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Outsourcing Service Processes to a Common Service Provider under Price and Time Competition

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In many industries, firms consider the option of outsourcing an important service process associated with the goods or services they bring to the market. Often, competing firms outsource this service process to one or more common service suppliers. When they outsource to a common service provider, this gives rise to a service supply chain. We develop analytical models to characterize the benefits and disadvantages of outsourcing in service industries in which the retailers compete with each other in terms of the price they charge and/or the waiting time expectations and standards which they adopt and sometimes advertise. We show that the benefits of outsourcing are affected by the supplier's ability to exploit the benefits of service pooling as well as differences in the cost rates themselves.

1. Introduction

In many industries, firms consider the option of outsourcing an important service process associated with the goods or services they bring to the market. Often, competing firms outsource this service process to one of more common service suppliers. For example, internet retailers such as Amazon, Barnes and Noble and other book distributers use common carriers to deliver the merchandise to their customers. After-sales support and maintenance services of appliances and electronic equipment are often outsourced by competing firms to a common maintenance or repair service provider. The same applies to (part of) the technical support function of software vendors. For make-to-order consumer goods, the potentially outsourced service process may refer to the final production or assembly stage of the product itself. Japanese automobile manufacturers, for example, have adopted make-to-order assembly systems to cut inventory costs for themselves and their dealers, and to reduce the need for rebates for slow selling vehicles.

The outsourced service process is often of critical strategic importance to the competing firms, as the above examples indicate: internet book distributers differentiate themselves to a large extent in terms of the delivery time guarantees or expectations they are specifying to their customers (along with price advantages.) In many industries, the product itself becomes increasingly commoditized and firms differentiate themselves in terms of the quality of the after-sales service they provide, even though maintenance, repair and technical support functions may fail to be part of the firms' core competency and, as a consequence, are often outsourced. A recent survey by AMR, (see Bijesse et al. (2002)) estimates that after-sales services represent, on average, 24% of the revenues and no less than 45% of the profits earned by consumer goods merchandisers. Dennis and Kambil (2003) and Wise and Baumgartner (1999) report similar results for a large verity of industries. (see also Cohen et al. (2006).) The attractiveness of the after-sales service process is, once again, often determined by the waiting time customers experience before their problem is solved, along with the price they pay for their service contract. Finally, in the automobile industry, customer waiting times become an increasingly important component of the firms' competitive strategies, along with the purchasing price of the automobile itself, in particular now that the quality gap between Japanese manufacturers and their American and European competitors has shrunk significantly. For example, Toyota's goal is to reduce the average waiting time between dealer order and delivery to fourteen days; see e.g. Fay (2004).

In this paper, we develop analytical models to characterize the benefits and disadvantages of outsourcing in service industries in which the retailers compete with each other in terms of the price they charge and/or the waiting time expectations and standards which they adopt and sometime advertise. When some or all of the firms outsource the service process to a common service provider, this gives rise to a *service supply chain*. The competing retailers may represent independent companies or divisions of the same corporation. Staying with the example of the automobile industry, we have observed that most domestic, European and Japanese manufacturers have chosen to "bring the market into the firm" by creating separate and competing divisions with broad managerial autonomy. Perhaps surprisingly, different divisions often sell similar cars with identical chassis and engines, often produced-to-order in a common plant. An example is Toyota which sells, the Sienna and Lexus Rx-30, with identical chassis and engines, via two independent divisions, which "outsource" the assembly process to common plants in West Virginia and Canada[0]. Several papers in the

[0] Baye et al. (1996) document similar practices in the mid nineties among the big three domestic manufacturers: "For example, General Motors produced the LeSabre (Buick Motor Division) and the Olds 88 (Oldsmobile Division) which, while differing in

economic literature provide theoretical frameworks to characterize the benefits of divisionalization, see e.g. Baye et al. (1996), Baker et al. (2001) and the references therein.

In the general strategy literature, several reasons are mentioned as to why firms may benefit from outsourcing: first, the service provider to whom the service process is outsourced, may enjoy lower operating cost rates, as the service process represents *his* core competency rather than that of the service retailers. A second explanation relates to possible *economies of scale* and *economies of scope*. The former consist of reduced per unit costs as the customer volume increases while the latter points at the benefits of pooling demand streams into a common service facility. Benson and Ieoronimo (1996) and Lacity and Hirschheim (1993), for example, identify cost efficiencies as the principle factor driving outsourcing decisions for maintenance functions and information systems, respectively. Gupta and Zhender (1994) emphasize economies of scale in the cost structure as a prime reason why outsourcing is often beneficial.

To assess the benefits of outsourcing strategies, we need to address the following questions:

(I) When are firms better off if all of them choose to outsource rather than perform the service in-house? Given a unique equilibrium under in-house service, the firms' profits are unambiguously specified. Under outsourcing, these depend on (i) what fraction $e \leq 1$ of first best aggregate performance the service chain achieves, and (ii) what (minimum) participation profits P the outside service provider demands. For the sake of brevity and simplicity only, we present all of our results where $e = 1$ and $P = 0$, thus portraying the outsourcing option in the best possible light. (In §7, we show that first best performance can indeed be achieved by a decentralized service chain, under specific pricing schemes, the structure of which depends on the type of competition the firms engage in.)

(II) when will a service chain in which all firms choose to outsource to a common provider be stable in the sense that no firm has an incentive to unilaterally abandon the chain and provide in-house service instead? More generally, assuming identical retailers, how large a service chain of outsourcing firms is stable? How do the answers to these questions depend on the intensity of the competition, the number of firms in the industry and the sales volume of the firms?

styling, are built on the same chassis and, comparable equipped, sell for virtually identical prices. Similarly, the Ford Motor Co. produced the Sable (Lincoln- Mercurey Division) and the Taurus (Ford Division), which are effectively the same car with different name plates, as are the Chrysler Corporation's Plymouth Voyager and Dodge Caravan. Indeed, in just about every price range, all the major domestic manufacturers have several divisions producing competing products."

(III) In what direction do equilibrium prices, waiting time standards and demand volumes change when all firms move from in-house service to outsourcing?

(IV) How do the answers to the questions raised in (I) -(III) depend on whether the outside supplier pools the service processes completely or in part, and whether it is able to operate at lower cost rates than the service retailers themselves?

Each of these above questions is analyzed under each of three types of competition: (i) *Price competition*, in which all waiting time standards are exogenously given and the firms compete on the basis of their prices only, (ii) *Waiting time* competition: here all prices are exogenously given and the competition is in terms of waiting time standards, and (iii) *Simultaneous competition*: all prices and waiting time standards are selected simultaneously by the various service retailers.

We represent a firm's demand rate as a general function of all prices and all waiting time standards in the industry. We focus primarily on two important classes of demand models: The first class uses a separable specification which, in addition, is linear in the price vector. The second class is that of the attraction models. Here, each firm is characterized by an attraction value given by a general function of its price and waiting time standard. A firm's market share, is given by the ratio of its attraction value and the attraction values of all firms in the industry, that of the no-purchase option included. These broad classes of demand models allow us to represent general tradeoffs for potential customers among the prices, waiting time standards and other service attributes. Price and waiting time are treated as truly independent attributes, in that, in general, a change in a firm's waiting time standard can *not* be compensated by a price change that leaves all market effects unchanged. [1]

The system of demand functions determines the rates at which customers arrive to the different service retailers. We assume all service retailers face Poisson demand processes while service times are exponentially distributed, with a rate to be determined by the service provider. Under outsourcing, the common service provider may employ general dynamic priority schemes to prioritize among customers referred by

[1] We refer to Allon and Federgruen (2007) for a discussion of why it is important to treat prices and waiting time standards as independent strategic instruments, in contrast to the traditional approach in the literature on service competition, in which the price and waiting time are aggregated into a single, so called, *full price* measure.

the different firms. Whoever provides the service incurs two types of costs: First, a cost per unit of time proportional to the adopted capacity level and a second, transaction/handling cost per customer served.

In §3-6, we analyze the special case where (a) all service retailers share identical characteristics, and (b) the demand functions are affine in both the price and service level. This special case permits analytical characterizations of the benefits of outsourcing.

The following represent some of our main insights: starting with the case where the retail firms engage in price competition , under a given waiting time standard, we show that they may earn *lower* profits under outsourcing, even if the resulting service chain operates at maximum efficiency. The potential for such profit reductions arises, even when the service chain under outsourcing operates at maximum efficiency ($e = 1$) and even if economies of scope are exploited under outsourcing, by pooling of the service processes, and as long as the cost rates of the external supplier are not excessively below those faced under in-house service. More specifically, assuming the supplier operates under identical cost rates as the retailers, they benefit from outsourcing if and only if their sales volume, when pricing at cost, falls below a given threshold. This threshold is a decreasing function of the waiting time standard, i.e., the more ambitious the firms are regarding their service level, the more likely it is that the firms benefit from outsourcing. Only in the extreme case, where the retailers reap *all* the profits of the service chain - and the supplier merely breaks even ($P = 0$) - are the retailers guaranteed to benefit from outsourcing. Moreover, even under this best case scenario for the outsourcing option, the service chain may fail to be immune to unilateral defections by one or more retailers. Such defections will not occur when there are up to 3 firms in the industry. When the number of firms $N \geq 4$, defections can be avoided if a certain index which measures the intensity of the price competition and varies between 0 and 1, is above a threshold, given by a function of N alone. This threshold is increasing in N and converges to the maximum value of one as N goes to infinity. If the competitive intensity is below the threshold value, defection can be avoided if and only if the firms' demand volume, when pricing at cost, is below a critical value, which increases with the degree to which benefits of service pooling are exploited under outsourcing. In other words, in an industry with a large number of firms, there is an incentive for a single firm to leave the service chain, even under this most favorable service chain design, unless the price competition is very intense or the firm's sales volume value sufficiently small. We also show that there is always a chain size m^o, such that a service chain with m^o firms is stable in the sense that no firm outside the chain has an incentive to join the chain and no firm in the chain has an incentive to bring service in-house.

When the supplier's marginal cost rate per customer is *lower* than under in-house service, the increased benefits of outsourcing manifest themselves, for example, in that the break-even sales volume (between outsourcing and in-house service) shifts upwards as a roughly affine function of the supplier's cost rate advantage.

Similar conclusions prevail when the firms compete in terms of their waiting time standards and under simultaneous price and waiting time competition. In the absence of service pooling or the supplier enjoying lower cost rates, whether the retailers benefit from outsourcing depends on which fee combination is used, but sometimes the retailers are worse off under any of these coordinating schemes (which avoid periodic fixed transfer payments). For example, under waiting time competition, if the external supplier services the different retailers with dedicated facilities, the service chain is, even in the above best case scenario for outsourcing, *always* stable when $N \leq 3$, *never* stable when $N \geq 5$, and under $N = 4$, it is stable if and only of a similar index of the intensity of the waiting time competition is above 0.96. In contrast, if the service chain *fully* exploits the benefits of service pooling, the chain is *always* stable. More generally, under *partial* exploitation of the benefits of service pooling, there exists, as in the case of price competition, a chain size m^o, such that a chain with m^o firms is stable. We give a general condition under which this chain size is unique, and show that it only depends on the number of firms in the industry and the competitive intensity.

In §7, we identify for the general model, payment schemes for the retailers such that, under outsourcing, the service supply chain can operate at a first best level ($e = 1$), as assumed in many of the companies in §3-6. Under price competition, it suffices to charge each retailer a constant *fee per customer*, but under waiting time competition and simultaneous competition, it is necessary to add a second periodic fee which is inversely proportional to the waiting time standard requested. (We also show how the exact fee levels can be determined.) Alternatively, the second periodic fee may be set to be proportional to the capacity level which the supplier would need to adopt if he were to serve the retailers' customers in a dedicated M/M/1 facility. In the case of simultaneous competition, a unique volume based fee and capacity based fee are required to achieve first best profits but under waiting time competition, a continuum of such fee pairs may coordinate the service chain. The above fee structures may be complemented with a periodic fixed fee to be paid by the retailers to the supplier (or vice versa). Outsourcing contracts often include volume based and capacity based fees, see K'Djah.com and Jackson (1999), and Hasija et al. (2006) and Chander (2007).

In §8 we use a numerical study to demonstrate how the above findings carry over in asymmetric industries and when the demand functions are given by an attraction model. §9 completes the paper with a brief outline of how our analysis can be extended to systems with more general service processes, represented by general queueing systems.

The literature on outsourcing in service industries is recent. Aksin et al. (2006) and Gans and Zhou (2003) consider a single server retailer (e.g. call center) who can outsource part of its business to an outside supplier. In Aksin et al. (2006), both firms incur costs proportional to their selected capacity level, but the outside supplier enjoys a lower capacity cost rate. Demand in any given time interval can be satisfied if it falls below the total available capacity, with any excess being lost. The authors state that for tractability reasons they do not model the service facility as a queue. Aksin et al. (2006) consider the following two arrangements between the outside supplier and the service retailer which are closely related to the above volume- and capacity- based fees: in the *first*, the retailer buys a certain capacity level from the supplier, for which it is charged a given fee per *unit of capacity*. All demand is first directed to the supplier with any excess "overflowing" to the retailer's own facility. In the *second* arrangement, demand is first directed to the retailer's own facility while any excess is handled by the outside retailer, at a given fee per customer referred. Under this arrangement the service retailer and outside supplier choose their capacity levels non-cooperatively and the authors establish the existence of a unique Nash equilibrium. Gans and Zhou (2003), model the service facilities as queueing processes, but assume that both the capacity level of the retailer and that of the outside supplier are determined by the former. (Two customer classes are considered; outsourcing is an option for *one* of them according to a queue-size dependent strategy.)

To our knowledge, Cachon and Harker (2002) is the only paper which addresses an industry of two *competing* service retailers, modeled as M/M/1 systems. In the outsourcing part of the paper, demand rates are linear functions of the firms' full price (= price + multiple of the expected sojourn time). Moreover, the demand of a firm is equally sensitive to a change in its own price as to that of its competitor. Both firms may outsource to a common supplier, who faces the same cost structure as the retailers themselves, and who serves the customers of each retailer in a dedicated facility, for a given fee per customer. The authors show that both firms benefit when they *both* outsource, compared to when they service the customers in-house, irrespective of the fee charged by the supplier. But, if this fee is set at a sufficiently high level, one of the retailers may benefit by keeping its service process in-house, assuming the competitor continues to outsource. As mentioned, these results contrast with ours, where firms *do not necessarily* benefit from outsourcing, even when the service chain operates at a first best level ($e = 1$), unless the supplier's profit is reduced to zero ($P = 0$). Hassin and Haviv (2003) and Allon and Federgruen (2007) offer surveys of competition models in service industries. See also the nascent literature on competition models in which waiting time sensitive customers are segmented into multiple classes. e.g. Loch (1991), Lederer and Li (1997), Armony and Haviv (2001), Afeche (2004), and Allon and Federgruen (2004).

2. Model and Notation

We consider a service industry with N competing service retailers each acting as an M/M/1 facility when providing in-house service. Each firm i differentiates itself in the market by selecting a price p_i, as well as a waiting time standard w_i. The waiting time standard is defined as the expected steady state waiting time experienced by the customers, i.e. $w_i = \mathbb{E}(W_i)$. (Alternatively, the waiting time standard may be specified as a given *fractile* of the waiting time distribution. All of our results continue to apply, since the structural form of all profit functions remains unchanged.) When firm i serves its customers in-house, it faces two types of cost: first, it incurs a cost c_i per customer served, and, second, it incurs a capacity cost, at a rate γ_i per unit of capacity installed. Similarly, when the service processes are outsourced to an outside supplier, this supplier incurs the same two types of costs, at rates c_0 and γ_0 respectively. The capacity of a service facility is defined as the facility's service rate. Let μ_i, μ_0 denote the capacity levels chosen by firm i and the outside supplier respectively. Under in-house service, the following simple relationship exists between μ_i, w_i and λ_i, the firm's demand rate. Assuming $\lambda_i > 0$, we have

$$\mu_i = \lambda_i + \frac{1}{w_i} \tag{1}$$

(When $\lambda_i = 0, \mu_i = 0$ as well). The first term in this expression is the *base capacity level*, ensuring stability of the system, while the second term is the *service based capacity* required to guarantee a given waiting time standard.

If an outside supplier services the customers of the different firms in separate (dedicated) facilities, the required capacity is, therefore given by: $\mu_0^B = \sum_{i=1}^N \left(\lambda_i + \frac{1}{w_i}\right)$. Allon and Federgruen (2004) have shown that, under *pooled* service, the minimum required capacity level, considering all non-anticipating dynamic priority schemes, is given by:

$$\mu_0^P = \max_{S\subset\{1,\ldots,N\}} \left\{ \sum_{i\in S} \lambda_i + \frac{\sum_{i\in S}\lambda_i}{\sum_{i\in S}\lambda_i w_i} \right\} = \max_{S\subset\{1,\ldots,N\}} \left\{ \sum_{i\in S} \lambda_i + \frac{1}{\sum_{i\in S} \frac{\lambda_i}{\sum_{j\in S}\lambda_j} w_i} \right\} \tag{2}$$

(Note, under non-identical waiting times, it is no longer feasible to serve all customers on a FIFO basis) Proposition 1(a) ibid show that $\mu_0^P \le \mu_0^B$, reflecting *economies of scope*. (More generally, the pooling of any two groups results in cost savings.) This observation, in itself, bodes well for the increased benefits of outsourcing under pooled service versus service with dedicated facilities. The presence of *economies of scope* do not imply that the cost structure (2) exhibits *economies of scale* as well: as demonstrated in Allon and Federgruen (2004), both marginal and average costs per customer may increase if the demand volume of a *single* firm is increased.

The price p_i is chosen from an interval $[p_i^{min}, p_i^{max}], i = 1, \ldots, N$. Clearly, firm i selects a price p_i which results in a non-negative gross profit margin $p_i - c_i - \gamma_i$. (By (1), $c_i + \gamma_i$ is the marginal cost per unit of demand.) Thus, without loss of generality, we select $p_i^{min} = c_i + \gamma_i, \quad i = 1, \ldots, N$. As to p_i^{max}, it is chosen to be sufficiently large as to have no impact on the equilibrium behavior.

