Outsourcing Service Processes to a Common Service Provider under Price and Time Competition

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In many industries, firms consider the option of outsourcing an important service process associated with the goods or services they bring to the market. Often, competing firms outsource this service process to one or more common service suppliers. When they outsource to a common service provider, this gives rise to a service supply chain. We develop analytical models to characterize the benefits and disadvantages of outsourcing in service industries in which the retailers compete with each other in terms of the price they charge and/or the waiting time expectations and standards which they adopt and sometimes advertise. We show that the benefits of outsourcing are affected by the supplier's ability to exploit the benefits of service pooling as well as differences in the cost rates themselves.

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1. Introduction

In many industries, firms consider the option of outsourcing an important service process associated with the goods or services they bring to the market. Often, competing firms outsource this service process to one or more common service suppliers. For example, after-sales support and maintenance services of appliances and electronic equipment are often outsourced by competing firms to a common maintenance or repair service provider. The same applies to (part of) the technical support function of software vendors or the back office operations of banks[0] . For make-to-order consumer goods, the potentially outsourced service process may refer to the final production or assembly stage of the product itself. Japanese automobile manufacturers, for example, have adopted make-to-order assembly systems to cut inventory costs for themselves and their dealers, and to reduce the need for rebates for slow selling vehicles. The outsourcing firms may represent independent companies or divisions of the same corporation. Staying with the example of the automobile

[0] An example of the former is the Israeli firm Calanit to whom both Microsoft and Sun Microsystems outsource their software support. Back office operations represent the bulk of the services banks provide in managing checking and money markets accounts and a variety of other asset funds and hence the justification for the transfer fees and interest spread charged to the customer. Again, many competing banks outsource to the same IT service provider, such as IBM Global Services, who handles these different banks sometimes with dedicated and sometimes with shared facilities.

industry, we have observed that most domestic, European and Japanese manufacturers have chosen to "bring the market into the firm" by creating separate and competing divisions with broad managerial autonomy. Perhaps surprisingly, different divisions often sell similar cars with identical chassis and engines, often produced-to-order in a common plant. An example is Toyota which sells, the Sienna and Lexus Rx-30, with identical chassis and engines, via two independent divisions, which "outsource" the assembly process to common plants in West Virginia and Canada[1]. Several papers in the economic literature provide theoretical frameworks to characterize the benefits of divisionalization, see e.g. Baye et al. (1996), Baker et al. (2001) and the references therein.

The outsourced service process is often of critical strategic importance to the competing firms, as the above examples indicate. In many industries, the product itself becomes increasingly commoditized and firms differentiate themselves in terms of the quality of the after-sales service they provide, even though maintenance, repair and technical support functions may fail to be part of the firms' core competency and, as a consequence, are often outsourced. [2] The attractiveness of the after-sales service process is, once again, often determined by the waiting time customers experience before their problem is solved, along with the price they pay for their service contract. Finally, in the automobile industry, customer waiting times become an increasingly important component of the firms' competitive strategies, along with the purchasing price of the automobile itself, in particular now that the quality gap between Japanese manufacturers and their American and European competitors has shrunk significantly. For example, Toyota's goal has been to reduce the average waiting time between dealer order and delivery to fourteen days; see e.g. Fay (2004).

We develop analytical models to characterize the benefits and disadvantages of outsourcing in service industries in which the retailers compete with each other in terms of the price they charge and/or the waiting time expectations and standards which they adopt and sometimes advertise. When some or all of the firms outsource the service process to a common service provider, this gives rise to a *service supply chain*.

In the general strategy literature, several reasons are mentioned as to why firms may benefit from out-

[1] Baye et al. (1996) document similar practices in the mid nineties among the big three domestic manufacturers: "For example, General Motors produced the LeSabre (Buick Motor Division) and the Olds 88 (Oldsmobile Division) which, while differing in styling, are built on the same chassis and, comparably equipped, sell for virtually identical prices. Similarly, the Ford Motor Co. produced the Sable (Lincoln- Mercurey Division) and the Taurus (Ford Division), which are effectively the same car with different name plates, as are the Chrysler Corporation's Plymouth Voyager and Dodge Caravan. Indeed, in just about every price range, all the major domestic manufacturers have several divisions producing competing products."

[2] A recent survey by AMR, (see Bijesse et al. (2002)) estimates that after-sales services represent, on average, 24% of the revenues and no less than 45% of the profits earned by consumer goods merchandisers. Dennis and Kambil (2003) and Wise and Baumgartner (1999) report similar results for a large verity of industries. (see also Cohen et al. (2006).)

sourcing: first, the service provider to whom the service process is outsourced, may enjoy lower operating cost rates, as the service process represents *his* core competency rather than that of the service retailers. A second explanation relates to possible *economies of scale* and *economies of scope*. The former consist of reduced per unit costs as the customer volume increases while the latter points at the benefits of pooling demand streams into a common service facility. Benson and Ieoronimo (1996) and Lacity and Hirschheim (1993), for example, identify cost efficiencies as the principle factor driving outsourcing decisions for maintenance functions and information systems, respectively. Gupta and Zhender (1994) emphasize economies of scale in the cost structure as a prime reason why outsourcing is often beneficial. A *third*, less recognized driver of the benefits of outsourcing (to a common supplier) results from the altered competitive dynamics when a decentralized (but coordinated) service chain competes with each of the remaining firms with in-house service. Under price competition, for example, we show that even in the absence of the first two potential benefits of outsourcing, the aggregate equilibrium profits achieved by a service chain are higher than those earned by the participating retail firms when they keep service in-house; however, part of these incremental profits need to be shared with the outside supplier.

To assess the benefits of outsourcing strategies, we need to address the following questions:

(I) When are firms better off if all of them choose to outsource rather than perform the service in-house? Given a unique equilibrium under in-house service, the firms' profits are unambiguously specified. Under outsourcing, these depend on (i) what fraction $e \leq 1$ of first best aggregate performance the service chain achieves, and (ii) how the aggregate profits in the chain are divided among the outside service provider and the retailers. As far as (i) is concerned, for the sake of brevity and simplicity only, we present all of our results assuming $e = 1$, thus portraying the outsourcing option in the best possible light. (In §7, we show that first best performance can indeed be achieved by a decentralized service chain, under specific pricing schemes of the types most commonly used in practice.)

