holmes (1992, 1993).[7,8] Purchasing in advance requires consumers to give up flexibility in their purchase decision, departure time, or destination. Consider the following simple model, which is inspired

[6] This literature began with Naor (1969). Other papers in this literature, besides those discussed below, include Mendelsen and Whang (1990) and Ha (2001).

[7] Advance purchase requirements can help the firm extract greater surplus from heterogeneous consumers (see Shugan and Xie, 2001, Courty and Li, 2000) and also enable the firm to increase capacity utilization (see Gale and Holmes, 1992, 1993, and Dana, 1998, 1999).

[8] Advance purchase discounts can also benefit the firm in other ways. First, advance purchase discounts can be used to improve production efficiency of production by giving the firm better forecast of spot market demand (Tang et. al. 2004, and McCardle et. al. 2004). Also, firms may find it more profitable to sell in advance when consumers have an imperfect forecast of their spot market preferences (Shugan and Xie 2001, and Courty 2003).

by Shugan and Xie (2001) and Courty and Li (2000). Assume the firm can set one price, p_0, for travel if the ticket is purchased at time 0 (e.g., 14-days in advance) and another price, p_1, for travel if the ticket as purchased at time 1 (e.g., one day in advance). Assume two types of consumers, business travelers and leisure travelers, who differ in their valuations for the product and in their cost of planning. Specifically, consumers value for travel is $v_B = v + \varepsilon$ and $v_L = v$ if they buy in the spot market and is $v_B - x_B$ and $v_L - x_L$ if they buy in advance.[9] We also assume $x_B > x_L$ and zero marginal product cost.

The firm has three pricing options. It can sell to all the business travelers at price v_B ($p_0 = p_1 = v_B$), sell to all buyers at price v_L ($p_0 = p_1 = v_L$), or sell to leisure travelers at price $p_0 = (v_L - x_L)$ at time 0 and sell to business travelers at price $p_1 = p_0 + x_B$ at time 1. By Proposition 1, the price discrimination is the most profitable option if and only if $x_B/(v+\varepsilon-x_B) > x_L/(v-x_L)$. Clearly this is more likely to be satisfied if consumer valuations, *v*, increase. Thus, our increasing percentage differences condition is more likely to be satisfied in markets where the average product valuation is greater.

F. Coupons

Finally, we apply our results to the literature on coupons (Anderson and Song 2004, Nevo and Wolfram 2002, Gerstner and Hess 1991). A commonly expressed intuition about coupons is that they are profitable when consumers' product valuations and the disutility of consuming the inferior good are positively correlated. We now consider a simple model of coupon-based price discrimination, based on Anderson and Song (2004) that demonstrates that this correlation is *not* sufficient.

Assume that consumers are uniformly distributed on $\left[\underline{\theta},\overline{\theta}\right]$ the unit interval and that their utility is $V(\theta,N) = \alpha + \theta\varphi$ if they do not use a coupon and $V(\theta,C) = \alpha + \theta\varphi - H(\theta)$ if they do use a coupon. The function $H(\theta)$ represents the cost of using a coupon and is assumed to be

[9] The literature on advance purchase discounts, consumers who buy in advance are either uncertain about their spot market valuations (Courty and Li, 2000, Dana 1998, and Shugan and Xie 2000) or about their departure time preferences (Gale and Holmes 1992, 1993 and Dana 1999).

increasing in the consumer's type. The parameters α and φ are positive scalars. The firm chooses, *d,* the face value of the coupon, and *p,* the shelf price. The constant marginal cost of the good is *c,* and the cost of printing the coupons is λ per coupon user.