The demand rate λ_i may depend on all of the industry's prices and waiting time standards, according to a general set of twice differentiable functions:$\lambda_i = \lambda_i(p, w),\ i = 1, \ldots, N$, with $\frac{\partial \lambda_i}{\partial p_i} \le 0, \frac{\partial \lambda_i}{\partial w_i} \le 0$ and λ_i varying concavely with w_i, i.e. $\frac{\partial^2 \lambda_i}{\partial w_i^2} \le 0$. In our base models, we focus on the following class of demand functions which are linear in the prices: (In §8, when addressing the general model, we do consider demand functions given by attraction models.)

$$\lambda_i = \left[a_i(w_i) - \sum_{i \neq j} \alpha_{ij}(w_j) - b_i p_i + \sum_{i \neq j} \beta_{ij} p_j \right]^+, \ i = 1, \ldots, N \tag{3}$$

where $x^+ = \max(x, 0)$. a_i is a decreasing concave function, reflecting the fact that reductions of a firm's waiting time standard result in increases of its demand volume; however, these increases become progressively smaller as the waiting time standard continues to be cut. The functions α_{ij} are *general* decreasing functions, since a reduction of a competitor's waiting time standard results in a decrease of the firm's demand volume. The price coefficients $b_i > 0$, $\beta_{ij} > 0, i \neq j$ satisfy the well known dominant diagonal conditions

$$(D) \quad b_i > \sum_{j \neq i} \beta_{ij}, \qquad (D') \quad b_i > \sum_{j \neq i} \beta_{ji}, \qquad i = 1, \ldots, N$$

These conditions are generally satisfied; they merely stipulate that a *uniform* price increase by all N firms cannot result in an increase in any firm's demand volume, and that a price increase by a given firm cannot result in an increase of the industry's aggregate demand. [2]

Let $\overline{b}_i = b_i - \sum_{j \neq i} \beta_{ij} > 0$ denote the *total* price sensitivity of firm i's demand ,i.e., the (absolute value of the) marginal change in firm i's demand volume due a to uniform price increase by all firms.

In contrast, (3), in addition to enjoying analytical simplifications, specifies a firm's demand to be zero under such extreme choices. Allon and Federgruen (2007) show that under (3) the firms' *equilibrium* choices induce a positive market share for each. To guarantee that this is the case, it suffices in the Price Competition model to assume,

$$\lambda_i(c + \gamma, w) > 0, i = 1, \ldots, N, \forall w \in [0, w^{max}]^N, \tag{4}$$

i.e., any firm i can achieve a positive market share at least when willing to operate with zero variable profit margin, i.e. when $p_i = p_i^{min} = c_i + \gamma_i$. (4) guarantees that under *this* price, $\lambda_i > 0$, regardless of the competitors' choices.[3] However, other price-service level combinations may result in zero demand. In the

[2] As is well known from the literature on oligopoly models with *product differentiation*, systems of demand equations need not, but often can be obtained from one of several underlying consumer utility models, in particular the *representative consumer model*, the *random utility model* and the *address model*. Similarly, (3) may, e.g. be derived from a representative consumer model with utility function $U(\lambda, \theta) \equiv C + \frac{1}{2}\lambda^T B^{-1} \lambda + \lambda^T B^{-1} \overline{a}(w)$ where the $N \times N$ matrix B has $B_{ii} = -b_i$ and $B_{ij} = \beta_{ij}, i \neq j$, $\overline{a}(w) \equiv a_i(w_i) - \sum_{j \neq i} \alpha_{ij}(w_j)$ and $C > 0$. ((D) ensures that B^{-1} exists and is negative semi-definite, giving rise to a jointly concave utility function). The demand functions (3) arise by optimizing the utility function subject to a budget constraint.

[3] (4) reduces to lower bounds for the intercept values: $a_i(0) > \sum_{j \neq i} \alpha_{ij}(\overline{w}) + b_i(c_i + \gamma_i) - \sum_{j \neq i} \beta_{ij}(c_j + \gamma_j)$.

two remaining competition models (Waiting Time Competition and Simultaneous Competition),Allon and Federgruen (2007) show that a somewhat stronger condition is needed, namely

$$\lambda_i(p,w) > 0, \forall p \in \times_{i=1}^N \left[p_i^{min}, p_i^{max}\right], w \in [0, w^{max}]^N \text{ [4]} \tag{5}$$

3. Identical Retailers: Price Competition

We initially focus on a basic class of models with *identical* service retailers, and demand functions of the quasi-separable type (3). This allows for analytical comparisons of the equilibria and coordinating pricing schemes under each of the three types of competition and each of the various outsourcing options.

Thus, assume the firms face identical cost parameters $\{c_i, \gamma_i\}_{i=1}^N = (c, \gamma)$. Consider firm independent demand functions with price linearity as in (3), and assume that the functions $a_i(\cdot)$ and $\alpha_{ij}(\cdot)$ are affine as well. Thus, for positive constants a^0, a, α, b and β:

$$\lambda_i = \left[a^0 - aw_i + \alpha \sum_{j \neq i} w_j - bp_i + \beta \sum_{j \neq i} p_j\right]^+, i = 1, \ldots, N. \tag{6}$$

Similar to condition (D), we assume without practical loss of generality, that

$$a > (N-1)\alpha \tag{7}$$

,i.e., no firm experiences an increase in its demand volume when all firms increase their waiting time standards by the same amount. Define the intensity of the price competition by $\rho \equiv \frac{(N-1)\beta}{b}$ and the intensity of the waiting time competition by $\theta \equiv \frac{(N-1)\alpha}{a}$. Note that both ρ and θ are dimensionless indices, with $0 < \rho < 1$ and $0 < \theta < 1$, by (D) and (7). In the symmetric model, condition (5) can be replaced by a considerably weaker requirement, namely:

$$\lambda_i(p, \ldots, p; w, \ldots, w) > 0 \text{ for all } c + \gamma \leq p \leq p^{max} \text{ and } w^{min} \leq w \leq w^{max} \tag{8}$$

In other words, when the firms offer identical terms to the customers, each faces an identical and positive

[4] (5) reduces to similar lower bounds for the intercept values $a_i(0)$.

demand volume. ((8) is identical to the following lower bound for $a^0 : a^0 \geq b(1-\rho)p^{max} + a(1-\theta)w^{max}$.)

When all N firms provide *in-house* service, the profit function for firm i is, by 1, given by

$$\pi_i = \lambda_i(p_i - c) - \gamma\mu_i = \lambda_i(p_i - c) - \gamma\left(\lambda_i + \frac{1}{w_i}\right). \tag{9}$$

Assume first that the firms engage in price competition, under identical waiting time standards $w_i = w$. Theorem 1 in Allon and Federgruen (2007) establishes that, like with all price-linear demand functions, for identical as well as non-identical firms, the price competition model has a unique price equilibrium p^* which satisfies the set of linear equations

$$\lambda_i^* = b(p_i^* - c - \gamma), i = 1\ldots, N \tag{10}$$

where $\lambda_i^* = \lambda_i(p^*, w) > 0$. It is easily verified, by substitution in (6) and (9), that

$$p_i^* = p^* \equiv \frac{a^0 - w(a-(N-1)\alpha) + b(c+\gamma)}{2b-(N-1)\beta}; \quad \pi_i^* = b_i(p^* - c - \gamma)^2 - \frac{\gamma}{w}. \tag{11}$$

In particular, by (10), the model has a unique symmetric equilibrium, with $p_i^* = p^* > c + \gamma$.

We now compare the decentralized system with in-house service at the service retailers, with various outsourcing scenarios. We start with a *base* model in which a common supplier faces the *same* cost rates as the retailers encounter when providing in-house service. To put the outsourcing option in the best possible light, we assume the resulting service chain, while continuing to be decentralized, operates at maximum efficiency. The profits in a *centralized* chain, under outsourcing, are unaffected by transfer payments between the retailers and the supplier, and are given by $\Pi(p) = \sum_{i=1}^N \lambda_i p_i - c\sum_{i=1}^N \lambda_i - \gamma\left(\sum_{i=1}^N \lambda_i + \frac{\nu}{w}\right)$. Here $1 \leq \nu \leq N$ represents an inefficiency index. The extreme values $\nu = 1$ and $\nu = N$ arise when the suppliers exploit economies of scope maximally, or not at all. When serving the customers of different retailers in dedicated facilities, $\nu = N$, see (1); when pooling all service processes into a *single* facility, $\nu = 1$. (When the same waiting time standards and the service time distributions apply to all customers, there is no advantage to give priority to some firms over others. i.e., customers are optimally served on a FIFO basis and the supplier's service process is an M/M/1 system with arrival rate $\sum_{i=1}^{N}\lambda_i$. The capacity related cost is thus given by $\gamma\left(\sum_{i=1}^{N}\lambda_i + \frac{1}{w}\right)$.) More generally, the potential for economies of scale may only be exercised in part, giving rise to an intermediate value for the inefficiency index.

The function Π is a strictly jointly concave quadratic function of p. (In its Hessian, the diagonal elements equal $-2b$, while the off-diagonal elements equal β. This matrix is negative semi-definite ,since by (D), the absolute value of the diagonal element dominates the sum of the absolute values of the off diagonal elements in each row.) Thus, the *unique* optimal price vector p^{CB} is the one that satisfies the set of first order conditions:

$$-b(p_i - c - \gamma) + \lambda_i + \beta\sum_{j\neq i}(p_j - c - \gamma) = 0, \tag{12}$$

This system of equations has a *symmetric* solution, p^{CB}, which is independent for

$$p_i^{CB} = p^{CB} \equiv \frac{a^0 - w(a-(N-1)\alpha)}{2(b-(N-1)\beta)} + \frac{c+\gamma}{2} = \frac{a^0 - w(a-(N-1)\alpha)}{2\overline{b}} + \frac{c+\gamma}{2}. \tag{13}$$

Let $\lambda^{CB} = \lambda(p^{CB}, w)$ and $\Pi^{CB} = \Pi(p^{CB})$ denotes the *optimal* chain-wide profits in the service chain. Note that p^*, p^{CB}, λ^* and λ^{CB} are all independent of the inefficiency index. Proposition 3.1 shows that each firm's optimal price level is higher and its demand volume lower than under in-sourcing.

Proposition 3.1*(Comparing decentralized system vs centralized system) Assume (4). (a) $p^* < p^{CB}$; (b) $\lambda^* > \lambda^{CB}$, (c) For a fixed value of $\overline{b}$, the total price sensitivity of demand, Π^{CB} is independent of ρ, while, under in-house service, each retailer's profits decreases with ρ: $\pi_i^* = \frac{1}{b}\left([a^0 - w(a-(N-1)\alpha)] - (c+\gamma)\overline{b}\right)^2 \frac{1-\rho}{(2-\rho)^2}$.*

As shown in §7 the service chain, under outsourcing, may achieve first best level aggregate profits ($e = 1$), whenever a specific per customer fee c^{WB} is charged to each retailer. The retailers' share of these profits depends on whether a fixed per period payment or subsidy K is added to the volume based fee. Hasija et al. (2006)'s survey shows that such fixed payments are added to some of the contracts, but not to others. In the next theorem, we therefore consider three potential values for a fixed payment K: (i) K=0, (ii) K is set to compensate the supplier for the cost of the service based capacity, and (iii) K is set to ensure that the supplier merely breaks even. We are particularly interested in (iii) as it reflects a *best case scenario* for outsourcing: a maximally efficient service chain in which the retailers' share of the profits is maximized.

Theorem 3.1*(Profit comparisons between in-house service and outsourcing) Assume the service chain under outsourcing achieves first best aggregate profits. For sufficiently large w_i^{max}:*

(a) $\sum_{i=0}^{N} \pi_i^{*OB} \geq \sum_{i=1}^{N} \pi_i^*$ regardless of what fixed transfer payments are used. Under any fixed payment K, π_i^{*OB} is independent of the supplier's inefficiency index ν, $\forall i = 1, \ldots, N$. π_0^{*OB} decreases with ν.

(b) In the absence of fixed transfer payments $(K = 0)$

(i) $\pi_i^{*OB} \geq \pi_i^*$ iff $\lambda(c+\gamma, w) \leq \lambda^0 = \frac{2-\rho}{\sqrt{1-\rho}\sqrt{4\rho-\rho^2}} \sqrt{\frac{4\gamma\bar{b}}{w}}$

(ii) $\pi_0^{*OP} \leq 0$ for w sufficiently small, i.e., $w < \underline{w}(\nu)$, with $\underline{w}(\nu)$ increasing in ν.

(c) Assume $K = \frac{\gamma}{w}$, i.e., each retailer compensates the supplier periodically for the service based capacity he imposes (if served by herself): $\pi_0^{*OB} > 0$, $\pi_i^{*OB} \leq \pi_i^*$ irrespective of the waiting time standard w.

(d) Assume the fixed payment K is set to ensure that the supplier breaks even. Then $0 \leq \pi_i^{*OB}(\nu) - \pi_i^* = \frac{\lambda^2(c+\gamma,w)}{\bar{b}} \frac{\rho^2}{4(2-\rho)^2} + \frac{(N-\nu)\gamma}{Nw}$

Thus, without fixed transfer payments, whether or not the retail firms benefit from outsourcing depends on the given waiting time standard. For any given value of w, outsourcing is beneficial if and only if the firms' demand volume, when pricing at cost $(p = c+\gamma)$, is less than a given break even value, and this break-even volume is inversely proportional with the square-root of the waiting time standard. Outsourcing is beneficial only when the demand volume falls below a given break-even point, since the firm's cost function , under in house service, is affine with a per unit cost given by $c+\gamma+\left(\frac{\gamma}{w}\right)\frac{1}{\lambda}$. In other words, the *average* cost function exhibits *economies of scale*, even though the marginal cost is constant. In addition, the smaller the waiting time standard w, the more pronounced the economies of scale are, and the larger the break even volume. In the absence of fixed transfer payments, the break even volume is independent of the supplier's inefficiency index ν.

If the coordinating volume based fee c^{WB} is complemented with a periodic fee $K = \frac{\gamma}{w}$, i.e., the cost of the service based capacity which the retailer imposes on the supplier, if served by himself, the retailers are always worse off under outsourcing, *irrespective* of their demand volume. However, Theorem 3.1(d) shows that there are payment schemes under which all retailers benefit from outsourcing and the supplier breaks at least even. The expression, there, denotes the *maximum* benefit retailers can reap when outsourcing, i.e., assuming the service chain, under outsourcing, is maximally efficient *and* the retailers are able to squeeze the supplier's profits down to zero. The expression for these maximum benefits consists of two parts. The first term represents the benefits, if the supplier makes *no* use of the potential for service pooling, while the second term denotes the additional benefits that accrue when the supplier applies full or partial service pooling. The benefits increase as the competitive intensity ρ increases, for a fixed simple price sensitivity b, and even, for a fixed *total* price sensitivity $\overline{b}$ and hence for a fixed demand volume, when pricing at cost. They grow in proportion to the *square* of this demand volume. Not surprisingly, the more efficient the supplier, the larger the maximum benefits associated wuth outsourcing: indeed the maximum benefits depend linearly on the inefficiency index ν.

As mentioned in the introduction, beyond the outside supplier's ability to exploit economies of scope by pooling the service processes for the different retail firms, a *second* driver behind the benefits of outsourcing often results from the supplier's ability to operate with lower cost rates than the retailers. (The service process often represents the supplier's core competency rather than that of the retailers.) We refer to the on-line Appendix for Theorem A.1, which generalizes Theorem 4.1 to allow for arbitrary cost differentials between the supplier and the retailers. (To simplify the exposition, we confine ourselves, there, to the case where the supplier uses a dedicated facility for each of the retailers. All results can be extended to the general case with a general efficiency index ν.) Theorem 4.1 showed that in the base outsourcing model without fixed transfer payments, the retailing firms are better off outsourcing iff their demand volume, when pricing at cost, is below a critical value. The same characterization applies to the case where the supplier enjoys lower cost rates. The break even value for this demand volume increases with Δ, the differential in the total cost rate enjoyed by the supplier. The break even value is also decreasing in the waiting time standard w. Thus, the higher the service level, the higher the minimum demand under which a retailer prefers preforming service in-house. While this phenomenon occurs in the base model (see Theorem 1(b)), it is all the more pronounced when the supplier operates under lower cost rates.

4. Price Competition: Stability of Service Chain

Even if all retailers benefit from *collective* outsourcing compared to all of them providing in-house service, it is not clear whether the service chain, under outsourcing, is immune to defections by individual retailers, or, whether collective outsourcing arises as an equilibrium, when each firm has an upfront choice whether to outsource or to keep service in-house. This applies even when outsourcing is portrayed in the best possible light, i.e., the service chain achieves first best level profits, all of which are earned by the retailers.

Thus, consider the following two-stage game: in the first stage each retailer decides whether to outsource the service process or to perform it in-house. For the firms who opt for outsourcing, the resulting chain is assumed to guarantee maximum possible profits for its participating retailers, i.e., it operates under a perfectly coordinating pricing scheme with fixed transfer payments, reducing the supplier's profits to zero. In the second stage game, the service chain competes with each of the retailers that have chosen to perform the service in-house, once again ensuring the firms inside the chain of first-best level profits, given the competition of the firms outside the chain.

Theorem 4.1 characterizes under what conditions the solution where all retailers decide to outsource in the first stage, is a Sub game Perfect Nash Equilibrium (SPNE).

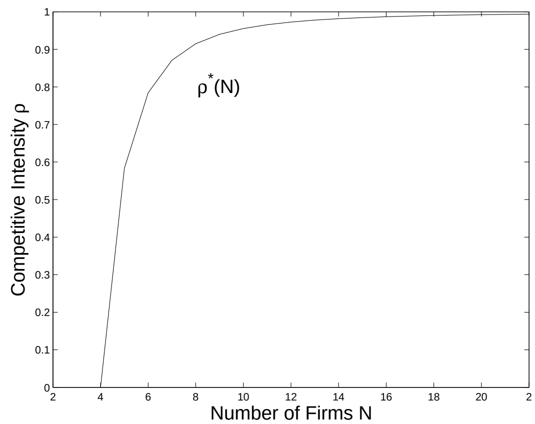
Theorem 4.1*(Sustainability of service chains under price competition)*

(a) Let $0 \leq \rho^*(N) \equiv -2(N^2-4N+5)+2\sqrt{(N^2-4N+5)^2+(N-1)(N-3)} \leq 1$, *and* $\widehat{\lambda}(\rho,N,\nu) \equiv \sqrt{\frac{\gamma}{w}\left(1-\frac{\nu}{N}\right)}\left[\overline{b}\left(\frac{2\overline{b}+\frac{N+1}{N-1}\rho}{4\frac{\overline{b}^2}{\sqrt{1-\rho}}-\rho^2\frac{\sqrt{1-\rho}}{N-1}}\right)^2-\frac{1}{4\overline{b}}\right]^{-\frac{1}{2}}$*. The solution where all firms decide to outsource in stage one and adopt the price vector* p^{CB} *in stage two is a SPNE if and only if one of the following three conditions is satisfied: (i)* $N \leq 3$, *(ii)* $N \geq 4$, $\rho \geq \rho^*(N)$ *(iii)* $N \geq 4, \rho \leq \rho^*(N)$ *and* $\lambda(c+\gamma,w) \leq \widehat{\lambda}(\rho,N,\nu)$

(b) $\rho^*(N)$ *is non-decreasing in* N *with* $\lim_{N\uparrow\infty}\rho^*(N)=1$.