As to the division of aggregate profits in the service chain, we assume that it is given by a generalized or asymmetric Nash Bargaining solution, which takes into account the relative bargaining powers of the chain members, as well as minimum profit levels for each of the members. The minimum profit level for each retailer is given by his equilibrium profit under in-house service while that of the supplier is exogenously given. As shown by Roth (1979), the generalized Nash bargaining solution is the only bargaining concept to satisfy three intuitive axioms, the same axioms Nash postulated in 1951, with the omission of the symmetry requirement, which applies only when all chain members have equal bargaining powers[3]. In addition, experimental bargaining theory exhibits stronger empirical evidence of this bargaining concept than any other[4]. The (generalized) Nash bargaining solution is also universally used to model the bargaining outcome of profit sharing in supply chains, see for example Hanany and Gerchak (2008) and Nagarajan and Sošić (2008) for a survey.

(II) When will a service chain in which all firms choose to outsource to a common provider be stable in the sense that no firm has an incentive to unilaterally abandon the chain and provide in-house service instead? More generally, what sets of outsourcing retailers constitute a stable service chain in the sense that no firm within the chain has an incentive to unilaterally bring the service in-house and no-firm outside the chain to join it?. How do the answers to these questions depend on the intensity of the competition, the number of firms in the industry the sales volumes of the firms, the degree to which service pooling benefits are exploited by the supplier and the degree to which the supplier can operate at lower cost rates than the service retailers themselves?

Each of the above questions is analyzed under each of three types of competition: (i) *Price competition*, in which all waiting time standards are exogenously given and the firms compete on the basis of their prices only, (ii) *Waiting time* competition: here all prices are exogenously given and the competition is in terms of waiting time standards, and (iii) *Simultaneous competition*: all prices and waiting time standards are selected simultaneously by the various service retailers.

We represent a firm's demand rate as a general function of all prices and all waiting time standards in the industry. We focus primarily on two important classes of demand models: The first class uses a separable specification which, in addition, is linear in the price vector. The second class is that of the attraction models. Here, each firm is characterized by an attraction value given by a general function of its price and waiting time standard. A firm's market share, is given by the ratio of its attraction value and the attraction values of all firms in the industry, that of the no-purchase option included. These broad classes of demand models allow us to represent general tradeoffs for potential customers among the prices, waiting time standards and other service attributes. Price and waiting time are treated as truly independent attributes, in that, in general, a change in a firm's waiting time standard can *not* be compensated by a price change that leaves all market

[3] The three axioms merely require the bargaining outcome to be Pareto optimal, invariant under linear transformations of the expected profits, and independent of irrelevant alternatives

[4] see Roth (1995)

effects unchanged. [5]

The system of demand functions determines the rates at which customers arrive to the different service retailers. For most of the paper, we assume all service retailers face Poisson demand processes and we model each service facility as an M/M/1 system whose capacity, i.e, service rate is chosen to handle the demand volume(s) while meeting the waiting time standard(s). However, in §9, we discuss how our analysis can be extended to more general queueing systems. Under outsourcing, the common service provider may employ general dynamic priority schemes to prioritize among customers referred by the different firms. Whoever provides the service incurs three types of costs: first, there is a cost per unit of time proportional to the adopted capacity level and a second, transaction/handling cost per customer served. Finally, there may be fixed costs for operating a service facility.

In §4-6, we analyze the special case where (a) all service retailers share identical characteristics, and (b) the demand functions are affine in both the price and service level. This special case permits analytical characterizations of the benefits of outsourcing. The following represent some of our main insights:

Assume first that the firms engage in price competition, under given waiting time standards. The first question that needs to be resolved to determine the attractiveness of the outsourcing option, is whether the full service chain, (which arises when all retail firms outsource,) is *viable*, i.e., whether the chain permits a Nash bargaining solution under which all retail firms earn in excess of their equilibrium profits under in-house service, while ensuring the supplier a profit level at least equal to her minimum participation level. The necessary and sufficient conditions for viability can be expressed in terms of a simple minimum threshold for the firms' sales volume, when they set their prices to cover marginal costs only. We refer to this sales volume as the *benchmark* sales volume, it is easily shown to be an upper bound for the firms' sales volume under any pricing scheme which leaves the firms with non-negative variable profits. The minimum threshold value is a simple closed form expression of (some of) the model parameters. It increases with (i) an index measuring the competitive intensity in the market, (ii) the so-called total price sensitivity, i.e., the marginal decrease in sales volume when all prices increase by the same (marginal) amount , (iii) the supplier's minimum profit value, and (v) the adopted waiting time standard. At the same time, the minimum threshold decreases with an efficiency index which measures the degree to which economies of scope due

[5] We refer to Allon and Federgruen (2007) for a discussion of why it is important to treat prices and waiting time standards as independent strategic instruments, in contrast to the traditional approach in the literature on service competition, in which the price and waiting time are aggregated into a single, so called, *full price* measure.

to service pooling are exploited. (This provides an example of how economies of scope act as a driver of the benefits of outsourcing, see above.)

However, even when the full service chain is viable, it may fail to be *stable* in that an individual retail firm may be better off when defecting from the chain and bringing the service process in-house. The necessary and sufficient condition for the *full* service chain to be stable, can again be expressed in terms of a minimum threshold level for the firms' (benchmark) sales volume (, when pricing at cost), along with a (closed form) *industry power condition* involving only three industry characteristics: (a) the number of firms in the industry, (b) the aforementioned competitive intensity index, as well as (c) the relative bargaining power of the supplier defined as the ratio of her bargaining index and that of the retailers. This characterization applies to the most common setting where the supplier's minimum profit level is *larger* than any savings in terms of fixed operating and capacity costs, captured when all retail firms switch from in-house service to outsourcing. We refer to this as the case of a *demanding* supplier. In the alternative case of a *modest* supplier, i.e., when the supplier's minimum profit level is *exceeded* by these savings, the above industry power condition is sufficient by itself, i.e. without a minimum threshld level for the benchmark sales volume. In this case, to extend the industry power condition to a necessary and sufficient condition, one must allow for this power condition to be violated while the firms' benchmark sales volume is below a simple (closed form) *upper bound*. Once again, the bounds for the firms' benchmark sales volume are increasingly relaxed as the efficiency index grows, a further manifestation of economies of scope as a driver of the benefits of outsourcing.

It is of interest to analyze the industry power condition, in the extreme case when the supplier's relative bargaining power is negligible. Even in this best case scenario for a stable chain, the industry power condition is satisfied for all industries with $N = 2$ or 3 firms, but for industries with 4 or more firms, only if the competitive index is above a minimum level which increases very rapidly to the limiting value of 1. (The minimum value for the competitive index is 91% when $N \geq 7$.) The implication for the general case, when the supplier has bargaining power, is that the full service chain is stable if and only if the number of firms in the industry is small or the competitive intensity sufficiently high, while the benchmark sales volume is sufficiently high as well. For a given competitive intensity, the minimum threshold level for the benchmark sales volume increases with the price sensitivity and decreases with the efficiency index.