From Proposition 2, coupons are profitable only if $V(\theta,q)-c(q)$, $q\in\{C,N\}$, is log supermodular, and $V(\theta,q)-c(q)$, $q\in\{C,N\}$, is log supermodular if

$$\frac{\varphi}{\alpha+\theta\varphi-c} > \frac{\varphi-H'(\theta)}{\alpha+\theta\varphi-H(\theta)-c-\lambda}, \text{ or equivalently} \tag{15}$$

$$\frac{\varphi}{\alpha+\theta\varphi-c} < \frac{H'(\theta)}{H(\theta)+\lambda}.$$

If $H(\theta)=\theta H$ and $\lambda=\alpha=0$, then there is perfect positive correlation between hassle cost and product valuation but price discrimination is not optimal. Thus, positive correlation is not sufficient for a firm to price discriminate. In contrast, our increasing percentage differences condition is both intuitive and sufficient.[10]

6. Conclusion

Before deciding how to price discriminate a firm must understand whether it is optimal to price discriminate. In this paper, we present a general model of product line pricing that yields a single, intuitive condition that is both necessary and sufficient for price discrimination. We refer to this as an *increasing percentage differences* condition.

The academic literatures in marketing, economics, and operations management have considered many different applications of price discrimination. These include product line pricing, versioning of information goods, damaged goods, intertemporal price discrimination, service queues, advance purchase discounts and coupons. To a large extent, these applications have been developed in isolation and have ignored their common elements. We show that our *increasing percentage differences* condition can be used to interpret and generalize many of the

[10] In the coupon model a necessary condition for price discrimination when $\lambda=\alpha=0$ is that $H'(\theta)>H(\theta)/\theta$, which one might loosely interpret as marginal hassle cost is greater than average hassle cost.

results from these seemingly disparate literatures. For example, to the lay reader the question of whether a brand manager should offer a coupon or whether United Airlines should offer advance purchase discounts appear unrelated. Yet, at their most basic level, these are simply two applications of price discrimination. By showing how our model applies to many applications, we generalize and advance our knowledge of second-degree price discrimination.

A key feature of our analysis is that we explicitly recognize the role of quality constraints. In the absence of these constraints, price discrimination is always optimal. But in practice, quality constraints are often explicit or implicit. For example, past R&D efforts by Intel limit the set of available processors that they can offer. Alternatively, a firm may face implicit quality constraints when the cost of producing a high quality option is prohibitively large. We also recognize that the mechanisms available for reducing quality are extensive and quite different from the mechanisms for raising product quality. Consider for example how Intel chose to reduce the quality of its 486 processor (i.e., damaged goods) to create a product line. Similarly, delayed consumption, coupon hassle costs, and consumption uncertainty provide other mechanisms that facilitate price discrimination.

While our framework is quite general, there are some important limitations on our analysis. First, we consider only quasi-linear utility. Second, we consider only a single dimension of consumer heterogeneity. And finally we consider only monopoly pricing. Some generalizations to competition are possible (see Johnson and Myatt, forthcoming, for one potential approach), but the most natural models of competition would have both dimensions of horizontal and vertical consumer heterogeneity (see Stole, 2005). The last two issues are important because empirical tests of the theory are likely to be performed on firm behavior in competitive environments.

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8. Appendix

Properties of Log Supermodular Functions

1. If $f(x,y)$ is continuously differentiable, then it is log supermodular if and only if $f_{xy}f - f_x f_y > 0$.
2. If $f(x,y)$ is multiplicatively separable, i.e., equal to $g(x)h(y)$ then it is neither log supermodular nor log supermodular, that is $f_{xy}f - f_x f_y = 0$.
3. For any strictly increasing functions *g* and *h,* the function $k(u,v)$ is log supermodular if and only if $f(x,y) = k(g(x),h(y))$ is log supermodular.
4. For any strictly increasing function *g* and strictly decreasing function *h,* the function $k(u,v)$ is log supermodular if and only if $f(x,y) = k(g(x),h(y))$ is log submodular.
5. For any strictly positive, strictly increasing, differentiable function *g,* $xy - g(x)$ is log supermodular if and only if $g'(x) > g(x)/x$.