As mentioned, the profit values in Theorem 4.1(d) is arguably the best a participating service retailer can hope for, as it maximizes chain-wide profits while preserving all such profits for the service retailers and reducing those of the outside supplier to zero. When $N \leq 3$, the service chain, under outsourcing, is immune to defections. However, when $N \geq 4$ and the supplier fails to exploit economies of scope ($\nu = N$),

Figure 1 **Stability of the Service Chain**



an individual service retailer has an incentive to leave the chain and bring the service process in-house if and only if the competitive intensity is below the critical value $\rho^*(N)$. In other words, whether the chain is stable or not depends on whether the pair of industry characteristics (N,ρ) lies below or above the switching curve $\{\rho^*(N) : N = 2, 3, \ldots\}$ depicted in Figure 1. (Define $\rho^*(2) = \rho^*(3) = 0$.) This phenomenon occurs because the unmitigated competition a firm faces when defecting, acts less as a deterrent when the competitive intensity is lower. Cachon and Harker (2002), in their duopoly model, established the existence of a linear wholesale pricing scheme under which both service retailers benefit from outsourcing, the outside supplier earns a profit and the chain is immune for defections. Theorem 4.1 shows that stability of the chain, while guaranteed under a small number of competitors, $N \leq 3$, becomes increasingly more difficult to achieve as the number of competing firms increases.

The reason defection from the service chain may be beneficial is that the chain adopts a significantly larger price than, say, the price p^* in a decentralized system so as to drive the profits of the participating retailers to their maximal level. A defecting retailer, may, when $\rho < \rho^*(N)$ exploit this by adopting a significantly lower price and thus attracting a significantly larger market share. Paradoxically, the chain can prevent defections, by inducing the participating retailers to adopt the price p^* instead of p^{CB}, even though the resulting profits for the retailers are lower than under the coordination scheme of Theorem 4.1(d) (As shown in §7, the chain may induce the retailers to adopt *any* desired price $\geq c + \gamma$, by charging a specific volume based fee c^W.)

Stability of the service chain under outsourcing is enhanced to the extent the supplier exploits the benefits of service pooling: when $\nu < N$, the chain remains stable, even when $N \geq 4$, and $\rho \leq \rho^*(N)$ as long as the sales volume, when pricing at cost, is below a given threshold value $\widehat{\lambda}(\rho, N, \nu)$. The more efficient the supplier, the lower the inefficiency index ν, and the larger the threshold $\widehat{\lambda}$. Similarly, the stricter the waiting time standard or the larger the cost of capacity γ, the larger the sales volume threshold to ensure stability, when the supplier applies partial of complete service pooling.

Finally, it is easily verified that any cost rate advantages for the supplier, see Appendix B, reduce the potential for defections of individual firms from a service chain with outsourcing: when $N \leq 3$ or $N \geq 4$ and $\rho > \rho^*(N)$, such defections are precluded in the base model and continue to be precluded if the supplier enjoys cost rate advantages. At the same time, the stability of the service chain continues to prevail under such cost rate advantages, even when $N \geq 4$ and $\rho \leq \rho^*$ provided equilibrium demands are not too large.

Theorem 5.1 identifies the necessary and sufficient conditions for *universal* outsourcing, i.e., for a service chain with all N firms, to arise as a SPNE in the two-stage game. We now address the more general question, when a Nash equilibrium arises with m^i firms maintaining in house service, and the remaining $m^o = N - m^i$ firms outsourcing their service processes, for any $0 \leq m^i \leq N-1$. In other words, when is a service chain with m^o participating firms stable, in the sense that none of the firms inside the chain has an incentive to bring the service process in house, while any of the firms providing in house service incurs a *strict* profit loss by outsourcing and joining the service chain.[5] The following Theorem shows that a (pure) SPNE exists, for at least one value of m^o, and it provides a sufficient condition for m^o, the *size* of the outsourced service chain, to be *unique* among all Nash equilibria. (Clearly, if a Nash equilibrium exists with $1 \leq m^o \leq N$ firms in the chain, any of the $\binom{N^o}{m^o}$ nets of m^o firms among the industry of N, gives rise to a Nash equilibrium.) We have observed that, except when the number of firms in the industry is very small ($N \leq 3$), or when the competitive intensity ρ is very large, equilibria arise with *partial* outsourcing, i.e. $1 < m^o < N$; this, even in an industry where all firms have identical characteristics. Let

$$\pi^o(i, m^o)[\pi^o(o, m^o)] = \text{the profit of a firm currently in a service chain with } m^o \text{ firms,}$$
$$\text{when switching to in-house service [when continuing to outsource].}$$

[5] This asymmetric definition favors the outsourcing option, in case of a tie.

$\pi^i(i,m^i)[\pi^i(o,m^i)] =$ the profit of a firm, currently among m^i firms providing in house service, when continuing to do so [when switching to outsource].

Note, $\pi^o(i,m^o) = \pi^i(i,N-m^o+1)$; $\pi^o(o,m^o) = \pi^i(o,N-m^o+1)$, $\forall\ 1 \leq m^o \leq N$. A SPNE, with m^o firms outsourcing, exists iff

$$\pi^o(o,m^o) \geq \pi^o(i,m^o) \text{ and } \pi^i(i,N-m^o) > \pi(o,N-m^o) \tag{14}$$

Finally, let X be a lattice with partial order $\succcurlyeq$ and T an ordered set. Milgrom and Shannon (1994) define a function $f: X \times T \to \mathbb{R}$ as having the strict *single crossing point property* if for all $x \succeq x'$ and $t \in T$, $f(x,t) > f(x',t) \Rightarrow f(x,t') > f(x',t')$ if for all $t' \geq t$.

Theorem 4.2 *(a) The two stage outsourcing game has a (pure) SPNE*

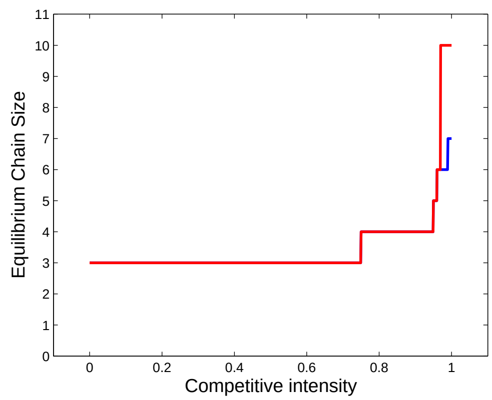
(b) Assume the function $\pi^o : X \times \mathbb{N}_0 \to \mathbb{R}$ with $X = \{i,o\}$ and $i \succeq o$ has the strict single crossing point property. Then, a SPNE exists for a single value of outsourcing firms m^o, which decreases with the inefficiency index ν.

Thus, under the single crossing point property, the more the service supply chain is able to exploit the benefits of service pooling, the larger a service chain arises in the two stage outsourcing game. We now show that the size m^o of a stable outsourcing chain, depends critically on N, the number of firms in the industry, and the competitive intensity ρ, similar to the stability conditions for universal outsourcing ($m^o = N$) in Theorem 5.3. To do so, we confine ourselves to the case where the common supplier services each of the participating firms in a dedicated facility. In this case, the condition for a stable chain with m^o firms reduces to a pair of inequalities, merely involving m^o, N, and ρ.

Theorem 4.3*Assume the supplier in the service chain serves each of the participating firms in a dedicated facility. In the two stage game, an SPNE with m^o outsourcing firm arises iff*

$$\left[\frac{2\left(1+\frac{\rho}{N+1}\right)-\frac{\rho}{N-1}(N-m)}{2\left(1-(N-m-1)\frac{\rho}{N-1}\right)\left(2-(m-1)\frac{\rho}{N-1}\right)-(N-m)m\left(\frac{\rho}{N-1}\right)^2}\right]^2 > \tag{15}$$

Figure 2 **Stability of the Service Chain**



$$\left(1-(N-m)\frac{\rho}{N-1}\right)\left[\frac{2+\frac{\rho}{N-1}}{2\left(1-(N-m)\frac{\rho}{N-1}\right)\left(2-(m-2)\frac{\rho}{N-1}\right)-\left(\frac{\rho}{N-1}\right)^2(m-1)(N-m+1)}\right]^2$$

$$\left[\frac{2\left(1+\frac{\rho}{N+1}\right)-\frac{\rho}{N-1}(N-m-1)}{2\left(1-(N-m-2)\frac{\rho}{N-1}\right)\left(2-m\frac{\rho}{N-1}\right)-(N-m-1)(m+1)\left(\frac{\rho}{N-1}\right)^2}\right]^2 < \qquad (16)$$

$$\left(1-(N-m-1)\frac{\rho}{N-1}\right)\left[\frac{2+\frac{\rho}{N-1}}{2\left(1-(N-m-1)\frac{\rho}{N-1}\right)\left(2-(m-1)\frac{\rho}{N-1}\right)-\left(\frac{\rho}{N-1}\right)^2 m(N-m)}\right]^2$$

We have observed that for almost all values of N and ρ, the function π^o has the single crossing point property so that the equilibrium size of the service chain is unique. However, for some very high competitive intensities, the property may fail to hold, and multiple equilibrium chain sizes may arise. Figure 2 shows for an industry with $N = 10$ firms, how the equilibrium value(s) of m^o vary as a function of ρ; even under a low competitive intensity, $m^{o^*} = 3$; as ρ increases so does m^{o^*}. When $\rho \geq \rho^*(N)$, a chain with $m^o = N$ firms is stable, but so is one with $m^o = 6$ firms.

5. Identical Retailers: Waiting Time Competition

In this section we assess the benefits of outsourcing when the retailers compete by selecting their waiting time standards, under a given, exogenously specified, price level p.

Corollary 2 in Allon and Federgruen (2007) establishes for general systems with possibly non-identical retailers, that a unique equilibrium vector w^* exists. In our case, this equilibrium satisfies the first order conditions $\frac{\partial \pi_i}{\partial w_i} = -a(p-c-\gamma) + \frac{\gamma}{w_i^2} = 0$, i.e. $w_1^* = \cdots = w_N^* = \sqrt{\frac{\gamma}{a(p-c-\gamma)}}$. Allon and Federgruen (2007) show, in fact, that this equilibrium is a *dominant* solution, i.e. it is optimal for each firm i to adopt this waiting time standard regardless of the choices made by its competitors.

In comparing the case of in-house service with the equilibrium under outsourcing we portray the latter again in the best possible light, i.e., we assume the service chain under outsourcing operates in the most efficient possible way. We first need to following Lemma:

Lemma 5.1*A service chain achieves first-best performance, when all firms adopt a common waiting time standard* w^{CB}*, both when the supplier serves the different firms in dedicated facilities and when it pools the service process. In the former case* $w^{CB} = \frac{1}{\sqrt{1-\theta}} w^* = \sqrt{\frac{1}{1-\theta}\frac{\gamma}{a(p-c-\gamma)}}$*, in the latter case* $w^{CB} = \frac{1}{\sqrt{N}}\frac{1}{\sqrt{1-\theta}} w^* = \sqrt{\frac{1}{N(1-\theta)}\frac{\gamma}{a(p-c-\gamma)}}$

Thus, aggregate profits in the service chain, under an optimal (common) waiting time standard w, are given by $\Pi^{CB}(w) = (p-c-\gamma)\sum_{i=1}^N \lambda_i - \gamma\frac{N}{w}$ when dedicated facilities are used, and by $\Pi^{CB}(w) = (p-c-\gamma)\sum_{i=1}^N \lambda_i - \frac{\gamma}{w}$, under service pooling. As in the case of price competition, we give a unified treatment to both settings, as well as intermediate ones with *partial* pooling, giving rise to profit function $\Pi^{CB}(w) = (p-c-\gamma)\sum_{i=1}^N \lambda_i - \gamma\frac{\nu}{w}$, for a general inefficiency index $1 \le \nu(N) \le N$ with a corresponding optimal waiting time standard $w^{CB} = \sqrt{\frac{\gamma\nu(N)}{N(1-\theta)a(p-c-\gamma)}}$. Thus, $w^{CB} \le w^*$ iff $\theta \le 1 - \frac{\nu}{N}$ and w^{CB} increases in proportion to the square root of the inefficiency index. In §7, we show that the service chain can achieve first best level aggregate profits under a continuum of volume based and capacity based fee pairs. We also show how the retailers' and the supplier's profits are affected by the choice of a coordinating fee pair.

We now show that, even when the retailers reap maximum benefits from outsourcing the service chain may fail to be immune to defections. Even more pronounced than in the case of price competition, the ability of the supplier in a service chain to exploit the benefits of service pooling , has fundamental implications for the stability of the chain: without service pooling the chain is stable only when $N \le 4$ (and for $N = 4$ only when the competitive intensity $\theta \ge 0.96$). Under full service pooling, the chain is *always* stable.

Theorem 5.2*(Sustainability of service chains under waiting time competition)* *(a) The service chain is stable, i.e., the solution where all firms decide to outsource in stage one of the outsourcing game, and adopt the waiting time standard w^{CB} in stage 2 is an SPNE iff*

$$\frac{\theta\sqrt{\nu(N-2)}}{\sqrt{(N-1)-(N-2)\theta}}+2\sqrt{\frac{(1-\theta)\nu(N)}{N}}\leq 2 \tag{17}$$

In particular, stability is enhanced whenever the inefficiency index $\nu(\cdot)$ is decreased.

(b) With dedicated facilities $(\nu = N)$, the chain is stable if $N \leq 3$ or $N = 4$ and $\theta \geq 0.96$. The chain is never stable when $N \geq 5$.

(c) With full service pooling $(\nu = 1)$, the chain is always stable.

Finally, it is easily verified, along the lines of the above proof, that for example with dedicated facilities, an SPNE exists with m^o outsourcing firms iff $\theta\frac{N-m^o}{N-1}\sqrt{\frac{1}{1-\theta\frac{N-m^o-1}{N-1}}}+2\sqrt{1-\theta\frac{N-m^o}{N-1}}>2$ and $2>\theta\frac{N-m^o-1}{N-1}\sqrt{\frac{1}{1-\theta\frac{N-m^o-2}{N-1}}}+2\sqrt{1-\theta\frac{N-m^o-2}{N-1}}$

In general, Theorem 4.2 continues to apply, i.e. a pure SPNE always exists in the two-stage outsourcing game. It induces a *unique* chain size m^o if the function π^o has the single point crossing property.

6. Identical Retailers: Simultaneous Competition

When the retailers compete simultaneously in terms of their prices and waiting times, the benefits of outsourcing are somewhat more difficult to assess. For one, a sufficient condition is required to ensure that a decentralized system has a unique equilibrium which is symmetric and that the centralized system has a symmetric optimal solution as well. We start with the case where an outside supplier serves the customers of each firm in a dedicated facility. For $i = 1, \ldots, N$, let $\pi_i^*(p, w)$ denote the equilibrium profit for retailer i, under the price vector p and the vector of waiting time standards w, and let $\Pi^{CB}(p, w)$ denote the optimal aggregate profit of the service chain under outsourcing.

Theorem 6.1[6] *(Comparison of price and waiting time choices, with and without outsourcing, under simultaneous competition) There exists a value $\underline{\gamma}$ such that for all $\gamma \geq \underline{\gamma}$:*

[6] The parameter conditions are overly restrictive. They provide guarantees under the simplest verifiable condition that the Nash equilibrium under in-house service and the optimal solution under outsourcing are symmetric

(a) Assume $2b > a + (N-1)(\alpha+\beta)$. There exists a unique equilibrium (p^*, w^*) in the decentralized system, which is symmetric, i.e. $p_1^* = \cdots = p_N^*$ and $w_1^* = \cdots = w_N^*$, where

$$p^* = \frac{a^0 - w^* a(1-\theta) + b(c+\gamma)}{b(2-\rho)}$$
$$w^* = \begin{cases} \text{the unique root on } [0, w^{max}] \text{ of } C^*(w) \equiv w^3 - w^2 \frac{a^0-(c+\gamma)b(1-\rho)}{a(1-\theta)} + \frac{\gamma b(2-\rho)}{a^2(1-\theta)}, \\ \qquad if \gamma \le \frac{a^2(1-\theta)}{b(2-\rho)} (w^{max})^2 \left[\frac{a^0-(c+\gamma)b(1-\rho)}{a(1-\theta)} - w^{max}\right] \\ w^{max}, \qquad \text{otherwise} \end{cases}$$

The retailers' equilibrium sales volume. $\lambda^* = \frac{b\gamma}{a(w^*)^2}$ if $w^* < w^{max}$ and $\lambda^* \le \frac{b\gamma}{a(w^*)^2}$ if $w^* = w^{max}$.

(b) Assume $2b > a + (N-1)\alpha + 2(N-1)\beta$. There exists a unique optimal solution (p^{CB}, w^{CB}), in the centralized problem, which is symmetric, i.e. $p_1^{CB} = \cdots = p_N^{CB}$ and $w_1^{CB} = \cdots = w_N^{CB}$, where

$$p^{CB} = \frac{a^0 - w^{CB} a(1-\theta)}{2b(1-\rho)} + \frac{c+\gamma}{2}$$
$$w^{CB} = \begin{cases} \text{the unique root on } [0, w^{max}] \text{ of } C^C(w) \equiv (w^{CB})^3 - (w^{CB})^2 \frac{a^0-(c+\gamma)b(1-\rho)}{a(1-\theta)} + \frac{2\gamma b(1-\rho)}{a^2(1-\theta)^2}, \\ \qquad if \gamma \le \underline{\gamma} \equiv \frac{a^2(1-\theta)}{b(2-\rho)} (w^{max})^2 \left[\frac{a^0-(c+\gamma)b(1-\rho)}{a(1-\theta)} - w^{max}\right] \\ w^{max}, \qquad \text{otherwise} \end{cases}$$

Let λ^{CB} denote the retailers' optimal sales volume, in the centralized problem. Then $\lambda^{CB} = \frac{\gamma b(1-\rho)}{a(1-\theta)(w^{CB})^2}$ if $w^{CB} < w^{max}$ and $\lambda^{CB} < \frac{\gamma b(1-\rho)}{a(1-\theta)(w^{CB})^2}$ if $w^{CB} = w^{max}$

(c) Assume $2b > a + (N-1)\alpha + 2(N-1)\beta$. If $\theta \ge [<] \frac{\rho}{2-\rho}$ then $w^{CB} \ge [\le] w^*$.