Whether all, some or none of the firms outsource may be viewed as the outcome of a two-stage non-coopeative game: in the first stage, each of the retail firms choose whether to outsource the service process to the supplier; in the second stage, the firms select their price levels, with the understanding that those that have chosen to outsource pay the supplier a set of fees under which the service chain operates at maximum efficiency, while, in addition, aggregate equilibrium profits of the service chain are reallocated with periodic fixed fees to achieve the above mentioned Nash bargaining solution. In terms of this two-stage game, the outcome where all firms join a viable and stable chain, corresponds with a Subgame Perfect Nash Equilibrium (SPNE) in which all firms decide to outsource in the first stage game[6].

As detailed above, this solution is an SPNE only under special circumstances. We therefore provide a full characterization of the equilibrium in the two-stage game showing that it *always* has a pure SPNE, but often one in which only some of the firms outsource. Since the model is symmetric, if a SPNE exists with specific set of m^o firms outsourcing, then all other sets of m^o firms also give rise to a SPNE. In other words, the equilibrium outcome of the first stage game is completely characterized by the *number* m^o of firms choosing to outsource. We derive the necessary and sufficient conditions for a chain with m^o firms to arise as a SPNE. In the most common case of a *demanding* supplier, for $1 < m^o < N$, these conditions take for form of a *pair* of industry power conditions, in conjunction with a lower and possibly an upper bound for the benchmark sales volume. The industry power conditions, once again, depend only on the number of firms in the industry, the competitive intensity and the relative bargaining power of the supplier. [7] We also provide a simple condition for m^o, the number of outsourcing firms in the equilibrium to be unique. (In our numerical investigations, the condition is almost always, but not always satisfied). For a given industry size, the equilibrium service chain size is increasing in the competitive intensity.

The above results are obtained under the assumption that the supplier incurs the same customer handling and capacity cost rates as the retail firms. However, we systematically show how the necessary and sufficient conditions for viability or stability of a given service chain, i.e. for a given equilibrium outcome of the two-stage game, are affected by any cost rate advantage the supplier may enjoy. Clearly, the larger such cost rate advantages are, the more likely it is that any given service chain is viable and the more firms can be expected to outsource, thus confirming the supplier's cost rate advantages as a prime driver of the benefits of outsourcing.

[6] In general, if for a given chain composition, no Nash bargaining allocation exists for its equilibrium profits, i.e. if these fail to cover the minimum profit levels of the chain members, the service chain fails to be viable. Such an outcome, clearly, fails to be an SPNE

[7] A similar characterization of two necessary and sufficient equilibrium conditions is derived for the remaining case of a modest supplier

The above analyses are also pursued, assuming the firms compete by choosing their waiting time standards, under given prices. The necessary and sufficient condition for viability of the full chain now takes the form of a lower bound on the firms' gross profit margin, rather than the benchmark sales volume. The lower bound decreases to zero as the number of firms in the industry increases; it also depends on (i) the suppliers' minimum profit level net of savings in *fixed* capacity costs when the retail firms abandon their in-house service, (ii) an index measuring the intensity of the waiting time competition, (iii) the capacity cost rate, (iv) the firms' demand sensitivity to changes in their waiting time standard, and (v) the efficiency index. Among these monotonicities we show that the lower bound decreases with each of the last three parameters. The necessary and sufficient conditions for stability of the full service chain consists of a *single* industry power condition: an expression involving only (i) the number of firms in the industry, (ii) the competitive intensity index, (iii) the efficiency index, and (iv) the relative bargaining power of the supplier, needs to exceed a given threshold.

Inter alia, these results point at the importance of economies of scope as a driver of the benefits of outsourcing. This is further highlighted by the following observation. As in the case of price competition, the larger the efficiency index, i.e. the degree of service pooling, the more likely the full chain is stable. However, the impact of the degree of service pooling is considerably more drastic in the case of waiting time competition: under full service pooling, the chain is stable whenever it is viable, wheres in the absence of service pooling, the chain is never stable when $N \geq 5$, and only under a very high competitive intensity under $N = 4$. As in the case of price competition, we establish the existence of a SPNE in the two-stage game. We also show that the necessary and sufficient conditions for a SPNE with $m^o < N$ outsourcing firms, take the form of a *pair* of industry power conditions, similar to the above conditions for the case $m^o = N$.

Finally, we analyze the case of *simultaneous competition* where the firms compete by selecting prices as well as waiting time standards. This model no longer allows for closed form expressions of the equilibrium prices and waiting time standards under various outsourcing scenarios. However, we show that these continue to be *uniquely* determined (, under minor parameter conditions) and that they can be evaluated efficiently, as can the conditions for any given chain to be viable or stable. We, again, establish the existence of a pure SPNE in the two-stage game. Among other monotonicities, we show that the number of firms deciding to outsource increases with the price and the waiting time competitive intensity index.

In §7, we show for the general model and all three types of competition that under outsourcing, a decentralized service supply chain can operate at a first best level ($e = 1$), if the participating firms are charged a volume-based fee combined with a capacity based fee. The former consists of a constant fee per referred customer while the latter is proportional to the capacity level which the supplier needs to adopt if he were to serve the retailers' customers in a dedicated facility. The above fee structures may be complemented with a periodic fixed fee to be paid by the retailers to the supplier (or vice versa). Outsourcing contracts often include volume based and capacity based fees, see K'Djah.com and Jackson (1999), Hasija et al. (2006) and Chander (2007).

In §8 we use a numerical study to demonstrate how the above findings carry over in asymmetric industries and when the demand functions are given by an attraction model. §9 completes the paper with a brief outline of how our analysis can be extended to systems with more general service processes, represented by general queueing systems.

2. Literature Review

The literature on outsourcing in service industries is recent. Aksin et al. (2006) and Gans and Zhou (2003) consider a single server retailer (e.g. call center) who can outsource part of its business to an outside supplier. In Aksin et al. (2006), both firms incur costs proportional to their selected capacity level, but the outside supplier enjoys a lower capacity cost rate. Demand in any given time interval can be satisfied if it falls below the total available capacity, with any excess being lost. The authors state that for tractability reasons they do not model the service facility as a queue. Aksin et al. (2006) consider the following two arrangements between the outside supplier and the service retailer which are closely related to the above volume- and capacity- based fees: in the *first*, the retailer buys a certain capacity level from the supplier, for which it is charged a given fee per *unit of capacity*. All demand is first directed to the supplier with any excess "overflowing" to the retailer's own facility. In the *second* arrangement, demand is first directed to the retailer's own facility while any excess is handled by the outside retailer, at a given fee per customer referred. Under this arrangement the service retailer and outside supplier choose their capacity levels non-cooperatively and the authors establish the existence of a unique Nash equilibrium. Gans and Zhou (2003), model the service facilities as queueing processes, but assume that both the capacity level of the retailer and that of the outside supplier are determined by the former. (Two customer classes are considered; outsourcing is an option for *one* of them according to a queue-size dependent strategy.) We refer to Milner and Olsen (2008), Ren and Zhou (2008) and Shumsky and Pinker (2003) for papers studying a variety of pricing schemes for a service retailer who outsources part (or all) of his service requests to an outside supplier.