Proof of Proposition 1:

The seller selects quality levels, q_L and q_H, and transfers, t_L and t_H, subject to incentive compatibility and participation constraints:

$$\max_{q_L, q_H, t_L, t_H} I\left(V(q_L,\theta_L) - t_L\right) n_L \left(t_L - c\left(q_L\right)\right) + I\left(V(q_H,\theta_H) - t_H\right) n_H \left(t_H - c\left(q_H\right)\right) \tag{16}$$

subject to

$$V(q_H,\theta_H) - t_H \geq V(q_L,\theta_H) - t_L, \tag{IC-1}$$

$$V(q_L,\theta_L) - t_L \geq V\left(q_H,\theta_L\right) - t_H, \tag{IC-2}$$

and $q_L \leq q_H \leq 1$, where *I* is the indicator function (consumers purchase only if their surplus is non-negative).

Clearly any solution to (16) satisfies $q_H = 1$. Hence, the solution to (16) takes on one of three possible forms. The first, which we label strategy S1, is to sell a single quality, $q_H = 1$, to only the high type buyers at $t_H = V(1,\theta_H)$ and profit $n_H\left(V(1,\theta_H) - c\left(1\right)\right)$. The second, which we label strategy S2, is to sell a single quality, $q_L = q_H = 1$, to all buyer types at price $t_L = V\left(1,\theta_L\right)$ and profit $\left(n_L + n_H\right)\left(V(1,\theta_L) - c\left(1\right)\right)$.

The third, which we label strategy S3, is to offer multiple qualities and sell to both buyer types. The low-type buyer pays $t_L = V\left(q_L,\theta_L\right)$ for quality $q_L < 1$, and the high type buyer pays $t_H = V(1,\theta_H) - \left(V(q_L,\theta_H) - V(q_L,\theta_L)\right)$ for quality $q_H = 1$ and the firm earns a profit $n_L\left(V(q_L,\theta_L) - c(q_L)\right) + n_H\left(V(1,\theta_H) - c(1) - \left(V(q_L,\theta_H) - V(q_L,\theta_L)\right)\right)$. When the firm adopts strategy S3, the low quality level solves

$$\max_{\hat{q}} \quad n_L\left(V\left(\hat{q},\theta_L\right) - c\left(\hat{q}\right)\right) + n_H\left(V(1,\theta_H) - c\left(1\right) - \left(V(\hat{q},\theta_H) - V(\hat{q},\theta_L)\right)\right). \tag{17}$$

The first order condition,

$$G\left(q\right) = n_L\left(V_q\left(q,\theta_L\right) - c_q\left(q\right)\right) + n_H\left(V_q\left(q,\theta_L\right) - V_q\left(q,\theta_H\right)\right) = 0, \tag{18}$$

has a strictly maximal interior solution if and only if $G\left(0\right) > 0$ and $G\left(1\right) < 0$, even if (18) has multiple solutions. Let $\tilde{q}$ denote this maximum.

Comparing the three solution strategies, $\tilde{q} < 1$, or equivalently strategy S3 strictly dominates strategy S2, if and only if $G\left(0\right) > 0$ and $G\left(1\right) < 0$, the later of which can be written as

$$n_L\left(V_q\left(1,\theta_L\right) - c'\left(1\right)\right) + n_H\left(V_q\left(1,\theta_L\right) - V_q\left(1,\theta_H\right)\right) < 0. \tag{19}$$

Strategy S3 strictly dominates strategy S1 if and only if

$$\begin{aligned} &n_L\left(V\left(\hat{q},\theta_L\right) - c\left(\hat{q}\right)\right) + n_H\left(V(1,\theta_H) - c\left(1\right) - \left(V(\hat{q},\theta_H) - V(\hat{q},\theta_L)\right)\right) \\ &\qquad > n_H\left(V(1,\theta_H) - c\left(1\right)\right), \end{aligned} \tag{20}$$

or equivalently $n_L\left(V(\hat{q},\theta_L) - c\left(\hat{q}\right)\right) - n_H\left(V(\hat{q},\theta_H) - V(\hat{q},\theta_L)\right) > 0$, for some $\hat{q} < 1$. Note that (20) and $V(0,\theta) - c(\theta) < 0, \forall\theta$ imply $G\left(0\right) > 0$, so S3 dominates both S1 and S2 if and only if (19) and (20) hold, or equivalently