(d) Assume $2b > a + (N-1)\alpha + 2(N-1)\beta$. If $p^{CB} \le p^*$, then $w^{CB} \ge w^*$, and hence $\theta \ge \frac{\rho}{2-\rho}$.

(e) If $\rho = \theta$, $w^{CB} \ge w^*$ and $\lambda^{CB} \le \lambda^*$.

If the cubic functions $C^*(w)$ and $C^C(w)$ have a root, this root can, of course, be solved in closed form, see e.g, Abramovitz and Stegun (1965). Contrary to the waiting time competition model under a *given* price level p, it is no longer certain that the firms choose a lower waiting time standard when providing in-house service, as compared to when they outsource. However, if the intensity of the waiting time competition θ is sufficiently large compared to the intensity of the price competition (as specified by the condition $\theta \ge \frac{\rho}{2-\rho}$), and, in particular when $\theta \ge \rho$ this will be the case: the larger the price competitive intensity ρ, the smaller the minimal waiting time competitive intensity $\underline{\theta}$ for which outsourcing results in lower service, i.e., a higher waiting time. Part (d) shows that it is impossible for both prices *and* the waiting time standards under outsourcing to be *lower* than those selected under in-house service.

To compare the firms' profits with and without outsourcing, we consider again the best case scenario for the outsourcing option, i.e. where the service chain maximizes its aggregate profits by implementing a perfect coordination scheme. As will be shown in §7, a two-part scheme, with a (volume based) fee per customer served and a second (capacity based) fee, again, suffices to achieve perfect coordination. Clearly, the aggregate chain wide profit under outsourcing is higher than that obtained by the retailers when providing in house service, i.e. $\sum_{i=0}^{N} \pi_i^{*OB} - \sum_{i=1}^{N} \pi_i^* \geq 0$ and the aggregate benefits of outsourcing, can be allocated arbitrarily between the supplier and the retailers when adding a periodic fee (K) to the supplier, in conjunction with the above two-part payment scheme. Assuming no such fixed fees are used $(K = 0)$, and similar to our results for Price competition and Waiting Time competition, retailers do not always benefit from outsourcing:

Proposition 6.1*(Profit comparison between in-house service and outsourcing with dedicated facilities under simultaneous competition) Assume $w^*, w^{CB} < w^{max}$. Under the conditions of Theorem 6.1 and in the absence of periodic transfer payments $(K = 0)$:*

(a) $\pi_i^{*OB} - \pi_i^* \geq 0$ *iff* $\frac{(\lambda^{CB})^2}{b(1-\rho)} - \sqrt{\frac{1-\theta}{1-\rho}}\sqrt{\frac{\lambda^{CB} a\gamma}{b}} \geq \frac{(\lambda^*)^2}{b} - \sqrt{\frac{\lambda^* a\gamma}{b}}$

(b) Assume $\theta = \frac{\rho}{2-\rho}$, $\pi_i^{*OB} - \pi_i^* \geq 0$ *iff* $\rho \geq \frac{2}{3}$ *(and* $\theta \geq 0.5$*)*

In the case of Price Competition, Theorem 4.1(b) reveals that the retailers benefit from outsourcing if and only if the demand volume (when pricing at cost) falls below a given threshold. Under Simultaneous Competition, the necessary and sufficient condition does not reduce to a simple upper bound for *this* demand volume. For general combinations of (θ, ρ), whether outsourcing is beneficial for the retailers or not, depends both on the magnitude of the retailers' sales volume (under outsourcing) and how much in additional sales they can realize under in-house service. The special case where $\theta = \frac{\rho}{2-\rho}$ shows that outsourcing is more likely to be beneficial for the retailers when the intensity of the competition (measured by θ and ρ) increases; in this special case, outsourcing is beneficial if and only if $\rho \geq \frac{2}{3}$ and $\theta \geq 0.5$. This finding confirms, once again, that, in the absence of service pooling or cost rate advantages for the supplier, the main benefit of outsourcing arises from the ability to induce the retailers to adopt a first best solution.

As in the case where the firms compete along one dimension only, the service chain fails, in general, to be immune to defections, even when *all* of the benefits of outsourcing are assigned to the retailers. When $N = 2$, the chain is always immune to defections, since, after a defection the system returns to the original decentralized system and the defecting firm earns $\pi_i^* \leq \frac{1}{2}\sum_{i=0}^{2} \pi_i^{*OB}$. In the special where $\theta = \rho$, and $\frac{\alpha}{a} = \frac{\beta}{b}$, the demand model (6) reduces to a full price model, i.e. consumers aggregate each firm's price and waiting time standard into a *single*, so called full price measure $F_i = p_i + kw_i$, so that all demand volumes can be expressed as functions of the vector F only: $\lambda_i = a^0 - bF_i + \sum_{j\neq i} \beta F_j \; i = 1, \ldots, N$ (Here $k = \frac{a}{b}$). Cachon and Harker (2002) addressed the benefits of outsourcing for this specification, in an industry with $N = 2$ firms and $\beta = b$ i.e. $\theta = \rho = 1$. In stark contrast to our results, which do not include this limiting value, but cover a much larger parameter space, the authors conclude that outsourcing is *always* beneficial to the retailers (The condition in part (a) is indeed satisfied when $\rho = \theta \uparrow 1$. They also claim that the service chain is always immune to defections. As to the latter, we obtain the same conclusions when $N = 2$. For $N \geq 3$, the necessary and sufficient conditions for the service chain to be immune to defections no longer reduces to a single condition in terms of the measure(s) of competitive intensity and the number of firms in the industry. We have observed, however, that the chain is more likely to be immune when θ or ρ increases, i.e. when the competitive intensities act as deterrents for defections. Example 1 in Appendix A exhibits this phenomenon. As in the case of one-dimensional (Price or Waiting Time) Competition, the benefits of outsourcing are significantly larger, if the outside supplier is able to benefit from lower cost rates or by pooling the service processes of the individual firms. Pursuing the latter option, the cost saving from service pooling make it more likely that the waiting time standard w^{CP} and price p^{CP} adopted under outsourcing are lower than their counterparts w^* and p^*, under in-house service. Following the proof of Proposition 2, the profit function $\Pi^{CP}(p, w) = \sum_{i=1}^{N} \lambda_i(p_i - c - \gamma) - \gamma\frac{\sum_{i=1}^{N}\lambda_i}{\sum_{i=1}^{N}\lambda_i w_i}$. Following the proof of Theorem 1 in Allon and Federgruen (2004), it can be shown that the function is jointly concave in (p, w) if the demand values $\{\lambda_i\}$ are sufficiently large on the feasible price/waiting time space. Also, $\frac{\partial \Pi^{CP}}{\partial w_i} =$

$-a(p_i - c - \gamma) + \alpha \sum_{j \neq i}(p_j - c - \gamma) - \gamma \frac{(\alpha(N-1)-a)\sum_{j=1}^N(\lambda_j w_j) - (\sum_{j=1}^N \lambda_j)(\sum_{j=1}^N a w_j - w_i \lambda_i)}{(\sum_{j=1}^N)^2}$,, and $\frac{\partial \Pi^{CP}}{\partial p_i} = \lambda_i - b(p_i - c - \gamma) + \beta \sum_{j \neq i}(p_j - c - \gamma) - \gamma \frac{(\beta(N-1)-b)\sum_{j=1}^N(\lambda_j w_j) - (\sum_{j=1}^N \lambda_j)(\sum_{j=1}^N a w_j - w_i \lambda_i)}{(\sum_{j=1}^N)^2}$ Since Π^{CP} is jointly concave. it has a symmetric maximum with $p_1^{CP} = \cdots = p_N^{CP} = p^{CP}$ and $w_1^{CP} = \cdots = w_N^{CP} = w^{CP}$. Substituting these identities, we obtain that (p^{CP}, w^{CP}) is the *unique* solution to the system of equations: $-(a-(N-1)\alpha)(p-c-\gamma) + \frac{\gamma}{Nw^2} = 0$,, and $-(b-(N-1)\beta)(p-c-\gamma) + \lambda = 0$.

Eliminating p, we obtain, as in the proof of Theorem 6.1, that w^{CP} is the *unique* root of the cubic equation $w^3 - w^2 \frac{(a^0-(c+\gamma))(b-(N-1)\beta)}{a-(N-1)\alpha} + \frac{2\gamma(b-(N-1)\beta)}{N(a-(N-1)\alpha)^2} = 0$, unless $\gamma > N\underline{\gamma}$, in which case $w^{CP} = w^{max}$. We conclude that $w^{CP} \leq w^*$ iff $\theta \leq 1 - \frac{2(1-\rho)}{N(2-\rho)}$. Thus, for given competitive intensity values θ and ρ, it is increasingly likely that this condition is satisfied as the number of firms in the industry increases. It is also more likely, though still not guaranteed, that the retailers benefit from outsourcing under the again unique two part pricing scheme which induces perfect coordination. Finally, it is, also, more likely that the service chain is immune to defections, assuming the retailers reap all benefits from outsourcing. To illustrate this, when adapting Example 1 to the case of pooled service, the chain is stable when $N = 4$ and $\theta = \rho = 0.57$ (while it fails to be so under dedicated service). At the same time, the chain is again prone to defections for the same values of θ and ρ when $N = 5$.

7. Efficient Outsourcing: Pricing Schemes

In most of the comparisons in §3-6, we have assumed the *best case* scenario for outsourcing, where the resulting service chain operates at maximum efficiency. In this §, we show that this can be achieved via a coordinating pricing scheme which induces the retail firms to adopt a first-best price, waiting time standard or a combination thereof (, depending on the type of competition the firms engage in). We refer to Golany and Rothblum (2006) for general derivations of coordinating pricing schemes.

Price Competition:

Under this type of competition, a vector of waiting time standards w^0 is exogenously given. The retail firms can be induced to adopt *any* desired price vector p^I - for example the price vector which maximizes chain wide profits - via a simple volume based fee, i.e., by charging firm i a fee c_i^W for each of its customers.

The profit function of firm i is then given by $\pi_i^o = \lambda_i(p, w^0)(p_i - c_i^W)$, assumed to be strictly concave in the firm's own price p_i. Thus, firm i chooses the desired price p_i^I if $\frac{\partial\lambda_i(p^I,w)}{\partial p_i}(p_i^I - c_i^W) + \lambda_i(p^I, w^0) = 0$. The coordinating fee c_i^W is uniquely determined by the linear equation $c_i^W = p_i^I + \left[\frac{\partial\lambda_i(p^I,w^0)}{\partial p_i}\right]^{-1} \lambda_i(p^I, w^0) < p_i^I$ Under the demand model (6) in §3-6, this reduces to: $c^W = p^I - \frac{\lambda}{b}(p^I, w^0)$, such that the supplier's variable profit margin $c^W - c - \gamma = \rho(p^I - c - \gamma)$, see (12). Thus, ρ, the competitive intensity, denotes the fraction of the total variable profit margin earned by the suppler under the coordination scheme. As explained in Theorem 3.1, the coordinating volume based fee c^W may be complemented with a fixed payment K from each retailer to the supplier.

Waiting Time Competition

In this case, a vector of prices p^0 is exogenously given. To induce the firms to adopt any desired vector of waiting time standards $0 < w^I < w^{max}$ (e.g, the vector maximizing chain wide profits), a simple volume based fee c_i^W for each firm i, no longer suffices. Instead, this fee needs to be combined with a fee e_i^W for any unit of service based *capacity* firm i requires (under dedicated service). The profit function of firm i is now given by $\pi_i^o = \lambda_i(p^0, w)(p_i^0 - c_i^W) - e_i^W w_i$ which is strictly concave in w_i, since the demand function is concave in w_i.[7] Thus, firm i adopts the desired waiting time standard w_i^I if $0 = \frac{\partial\pi_i^o}{\partial w_i} = \frac{\partial\lambda_i(p^0,w^I)}{\partial w_i}(p_i^0 - c_i^W) + \frac{e^W}{(w_i^I)^2}$. In other words, perfect coordination is achieved as long as the fees (c_i^W, e_i^W) are chosen from the line segment: $\left\{(c_i^W, e_i^W) : e_i^W = p_i^0 + (w_i^I)^{-2}\left[\frac{-\partial\lambda_i(p^0,w^I)}{\partial w_i}\right]^{-1} e_i^W; e_i^W \geq 0\right\}$.

In the symmetric demand model (6) used in §5, this reduces to: $\{(c^W, e^W) : e^W = a(w^{CB})^2(p - c^W) : c^W \leq p\}$ induces perfect coordination within the service chain. As in the case of price competition, the volume-and capacity fees may be complemented with a fixed periodic transfer payment K, which does not affect the equilibrium choices. In Theorem 7.1 below we assume that $K = 0$. Let λ_i^{CB} and λ_i^* denote firm $i's$ demand volume, under outsourcing and in-house service.

Theorem 7.1*(Profit comparison between in-house service and outsourcing under waiting time competition)*

(a) $\sum_{i=0}^N \pi_i^{*OB}(\nu) \geq \sum_{i=1}^N \pi_i^*$; *aggregate profits* $\sum_{i=0}^N \pi_i^{*OB}(\nu)$ *decrease with the inefficiency index* ν.

[7] An alternative to the capacity based fee is to charge the retailers in proportion to *any* convexly decreasing function of the waiting time standard.

(b) Consider a coordination scheme, with a given choice for c^W. $e^W(\nu)$, the corresponding capacity fee and $\pi_i^{*OB}(\nu)$, the resulting profit for firm i, are, respectively increasing and decreasing in ν.

(c) As c^W varies from one side of the break even value interval to the other, a retailer's profit is either uniformly higher or lower under outsourcing as opposed to in-house service, or there exists a unique break even value $\underline{c}^W$ at which the retailers switch from benefitting to losing, due to outsourcing,

(d) As c^W varies from one side of the break even value interval to the other, the supplier either uniformly earns a profit or loses money or, there exists a unique break even value $\overline{c}^W$ such that the supplier switches from losing money to earning a profit

(e) $\frac{\partial \pi_i^{*OB}}{\partial c^W} \frac{\partial \pi_0^{*OB}}{\partial c^W} \leq 0$, i.e., the retailers' and the supplier's profits move in opposite directions as we move from one end of the spectrum of coordinating schemes to the other.

If a break even value c_R^W exists for the retailer's profit differential $\pi_i^{*OB} - \pi_i^*$, see part (c), and if a break even value c_S^W exists for the supplier's profit, see part (d), then, the supply chain's benefits from outsourcing may be divided continuously between the supplier and the retailers as one moves between the break even values. (When $c^W = c_R^W$, all benefits go to the supplier, and when $c^W = c_S^W$, all benefits go to the retailers.) However, situations where the retailers are worse off under outsourcing irrespective of which pair of coordinating fees (c^W, e^W) is employed may arise in particular when $\nu = N$, i.e. when the chain uses dedicated facilities for the firms. Note that in the affine function in c^W may have a negative slope (e.g. when a^0 is sufficiently large) so that $\pi_i^{*OB} - \pi_i^* < 0$ for $c^W > c + \gamma$, see part(b). Moreover, if $(p - c - \gamma)$, the variable profit margin, is sufficiently small, $\pi_i^{*OB} - \pi_i^* < 0$ even when $c^W = 0$. To allow for a full spectrum of possible allocations of the benefits of outsourcing, one may therefore need to add a fixed payment K to the compensation scheme.

Simultaneous Competition

To induce the firms to adopt any desired price vector p^I, along with any desired waiting time vector $0 < w^I < w^{max}$, a combined volume- and capacity base fee, again, suffices. As before, let c_i^W and e_i^W denote the fee per customer and per unit of service capacity, respectively. Perfect coordination now requires a *unique* pair of fee rates (c_i^W, e_i^W) obtained from the set of equations: $\frac{\partial \pi_i^o(p^I, w^I)}{\partial p_i} = 0$ and $\frac{\partial \pi_i^o(p^I, w^I)}{\partial w_i} = 0$. (Specific parameters restrictions may be required to ensure that the profit function is jointly concave in (p_i, w_i), see §6): $c_i^W = p_i^I + \left[\frac{\partial\lambda_i(p^I,w^I)}{\partial p_i}\right]^{-1}\lambda_i(p^I,w^I); e_i^W = (w_i^I)^2\left[\frac{-\partial\lambda_i(p^I,w^I)}{\partial w_i}\right](p_i^I - c_i^W) \geq 0$. Under the symmetric demand model (6) used in §6, this reduces to: $0 < c^W \equiv p^I - \frac{\lambda(p^I,w^I)}{b} < p^I$, and $e^W = a(w^I)^2\frac{\lambda(P^I,W^I)}{b} > 0$, (Note that $\lambda(p^I,w^I) = a^0 - (b-(N-1)\beta)p^I - (a-(N-1)\alpha)w^I b(1-\rho)(p^I - c - \gamma) < bp^I$.)

8. Asymmetric Models: Numerical Study

Existence and characterization of equilibria via first order conditions all carry over, under mild conditions, to the general model with non-identical retailers, both under in-house service and the various outsourcing options, see Allon and Federgruen (2004, 2007). However, with non-identical retailers, it is no longer possible to obtain closed form expressions for these equilibria. In appendix A we report on a numerical study aimed to investigate which of the insights obtained in §3-6 for the symmetric model, carry over to general asymmetric industries. Here we confine ourselves to a few general insights.

Our initial set of problems involves an industry with $N = 3$ firms and demand functions of the quasi-separable form in (3). Under price competition, we showed in §3, that under outsourcing with a maximally efficient service chain, prices are higher and demand volumes lower than when then firms provide service in-house. These patterns are, in general, maintained by our asymmetric instances, but a few exceptions arise. We showed that in a symmetric model with $N = 3$ firms, the service chain is always stable. With three asymmetric firms, the chain is ,again, stable in most, but not all, instances. As in the case of symmetric retailers, stability of the full chain is increasingly likely as the competition intensity ρ increases. Under pooled service, the full service chain (, with 3 asymmetric firms,) is always stable. We also report, for all instances, the equilibrium under simultaneous competition, and investigate when the full chain is stable. The above observations for the case of price competition continue to apply here.

In the second part of the study, we consider price competition instances with $N = 3$ firms, whose demand functions arise from an attraction model. In this case, we conclude, for example, that the full service chain under outsourcing, with $N = 3$ firms, is *always* stable.

9. Conclusions and Extensions

In many service industries, competing firms consider the option of outsourcing an important service process to a common service provider. When the firms choose to outsource, this gives rise to a service supply chain. We have developed analytical models to characterize the potential benefits of outsourcing. Two factors are critical determinants of these benefits: (i) the ability of the common service supplier to exploit economies of scope by complete or partial pooling of the service processes for the different firms, and (ii) the ability of the outside service supplier to operate at lower cost rates.