To our knowledge, Cachon and Harker (2002) is the only paper which addresses an industry of two *competing* service retailers, modeled as M/M/1 systems. In the outsourcing part of the paper, demand rates are linear functions of the firms' full price (= price + multiple of the expected sojourn time). Moreover, the demand of a firm is equally sensitive to a change in its own price as to that of its competitor. Both firms may outsource to a common supplier, who faces the same cost structure as the retailers themselves, and who serves the customers of each retailer in a dedicated facility, for a given fee per customer. The authors show that both firms benefit when they *both* outsource, compared to when they service the customers in-house, irrespective of the fee charged by the supplier. But, if this fee is set at a sufficiently high level, one of the retailers may benefit by keeping its service process in-house, assuming the competitor continues to outsource. Hassin and Haviv (2003) and Allon and Federgruen (2007) offer surveys of competition models in service industries. See also the nascent literature on competition models in which waiting time sensitive customers are segmented into multiple classes. e.g. Loch (1991), Lederer and Li (1997), Armony and Haviv (2001), Afeche (2004), and Allon and Federgruen (2004).

Our analysis of the stability of service chains is related to Nagarajan and Sosic (2007). For most of their paper, the authors consider the special case of our symmetric model in which firms do not incur any cost to sell their goods or services. Moreover the firms' demand functions only depend on the prices charged in the industry, as opposed to operational characteristics such as the waiting time standards considered here. More generally than in our model, firms may join any number of chains or "coalitions"; these do not involve any outside parties and are therefore irrelevant for outsourcing studies. The authors adopt the "farsighted stability" concept first introduced by Chwe (1994): a set of chains is unstable only if a coalition of firms may alter its allegiances by forming a new (set of) chain(s) or joining an existing chain, while improving profit levels for all defecting firms, and only if these profit requirements are sustained in any number of subsequent rounds of defections, each of which generate myopic profit improvements for the defecting set of firms. As the the authors note, this stability concept has been criticized as "too restrictive"[8]. Moreover, even in the above simple price competition model, full characterization of the set of stable coalitions is achievable only with $N = 3$ or $N = 4$ firms, where only a small number of sets of coalitions needs to be enumerated.

[8] The authors therefore also consider a second somewhat stronger stability concept called EPCF, see Nagarajan and Sosic (2007).

3. Model and Notation

We consider a service industry with N competing service retailers. Each firm i differentiates itself in the market by selecting a price p_i, as well as a waiting time standard w_i. The waiting time standard is defined as the expected steady state waiting time experienced by the customers, i.e. $w_i = \mathbb{E}(W_i)$. (Alternatively, the waiting time standard may be specified as a given *fractile* of the waiting time distribution. All of our results continue to apply, since the structural form of all profit functions remains unchanged.) When firm i serves its customers in-house, it faces three types of cost described by three parameters:

$$\begin{aligned} c_i &= \text{the variable transaction cost per customer demand and serviced}, i = 1, \ldots, N \\ \gamma_i &= \text{capacity cost (per unit of time,) per unit of capacity installed}, i = 1, \ldots, N \\ F_i &= \text{the fixed cost (per unit of time) of operating a service facility at firm } i, i = 1, \ldots, N \end{aligned}$$

Similarly, when the service process is outsourced to an outside supplier, it incurs the same volume-based transaction and capacity costs at rates c_0 and γ_0, respectively. In addition, the supplier may specify a minimum compensation level V, which may incorporate any fixed costs she incurs.

We model each service facility as an M/M/1 system whose capacity is specified by the facility's service rate. Under in-house service, let μ_i denote the capacity levels chosen by firm i. The following simple relationship exists between μ_i, w_i and λ_i, the firm's demand rate. Assuming $\lambda_i > 0$, we have

$$\mu_i = \lambda_i + \frac{1}{w_i} \tag{1}$$

(When $\lambda_i = 0, \mu_i = 0$ as well). The first term in this expression is the *base capacity level*, ensuring stability of the system, while the second term is the *service based capacity* required to guarantee a given waiting time standard.

If an outside supplier services the customers of the different firms in separate (dedicated) facilities, the required capacity is, of course, given by: $\mu_0^B = \sum_{i=1}^N \left(\lambda_i + \frac{1}{w_i}\right)$. Allon and Federgruen (2004) have shown that, under *pooled* service, the minimum required capacity level, considering all non-anticipating dynamic priority schemes, is given by:

$$\mu_0^P = \max_{S \subset \{1,\ldots,N\}} \left\{ \sum_{i \in S} \lambda_i + \frac{\sum_{i \in S} \lambda_i}{\sum_{i \in S} \lambda_i w_i} \right\} = \max_{S \subset \{1,\ldots,N\}} \left\{ \sum_{i \in S} \lambda_i + \frac{1}{\sum_{i \in S} \frac{\lambda_i}{\sum_{j \in S} \lambda_j} w_i} \right\} \tag{2}$$

(Note, under non-identical waiting times, it is no longer feasible to serve all customers on a FIFO basis) Proposition 1(a) ibid show that $\mu_0^P \leq \mu_0^B$, reflecting *economies of scope*. (More generally, the pooling of any two groups results in cost savings.) This observation, in itself, bodes well for the increased benefits of outsourcing under pooled service versus service with dedicated facilities. The presence of *economies of scope* do not imply that the cost structure (2) exhibits *economies of scale* as well: as demonstrated in Allon and Federgruen (2004), both marginal and average costs per customer may increase if the demand volume of a *single* firm is increased.

The price p_i is chosen from an interval $[p_i^{min}, p_i^{max}], i = 1, \ldots, N$. Clearly, firm i selects a price p_i which results in a non-negative gross profit margin $p_i - c_i - \gamma_i$. (By (1), $c_i + \gamma_i$ is the marginal cost per unit of demand.) Thus, without loss of generality, we select $p_i^{min} = c_i + \gamma_i, \quad i = 1, \ldots, N$. As to p_i^{max}, it is chosen to be sufficiently large as to have no impact on the equilibrium behavior.

