The Efficiency of Competitive Mechanisms Under Private Information

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Outline

The problem

- We look at large (but finite) exchange economies with
  1. private values
  2. private information
  3. strategic behavior
- Trade is organized according to a competitive mechanism
  - We view any such mechanism as a black box
  - We look for results that hold for all competitive mechanisms

Type of questions one can ask

1. Do players report their types truthfully when the economy is large?
2. Do competitive mechanisms lead to (approximately) efficient allocations?

1. It is clear that (1) cannot be guaranteed in general
2. We show that (2) obtains

Informal statement of the main result

Main (and only) Theorem of this paper

Given any desired degree of approximation

- There is $\bar{N}$ such that in any economy with $\bar{N}$ or more traders
- *Any* prior distribution on traders' types
- *Any* Bayesian-Nash equilibrium
- Of *any* competitive mechanism for this economy

.. the resulting allocation is approximately efficient.

Important features of the theorem

- No replica structure is invoked
- No need to make assumptions about the existence of a well defined limit economy
- Distribution on types may be correlated (within limits)
- Competitive mechanisms may be very sensitive to individuals' reports

Methodology

- Focus on a particular institution: *competitive markets*
- ..rather than search for the *optimal* institution.

- Consider all competitive mechnaisms..
- ..and equilibria of these mechanisms

Why all competitive mechanisms?

- We do not understand the process of price formation
- Walrasian eq. is a consistency requirement a reasonable outcome must satisfy; it is not a prediction of any particular outcome

Formal Model
Complete information economies

$$E^N = (\theta^N, w^N)$$

is an N-agent classical exchange economy with complete information where:

- $w^N = \{w_n,\ n \in N\}$ is the vector of endowments
- $\theta^N = \{\theta_n,\ n \in N\}$ is the vector of types
- Each θ represents a classical utility function.

Endowments

- Endowments are common knowledge
- There exist constants $\xi^+ > \xi^- > 0$ such that, uniformly across economies,

$$w_n \in W \equiv [\xi^-(1, \ldots, 1), \xi^+(1, \ldots, 1)].$$

$\Longrightarrow$ ..each player holds a negligible fraction of total endowments in the limit.

Preferences

- Finite set of types Θ, with $\theta \in \Theta$ a classical utility functions
- *Sorting condition*: Suppose that hyperplane H supports U_θ at x, and hyperplane H' supports $U_{\theta'}$ also at x. Then $H \neq H'$.

Competitive mechanisms

A competitive mechanisms is *any* selection

$$\sigma : \mathcal{E}^N \to \Delta \times (R^l_+)^N$$

of the competitive correspondence.

Motivation....

Efficiency

A feasible allocation x^N is (ex post) *η-efficient* if there is no feasible allocation $\check{x}^N$ such that

$$U_{\theta_n}(\check{x}_n) > U_{\theta_n}(x_n) + \eta, \ \ \forall n = 1 \ldots, N.$$

$\mathit{Eff}(\theta^N, w^N)$ and $\mathit{Eff}_\eta(\theta^N, w^N)$ will denote the sets of all efficient and η-efficient allocations respectively.

Type space: *Strategic vs. non-strategic types*

Strategic types: May report any type in $\bar{\Theta} = \Theta$

Non-strategic types: reports his true type $\hat{\Theta} = \Theta$

For each player there is a 0-1 random variable that determines whether he is strategic or not

Type space is: $[\hat{\Theta} \times \bar{\Theta} \times \{0, 1\}]^N$

Details on definition of the type space

Type distributions

Prior distribution on types: $\psi^N \in \Psi^N(\epsilon_c, \epsilon_\chi)$

- $\epsilon_c, \ \epsilon_\chi \in (0, 1]$
- $\psi_s^N, \ \psi_c^N, \ \psi_\chi^N$ are the marginals on $\hat{\Theta}^N, \ \bar{\Theta}^N$ and $\{0, 1\}^N$

Assumptions:

1. ψ_c^N and ψ_χ^N have independent marginals across n;
2. For all n, $\psi_c^N(\bar{\theta}_n = \theta) \geq \epsilon_c \ \ \forall \theta \in \bar{\Theta}$;
3. For all n, $\psi_\chi^N(\chi_n = 1) \geq \epsilon_\chi$.

A trader's true type is: $\theta_n = \hat{\theta}_n$ if $\chi_n = 0$ and $\theta_n = \bar{\theta}_n$ if $\chi_n = 1$.