Our models always allow for numerical comparisons of the equilibrium prices, waiting time standards, demand volumes and profit levels under in-house service and the various outsourcing options. Such numerical comparisons have been carried out in §8. For symmetric models, in which the retail firms have identical characteristics, these comparisons can be made analytically, resulting in important general insights. For example, we have shown that when the outside supplier operates with identical cost rates and when the service processes fail to be pooled, firms do not necessarily benefit from outsourcing even when the service chain can be designed to achieve first best level profits. Moreover, even when all firms benefit from collective outsourcing, the service chain may fail to be stable, i.e., it may not be immune to unilateral defections. We have shown how the benefits associated with the various outsourcing options and the stability of the service chain depend in simple ways on the number of firms in the industry, a single index ρ characterizing the intensity of the price competition under in-house service, a similar single index θ, characterizing the intensity of the waiting time competition and the inefficiency index ν. Similarly, we have shown how equilibrium prices, waiting times standards, demands, profits and the benefits of outsourcing depend on a benchmark demand volume- e.g, the demand volume a firm would face if all firms charged at cost- or the differential Δ in the marginal cost per customer between outsourced and in-house service. See the Introduction for a summery of the general conclusions we have been able to prove.

One important restriction of the model is the assumption that the service process can be modeled as an M/M/1 system. For the in-house service and outsourcing with dedicated facilities options, the ability to generalize our results to more general queueing systems, depends on the ability to obtain an analytical characterization of the dependence of the service rate (= average number of customers which can be served per unit of time) with respect to a firm's demand volume and waiting time standard. Allon and Federgruen (2003) have shown that such analytical characterizations are indeed possible for a wide variety of queueing systems. More specifically, the capacity function (1) can be extended to one of the type $\mu = B_1\lambda + B_2/w + \sqrt{B_3\lambda^2 + B_4\lambda/w + B_5/w^2}$ either exactly (e.g. for M/G/1 systems, Jackson networks) or as a close approximation (e.g. GI/GI/1 or GI/GI/S) systems. Similarly, extensions of our equilibrium results under *pooled* service require the ability to characterize the so-called achievable performance space, i.e., the region of the feasible demand volume - and waiting time vectors which can be achieved under a *given* (pooled) capacity level. Such characterizations are available for multi-class M/G/C and G/M/C systems, see for example, Federgruen and Groenevelt (1988) and Shanthikumar and Yao (1992). (For the former, the waiting time standard must be specified as the expected delay before service.) Bertsimas et al. (1994) have developed *approximate* characterizations of the achievable performance space for various multiclass queueing networks. Future work should investigate how the benefits of various outsourcing options depend on the characteristics of the queueing systems (for example, the coefficients of variation of service and interarrival times), and what priority disciplines should be used under pooled service.

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Appendix A: Examples

Example 1: Let $a^0 = 1000; a = 100, b = 50, \alpha = 0.75, \beta = 0.375$ and $c = \gamma = 1$. In an industry with $N = 4$ firms, the retail price and waiting time standard under outsourcing are $p^{CB} = 11.6$ and $w^{CB} = 0.0328$; when all benefits of outsourcing are assigned to the retailers, each earns $\pi_i^{*OB} = 4.305$. An individual firm can increase its profits (to 4.306) by switching to in-house service, simultaneously decreasing the price by \$0.22 and the waiting time standard by 1×10^{-3}. In this instance $\theta = \rho = 0.03$. It is only when the competitive intensities are increased to $\theta = \rho = 0.6$ that the possibility of defections from the service chain can be eliminated. At the same time, for the same value of $\theta = \rho = 0.6$ the service chain is again unstable when $N = 5$. Here $p^{CB} = \$25.95$ and $w^{CB} = 0.032$ and the firms earn a profit of 11.458. By switching to in-house service, an individual firm can increase its profits by more than 4%, again simultaneously decreasing the price to \$17.47 and the waiting time standard to 0.025.

Appendix B: Numerical Study

As mentioned in §8, our initial set of problem instances consider an industry with $N = 3$ firms, and demand function of the quasi-separable form, as follows: (3)

$$\begin{aligned}
\lambda_1(p,w) &= \left[405 - 0.1w_1 + 0.04w_2 + 0.04w_3 - \frac{7}{1-\rho}p_1 + \frac{3.5\rho}{1-\rho}p_2 + \frac{3.5\rho}{1-\rho}p_3\right]^+ \\
\lambda_1(p,w) &= \left[405 - 0.1w_2 + 0.04w_1 + 0.04w_3 - \frac{7}{1-\rho}p_2 + \frac{3.5\rho}{1-\rho}p_1 + \frac{3.5\rho}{1-\rho}p_3\right]^+ \\
\lambda_3(p,w) &= \left[825 - 0.1w_3 + 0.04w_2 + 0.04w_1 - \frac{7}{1-\rho}p_3 + \frac{3.5\rho}{1-\rho}p_2 + \frac{3.5\rho}{1-\rho}p_1\right]^+
\end{aligned}$$

Thus, the total price sensitivity $\overline{b} = 7$ for all three firms. As to the cost parameters, $c_1 = c_2 = 10$, and $\gamma_1 = \gamma_2 = 5$, while $c_3 = 5$ and γ_3 varies. We specify $c_0 = \frac{1}{3}\sum_{i=1}^{3} c_i$ and $\gamma_0 = \frac{1}{3}\sum_{i=1}^{3} \gamma_i$, i.e., the supplier fails to face systematic cost rate advantages or disadvantages compared to the in-house service option.

We start with 4 instances, corresponding with 4 values of γ_3 ($\gamma_3 = 5, 10, 15, 20$), a price competition intensity $\rho = 0.3$ and a common waiting time standard $w = 1$. This quartet is followed by two others, one with $\rho = 0.3$ replaced by $\rho = 0.9$, and one with w replaced by $w = 0.01$. These instances may, for example represent an industry with a well established domestic provider (firm 3) facing competition by two more recent and identical foreign entrants who are able to operate at lower capacity costs, but higher transaction costs per customer. (The fact that firm 3 is the more established or recognized provider is reflected by the larger intercept of its demand function.) Since the total marginal cost per customer is given by $c_i + \gamma_i$, firm 3 enjoys a cost advantage over its competitors when $\gamma_3 = 5$, while firms 1 and 2 have the advantage when $\gamma_3 \geq 10$. (It follows from (9) that if two firms have identical *total* marginal cost rates $c_i + \gamma_i$, the one with the lower γ value enjoys a cost advantage.)

In Table 1, we compare for all 12 instances the equilibrium under(i) in-house service, (ii) outsourcing to a supplier using a dedicated facility for each of the firms, and (iii) outsourcing to a supplier who pools the service processes. As to the firms' profit levels under outsourcing (in (ii) and (iii)), we continue to present the outsourcing option in the most favorable light, i.e., we continue to assume that the service chain is perfectly coordinated while *all* of its profits are earned by the retail firms. In the symmetric case, this uniquely characterizes each firm's profit level, but with asymmetric firms this is no longer the case. When dedicated facilities are used, it seems reasonable to assume that each firm in the service chain is awarded all revenues that are obtained from its customers and that it is charged the cost of the dedicated facility the supplier operates on its behalf. We therefore report *this* profit allocation along with the firms' price and demand volumes. The same performance measures are reported with respect to the in-house service option (i). In the case of service pooling, there is no "obvious" way to allocate aggregate profits; here we merely report the chain wide profits. (As we showed to be the case in symmetric models, under identical waiting times, the firms adopt identical prices and demand volumes when the service chain is perfectly coordinated, whether the service processes are pooled or not.)

The brand recognition firm 3 enjoys, allows it, under in-house service, to position itself in the market with a higher price than its competitors, irrespective of the value of γ_3 or the competitive intensity ρ. When $\rho = 0.3$, firm 3's price differential is approximately \$19. When $\rho = 0.9$, the much more intense price competition compels firm 3 to reduce the price difference to less than \$1.50. (Note, that all prices reduce sharply when ρ moves from 0.3 to 0.9; while variable profit margins are above 100% and sometimes 200% when $\rho = 0.3$, they shrink to 20% for firms 1 and 2 in case $\gamma_3 = 5$ and to approximately 20% for firm 3, in case $\gamma_3 = 20$.) When $\gamma_3 = 0.3$, firms 3's market share is above 50% (except when $\gamma_3 = 5$, where it is slightly below 50%), its large price differential notwithstanding. When $\rho = 0.9$, the intensive price competition causes firm 3's market share to shrink to the 42-45% range. Comparing corresponding instances, one notes that firm 3's profit reduces approximately by a factor of four, and those of its competitors by approximately tenfold, when going from $\rho = 0.3$ to $\rho = 0.9$. Finally, under $\rho = 0.3$, as γ_3 and hence the marginal cost rate per customer of firm 3 decreases in increments of \$5, approximately 60% of the cost saving (or \$2.94) is passed on to the customer, forcing firms 1 and 2 to reduce their prices by an almost identical amount. In contrast, when $\rho = 0.9$, 90% of the same cost rate reductions is passed on to the customer.

In the symmetric model of §4, we proved that, under outsourcing, prices are higher and demand volumes lower than when the firms perform service in-house. This pattern is maintained by our asymmetric instances, with the exception of two instances with $\gamma_3 = 20$ and $\rho = 0.3$, where, under outsourcing, firm 3 faces a slight *increase* in its demand volume, along with a slight price increase. (Note, however, that this demand increase is the result of the supplier operating at a lower capacity cost rate $\gamma_0 = 10$ compared with $\gamma_3 = 20$).

Note that the firms do not always benefit from outsourcing when the supplier uses dedicated facilities, even though all of the service chain's profits are assigned to the retailers. While this is different than what we proved to be the case for symmetric models, see Theorem 4.1, it should be recognized that (a) the employed profit allocation, while highly plausible, is by no means unequivocal under non-identical cost and demand structures, and (b) the supplier is assumed to operate with different cost rates than those pertaining to any specific firm. In 5 of the 12 instances, even the *aggregate* profits of the retailers are higher under in-house service; this can be explained by factor (b). At the same time, when $\rho = 0.9$, individual firms' profits are always higher under outsourcing, and the same applies, a fortiori, to aggregate profits. Here, the benefits of outsourcing clearly stem from the ability to circumvent the cut throat competition, by creating a perfectly coordinated service chain. For the first eight instances, only small differences arise between the outsourcing option with dedicated facilities versus pooled service. This is due to the cost of the service based capacity being very small compared to the other cost components, in these instances. (The cost savings that arise from service pooling are confined to the cost of the service based capacity.) Table 2 exhibits whether the service chain that arises under outsourcing is immune to defections. Given the ambiguity about how profits should be allocated among non-identical firms, we define a chain to stable if *some* profit allocation exists which deters defections. Clearly, this is the case if and only if the aggregate profits in the chain are bigger than or equal to the sum of the profits each firm can obtain when defecting. Recall that in symmetric models, we have shown that in instances with up to 3 firms, stability always prevails, but that the chain fails to be stable when the number of firms is larger and the competitive intensity is below a threshold $\rho^*(N)$, which is increasing in N. Table 2 shows that, with 3 asymmetric firms, the chain is stable in most instances, but fails to be in a few. However, consistent with our results for symmetric retailers, the chain is impeccably stable when ρ is large, and in two of the 3 instances where it is not, stability fails only under dedicated service and this only by a small amount. Furthermore, the lack of stability may well be due to the supplier operating with higher cost rates than some of the firms.

Table 1 Price Competition

Scenario		In-house			Dedicated			Pooled
	Firm	Price	Demand	Profit	Price	demand	profits	Profit
$\gamma 3 = 20, w = 1, \rho = 0.3$	1,2	40.25	152.52	3846.60	41.95	138.57	3262.49	
	3	59.79	347.87	12081.00	60.10	349.83	14601.35	
	total						21126.00	21141.00
$\gamma 3 = 15, w = 1, \rho = 0.3$	1,2	37.31	173.11	3857.40	41.40	142.42	3451.39	
	3	56.85	368.46	13561.00	59.55	353.68	15259.96	
	total						21141.00	22201.00
$\gamma 3 = 10, w = 1, \rho = 0.3$	1,2	34.37	193.70	3747.00	40.30	150.12	3791.24	
	3	53.90	389.05	15126.00	58.45	361.38	15695.29	
	total						23278.00	23290.00
$\gamma 3 = 5, w = 1, \rho = 0.3$	1,2	31.43	214.29	3515.50	39.75	153.97	4062.76	
	3	50.96	409.63	16775.00	57.90	365.23	16273.30	
	total						24397.00	24406.00
$\gamma 3 = 20, w = 1, \rho = 0.9$	1,2	28.99	279.21	3900.80	47.45	142.14	4128.69	
	3	31.44	450.63	2881.00	49.65	338.83	10601.14	
	total						18858.00	18874.00
$\gamma 3 = 15, w = 1, \rho = 0.9$	1,2	24.44	311.03	2932.10	46.35	149.84	4406.41	
	3	26.89	482.45	3310.20	48.55	346.53	11108.21	
	total						19919.00	19933.00
$\gamma 3 = 10, w = 1, \rho = 0.9$	1,2	19.90	342.85	1674.20	45.80	153.69	4726.98	
	3	22.35	514.27	3768.20	48.00	350.38	11555.87	
	total						21010.00	21023.00
$\gamma 3 = 5, w = 1, \rho = 0.9$	1,2	15.35	374.66	127.01	44.70	161.39	5057.80	
	3	17.80	546.09	4255.20	46.90	358.08	12015.75	
	total						22129.00	22139.00
$\gamma 3 = 20, w = .01, \rho = 0.3$	1,2	40.22	152.22	3839.00	41.95	138.57	2273.14	
	3	59.73	347.33	12064.00	60.10	349.85	13613.23	
	total						18157.00	20153.00
$\gamma 3 = 15, w = .01, \rho = 0.3$	1,2	37.28	172.80	3850.10	41.40	142.42	2626.97	
	3	56.79	367.92	13536.00	59.55	353.70	14436.15	
	total						19713.00	21377.00
$\gamma 3 = 10, w = .01, \rho = 0.3$	1,2	34.34	193.39	3740.00	40.30	150.12	3131.16	
	3	53.85	388.51	15094.00	58.45	361.40	15035.82	
	total						21299.00	22631.00
$\gamma 3 = 5, w = .01, \rho = 0.3$	1,2	31.40	213.98	3508.80	39.75	153.97	3568.02	
	3	50.91	409.10	16736.00	57.90	365.25	15779.18	
	total						22913.00	23912.00

Table 3 exhibits and discusses, for all eight combinations of ρ and γ_3, the equilibria which arise under *simultaneous* competition, both when the firms provide in-house service and when they outsource to a supplier using dedicated facilities.

Commenting briefly on the results in Table 3, under in-house service, firm 3 positions itself as the firm with the highest service level, when $\gamma_3 = 5$, i.e., when it operates with lower costs than its competitors. When $\gamma_3 = 10$ and all firms exhibit identical marginal costs, they offer virtually identical waiting time standards. Finally, firm 3's waiting time is higher when $\gamma_3 \geq 15$, i.e. when firm 3 is at a cost disadvantage. As in the case of price competition, firm 3 exploits its larger brand recognition to charge a higher price than its competitors, albeit that the price differential is considerably smaller under high values of ρ. Under simultaneous competition, the prices charged by firms 1 and 2 are much less sensitive to the cost value γ_3 and the corresponding price of firm 3 than under price competition.

Comparing the equilibria under in-house service and outsourcing, we note that the coordinated service chain is

Table 2 Price Competition, defections

Scenario	3rd firm defects				1st firm defects			
	Firm	Price	demand	profit	Firm	Price	demand	profit
$\gamma3 = 20, w = 1, \rho = 0.3$	1,2	38.1	171.29	3376	1	35.35	196.305	**3989.8**
	3	60.1	338.28	**11854**	2	39.2	152.04	8664
					3	57.35	363.315	8664
$\gamma3 = 15, w = 1, \rho = 0.3$	1,2	37	176.515	3581	1	35.35	194.655	**3956.2**
	3	57.35	362.48	**13524**	2	38.65	156.715	9098.6
					3	56.8	367.99	9098.6
$\gamma3 = 10, w = 1, \rho = 0.3$	1,2	35.9	181.74	379.2	1	35.35	191.355	**3889.1**
	3	54.6	386.68	**15303**	2	37.55	166.065	9544.6
					3	55.7	377.34	9544.6
$\gamma3 = 5, w = 1, \rho = 0.3$	1,2	34.8	186.965	4009	1	35.35	189.705	**3855**
	3	51.85	410.88	**17190**	2	37	170.74	10003
					3	55.15	382.015	10003
$\gamma3 = 20, w = 1, \rho = 0.9$	1,2	27.1	336.565	2941	1	22.7	488.63	**3757**
	3	30.95	365.78	**2156**	2	25.45	209.515	2628
					3	27.65	406.215	2628
$\gamma3 = 15, w = 1, \rho = 0.9$	1,2	24.9	317.315	2604.2	1	22.15	457.83	**3268.5**
	3	27.65	458.18	**3490.1**	2	24.35	234.54	3023.7
					3	26.55	431.24	3023.7
$\gamma3 = 10, w = 1, \rho = 0.9$	1,2	22.7	298.065	2288.4	1	22.15	388.53	**2773**
	3	24.35	550.58	**5137.9**	2	23.25	276.89	3610
					3	25.45	473.59	3610
$\gamma3 = 5, w = 1, \rho = 0.9$	1,2	20.5	261.49	1869	1	21.6	357.73	**2356**
	3	20.5	681.48	**7150.5**	2	22.15	301.915	4072.5
					3	24.35	498.615	4072.5
$\gamma3 = 20, w = .01, \rho = 0.3$	1,2	38.1	171.2999	2386	1	196.3248	35.35	**3495.2**
	3	60.1	338.2998	**9874.3**	2	152.0499	39.2	7674.3
					3	363.3249	57.35	7674.3
$\gamma3 = 15, w = .01, \rho = 0.3$	1,2	37	176.5249	2756	1	35.35	194.6748	**3461.6**
	3	57.35	362.4998	**12039**	2	38.65	156.7249	8273.9
					3	56.8	367.9999	8273.9
$\gamma3 = 10, w = .01, \rho = 0.3$	1,2	35.9	181.7499	3132	1	35.35	191.3748	**3394.5**
	3	54.6	386.6998	**14313**	2	37.55	166.0749	8884.9
					3	55.7	377.3499	8884.9
$\gamma3 = 5, w = .01, \rho = 0.3$	1,2	34.8	186.9749	3514	1	35.35	189.7248	**3360.9**
	3	51.85	410.8998	**16696**	2	37	170.7499	9508
					3	55.15	382.0249	9508

consistently able to charge higher prices and offer higher waiting time expectations. As in the case of price competition, the benefits of outsourcing are considerably higher when $\rho = 0.9$. Under outsourcing, firm 3 adopts a considerably lower price and service level, when $\rho = 0.9$ compared to $\rho = 0.3$, while firms 1 and 2 adopt a higher price and lower service.