The demand rate λ_i may depend on all of the industry's prices and waiting time standards, according to a general set of twice differentiable functions:$\lambda_i = \lambda_i(p, w),\ i = 1, \ldots, N$, with $\frac{\partial \lambda_i}{\partial p_i} \leq 0, \frac{\partial \lambda_i}{\partial w_i} \leq 0$ and λ_i varying concavely with w_i, i.e. $\frac{\partial^2 \lambda_i}{\partial w_i^2} \leq 0$. In our base models, we focus on the following class of demand functions which are linear in the prices: (In §8, when addressing the general model, we do consider demand functions given by attraction models.)

$$\lambda_i = \left[a_i(w_i) - \sum_{i \neq j} \alpha_{ij}(w_j) - b_i p_i + \sum_{i \neq j} \beta_{ij} p_j \right]^+, \ i = 1, \ldots, N \tag{3}$$

where $x^+ = \max(x, 0)$. a_i is a decreasing concave function, reflecting the fact that reductions of a firm's waiting time standard result in increases of its demand volume; however, these increases become progressively smaller as the waiting time standard continues to be cut. The functions α_{ij} are *general* decreasing functions, since a reduction of a competitor's waiting time standard results in a decrease of the firm's demand volume. The price coefficients $b_i > 0$, $\beta_{ij} > 0, i \neq j$ satisfy the well known dominant diagonal conditions

$$(D) \quad b_i > \sum_{j \neq i} \beta_{ij}, \qquad (D') \quad b_i > \sum_{j \neq i} \beta_{ji}, \qquad i = 1, \ldots, N$$

These conditions are generally satisfied; they merely stipulate that a *uniform* price increase by all N firms cannot result in an increase in any firm's demand volume, and that a price increase by a given firm cannot result in an increase of the industry's aggregate demand. [9]

Let $\bar{b}_i = b_i - \sum_{j \neq i} \beta_{ij} > 0$ denote the *total* price sensitivity of firm i's demand ,i.e., the (absolute value of the) marginal change in firm i's demand volume due a to uniform price increase by all firms.

Under a given type of competition (i.e. price–, waiting time–, or simultaneous competition), let

$\pi_i^* =$ the expected equilibrium profits of firm i, exclusive of fixed costs assuming each firm provides in-house service

$\overline{\pi_i}^* = \pi_i^* - F_i =$ the *net* expected equilibrium profits of firm i, assuming all firms provide in-house service.

When a group of retail firms I decide to outsource its service to an outside supplier, with aggregate chain wide profits Π^c, we assume that their profits are shared in accordance with the (asymmetric) generalized Nash Bargaining Solution, in which the so-called "disagreement" value for the supplier is given by V, her minimum acceptable profit value, and that of firm i, by the net profit value (under in-house service) $\overline{\pi_i}^*$, $i \in I$. These disagreement values may be interpreted as minimum acceptable profit values. The generalized Nash bargaining solution is further specified by bargaining power indices $\eta_0 > 0$ and $\eta_i > 0$ for the supplier and firm i respectively. A Nash bargaining solution exists if and only if the chain-wide profits Π^c are at least equal to the sum of the disagreement values, i.e.

$$\Pi^c \geq \sum_{i \in I} (\pi_i^* - F_i) + V. \tag{4}$$

In this case, the supplier receives a surplus payment x_0 (beyond V), and retailer i a payment x_i (beyond $\overline{\pi_i}^*$) so as to maximize: $\max x_0^{\eta_0} \Pi_{i \in I} x_i^{\eta_i}$ s.t. $x_0 + \sum_{i \in I} x_i = \Pi^c - \sum_{i \in I} \pi_i^* - V + \sum_{i \in I} F_i$. It is easily verified that the surplus payments are given by:

$$x_0 = \frac{\eta_0}{\eta_0 + \sum_{j \in I} \eta_j} \left(\Pi^c - \sum_{i \in I} \pi_i^* - V + \sum_{i \in I} F_i \right), \text{ and } x_i = \frac{\eta_i}{\eta_0 + \sum_{j \in I} \eta_j} \left(\Pi^c - \sum_{i \in I} \pi_i^* - V + \sum_{i \in I} F_i \right), \tag{5}$$

In other words, the surplus created by the service chain beyond the aggregate disagreement value is distributed to the members in proportion to their bargaining power indices.

[9] As is well known from the literature on oligopoly models with *product differentiation*, systems of demand equations need not, but often can be obtained from one of several underlying consumer utility models, in particular the *representative consumer model*, the *random utility model* and the *address model*. Similarly, (3) may, e.g. be derived from a representative consumer model with utility function $U(\lambda, \theta) \equiv C + \frac{1}{2}\lambda^T B^{-1}\lambda + \lambda^T B^{-1}\overline{a}(w)$ where the $N \times N$ matrix B has $B_{ii} = -b_i$ and $B_{ij} = \beta_{ij}, i \neq j$, $\overline{a}(w) \equiv a_i(w_i) - \sum_{j \neq i} \alpha_{ij}(w_j)$ and $C > 0$. ((D) ensures that B^{-1} exists and is negative semi-definite, giving rise to a jointly concave utility function). The demand functions (3) arise by optimizing the utility function subject to a budget constraint.

We initially focus on a basic class of models with *identical* service retailers, and demand functions of the quasi-separable type (3). This allows for analytical comparisons of the equilibria and coordinating pricing schemes under each of the three types of competition and each of the various outsourcing options.

Thus, assume the firms face identical cost parameters $\{c_i, \gamma_i\}_{i=1}^N = (c, \gamma)$. Consider firm independent demand functions with price linearity as in (3), and assume that the functions $a_i(\cdot)$ and $\alpha_{ij}(\cdot)$ are affine as well. Thus, for positive constants a^0, a, α, b and β:

$$\lambda_i = \left[a^0 - aw_i + \alpha \sum_{j \neq i} w_j - bp_i + \beta \sum_{j \neq i} p_j\right]^+, i = 1, \ldots, N. \tag{6}$$

Similar to condition (D), we assume without practical loss of generality, that

$$a > (N-1)\alpha, \tag{7}$$

,i.e., no firm experiences an increase in its demand volume when all firms increase their waiting time standards by the same amount. Define the intensity of the price competition by $\rho \equiv \frac{(N-1)\beta}{b}$ and the intensity of the waiting time competition by $\theta \equiv \frac{(N-1)\alpha}{a}$. Note that both ρ and θ are dimensionless indices, with $0 < \rho < 1$ and $0 < \theta < 1$, by (D) and (7).