$$\frac{V_q\left(1,\theta_L\right) - c_q\left(1\right)}{V_q\left(1,\theta_H\right) - c_q\left(1\right)} < \frac{n_H}{n_H + n_L}, \text{ and} \tag{21}$$

$$\frac{V(\hat{q},\theta_L)-c(\hat{q})}{V(\hat{q},\theta_H)-c(\hat{q})}>\frac{n_H}{n_H+n_L}, \tag{22}$$

for some $\hat{q}<1$. Clearly a necessary condition for (21) and (22) to hold simultaneously is

$$\frac{V_q(1,\theta_L)-c_q(1)}{V_q(1,\theta_H)-c_q(1)}<\frac{V(\hat{q},\theta_L)-c(\hat{q})}{V(\hat{q},\theta_H)-c(\hat{q})}, \tag{23}$$

for some $\hat{q}$. Equation (23) defines the interval $N^*(\hat{q})$.

If $V(q,\theta)-c(q)$ is everywhere log submodular, then since $S(q,\theta_L)/S(q,\theta_H)$ is increasing in q and $S_q(1,\theta)/S(1,\theta)$ is increasing in θ, it follows that

$$\frac{S_q(1,\theta_L)}{S_q(1,\theta_H)}>\frac{S(1,\theta_L)}{S(1,\theta_H)}>\frac{S(\hat{q},\theta_L)}{S(\hat{q},\theta_H)} \tag{24}$$

for all $\hat{q}<1$ and (23) cannot hold. So $N^*(\hat{q})$ is empty, conditions (21) and (22) cannot both be satisfied, and either strategy S1 or S2 dominates strategy S3. That is, the firm produces only a high quality product.

If $V(q,\theta)-c(q)$ is everywhere log supermodular, then since $S(q,\theta_L)/S(q,\theta_H)$ is decreasing in q and $S_q(1,\theta)/S(1,\theta)$ is decreasing in θ, it follows that (24) holds with the inequalities reversed for all $\hat{q}<1$ and so (23) does hold. That is, $N^*(\hat{q})$ is non-empty, and (21) and (22) both hold for all n_L and n_H such that $n_H/(n_H+n_L)\in N^*(\hat{q})$, and strategy S3 dominates both strategies S1 and S2. That is, the firm offers both a high and low quality product. Finally, it is clear that $N^*(\tilde{q})$ is the largest such interval.

Proof of Proposition 4:

If a seller is restricted to offering a single quality, it will sell only high quality. Also, it will sell exclusively to the high types if and only if

$$\bar{n}\left(V(1,\bar{\theta})-c(1)\right)>\left(\bar{n}+\underline{n}\right)\left(V(1,\underline{\theta})-c(1)\right), \text{ or}$$

$$\frac{V(1,\underline{\theta})-c(1)}{V(1,\bar{\theta})-c(1)}<\frac{\bar{n}}{\bar{n}+\underline{n}}. \tag{25}$$

Log supermodularity implies

$$\frac{\partial V(1,\underline{\theta})/\partial q - c'(1)}{\partial V(1,\overline{\theta})/\partial q - c'(1)} < \frac{V(1,\underline{\theta}) - c(1)}{V(1,\overline{\theta}) - c(1)} < \frac{V(\hat{q},\underline{\theta}) - c(\hat{q})}{V(\hat{q},\overline{\theta}) - c(\hat{q})}, \tag{26}$$

so in the subinterval

$$\left(\frac{V(1,\underline{\theta}) - c(1)}{V(1,\overline{\theta}) - c(1)}, \frac{V(\hat{q},\underline{\theta}) - c(\hat{q})}{V(\hat{q},\overline{\theta}) - c(\hat{q})} \right)$$

of N^* allowing price discrimination results in a Pareto improvement. That is, it weakly increases seller profits by revealed preference, weakly increases type $\underline{\theta}$ buyers' consumer surplus because they were not previously served, and strictly increases type $\overline{\theta}$ buyers' consumer surplus from zero to something positive because their incentive compatibility constraint strictly binds. QED.