Table 4 exhibits the equilibria when either firm 3 or one of its two competitors defects from the service chain under outsourcing (, with dedicated facilities). This allows us to verify whether the chain is immune to defections. As in the case of price competition, the chain is sometimes unstable.

We conclude this section with a set of instances with demand functions from an attraction model.

(B) ***Demand functions given by an attraction model***

Here, there is a given potential number of customers M in thr market. Each firm's market share is determined by the so called attractiveness value z_i, itself a general function of the firm's price p_i and waiting time standard w_i, i.e.

Table 3 Simultaneous Competition

Scenario		In-house				Dedicated			
		Price	Waiting Time	demand	profit	Price	Waiting Time	demand	profit
$\gamma 3 = 20, \rho = 0.3$	1,2	34.788	1.5896	197.8801	3913	42	10	137.504	3253.72
	3	58.9724	2.4263	339.724	11533	60	2.08	351.592	14646.03
	total								21151.00
$\gamma 3 = 15, \rho = 0.3$	1,2	34.582	1.5979	195.8201	3831	42	10	137.504	3483.52
	3	56.4435	2.0288	364.4354	13274	60	2.08	351.592	15233.99
	total								22197.00
$\gamma 3 = 10, \rho = 0.3$	1,2	34.3759	1.6064	193.7595	3751	40	10	151.504	3786.93
	3	53.9148	1.603	389.1482	15137	58	2.08	365.592	15717.25
	total								23291.00
$\gamma 3 = 5, \rho = 0.3$	1,2	34.1697	1.615	191.697	3672	40	10	151.471	4039.23
	3	51.3865	1.0991	413.8645	17124	58	1.42	365.658	16330.42
	total								24407.00
$\gamma 3 = 20, \rho = 0.9$	1,2	21.5737	2.7579	460.1624	3023.2	46.6667	4.72	162.3521	4598.40
	3	28.0988	8.0337	216.9172	669.7	49.3333	6.04	311.4403	9654.02
	total								18849.00
$\gamma 3 = 15, \rho = 0.9$	1,2	20.7408	2.9518	401.6838	2303.3	46	4.06	146.0914	4284.27
	3	25.225	5.358	365.7469	1908.2	48	6.7	362.6548	11364.36
	total								19929.00
$\gamma 3 = 10, \rho = 0.9$	1,2	19.9032	3.1933	343.2239	1681.3	46	3.4	146.098	4527.076235
	3	22.3505	3.6884	343.413	3779.4	48	6.04	362.668	11966.9397
	total								21021
$\gamma 3 = 5, \rho = 0.9$	1,2	19.0647	3.5073	284.527	1155.1	44	3.4	160.032	4906.710852
	3	19.4739	2.2973	663.1725	6280.6	46	4.72	376.8	12308.99668
	total								22120

Table 4 Simultaneous Competition, defections

Scenario	3rd firm defects					1st firm defects				
	Firm	Price	Waiting time	demand	profit	Firm	Price	Waiting time	demand	profit
$\gamma 3 = 20, w = 1, \rho = 0.3$	1,2	42	10	137.6515	3257	1	35.4263	1.5646	204.2634	**4169.2**
	3	60.0881	2.3875	350.8806	**12303**	2	42	10	127.3009	8616.2
						3	60	2.08	341.4097	8616.2
$\gamma 3 = 15, w = 1, \rho = 0.3$	1,2	39	10	158.7104	3544	1	34.9762	1.5821	199.7625	**3987.3**
	3	57.14	2.0097	371.3995	**13786**	2	39	10	152.1267	9082.1
						3	57	2.08	366.2355	9082.1
$\gamma 3 = 10, w = 1, \rho = 0.3$	1,2	39	10	154.9425	3718	1	34.9762	1.5821	199.7625	**3987.3**
	3	54.6421	1.5883	396.4206	**15709**	2	39	10	152.1267	9514.5
						3	57	2.08	366.2355	9514.5
$\gamma 3 = 5, w = 1, \rho = 0.3$	1,2	39	10	151.1713	3880	1	34.9743	1.5822	199.7427	**3986.6**
	3	52.1446	1.0892	421.4455	**17757**	2	39	10	152.0841	9947.4
						3	57	1.09	366.3315	9947.4
$\gamma 3 = 20, w = 1, \rho = 0.9$	1,2	30	5.05	254.606	2968.4	1	24.5708	2.2857	2.2857	**6409.9**
	3	31.8933	5.3864	482.5327	**3322.5**	2	30	7.03	7.03	2402.5
						3	33	4.06	4.06	2402.5
$\gamma 3 = 15, w = 1, \rho = 0.9$	1,2	30	4.06	175.8594	2342.7	1	24.57	2.2858	669.8981	**6408.7**
	3	29.3938	3.996	657.5626	**6173.2**	2	30	5.05	118.1862	2696.4
						3	33	3.07	233.9634	2696.4
$\gamma 3 = 10, w = 1, \rho = 0.9$	1,2	30	4.06	97.0792	1454.5	1	23.8943	2.371	622.6043	**5535.5**
	3	26.8945	2.8995	832.6174	**9900.1**	2	30	3.07	2.6063	3186.9
						3	30	3.07	422.6063	3186.9
$\gamma 3 = 5, w = 1, \rho = 0.9$	1,2	30	3.07	18.3	3.04E+02	1	23.8943	2.371	622.6043	**5535.5**
	3	24.3947	1.8637	1007.6	**14502**	2	30	3.07	2.6063	3541.8
						3	30	3.07	422.6063	3541.8

$z_i = z_i(p_i, w_i)$. For a given positive constant z_0, the demand rates of the firms are thus given by the system of equations

$$\lambda_i = M \frac{z_i(p_i, w_i)}{\sum_{i=1}^{N} z_j(p_j, w_j) + z_0}, \ i = 1, \dots, N \tag{18}$$

Without loss of generality, we assume

$$\frac{\partial z_i}{\partial p_i} \le 0 \ , \ \frac{\partial z_i}{\partial w_i} \le 0. \tag{19}$$

Table 5 Attraction demand model

Scenario		In-house			Dedicated		
		Price	demand	profit	Price	demand	profit
$\gamma 3 = 20$	1,2	75.74	2371.1	120310	87.3333	2058.1	142005.69
	3	93.63	2858.2	181870	100.6667	2860.4	235505.90
	total						519517.28
$\gamma 3 = 15$	1,2	75.71	2354.7	119400	86.8	2079.1	145819.74
	3	91.61	2947.6	196340	100.4	2861.2	239588.56
	total						531228.05
$\gamma 3 = 10$	1,2	75.68	2338.6	118520	86	2106.8	149576.13
	3	89.61	3034.8	211250	99.6	2881.5	243768.23
	total						542920.49
$\gamma 3 = 5$	1,2	75.65	2322.9	117650	86	2106.8	153096.156
	3	87.62	3120.3	226590	99.6	2881.5	248582.005
	total						554774.317

Table 6 Attraction demand model, defections

Scenario	3rd firm defects				1st firm defects			
	Firm	Price	demand	profit	Firm	Price	demand	profit
$\gamma 3 = 20$	1,2	79.6	2148	131590	1	74	2611.8	**154090**
	3	94	2995.6	**206680**	2	82	2003.5	168910
					3	95.6	2721.5	168910
$\gamma 3 = 15$	1,2	78.8	2166.7	134610	1	74	2596.5	**153190**
	3	92.4	3048.1	**220670**	2	81.2	2035.6	173080
					3	94.8	2749.3	173080
$\gamma 3 = 10$	1,2	78	2178.6	137250	1	74	2589	**152750**
	3	90	3133.6	**235010**	2	81.2	2029.6	177180
					3	94	2784.9	177180
$\gamma 3 = 5$	1,2	77.2	2196.5	140280	1	74	2581.5	**152300**
	3	88.4	3183.1	**249550**	2	80.4	2067.2	181310
					3	94	2776.8	181310

See Bell et al. (1980) and Leeflang et al. (2000) for an axiomatic foundation of the class of attraction models and various specifications of the attraction functions. The MultiNomial Logit specification arises with the choice $z_i = e^{a_i(w_i)-b_ip_i}$.

Under the attraction model (18), a firm maintains a *positive* market share irrespective of how extreme and uncompetitive its price and service level choices are.

In our numerical study, we assume again an industry with $N = 3$ firms operating with the same cost rates as before, $M = 15000$, $z_1(p,w) = z_2(p,w) = 1800 - 20p + 15\log(10/w)$ and $z_3(p,w) = 220 - 20p + 15\log(10/w)$. As with the previous instances, firm 3's larger brand recognition manifests itself in the larger intercept of its attraction function. In Table 5 we compare, for each of the four γ_3 values, the equilibrium under in-house service with that arising under outsourcing *without* service pooling. Table 6 describes the equilibrium when either firm 3 or one of its two (identical) competitors defects from the service chain. As with the previous instances, firm 3 is able to exploit its superior brand recognition to charge a significantly higher price than its competitors, while continuing to be the largest provider in the market. These observations hold, both when service is provided in-house and when it is outsourced and irrespective of firm 3's capacity cost rate γ_3. Under in-house service, firm 3's price is rather sensitive to the value of γ_3, with roughly

40% of the cost savings due to a reduction of γ_3, being passed on to the customer. In contrast, under outsourcing, the prices are quite insensitive to the capacity cost rate. In this case, outsourcing enables an increase of aggregate profits by more than 20%. The chain is immune to defections.

Appendix C: Benefits of outsourcing when the supplier enjoys cost rate advantage

In this Appendix, we characterize the benefits of outsourcing when the outside supplier is able to operate with lower cost rates than the retailers. Let c_0 and γ_0 denote the suppliers cost rates by $\Delta \equiv c+\gamma-(c_0+\gamma_0) > 0$. To simplify the exposition, assume, that the outside supplier utilizes a dedicated facility for each of the retail firms he services. (All results can be extended to the general case where the suppliers operate with an inefficiency index $1 \leq \nu \leq N$.) Thus, let p^{CC} denote the optimal price level for the centralized profit function $\Pi^C(p) = \sum_{i=1} N \left\{\lambda_i(p_i - c_0 - \gamma_0) - \frac{\gamma_0}{w}\right\}$. (As in the base model, it is easily verified that profits are optimized by using identical price levels at each firm.)

Let c^{WC} denote the suppliers' charge per customer which induces the retailers to adopt the price level p^{CC}, recall from section §7 that

$$c^{WC} c^{WC} = p^{CC} - \frac{\lambda^{CC}}{b}, \tag{20}$$

where $\lambda^{CC} = \lambda(p^{CC}, w)$.

Finally, let $\pi_i^{*OC}(K)$ denote the equilibrium profit retailer i obtains under outsourcing when charged the coordinating per customer fee c^{WC} along with a fixed franchise fee K $(-\infty < K < \infty)$.

Theorem C.1*(Profit comparison between in-house service and outsourcing with lower cost rates for the supplier.)*

(a)$\sum_{i=0}^N \pi_i^{*OC}(K) \geq \sum_{i=0}^N \pi_i^{*OB}(K') \geq \sum_{i=1}^N \pi_i^*$ *for any pair of fixed payments* K, K'

(b)In the absence of fixed transfer payments

(i) $\pi_i^{*OC}(0) > \pi_i^{*OB}(0)$; $\pi_i^{*OC}(0) > \pi_i^*$ *iff* $\lambda(c+\gamma, w) \leq \frac{\bar{b}\Delta + 2\sqrt{\frac{\bar{b}^2\Delta^2}{(2-\rho)^2} + \left(\frac{4}{(2-\rho)^2} - 1\right)\frac{\bar{b}\gamma}{w(1-\rho)}}}{\frac{4}{(2-\rho)^2} - 1}$

(ii) There exists a minimal waiting time $\underline{w}^C \leq \underline{w}^B$ *such that* $\pi_0^{*OC}(0) < 0$ *if* $w \leq \underline{w}^C$

(c)Assume $K = \frac{\gamma_0}{w}$*, i.e., each retailer compensates the supplier periodically for the service based capacity he imposes on her:* $\pi_0^{*OC}(K) > 0$ *regardless of the waiting time standard* w*, while* $\pi_i^{*OC}(K) > \pi_i^*$ *iff* $\lambda(c+\gamma, w) \leq \frac{\bar{b}\Delta + 2\sqrt{\frac{\bar{b}^2\Delta^2}{(2-\rho)^2} + \left(\frac{4}{(2-\rho)^2} - 1\right)\frac{\bar{b}(\gamma-\gamma^0)}{w(1-\rho)}}}{\frac{4}{(2-\rho)^2} - 1}$

(d)Let $K_b^C[K_b^B]$ *denote the break even value for the fixed transfer payments to the suppler such that* $\pi_0^{*OC}(K_b^C) = 0[\pi_0^{*OB}(K_b^B) = 0]$*. Then* $\pi_i^{*OC}(K_b^C) \geq \pi_i^{*OB}(K_b^B) \geq \pi_i^* \forall i = 1, \ldots, N$

Proof:

*(a)*The first inequality follows from $\sum_{i=1}^{N}\pi_i^{*OC}(K)=\Pi^{*C}=\max_p\Pi^C(p)\geq\max_p\Pi^B(p)=\Pi^{*B}=\sum_{i=0}^{N}\pi_i^{*OB}$. The second inequality was shown in Theorem 3.1.

(b) *(i)* From the proof of Theorem 3.1 $\pi_i^{*OC}(0)=\frac{\lambda^2(c_0+\gamma_0,w)}{4b}>\frac{\lambda^2(c+\gamma,w)}{4b}=\pi_i^{*OB}(0)$. Also, $\pi_i^{*OC}(0)-\pi_i^*=\frac{\lambda^2(c_0+\gamma_0,w)}{4b}-\frac{b\lambda^2(c+\gamma,w)}{(2b-(N-1)\beta)^2}+\frac{\gamma}{w}$ so that $4b(\pi_i^{*OC}(0)-\pi_{i*})=\lambda^2(c_0+\gamma_0,w)-\frac{4\lambda^2(c+\gamma,w)}{(2-\rho)^2}+\frac{4b\gamma}{w}$. Substituting the identity $\lambda(c_0+\gamma_0,w)=\lambda(c+\gamma,w)+b(1-\rho)\Delta$, we obtain that $4b(\pi_i^{*OC}(0)-\pi_{i*})=[\lambda(c+\gamma,w)+b(1-\rho)\Delta]^2-\frac{4\lambda^2(c+\gamma,w)}{(2-\rho)^2}+\frac{4b\gamma}{w}$ and this quadratic expression in $\lambda(c+\gamma)$ is negative for λ below it's positive root.

(ii) Again, from the proof of Theorem 3.1 $\pi_0^{*OC}(0)=\frac{\lambda^2(c_0+\gamma_0,w)\rho}{4b(1-\rho)}-\frac{N\gamma_0}{w}$ such that $\lim_{w\downarrow 0}\pi_0^{*OC}(0)=-\infty$ and there exists a value $\underline{w}^C$ such that $\pi_0^{*OC}(0)<0$ for all $w<\underline{w}^C$. Also, for $w<\underline{w}^C$, $\pi_0^{*OB}(0)=\frac{\lambda^2(c+\gamma,w)\rho}{4b(1-\rho)}-\frac{N\gamma}{w}=\frac{[\lambda(c_0+\gamma_0,w)-b(1-\rho)\Delta]^2\rho}{4b(1-\rho)}-\frac{N\gamma_0}{w}<\frac{\lambda^2(c_0+\gamma_0,w)\rho}{4b(1-\rho)}-\frac{N\gamma_0}{w}=\pi_0^{*OC}(0)<0$ so that $\underline{w}^C<\underline{w}^B$.

(c)$\pi_0^{*OC}\left(\frac{\gamma_0}{w}\right)=\pi_0^{*OB}(0)+\frac{N\gamma_0}{w}=\frac{\lambda^2(c_0+\gamma_0,w)\rho}{4b(1-\rho)}>0$, following the proof of Theorem 1(a). By the proof of part (a), $\pi_i^{*OC}\left(\frac{\gamma_0}{w}\right)-\pi_i^*=\frac{\lambda^2(c_0+\gamma_0,w)}{4b}-\frac{b\lambda^2(c+\gamma,w)}{(2b-(N-1)\beta)^2}+\frac{\gamma-\gamma_0}{w}$. The rest of the proof is analogous to that of part (b).

*(d)*Immediate from part (a) and the fact that all centralized models have symmetric solutions.