4. Identical Retailers: Price Competition

In this section we consider settings where the firms engage in price competition, under identical waiting time standards $w_i = w$. As our initial benchmark we characterize the equilibrium prices, sales volumes and profit levels when all firms keep their service in-house. We then consider the other extreme, where *all* firms outsource their service process to a common outside supplier; we derive the necessary and sufficient conditions for the resulting service chain to be viable, i.e., for its equilibrium aggregate profits to cover the minimum acceptable profit level V of the supplier as well as those demanded by the retailers. (Recall, we assume that the minimum participation levels of the retailers are given by their equilibrium profits under in-house service). We also compare the equilibrium prices and sales volumes with those achieved under in-house service. A *viable* service chain implies that all participating retail firms are better off under outsourcing; however, this does not guarantee the chain's *stability*, i.e. it does not preclude the possibility that an individual firm could improve its equilibrium profits when defecting from the chain and bringing the service in-house. We therefore proceed with the derivation of the necessary and sufficient condition for the full service chain to be stable. When the full service chain is stable, this is equivalent to the two-stage outsourcing game having an SPNE in which all firms choose to outsource (in the first stage game). The final subsection of this section is dedicated to the full characterization of the SPNE in the two-stage game, in particular the derivation of the necessary and sufficient conditions for an SPNE in which a specific number of firms decide to outsource, while all others choose to keep the service in-house.

In-House Service

When all N firms provide *in-house* service, the profit function for firm i is, by (1), given by

$$\pi_i = \lambda_i(p_i - c) - \gamma\mu_i = \lambda_i(p_i - c) - \gamma\left(\lambda_i + \frac{1}{w_i}\right). \tag{8}$$

Theorem 1 in Allon and Federgruen (2007) establishes that, like with all price-linear demand functions and affine cost functions, for identical as well as non-identical firms, the price competition model has a *unique* price equilibrium p^* which satisfies the set of linear equations

$$\lambda_i^* = b(p_i^* - c - \gamma), i = 1\ldots, N \tag{9}$$

where $\lambda_i^* = \lambda_i(p^*, w) > 0$. It is easily verified, by substitution in (6) and (8), that

$$p_i^* = p^* \equiv \frac{a^0 - w(a-(N-1)\alpha) + b(c+\gamma)}{2b-(N-1)\beta}; \quad \overline{\pi_i}^* = b(p^* - c - \gamma)^2 - \frac{\gamma}{w} - F. \tag{10}$$

In particular, by (9), the model has a unique symmetric equilibrium, with $p_i^* = p^* > c + \gamma$.

Universal Outsourcing

We now compare the decentralized system with in-house service at all service retailers, with the opposite scenario where they all decide to outsource. We first assume that the common supplier faces the *same* cost rates as the retailers encounter where providing in-house service. To put the outsourcing option in the best possible light, we assume the resulting service chain, while continuing to be decentralized, operates at maximum efficiency, i.e., it achieves the same aggregate profits as a centralized chain. (In §7, we show that the decentralized service chain achieves maximum profits, identical to those in a centralized chain under various combinations of volume– and capacity based fees.) The latter are unaffected by transfer payments between the retailers and the supplier, and are given by $\Pi^c(p) = \sum_{i=1}^N \lambda_i p_i - c\sum_{i=1}^N \lambda_i - \gamma\sum_{i=1}^N \left(\lambda_i + \frac{1}{w}\right) -$

$\frac{\nu\gamma}{w}$. Here $0 \le \nu \le N-1$ represents an efficiency index. The extreme values $\nu = 0$ and $\nu = N-1$ arise when the suppliers do not exploit economies of scope at all or do so in a maximal way. When serving the customers of different retailers in dedicated facilities, $\nu = 0$, see (1); when pooling all service processes into a *single* facility, $\nu = N-1$. (When the same waiting time standards and the service time distributions apply to all customers, there is no advantage to give priority to some firms over others. i.e., customers are optimally served on a FIFO basis and the supplier's service process is an M/M/1 system with arrival rate $\sum_{i=1}^{N} \lambda_i$. The capacity related cost is thus given by $\gamma \left(\sum_{i=1}^{N} \lambda_i + \frac{1}{w} \right)$.) More generally, the potential for economies of scope may only be exercised in part, giving rise to an intermediate value for the efficiency index.

The function Π^c is a strictly jointly concave quadratic function of p. (In its Hessian, the diagonal elements equal $-2b$, while the off-diagonal elements equal β. This matrix is negative semi-definite ,since by (D), the absolute value of the diagonal element dominates the sum of the absolute values of the off diagonal elements in each row.) Thus, the *unique* optimal price vector p^c is the one that satisfies the set of first order conditions:

$$-b(p_i - c - \gamma) + \lambda_i + \beta \sum_{j \neq i} (p_j - c - \gamma) = 0, \tag{11}$$

This system of equations has a *symmetric* solution, p^c

$$p_i^c = p^c \equiv \frac{a^0 - w(a-(N-1)\alpha)}{2(b-(N-1)\beta)} + \frac{c+\gamma}{2} = \frac{a^0 - w(a-(N-1)\alpha)}{2\overline{b}} + \frac{c+\gamma}{2}. \tag{12}$$

Let $\lambda^c = \lambda(p^c, w)$ and let $\Pi^c = \Pi(p^c)$ denote the *optimal* chain-wide profits in the service chain. Note that p^*, p^c, λ^* and λ^c are all independent of the efficiency index. However, while for a given total price sensitivity $\overline{b}$, the price level p^c is independent of the competitive price intensity ρ, the price p^*, in a decentralized system with in-house service, decreases with ρ. Proposition 4.1 shows that under outsourcing each firm's optimal price level is higher and its demand volume lower than under in-sourcing.

Lemma 4.1*(a) $p^* < p^c$; (b) $\lambda^* > \lambda^c$, (c) For a fixed value of $\overline{b}$, the total price sensitivity of demand,* $\Pi^c = \frac{N}{4\overline{b}} \lambda^2(c+\gamma, w) - \frac{(N-\nu)\gamma}{w}$ *is independent of ρ, while, under in-house service, each retailer's profits decreases with ρ:* $\overline{\pi_i}^* = \frac{1}{b} \left([a^0 - w(a-(N-1)\alpha)] - (c+\gamma)\overline{b} \right)^2 \frac{1-\rho}{(2-\rho)^2} - \frac{\gamma}{w} - F = \frac{\lambda^2(c+\gamma, w)}{b(2-\rho)^2} - \frac{\gamma}{w} - F$

Proof: (a) and (b) $p^c = \frac{a^0 - w(a-(N-1)\alpha)}{2b(1-\rho)} + \frac{c+\gamma}{2} > \frac{a^0 - w(a-(N-1)\alpha)}{2b-\rho} + \frac{c+\gamma}{2-\rho} = p^*$, and $\lambda^* > \lambda^c$.