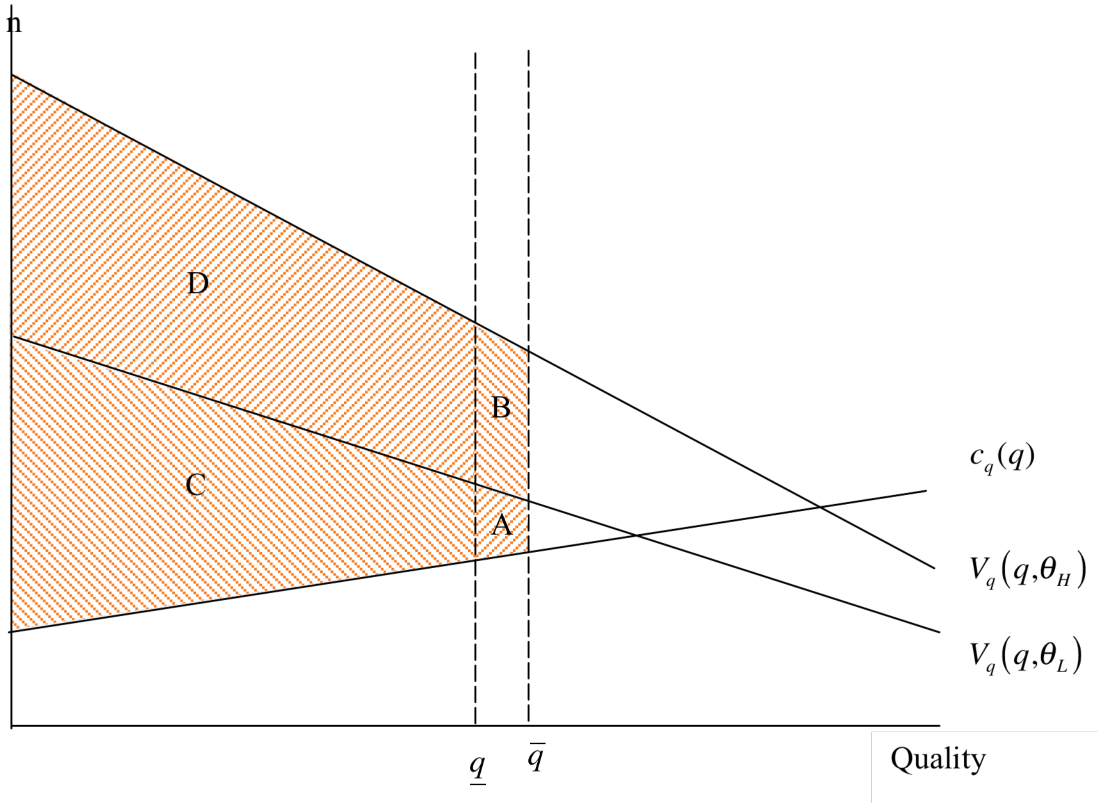

Figure 1

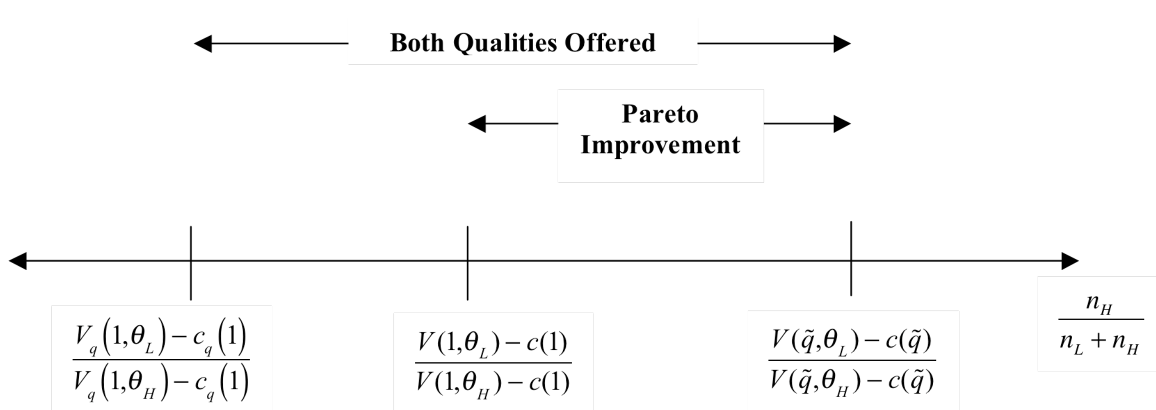

Figure 2: Regions of Price Discrimination and Pareto Improvement

Technical Appendix

This appendix is quite standard. It is included for the benefit of the reader who might not be familiar with it. This is not intended to be part of the final published paper.

Our proof follows almost identically the proof of Hermalin (2006). Hermalin does not consider a quality constraint, but it is clear the proof of the lemma is unaffected by the addition of a constraint.

Proof of Lemma

I will first argue that any pricing scheme $p(\theta), q(\theta)$ that satisfies the incentive compatibility and participation constraints must have the property that $q(\theta)$ is non-decreasing and must imply that

$$v(\theta) = \int_{\theta_L}^{\theta_1} V_\theta(q(t), t)dt. \tag{A1}$$

First note that it must be that $v(\theta_L) = 0$ since otherwise the incentive constraint would be violated. That is, some consumers for whom $\theta < \theta_L$ would prefer to announce they were type θ_L.

Consider two arbitrary consumer types, θ_a and θ_b. One must be larger than the other, so there is no loss of generality in assuming it is θ_b. Incentive compatibility requires that