■

Appendix D: Proofs

Proof of Proposition 4.1 Consider a setting where all N firms are part of a common chain which operates under the above described perfect coordination scheme. Assume now that one of the firms, without loss of generality firm N, decides to leave the chain and to service its customers in-house. After the "defection", the industry operates as a two player game: the chain comprised of firms $1,\ldots,N-1$ is the first player, deciding on a vector $(p_1,\ldots,p_{N-1})$ and firm N is the second player, selecting a single price $p_N=p^i$ for its *in-house* service. For all $i=1,\ldots,N-1$, $\lambda_i=a_i^0+\beta p^i+\alpha w-w(a-(N-2)\alpha)-bp_i+\beta\sum_{j\neq i}^{N-1}p_j$. Thus, for any price level p^i, the best response problem faced by the chain is equivalent to the centralized problem in a system with $N-1$ firms and no outside competitors, and an intercept for the firms' demand function given by $\widehat{a}^0=a^0+\beta p^i+\alpha w$. As shown above, it is optimal in the centralized system to select identical prices for all $N-1$ firms. Thus, with p^o denoting the common price among all firms $1,\ldots,N-1$, for their *outsourced* service, we obtain from (13) that

$$p^o=\frac{a^0-w(a-(N-1)\alpha)}{2(b-(N-2)\beta)}+\frac{c+\gamma}{2}+\frac{p^i\beta}{2(b-(N-2)\beta)}. \tag{21}$$

Firm N's profit $\pi_N=\lambda^i(p^i-c-\gamma)-\frac{\gamma}{N}$ with $\lambda^i=a^0-w(a-(N-1)\alpha)-bp^i+(N-1)\beta p^o$, so

$$p^i=\frac{a^0-w(a-(N-1)\alpha)}{2b}+\frac{c+\gamma}{2}+\frac{p^o\beta(N-1)}{2b}. \tag{22}$$

Substituting (21) into (22) we obtain $p^i\left[\frac{4b(b-(N-2)\beta)-\beta^2(N-1)}{4b(b-(N-2)\beta)}\right] = \frac{a^0-w(a-(N-1)\alpha)}{2b}\left[\frac{2(b-(N-2)\beta)+\beta(N-1)}{2(b-(N-2)\beta)}\right] + \frac{c+\gamma}{2}\left[\frac{2b+\beta(N-1)}{2b}\right]$ and $p^i = (a^0-w(a-(N-1)\alpha))\frac{2(b-(N-2)\beta)+\beta(N-1)}{4b(b-(N-2)\beta)-\beta^2(N-1)} + (c+\gamma)\frac{(2b+(N-1)\beta)(b-(N-2)\beta)}{4b(b-(N-2)\beta)-\beta^2(N-1)}$. By (10)

$$\begin{aligned}\pi_N &= b(p^i-c-\gamma)^2-\frac{\gamma}{w}\\ &= b\left[(a^0-w(a-(N-1)\alpha))\frac{2(b-(N-2)\beta)+\beta(N-1)}{4b(b-(N-2)\beta)-\beta^2(N-1)}+(c+\gamma)\left[\frac{(2b+(N-1)\beta)(b-(N-2)\beta)}{4b(b-(N-2)\beta)-\beta^2(N-1)}-1\right]\right]^2-\frac{\gamma}{w}\\ &= b\left[\frac{2b-\beta(N-3)}{4b(b-(N-2)\beta)-\beta^2(N-1)}\right]^2\left[a^0-w(a-(N-1)\alpha)-(c+\gamma)(b-(N-1)\beta)\right]^2-\frac{\gamma}{w}\\ &= b\left[\frac{2b-\beta(N-3)}{4b(b-(N-2)\beta)-\beta^2(N-1)}\right]^2\lambda(c+\gamma,w)^2-\frac{\gamma}{w}.\end{aligned}\tag{23}$$

In contrast, when firm N remains part of the chain, it obtains π_i^{*OB}, as in (25). Since the second term in (21) is always larger than the second term in (22), firm N has no incentive to defect from the chain if $b\left[\frac{2b-\beta(N-3)}{4b(b-(N-2)\beta)-\beta^2(N-1)}\right]^2 \leq \frac{1}{4(b-(N-1)\beta)} \Leftrightarrow 4b^2(N-3)-4\beta b(N^2-4N+5)-\beta^2(N-1)\leq 0 \Leftrightarrow Q(\rho,N)\equiv 4(N-1)(N-3)-4\rho(N^2-4N+5)-\rho^2\leq 0$, where the second inequality follows by multiplying both sides with the product of the denominators and the third inequality follows by multiplying both sides by $(N-1)/b^2$. When $N\leq 3$, the quadratic function $Q(\rho,N)$ is negative for all $\rho\geq 0$, i.e., it is more advantageous for firm N to stay in the chain, see (i). At the same time, for $N\geq 4$, $Q(\rho,N)$ viewed as a quadratic function in ρ, is non-positive for $\rho\geq\rho^*(N)$, its positive root. Clearly $\rho^*(N)>0$, since $(N-1)(N-3)$ for $N\geq 4$. Also, $\rho^*(N)\leq 1 \Leftrightarrow 2\sqrt{(N^2-4N+5)^2+(N-1)(N-3)}\leq 1+2(N^2-4N+5) \Leftrightarrow 4(N-1)(N-3)<1+4(N^2-4N+5)$, which holds for all N.

(b) Note, since $\rho^*(N)\leq 1$,

$$Q(\rho^*(N),N+1)=4(2N-3)(1-\rho^*(N))+Q(\rho^*(N),N)\geq 0. \tag{24}$$

Equation (24) implies that $\rho^*(N+1)$, the positive root of the quadratic equation $Q(\rho,N+1)=0$, is larger than $\rho^*(N)$ .Finally, the positive root of the quadratic equation can be written as $\frac{4(N-1)(N-3)}{\sqrt{4(N^2-4N+5)^2+4(N-1)(N-3)}+2(N^2-4N+5)}$, from which $\lim_{N\uparrow\infty}\rho^*(N)=1$ is immediate. Finally, if $\pi_N^{*OB}-\pi_N^*\geq 0 \Leftrightarrow \lambda^2(c+\gamma,w)\left[\frac{1}{4(b-(N-1)\beta)}-b\left(\frac{2b-\beta(N-3)}{4b(b-(N-2)\beta)-\beta^2(N-1)}\right)^2\right]\geq \frac{\gamma}{w}\left(\frac{\nu}{N}-1\right)$. When $N\geq 4$ and $\rho\leq\rho^*(N)$, the expression within squared brackets is non-positive and $\lambda(c+\gamma,w)\leq \widehat{\lambda}(\rho,N,\nu)$ is the necessary and sufficient condition to prevent defections, see (iii).

■

Proof of Proposition 4.2

*(a)*Note $\pi^o(o,1)=\pi^o(i,1)$. Let $m^*=\max\{1\leq m\leq N:\pi^o(o,m)\geq\pi^o(i,m)\}$. If $m^*=N$, a SPNE exists with $m^o=N$ since all firms are part of the service chain and none want to switch to in-house service. If $m^*<N$, $\pi^i(o,N-m^*)=\pi^o(o,m^*+1)<\pi^o(i,m^*+1)=\pi^i(i,N-m^*)$, so that a chain with m^* firms is in equilibrium: all firms inside (outside) the chain are better off continuing to outsource (provide in-house service).

(b)$\pi^o(i,m^*+1) > \pi^o(o,m^*+1)$, and by the strict single crossing point property, $\pi^o(i,m^*+2) > \pi^o(o,m^*+2),\dots,\pi^o(i,N) > \pi^o(o,N)$, so that no chain with $m^o > m^*$ is stable. Similarly, for a chain with $m^o < m^*$ firms, $\pi^i(o,N-m^o) = \pi^o(o,m^o+1) \geq \pi^o(i,m^o+1) = \pi^i(i,N-m^o)$, where the inequality follows from the strict single crossing point property. (If $\pi^o(i,m^o+1) > \pi^o(o,m^o+1)$, then $\pi^o(i,m^*) > \pi^o(o,m^*)$, contradicting the definition of m^*.) This implies that a firm outside the chain can improve its profits (or keep it equal) by outsourcing and will therefore defect, contradicting the stability of the chain. The monotonicity of m^* in ν follows from the fact that $\pi^o(i,m)$ is independent of ν, while $\pi^o(o,m)$ decreases with ν. These properties are easily verified, since the equilibrium prices of the firms inside and outside of the chain do *not* depend on ν, see the proof of Theorem 5.3.

■

Proof of Proposition 4.1

(a)$p^{CB} > p^* \Leftrightarrow \frac{a^0-w(a-(N-1)\alpha)}{2(b-(N-1)\beta)} + \frac{c+\gamma}{2} > \frac{a^0-w(a-(N-1)\alpha)+b(c+\gamma)}{2b-(N-1)\beta} \Leftrightarrow (a^0-w(a-\alpha(N-1)))\left[\frac{1}{2(b-(N-1)\beta)} - \frac{1}{2b-(N-1)\beta}\right] > (c+\gamma)\left[\frac{1}{2b-(N-1)\beta} - \frac{1}{2}\right] \Leftrightarrow (a^0-w(a-\alpha(N-1)))\frac{(N-1)\beta}{b-(N-1)\beta} > (c+\gamma)(N-1)\beta \Leftrightarrow 0 < \lambda(c+\gamma,w) = a^0 - w(a-\alpha(N-1)) - (b-(N-1)\beta)(c+\gamma)$. But, the last inequality holds by (8).

(b)$\lambda^{CB} = a^0 - w(a-(N-1)\alpha) - (b-(N-1)\beta)p^{CB} < a^0 - w(a-(N-1)\alpha) - (b-(N-1)\beta)p^* = \lambda^*$ by part (a) and $b > (N-1)\beta$.

*(c)*It follows from $\Pi^{CB} = N\lambda^{CB}(p^{CB}-c-\gamma) - \frac{\nu\gamma}{w}$ and (12) that $\Pi^{CB} = N\overline{b}(p^{CB}-c-\gamma)^2 - \frac{N\gamma}{w}$ which is independent of ρ, by (13). By (10), $\pi_i^* = b(p_i^*-c-\gamma)^2 - \frac{\gamma}{w}$. The expression for π_i^* follows by substituting the expression for p^* in (10). It is decreasing in ρ, since the funcition $\frac{1-\rho}{(2-\rho)^2}$ is.

■

Proof of Theorem 4.1

(a)$\sum_{i=0}^N \pi_i^{*OB} = \Pi(p^{CB}) \geq \Pi(p^*) = \sum_{i=0}^N \pi(p^*) = \sum_{i=0}^N \pi_i^*$. π_i^{*OB} is independent of ν since both the retail price p^{CB} and the customer fee c^{WB} are. However, the supplier's profits $c^{WB}\sum_{i=1}^N \lambda_i^{CB} - (c+\gamma)\left[\sum_{i=1}^N \lambda_i^{CB} + \frac{\nu}{w}\right] - K$ is a decreasing affine function of ν, since c^{WB} and λ^{WB} are independent of ν.

*(b)*Using (20), (12), and (13),

$$\begin{aligned}\pi_i^{*OB} &= \lambda^{CB}(p^{CB}-c^{WB}) = \frac{(\lambda^{CB})^2}{b} = \frac{(p^{CB}-c-\gamma)^2(b-(N-1)\beta)^2}{b} \\ &= \frac{1}{4b}\left[a^0 - w(a-(N-1)\alpha) - (c+\gamma)(b-(N-1)\beta)\right]^2 = \frac{\lambda^2(c+\gamma,w)}{4b}. \end{aligned} \tag{25}$$

Similarly, from (11), $p^* - c - \gamma = \frac{\lambda(c+\gamma,w)}{b(2-\rho)}$ and

$$\pi_i^* = \frac{\lambda^2(c+\gamma,w)}{b(2-\rho)^2} - \frac{\gamma}{w}. \tag{26}$$

Thus, $\pi_i^{*OB} - \pi_i^* = \frac{\lambda^2(c+\gamma,w)}{b}\left[\frac{1}{4} - \frac{1}{(2-\rho)^2}\right] + \frac{\gamma}{w}$, and part (i) follows. Part (ii) is derived from $\pi_0^{*OB} = N(c^{WB} - c - \gamma)\lambda^{CB} - \frac{\nu\gamma}{w}$ since c^{WB} and λ^{CB} are independent of w.

*(c)*In this case, $\pi_i^{*OB} - \pi_i^* = \frac{\lambda^2(c+\gamma,w)}{b}\left[\frac{1}{4} - \frac{1}{(2-\rho)^2}\right] < 0$, while $\pi_0^{*OB} = N(c^{WB} - c - \gamma)\lambda^{CB} + \frac{(N-\nu)\gamma}{w} > 0$

*(d)*Consider first the case where $\nu = N$, i.e, the supplier services the customers of each retailer in a dedicated facility. In this case, the optimal aggregate profit of the service chain, under outsourcing, equals the sum of the retailers' profits under in-house service, assuming the price vector p^{CB} is adopted. Thus, by (20), $\pi_i^{*OB} = \lambda^{CB}(p^{CB} - c - \gamma) - \frac{\gamma}{w} = (p^{CB} - c - \gamma)^2(b - (N-1)\beta) - \frac{\gamma}{w} = \frac{\lambda^2(c+\gamma,w)}{4b(1-\rho)} - \frac{\gamma}{w}$, by (25). By (26) $\pi_i^{*OB} - \pi_i^* = \frac{\lambda^2(c+\gamma,w)}{\bar{b}}\left[\frac{1}{4} - \frac{1-\rho}{(2-\rho)^2}\right] = \frac{\lambda^2(c+\gamma,w)}{4\bar{b}}\frac{\rho^2}{(2-\rho)^2}$. When $\nu < N$, $\pi_0^{*OB} = N\lambda^{CB}(c^{WB} - c - \gamma) - \frac{\nu\gamma}{w} + NK$, where λ^{CB} and c^{WB} are independent of ν. Thus, when $\nu < N$ each retailer's periodic fee can be $\frac{(N-\nu)\gamma}{Nw}$ lower than when $\nu = N$, and its profit correspondingly higher, without affecting the supplier's profit level. Thus,

$$\pi_i^{*OB} = \frac{\lambda^2(c+\gamma,w)}{4b(1-\rho)} - \frac{\nu\gamma}{Nw}. \tag{27}$$

Subtracting π_i^* we obtain the desired expression.

■

Proof of Theorem 5.3

Assume $m^o \equiv N - m^i$ firms outsource, without loss of generality, firms $\{m^i + 1, \ldots, N\}$. For given prices $\{p_1^i, \ldots, p_{m^i}^i\}$ of the firms outside the chain, the aggregate profit of the chain is given by $\Pi^c = \sum_{i=m^i+1}^N \left[\lambda_i(p_i - c - \gamma) - \frac{\gamma}{w}\right]$, where $\lambda_i = a^0 - [a - (N-1)\alpha]\, w - bp_i + \beta\sum_{j=m^i,j\neq i}^N p_j + \beta\sum_{j=1}^{m^i} p_j^i$. It is easily verified that the chain's aggregate profit function is a concave quadratic function, such that the optimal prices for the participating firms satisfy the first order conditions

$$\begin{aligned} 0 \;=\; \frac{\partial \Pi^c}{\partial p_i} &= \lambda_i - b(p_i - c - \gamma) + \beta \sum_{j=m^*+1,j\neq i}^{N} (p_j - c - \gamma) \\ &= a^0 - [a - (N-1)\alpha]\, w - bp_i + \beta\sum_{j=1}^{m^i} p_j^i + \beta \sum_{j=m^i,j\neq i}^{N} p_j + \beta\sum_{j=1}^{m^i} p_j \\ &- b(p_i - c - \gamma) + \beta \sum_{j=m^*+1,j\neq i}^{N} (p_j - c - \gamma), i = m^i + 1, \ldots, N \end{aligned} \tag{28}$$

This represents a linear system of $(N - m^i)$ equations in as many unknown prices $\{p_{m^i+1}, \ldots, p_N\}$. By writing the system in matrix form, one easily verifies that this system of equations has a unique solution which is symmetric, i.e. there exists a price p^o, such that $p_j = p^o$ for all $j = m^i + 1, \ldots, N$

$$p^o = \frac{a^0 - w(a-(N-1)\alpha)}{2[b-(N-m^i+1)\beta]} + \frac{c+\gamma}{2} + \frac{\beta \sum_{j\neq i}^{m} p_j^i}{2(b-(N-m^i+1)\beta)} \tag{29}$$

It follows from (28) that $\lambda_i = [b-(N-m^i+1)\beta](p^o - c - \gamma)$ for all $i = m^i+1, \ldots, N$. Thus

$$\pi^o(o, N-m^i) = \frac{\Pi^c}{N-m^i} = \frac{1}{N-m^o} \sum_{j=m^i+1}^{N} \lambda_j(p^o - c - \gamma) - \frac{\gamma}{w} = [b-(N-m^i+1)](p^o-c-\gamma)^2 - \frac{\gamma}{w} \tag{30}$$

The firms $i = 1, \ldots, m^i$ performing in-house service, have, under a given (common) price p^o for the outsourcing firms, a profit function $\pi_i = \lambda_i(p_i - c - \gamma) - \frac{\gamma}{w}$, with $\lambda_i = a^0 - [a-(N-1)\alpha]\, w - bp_i + \beta \sum_{j=1, j\neq i}^{m^i} p_j + \beta(N - m^i)p^o$. For a given price p^o for the outsourcing firms, the remaining m^i firms engage in a m^i-person game in which the Nash equilibrium satisfies the first order conditions:

$$0 = \frac{\partial \pi_i}{\partial p_i} = \lambda_i - b(p_i - c - \gamma) = [a^0 - w(a-(N-1)\alpha)] + (N-m^i)\beta p^o - 2bp_i + \beta \sum_{j=1, j\neq i}^{m^i} p_j + b(c+\gamma), i = 1, \ldots, m^i. \tag{31}$$

Once again, this system of linear equation has a unique solution, which is symmetric. In other words, for any price p^o selected by the outsourcing firms, the remaining firms choose a common price p^i, i.e. $p_j = p^i, j = 1, \ldots, m^i$, where

$$p^i = \frac{a^0 - w(a-(N-1)\alpha) + (N-m)\beta p^o + b(c+\gamma)}{2b - (m^o - 1)\beta}. \tag{32}$$

Also, by (34), $\lambda_i = b(p^i - c - \gamma)$ for all $i = 1, \ldots, m^i$, and

$$\pi^i(i, m) = \lambda_i(p^i - c - \gamma) - \frac{\gamma}{w} = b(p^i - c - \gamma)^2 \frac{\gamma}{w} \tag{33}$$

An overall Nash equilibrium in the $m^i + 1$ person game, consisting of the service chain and m^i firms with in house service, uses a price pair $\{p^i, p^o\}$ which satisfies (33) and (30). This system of two equations has the following solution

$$\begin{aligned}
p^i &= (a^0 - w(a-(N-1)\alpha)) \frac{2(b+\beta) - \beta(N-m)}{2(b-(N-m-1)\beta)(2b-(m-1)\beta) - (N-m)m\beta^2} \\
&+ (c+\gamma) \frac{(b-(N-m-1)\beta)(2b+(N-m)\beta)}{2(b-(N-m-1)\beta)(2b-(m-1)\beta) - (N-m)m\beta^2} \\
p^o &= (a^0 - w(a-(N-1)\alpha)) \frac{2b+\beta}{2(b-(N-m-1)\beta)(2b-(m-1)\beta) - (N-m)m\beta^2} \\
&+ (c+\gamma) \frac{(b-(N-m-1)\beta)(2b-(m-1)\beta) + b\beta m}{2(b-(N-m-1)\beta)(2b-(m-1)\beta) - (N-m)m\beta^2}
\end{aligned}$$

Substituting these into (34) and (37), we get: $\pi^i(i, m^i) > \pi^i(o, m^i) = \pi^o(o, N-m^i+1) \Leftrightarrow$

$$b \left\{ (a^0 - w(a-(N-1)\alpha)) \frac{2(b+\beta) - \beta(N-m)}{2(b-(N-m-1)\beta)(2b-(m-1)\beta) - (N-m)m\beta^2} \right.$$

$$+(c+\gamma)\left[\frac{(b-(N-m-1)\beta)(2b+(N-m)\beta)}{2(b-(N-m-1)\beta)(2b-(m-1)\beta)-(N-m)m\beta^2}-1\right]\Bigg\}^2 >$$
$$(b-(N-(m-1)-1)\beta)\Bigg\{(a^0-w(a-(N-1)\alpha))\frac{2b+\beta}{2(b-(N-m-1)\beta)(2b-(m-1)\beta)-(N-m)m\beta^2}$$
$$+(c+\gamma)\left[\frac{(b-(N-m-1)\beta)(2b-(m-1)\beta)+b\beta m}{2(b-(N-m-1)\beta)(2b-(m-1)\beta)-(N-m)m\beta^2}-1\right]\Bigg\}^2$$