(c) It follows from (12) that $\Pi^c = N\lambda^c(p^c - c - \gamma) - \frac{(N-\nu)\gamma}{w} = N(b-(N-1)\beta)(p^c - c - \gamma)^2 - \frac{(N-\nu)\gamma}{w} =$

$\frac{N(b-(N-1)\beta)}{4}\left(\frac{a^0-w(a-(N-1)\alpha)}{b-(N-1)\beta}-(c+\gamma)\right)-\frac{(N-\nu)\gamma}{w}=\frac{N}{4b}\lambda^2(c+\gamma,w)-\frac{(N-\nu)\gamma}{w}$. ■

Thus, both the firms' equilibrium profits under in-house service and the aggregate chain-wide profits under outsourcing can be represented as a simple quadratic function of $\lambda(c+\gamma,w)$, a benchmark demand volume, i.e. the demand volume when each of the firms prices at cost ($p=c+\gamma$). Note that the aggregate chain-wide profits grow with this benchmark demand volume at a faster rate than the firms' aggregate profits under in-house service. Substituting the expressions for Π^c and π_i^* into (4), we obtain the necessary and sufficient conditions for the existence of a Nash bargaining solution, i.e. for the full service chain to be viable:

$$N\lambda^2(c+\gamma,w)\left[\frac{1}{4b(1-\rho)}-\frac{1}{b(2-\rho)}\right]\geq V-NF-\frac{\gamma}{w}\nu,\text{or} \tag{13}$$

$$\lambda(c+\gamma,w)\geq\lambda^0\equiv\frac{2(2-\rho)}{\rho}\sqrt{b}\sqrt{\left[\frac{V}{N}-F-\frac{\gamma}{w}(\frac{\nu}{N})\right]^+}$$

In other words, a Nash bargaining solution exists if and only if the demand volume when pricing at cost exceeds a minimum threshold, which increases with (i) the total price sensitivity, (ii) the supplier's minimum profit value V and (iii) the waiting time standard and (decrease with) the efficiency index. To explain (ii), the larger the supplier's minimum profit value, the larger the benchmark demand volume must be to generate a sufficiently large profit surplus in the service chain. Similarly, the larger the waiting time standard or the lower the efficiency index, the less significant the capacity savings which arise in the service chain and hence the larger the benchmark demand volume must be to generate a sufficiently large profit surplus. For a given total price sensitivity, the threshold demand volume *decreases* with the competitive intensity ρ and the number of firms N: an increase in ρ impacts the aggregate firms' profits under in-house service more than under outsourcing; also the more retail firms participate in the service chain, the larger the profit surplus generated by the service chain for any *given* benchmark demand volume. Finally, substituting the expressions for Π^c and π_i^* in Proposition 4.1, into (5), we obtain the following expression for each firm's profit under outsourcing:

$$\pi_i^o=\frac{\lambda^2(c+\gamma,w)}{b}\left[\frac{N\eta}{N\eta+\eta_0}\frac{1}{4(1-\rho)}+\frac{\eta_0}{N\eta+\eta_0}\frac{1}{2-\rho}\right]-\frac{\eta}{N\eta+\eta_0}\left[V-NF-\frac{\gamma}{w}\nu\right] \tag{14}$$

Remark Assuming (4) holds, each retail firm's profit under outsourcing is larger than that earned under (universal) in-house service. This is, of course, a direct consequence of the latter being used as the disagreement value in the Nash bargaining solution. As an alternative one may consider a variant in which the retail firms' disagreement values are (exogenously specified) constant minimum profits values. It follows from (4) that the condition for the existence of a Nash bargaining solution again reduces to the benchmark demand volume being in excess of a given threshold value. At the same time, the retail firms are *better off* under outsourcing when their (benchmark) demand volume is *below* a given threshold value, if the allocation of revenues and profits is handled by a traditional two-part tariff, consisting of a volume-or capacity based fee, combined with a periodic fixed payment K (see §7) . This reversal arises because under the two-part tariff scheme, the retailers' profits grow with the demand volume at a *slower* rate under outsourcing than under in-house service, while they grow at a *faster* rate under a Nash bargaining solution.

As mentioned in the introduction, beyond the outside supplier's ability to exploit economies of scope by pooling the service processes for the different retail firms, a *second* driver behind the benefits of outsourcing often results from the supplier's ability to operate with lower cost rates than the retailers. (The service process often represents the supplier's core competency rather than that of the retailers.) We refer to the on-line Appendix for Theorem A.1, which generalizes the above analysis to allow for arbitrary cost differentials between the supplier and the retailers. The analysis above showed that in the absence of cost rate advantages for the supplier, the retailing firms are better off outsourcing iff their demand volume, when pricing at cost, is above a critical value. The same characterization applies to the case where the supplier enjoys lower cost rates. The break even value for this demand volume decreases with Δ, the differential in the total cost rate enjoyed by the supplier. The break even value is also increasing in the waiting time standard w, and the inefficiency index ν. Thus, the higher the service level and the more the outside supplier is able to exploit economies of scope due to pooling, the lower the minimum benchmark demand volume under which retailers prefer outsourcing. While this phenomenon occurs in the base model, (see 13), it is all the more pronounced when the supplier operates under lower cost rates.

Stability of the Full Service Chain

Even if all retailers benefit from *collective* outsourcing compared to all of them providing in-house service, it is not clear whether the service chain, under outsourcing, is immune to defections by individual retailers, or, whether collective outsourcing arises as an equilibrium, when each firm has an upfront choice whether to outsource or to keep service in-house.

Thus, consider the following two-stage game: in the first stage each retailer decides whether to outsource the service process or to perform it in-house. For the firms who opt for outsourcing, the resulting chain is assumed to operate, in the second stage of the game, at maximum efficiency, while competing with the remaining firm that have opted for in-house service. The equilibrium aggregate profits earned by the service chain are again shared among the participating chain members in accordance with the Nash bargaining solution described in section 4.