$$v(\theta_b) \geq V(q(\theta_a), \theta_b) - p(\theta_b) \tag{A2}$$

and

$$v(\theta_a) \geq V(q(\theta_b), \theta_a) - p(\theta_a). \tag{A3}$$

Using $v(\theta)$ from the participation constraint to substitute out for $p(\theta_a)$ and $p(\theta_b)$, we can these as

$$v(\theta_b) \geq v(\theta_a) + V(q(\theta_a), \theta_b) - V(q(\theta_a), \theta_a) \tag{A4}$$

and

$$v(\theta_a) \geq v(\theta_b) + V(q(\theta_b), \theta_a) - V(q(\theta_b), \theta_b), \tag{A5}$$

or

$$v(\theta_b) \geq v(\theta_a) + \int_{\theta_a}^{\theta_b} V_\theta(q(\theta_a), t)dt \tag{A6}$$

and

$$v(\theta_a) \geq v(\theta_b) + \int_{\theta_b}^{\theta_a} V_\theta(q(\theta_b), t)dt. \tag{A7}$$

These imply

$$\int_{\theta_a}^{\theta_b} V_\theta(q(\theta_b), t)dt \geq v(\theta_b) - v(\theta_a) \geq \int_{\theta_a}^{\theta_b} V_\theta(q(\theta_a), t)dt \tag{A8}$$

and ignoring the middle term we this implies

$$\int_{\theta_b}^{\theta_a} V_\theta(q(\theta_b), t) - V_\theta(q(\theta_a), t)dt \geq 0 \tag{A9}$$

and using the fundamental theorem of calculus

$$\int_{\theta_b}^{\theta_a} \int_{q(\theta_a)}^{q(\theta_b)} V_{q\theta}(z, t)dzdt \geq 0 \tag{A10}$$

which implies that $q(\theta)$ is non-decreasing (that is, $q(\theta_b) \geq q(\theta_a)$ for all θ_a and θ_b such that $\theta_b \geq \theta_a$).