After rearranging the terms, the above can be shown to be equivalent to

$$\left[\frac{2(b+\beta)-\beta(N-m)}{2(b-(N-m-1)\beta)(2b-(m-1)\beta)-(N-m)m\beta^2}\right]^2\lambda^2(c+\gamma,w)b >$$
$$(b-(N-m)\beta)\left[\frac{2b+\beta}{2(b-(N-m-1)\beta)(2b-(m-1)\beta)-(N-m)m\beta^2}\right]^2\lambda(c+\gamma,w)$$

By dividing both sides of the inequality by b, we obtain (32). By (30) and (31), the necessary and sufficient condition for an SPNE with m^o outsourcing firms consists of (32) and $\pi^o(o,m^o)\geq\pi^o(i,m^o)=\pi^i(i,N-m^o+1)$, which, by the above argument is equivalent to (33). ■

Proof of Lemma 6.1

If firms are served in dedicated facilities, aggregate profits are given by $\Pi^{CB}(w)=(p-c-\gamma)\sum_{i=1}^N\lambda_i-\gamma\sum_{i=1}^N\frac{1}{w_i}$, a jointly concave function of w, whose optimum is symmetric and satisfies the first order conditions $\frac{\partial\Pi^{CB}}{\partial w_i}=(a-(N-1)\alpha)(p-c-\gamma)-\frac{\gamma}{w_i}=0$. Hence , the expressions for w^{CB}. Under service pooling $\Pi^{CB}(w)=(p-c-\gamma)\sum_{i=1}^N\lambda_i-\gamma\mu^*$, with μ^* given by (2). Note first that under an optimal vector of waiting times, the maximum in (2) is achieved for $S=\{1,\ldots,N\}$. Allon and Federgruen (2004) show that a *largest* set S^* exists which achieves the maximum. Assuming to the contrary that $S^*\neq\{1,\ldots,N\}$: the waiting times $\{w_i^{CB}:i\notin S^*\}$ could be reduced without increasing the required capacity while increasing the first term in the profit function under pooling; this contradicts the optimality of w^{CB}. $\left(\frac{\partial(\sum_{l=1}^N\lambda_l)}{\partial w_i}=-a+(N-1)\alpha<0\right)$. Thus

$$\Pi^{CB}(w)=(p-c-\gamma)\sum_{i=1}^N\lambda_i-\gamma\frac{\sum_{i=1}^N\lambda_i}{\sum_{i=1}^N\lambda_i w_i} \tag{34}$$

and

$$\begin{aligned}\max_w \Pi^{CB}(w) &= \max_{w\geq 0,W>0}\left\{\sum_{i=1}^N(p-c-\gamma)\lambda_i-\gamma\frac{\sum_{i=1}^N\lambda_i}{W}\,|\sum_{i=1}^N\lambda_i w_i\geq W\right\} \\ &= \max_{W\geq 0}\min_{\delta\geq 0}\max_{w\geq 0}\left\{\sum_{i=1}^N(p-c-\gamma)\lambda_i-\gamma\frac{\sum_{i=1}^N\lambda_i}{W}+\delta\left(\sum_{i=1}^N\lambda_i w_i-W\right)\right\}\end{aligned} \tag{35}$$

where the second equality follows from the strong duality theorem of convex programming, as $\sum_{i=1}^N\lambda_i w_i$ is a strictly concave quadratic function of w. Note that the inner maximum (for any given $W,\delta\geq 0$) is achieved by a vector with *identical* components since the objective within curled brackets is a concave (quadratic) function of w, which is invariant to permutations of the vector $(w_1, \ldots, w_N)$. Thus, if a vector $w^{(1)}$ with non-identical components obtains the maximum, so would all of its permutations $\left\{w^{(l)}, l = 1, \ldots, N!\right\}$, but their average $\widehat{w} = \frac{1}{N!}\sum_{i=1}^{N!} w^{(l)}$ with identical components achieves a higher objective value than each these optima. The expression for w^{CB} follows, again, from $\frac{\partial \Pi^{CB}(w)}{\partial w_i} = 0$ using (40) and the fact that at the optimum point $\lambda_1 = \cdots = \lambda_N$. ■

Proof of Theorem 6.2

*(a)*Immediate from $\sum_{i=0}^{N} \pi_i^{*OB} = \Pi^B(w^{CB})$.

*(b)*It follows from (40) that $\frac{e^W(\nu)}{w^{CB}(\nu)} = aw^{CB}(\nu)(p - c^W)$. Since $w^{CB}(\nu)$ is increasing in ν, so is $e^W(\nu)$ and $e^W(\nu)/w^{CB}(\nu)$. $\lambda^{CB}(\nu)$ and $\pi_i^{*OB}(\nu) = (p - c^W)\lambda_i^{CB}(\nu) - \frac{e^W}{w^{CB}(\nu)}$ are decreasing in ν.

*(c)-(e)*To prove parts (b)-(e), note that $\pi_i^{*OB} = \lambda^{CB}(p - c^W) - \frac{e^W}{w^{CB}}$ and $\pi_i^* = \lambda_i^*(p - c - \gamma) - \frac{\gamma}{w_i^*}$. Substituting the above expressions for w^{CB} and w_i^* into (3) we obtain expressions for λ^* and λ^{CB}, and one for e^W. After some algebra we conclude:

$$\begin{aligned}\pi_i^{*OB} \, - \, \pi_i^* = & \left(a^0 - p(b - (N-1)\beta)\right)(c + \gamma - c^W) \\ & - (p - c^W)(2a - (N-1)\alpha)\sqrt{\frac{\gamma\nu(N)}{N(a - (N-1)\alpha)(p - c - \gamma)}} \\ & + (a - (N-1)\alpha)\sqrt{\frac{\gamma(p - c - \gamma)}{a}} + \sqrt{a\gamma(p - c - \gamma)},\end{aligned} \tag{36}$$

an affine function of c^W which takes on a negative value for $c^W = c + \gamma$, thus verifying part(b). Since $\pi_0^{*OB} + \sum_{i=1}^{N}(\pi_i^{*OB} - \pi_i^*) = \Pi^B(w^{CB}) - \sum_{i=1}^{N}\pi_i^* > 0$ does not depend on c^W, it follows that π_0^{*OB} is an affine function of c^W as well, the slope of which has the opposite sign from that of the function (36), thus verifying part(c) and (d). ■

Proof of Theorem 6.3

*(a)*Assuming the best case scenario for outsourcing, i.e. a maximally efficient chain with the retailers obtaining a maximum share of the chain-value profits, we get $\pi_i^{*OB} = \frac{1}{N}\Pi^{CB}(w^{CB}) = \lambda^{CB}(p - c - \gamma) - \frac{\nu(N)\gamma}{Nw^{CB}} = [a^0 - pb(1 - \rho)](p - c - \gamma) - 2\sqrt{\frac{\nu(N)}{N}}\sqrt{\gamma a(1 - \theta)(p - c - \gamma)}$ If one of the firms, without loss of generality, firm N, leaves the chain, the industry operates as a duopoly: the remaining chain of firms $1, \ldots, N-1$, is the first player who decides on the vector $(w_1, \ldots, w_{N-1})$, and firm N is the second player choosing the single "in-house" waiting time standard w^i. It is easily verified that irrespective of firm N's waiting time choice w^i, it is optimal for the firms in the remaining chain to make identical waiting time choices $w^o = \sqrt{\frac{\nu(N-1)\gamma}{(N-1)(a-(N-2)\alpha)(p-c-\gamma)}}$, while $w^i = w^*$ continues to be the dominant choice for firm N. Thus $\lambda^i = (a^0 - p(b - (N-1)\beta)) - aw^i + (N-1)\alpha w^o$ represents the demand volume

of the defecting firm N. Substituting w^i and w^o we obtain firm N's profit π^i when providing in-house service $\pi^i = (a^0 - p(b-(N-1)\beta))\,(p-c-\gamma) - 2\sqrt{\gamma a(p-c-\gamma)} + (N-1)\alpha\sqrt{\frac{\nu(N-1)}{N-1}\frac{\gamma(p-c-\gamma)}{a-(N-2)\alpha}}$. Thus, firm N doesn't benefit from defection if and only if

$$\pi^{*i} - \pi_N^{*OB} = \sqrt{\gamma(p-c-\gamma)}\left\{2\sqrt{a(1-\theta)\frac{\nu(N)}{N}} - 2\sqrt{a} + (N-1)\alpha\sqrt{\frac{\nu(N-1)}{N-2}\frac{1}{a-(N-1)\alpha}}\right\} \le 0$$

which is equivalent to (17).

*(b)*In this case, the stability condition can be written as

$$2(1-\sqrt{1-\theta}) > \theta\sqrt{\frac{1}{1-\theta\frac{N-2}{N-1}}} \Leftrightarrow \frac{2\theta}{1+\sqrt{1-\theta}} \ge \frac{\theta}{\sqrt{1-\theta\frac{N-2}{N-1}}} \Leftrightarrow 2\sqrt{1-\theta\frac{N-2}{N-1}} \ge 1+\sqrt{1-\theta}$$

. This inequality is always satisfied when $N=2$ and $N=3$. When $N \ge 5$, $2\sqrt{1-\theta\frac{N-2}{N-1}} \le 2\sqrt{1-\frac{3\theta}{4}} \le 1+\sqrt{1-\theta}$, since the last inequality is equivalent to $\theta + \sqrt{1-\theta} > 1$ for all $0 \le \theta < 1$, as can be seen by squaring both sides and grouping terms. Thus, the chain is never stable when $N \ge 5$. Finally, when $N=4$, the condition $2\sqrt{1-\frac{2}{3}\theta} \ge 1+\sqrt{1-\theta}$ is equivalent to a quadratic inequality, which on the interval $[0,1)$ is satisfied only for $\theta \in [0.96, 1)$.

*(c)*In this case, the stability condition reduces to $2\sqrt{\frac{1-\theta}{N}} + \frac{\theta}{\sqrt{(N-1)(1-\theta)+\theta}} \le 2$. But $H(\theta,N)$ is decreasing in N, so that $H(\theta,N) \le H(\theta,2) = \sqrt{2}\sqrt{1-\theta}+\theta \le 1.5$ for all $\theta \le 1$. (To verify the last inequality, note that the function to the left is concave, achieving its maximum for $\theta = 0.5$, since its derivative $1-\frac{\sqrt{2}}{2}(1-\theta)^{-\frac{1}{2}}$, has $\theta = 0.5$ as its root.)

■

Proof of Theorem 7.1 Assume $\gamma \ge a(w^{max})^3 \max\{\frac{a}{4b}, \frac{1+\theta}{2}\}$.

*(a)*Let $\pi_i = \lambda_i(p_i - c - \gamma) - \frac{\gamma}{w_i}$. Note,

$$\frac{\partial \pi_i}{\partial p_i} = -b(p_i - c - \gamma) + \lambda_i; \quad \frac{\partial \pi_i}{\partial w_i} = -a(p_i - c - \gamma) + \frac{\gamma}{w_i^2}. \tag{37}$$

π_i is jointly concave in the pair (p_i, w_i) on $[c+\gamma, p^{max}] \times [0, w^{max}]$, since its Hessian, on this rectangle, has diagonal elements $(-2b)$ and $-\frac{2\gamma}{w_i^3}$, and a determinant $\frac{4b\gamma}{w_i^3} - a^2 > 0$, since $\gamma \ge (w^{max})^3\frac{a^2}{4b}$. It follows that a Nash equilibrium exists. Moreover, the Nash equilibrium is *unique* since the Jacobian of the system of equations

$\left\{\frac{\partial\pi_i}{\partial p_i}=0; \frac{\partial\pi_i}{\partial w_i}=0, i=1,\dots,N\right\}$ is negative definite. (The Jacobian $J=\begin{pmatrix} -2b & \beta & \cdots & \beta & -a & \alpha & \cdots & \alpha \\ & \ddots & & & & \ddots & & \\ \beta & \cdots & \beta & -2b & \alpha & \cdots & \alpha & -a \\ -a & \alpha & \cdots & \alpha & \frac{-2\gamma}{w_i^3} & 0 & \cdots & 0 \\ & \ddots & & & & \ddots & & 0 \\ \alpha & \cdots & \alpha & -a & 0 & \cdots & 0 & \frac{-2\gamma}{w_i^3} \end{pmatrix}$ is dominant diagonal, since $2b > a+(N-1)(\alpha+\beta)$ and $\frac{2\gamma}{w_i^3} \geq a+(N-1)\alpha$. Since its diagonal elements are negative, J is negative definite.) Thus any pair (p,w), which satisfies the first order conditions:

$$\frac{\partial\pi_i}{\partial p_i}=0; \frac{\partial\pi_i}{\partial w_i}=0, \text{if } w_i < w^{max} \text{ and } \frac{\partial\pi_i}{\partial w_i} \geq 0, \text{ if } w_i = w^{max}, i=1\dots,N \tag{38}$$

is the *unique* Nash equilibrium. We show that (38) is indeed satisfied by a symmetric vector, i.e. $p_1=\cdots=p_N=p$ and $w_1=\cdots=w_N=w$. Using (37) this implies:

$$p=\frac{a^0-w(a-(N-1)\alpha)+b(c+\gamma)}{2b-(N-1)\beta} \tag{39}$$

(Substituting this identity into the expression for $\frac{\partial\pi_i}{\partial w_i}$ in (37), we obtain after some algebra and $C=\frac{2b-(N-1)\beta}{a[a-(N-1)\alpha]}$:

$$\frac{\partial\pi_i}{\partial w_i}\frac{w_i^2}{C}=w^3-w^2\frac{a^0-(c+\gamma)(b-(N-1)\beta)}{a-(N-1)\alpha}+\frac{\gamma(2b-(N-1)\beta)}{a(a-(N-1)\alpha)} \tag{40}$$

Clearly, $\frac{\partial\pi_i}{\partial w_i}>0$ when evaluated at $w=0$ (and $p=\frac{a^0-b(c+\gamma)}{2b-(N-1)\beta}$). Thus, if $\frac{\partial\pi_i}{\partial w_i}>0$ when evaluated at $w=w^{max}$, the pair $[\frac{a^0-w^{max}(a-(N-1)\alpha)+b(c+\gamma)}{2b-(N-1)\beta}, w^{max}]$ is a Nash equilibrium. Otherwise, (40) has a unique root on $[0,w^{max}]$, since $\frac{\partial\pi_i}{\partial w_i}$ is monotone and the characterization of the unique equilibrium (p^*,w^*) follows. Finally, using (37) and (38), we obtain $-\frac{a\lambda^*}{b}+\frac{\gamma}{(w^*)^2}=[\geq]0$, if $w^*<[=]w^{max}$.

*(b)*The proof is analogous to that of part (a), after establishing that the profit function $\Pi(p,w) = \sum_{i=1}^N\left[\lambda_i(p_i-c-\gamma)-\frac{\gamma}{w_i}\right]$ is jointly concave in the vector (p,w). This follows from its Hessian being negative dominant diagonal under the stated conditions.

(c)$C^C(w)$ and $C^*(w)$ differ by +a constant. If $\theta \geq [>]\frac{\rho}{2-\rho}$, $C^C(w) \geq C^*(w) \geq [>0]0$ for all $w \leq w^*$, so that $w^{CB} \geq [>]w^*$. Similarly, if $\theta < \frac{\rho}{2-\rho}$, $C^*(w) > C^C(w) \geq 0$, for all $w \leq w^{CB}$, so that $w^* > w^{CB}$.

*(d)*Since (p^*,w^*) is a Nash equilibrium under Simultaneous Competition, w^* is an equilibrium in the Waiting Time competition model, when the price level is fixed at $p=p^*$. Thus $w^*=\sqrt{\frac{\gamma}{a(p^*-c-\gamma)}}$, as shown in §4. Similarly, w^{CB} is the optimal waiting time standard in the centralized problem when the price level is fixed at p^{CB}, so that $w^{CB}=$

$\sqrt{\frac{\gamma}{(a-(N-1)\alpha)(p^{CB}-c-\gamma)}}$. Thus, if $p^{CB} \leq p^*$, then $w^{CB} \geq w^*$.

(e)$w^{CB} \geq w^*$ follows from part (c). We distinguish between two cases: (i) $w^* < w^{max}$; (ii) $w^* = w^{max}$. In case (i), $\lambda^* = \frac{b\gamma}{a(w^*)^2} \geq \frac{b\gamma}{a(w^{CB})^2} \geq \lambda^{CB}$, by parts (a) and (b). In case (ii), $w^{CB} = w^*$, and p^* and p^{CB} are, respectively, the equilibrium price and the optimal price level in a centralized system, where the waiting time standard is fixed at $w = w^{max} = w^{CB}$. $\lambda^* \geq \lambda^{CB}$ now follows from Proposition 1(b).

■

Proof of Proposition 7.1

*(a)*It follows from (37) that $p^* - c - \gamma = \frac{\lambda^*}{b}$. Employing the relationships between w^* and λ^*, and w^{CB} and λ^{CB} obtained in Theorem 6.1(a) and (b), we get $\pi_i^* = \lambda^*(p^* - c - \gamma) - \frac{\gamma}{w^*} = \frac{(\lambda^*)^2}{b} - \sqrt{\frac{a\gamma\lambda^*}{b}}$ and $\pi^{CB} = \lambda^{CB}(p^{CB} - c - \gamma) - \frac{\gamma}{w^{CB}} = \frac{(\lambda^{CB})^2}{b(1-\rho)} - \sqrt{\frac{1-\theta}{1-\rho}}\sqrt{\frac{\lambda^{CB}a\gamma}{b}}$.

*(b)*If $\theta = \frac{\rho}{2-\rho}$, $1-\theta = 2\frac{1-\rho}{2-\rho}$ and $C^*(w)$ and $C^C(w)$ coincide and have an identical (unique) root w^0 on $[0, w^{max}]$. Employing the relationships between λ^* and w^* and λ^{CB} and w^{CB} in parts (a) and (b) of Theorem 8, the condition in part (a) reduces to $\frac{\gamma^2 b(2-\rho)}{4a^2(1-\rho)(w^0)^4} - \frac{\gamma}{w^0} \geq \frac{b\gamma}{a^2(w^0)^4} - \frac{\gamma}{w^0}$ which holds iff $\frac{2-\rho}{4-4\rho} \geq 1 \Leftrightarrow \rho \geq \frac{2}{3}$.

■