The Market Game

We define an N-player game $\Gamma(\sigma, \psi^N, w^N)$ as follows:

- Types are drawn according to ψ^N
- Strategies are reporting functions

$$\tilde{\theta}_n : \hat{\Theta} \times \bar{\Theta} \times \{0, 1\} \to \Delta(\tilde{\Theta})$$

  such that non-strategic types report truthfully
- The competitive mechanism σ picks a competitive equilibrium $\sigma(\tilde{\theta}^N, w^N)$.
- Payoffs are $U_{\theta_n}(\sigma_{x_n}(\tilde{\theta}^N, w^N))$

Main Theorem

For any $\eta > 0$, there is $\bar{\epsilon} > 0$ such that for any $0 < \epsilon < \bar{\epsilon}$ and any pair $(\epsilon_c, \epsilon_\chi)$, satisfying $\epsilon_c \cdot \epsilon_\chi \geq \epsilon$:

- there exists $\bar{N}$ such that for any $N > \bar{N}$
- any private information economy (ψ^N, w^N), with $\psi^N \in \Psi^N(\epsilon_c, \epsilon_\chi)$
- any competitive mechanism σ
- and any Bayesian-Nash equilibrium of $\Gamma(\sigma, \psi^N, w^N)$

$$P\left\{\sigma_{x^N}(\tilde{\theta}^N, w^N) \in Eff_\eta(\theta^N, w^N)\right\} > 1 - \eta$$

What we do and don't do

We show that any competitive equilibrium of the reported economy must be approximately efficient relative to the true economy

...but we do NOT show that all agents truthfully report their types or that agents' influence on prices vanishes with large N

The literature: the mechanism design approach

Much of the literature is concerned with designing *good mechanisms*, *i.e.*, ones that possess *at least one* equilibrium in which *all agents* truthfully report their information

- Gul-Postlewaite (Econometrica 92)
- McLean-Postlewaite (several papers)
- Mas-Colell & Vives (RES 93)

The literature: focus on specific institutions

Other papers consider specific auction mechanisms, typically auctions, and establish their properties as *the* $N \to \infty$:

- Gresik-Satterthewaite, Rustichini-Satterthewaite-Williams
- Swinkels, Cripps-Swinkels
- Reny-Perry
- ...

We also look at the properties of a specific institution, but competitive markets rather than auctions.

The price: Types must be finite

This ensures that: “if a player has small influence, then he must report truthfully.”

This issue appears in:

- Gul-Postlewaite
- Jackson-Manelli

The price: Sorting condition

Needed if we want to conclude something about all Bayes-Nash equilibria.

compare with Gul-postlewaite..

The price: non-strategic types

We need enough strategic uncertainty about the opponents' reports and about the outcome of the mechanism that is bounded away from zero uniformly in the number of traders and the strategies they play

Using the terminology we introduced in Al-Najjar and Smorodinsky (2000), define the "influence" of a trader on an abstract outcome function as the maximum change in the expected outcome that can be caused by changes in this trader's actions.

For related ideas, so Fudenberg-Levine-Pesendorer (1998) and Swinkels (2001)for related concepts and results

Why is this true?

Stopped here!!

- Results on influence in abstract environments in Al-Najjar and Smorodinsky. We use the version for correlated environments.
- Role of noise
- Role of the assumption of finite types

HOWEVER: the fraction of influential agents decreases as N grows

$\Longrightarrow$ the distribution of characteristics in the true and reported economies must be close

$\Longrightarrow$ an equilibrium of the reported economy is an approximate equilibrium of the true economy

Outline of the proof: *1-Influence arguments*

Key lemma

Given any $\delta > 0$ and $\epsilon > 0$ there exist integers M and $\bar{N}$ such that for

- *any* economy E^N, with $N > \bar{N}$
- *any* competitive mechanism σ
- *any* (Bayes-Nash) equilibrium of $\Gamma(\sigma, \epsilon, N)$
- *any* prior distribution ψ on types (redundant)

$$P\left\{\tilde{\theta}^N : \#\{n : \tilde{\theta}_n \neq \theta_n\} < M \cdot |\Theta| \cdot N^{3/4} \quad | \, \theta^N\right\} > 1 - \delta$$

Details on definition of influence

Outline of the proof: *2-Continuity*

1. Use a standard metric on economies
2. standard result in general equilibrium is that the excess demand function is continuous in prices and economies
3. The influence arguments show that E^N and $\tilde{E}^N$ are close
4. law of large numbers implies that for small ϵ, $\tilde{E}^N$ and $\tilde{E}^N$ are also close
5. The outcome of the game is always a competitive equilibrium for $\tilde{E}^N$

PROPOSITION 2: Given Proposition 1, the outcome of the mechanism is an approximate equilibrium of the true economy, hence approximately efficient.

Concluding thoughts: *Why not require competitive mechanisms to be continuous?*

- Mas-Colell and Vives consider anonymous and continuous mechanisms in a continuum economy
- This requirement makes little sense in finite economies
- Even it makes sense, our ignorance of the process of price formation makes it unreasonable in our view to require the selection to be continuos

Concluding thoughts: *Common values*

HAS TO BE REVIEWED THOROUGHLY

Key issue: *How much information do prices reveal?*

- Interpretation of the fully revealing REE literature: Find σ so that information about θ^N is fully revealed

THE END !!

Appendix

Influence idea

A generic type realization $(\hat{\theta}, \bar{\theta}, i)$ means that:

- There is a random draw of the player's strategic type, which turned up to be $\hat{\theta}$;
- ..a separate draw of this player's non-strategic type $\bar{\theta}$;
- ..and finally a draw of whether the player is strategic or not.

This is a more complicated type space than: $\Theta \times \{0, 1\}$, but we cannot see see why this is substantively different from the type space we use, which is $\Theta \times \Theta \times \{0, 1\}$.

The latter is what we use in our proofs.