Equation (A8) also implies

$$\frac{1}{\epsilon} \int_{\theta_b}^{\theta_a} V_\theta(q(\theta_b), t)dt \geq \frac{v(\theta_b) - v(\theta_a)}{\epsilon} \geq \frac{1}{\epsilon} \int_{\theta_a}^{\theta_b} V_\theta(q(\theta_a), t)dt \tag{A11}$$

holds for all ϵ. So taking taking the limit as $\epsilon \to 0$, we have

$$V_\theta(q(\theta), \theta) + \int_\theta^\theta V_{\theta q}(q(\theta), t)q'(\theta)dt \geq v'(\theta) \geq V_\theta(q(\theta), \theta) \tag{A12}$$

or

$$v'(\theta) = V_\theta(q(\theta), \theta) \tag{A13}$$

almost everywhere. Finally, since $v(\theta_L) = 0$,

$$v(\theta) = \int_{\theta_L}^{\theta} V_\theta(q(t), t)dt. \tag{A14}$$

It also follows that any pricing scheme $p(\theta), q(\theta)$ that satisfies (A1), and has the property that $q(\theta)$ is everywhere non-decreasing and less than 1, must satisfy the incentive compatibility and participation constraints.

Clearly (A1) implies that the participation constraint is satisfied. Incentive compatibility (see A4 and A5) requires that

$$V(q(\theta_b), \theta_b) - V(q(\theta_b), \theta_a) \geq v(\theta_b) - v(\theta_a) \tag{A15}$$

for all θ_a and θ_b. First suppose $\theta_b > \theta_a$, so that $q(\theta_b) \geq q(t)$ for all $t \in [\theta_a, \theta_b)$. Using (A1) and the fundamental theorem of calculus, (A15) can be written

$$\int_{\theta_a}^{\theta_b} V_\theta(q(\theta_b), t)dt \geq \int_{\theta_L}^{\theta_b} V_\theta(q(t), t)dt - \int_{\theta_L}^{\theta_a} V_\theta(q(t), t)dt \tag{A16}$$

where the right hand side equals

$$\int_{\theta_a}^{\theta_b} V_\theta(q(t), t)dt \tag{A17}$$

so (A15) becomes

$$\int_{\theta_a}^{\theta_b} V_\theta(q(\theta_b), t) - V_\theta(q(t), t)dt \geq 0 \tag{A18}$$

which must hold because $q(\theta_b) \geq q(t)$ for all $t \in [\theta_a, \theta_b)$ and $V_{q\theta} > 0$ implies $V_\theta(q(\theta_b), t) - V_\theta(q(t), t) \geq 0$.

Now suppose $\theta_b \leq \theta_a$. Then (A15) can be rewritten as

$$\int_{\theta_b}^{\theta_a} V_\theta(q(\theta_b), t) - V_\theta(q(t), t)dt \leq 0 \tag{A19}$$

which holds because now $q(\theta_a) \geq q(t)$ for all $t \in [\theta_b, \theta_a)$ and $V_{q\theta} > 0$.

Therefore any pricing scheme $p(\theta), q(\theta)$ satisfies (A1) and has the property that $q(\theta)$ is everywhere non-decreasing if and only if it satisfies the incentive compatibility and participation constraints.

So substituting $v(\theta)$ from the participation constrain into the objective function, (4), we can write the firm's problem as

$$\max_{\theta_L, q(\theta), v(\theta)} \int_{\theta_L}^{\theta_1} [V(q(\theta), \theta) - c(q(\theta)) - v(\theta)]\, dF(\theta) \tag{A20}$$

subject to (A1), $q(\theta)$ being non-decreasing, and $q(\theta) < 1$. Finally, substituting for $v(\theta)$ using (A1) and applying intergration by parts, we can write the firm's problem as

$$\max_{\theta_L, q(\theta)} \int_{\theta_L}^{\theta_1} \left[V(q(\theta), \theta) - c(q(\theta)) - J(\theta) V_\theta(q(\theta), \theta) \right] dF(\theta) \tag{A21}$$

subject to $q(\theta)$ being non-decreasing and $q(\theta) < 1$.