Let $A \equiv V - NF - \frac{\nu\gamma}{w}$ denote the *minimum surplus* in aggregate profits, net of savings of fixed operating costs at the retailers and capacity costs due to (partial) pooling of the service processes, which the service chain must generate to ensure the participation of the supplier and the retailer firms. Recall from (14) that if $A \leq 0$, the retail firms are better off outsourcing, irrespective of their (benchmark) demand volume. At the same time, when $A > 0$, this is only true when the benchmark demand volume is in excess of a minimum threshold λ^0. Whether the service chain is stable or not, depends on *two* factors:

(a) the competitive intensity ρ, in relationship to the number of retailers N, as well as the relative bargaining power of the supplier, characterized by η_0/η,

(b) a bound for the retailer's benchmark demand volume $\lambda(c+\gamma, w)$

Theorem 4.2 below, shows, in the most common case where $A \geq 0$ (, i.e. the supplier's minimum acceptable profit level exceeds the savings in the retailer's fixed costs and any capacity cost savings due to (partial) pooling,) the necessary and sufficient condition for stability consists of a lower bound for the benchmark demand volume, i.e. $\lambda(c+\gamma, w) \geq \lambda^1 \equiv \sqrt{\frac{A\eta b}{N}} / \sqrt{\left[\frac{1}{4(1-\rho)} - \frac{1}{(2-\rho)^2}\right] + \left(1 + \frac{\eta_0}{\eta}\right)\left[\frac{1}{(2-\rho)^2} - \left(\frac{2(N-1)-\rho(N-3)}{4(N-1)-4(N-2)\rho-\rho^2}\right)^2\right]^+}$ along with an industry power condition

$$Q(\rho, N, \eta_0/\eta) = \left[\frac{2(N-1)-\rho(N-3)}{4(N-1)-4(N-2)\rho-\rho^2}\right]^2 - \frac{1}{4(1-\rho)} + \frac{\eta_0/\eta}{N+\eta_0/\eta}\left[\frac{1}{4(1-\rho)} - \frac{1}{(2-\rho)^2}\right] \leq 0 \tag{15}$$

In the alternative case where $A < 0$, the necessary and sufficient condition for stability consists of (i) $Q(\rho, N, \eta_0/\eta) \leq 0$ or (ii) $Q(\rho, N, \eta_0/\eta) > 0$ but $\lambda(c+\gamma, w)$ is below a given maximum threshold value $\lambda^2 \equiv \sqrt{\frac{|A|b\eta}{N\eta+\eta_0}} / \sqrt{(1-\rho)\left(\frac{2(N-2)-\rho(N-3)}{4(N-1)-4(N-2)\rho-\rho^2}\right)^2 - \left[\left(\frac{N\eta}{N\eta+\eta_0}\right)\frac{1}{4(1-\rho)} + \left(\frac{\eta_0}{N\eta+\eta_0}\right)\frac{1}{(2-\rho)^2}\right]}$. As we will see, the condition $Q(\rho, N, \eta_0/\eta) \leq 0$ is satisfied only when ρ is sufficiently large, i.e. $\rho \geq \underline{\rho}(N)$, a lower bound which is tight when the relative bargaining power of the supplier is negligible ($\eta_0/\eta = 0$). Moreover, the condition $Q(\rho, N, \eta_0/\eta) \leq 0$ is increasingly difficult to satisfy as the relative bargaining power of the supplier increases; when it is sufficiently large, it cannot be satisfied for any competitive intensity ρ or any number of firms N.

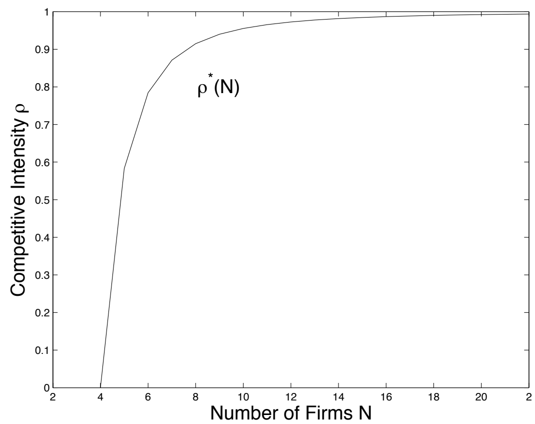
Theorem 4.2*(a) Assume $A \geq 0$. The solution where all firms choose to outsource in stage one, and adopt the price vector p^c in stage two is an SPNE iff $Q(N,\rho,\eta_0/\eta) \leq 0$ and $\lambda(c+\gamma,w) \geq \lambda^1$.*
(b) Assume $A < 0$. The above solution is a SPNE in the two-stage game iff (i) $Q(N,\rho,\eta_0/\eta) \leq 0$ or (ii) $Q(N,\rho,\eta_0/\eta) \geq 0$ and $\lambda(c+\gamma,w) \leq \lambda^2$
(c) Let $0 < \underline{\rho}(N) \equiv -2(N^2-4N+5)+2\sqrt{(N^2-4N+5)^2+(N-1)(N-3)} \leq 1$. $Q(\rho,N,\eta_0/\eta) \leq 0$ only if $\rho \geq \underline{\rho}(N)$ and $\underline{\rho}(N)$ is non-decreasing in N with $\lim_{N\to\infty}\underline{\rho}(N) = 1$

As a benchmark, consider the extreme case where the supplier's relative bargaining power is negligible ($\eta_0/\eta = 0$), with the retailers capable of extracting the full profits of the chain. In this case, when $N \leq 3$, the service chain is immune to defections, irrespective of the competitive intensity ρ and the retailers' benchmark demand volume. However, when $N \geq 4$, and the common case where $A \geq 0$ (i.e., the supplier's minimum profit profit level exceeds automatic fixed cost and capacity savings due to outsourcing), an individual service retailer has an incentive to leave the chain and bring the service process in-house unless the competitive intensity is above the critical value $\underline{\rho}(N)$. In other words, whether the chain is stable, for any benchmark demand volume, or not, depends on whether the pair of industry characteristics (N,ρ) lies below or above the switching curve $\{\underline{\rho}(N) : N = 2,3,\dots\}$ depicted in Figure 1. (Define $\underline{\rho}(2) = \underline{\rho}(3) = 0$.) This phenomenon occurs because the unmitigated competition a firm faces when defecting, acts less as a deterrent when the competitive intensity is lower. Cachon and Harker (2002), in their duopoly model, established the existence of a linear wholesale pricing scheme under which both service retailers benefit from outsourcing, the outside supplier earns a profit and the chain is immune from defections. Theorem 4.2 shows that stability of the chain, while guaranteed under a small number of competitors, ($N \leq 3$,) becomes increasingly more difficult to achieve as the number of competing firms increases.

As the supplier's relative bargaining power (η_0/η) increases, the stability condition (15) is increasingly difficult to satisfy, since $Q(\rho,N,\eta_0/\eta)$ is increasing in η_0/η while λ^2 is decreasing in this measure. We conjecture that as with the special case in which $\eta_0/\eta = 0$, in the general model, condition (15) is satisfied if and only if $\rho \geq \rho^*(N)$ for some switching curve $\{\rho^*(N), N = 3,\dots\}$ (with $\rho^*(N) \geq \underline{\rho}(N)$)[10]. Note that when $A > 0$, stability requires, in addition to (15) that the retailers' benchmark sales volume be in

[10] Extensive numerical investigations have verified that this switching curve property always applies. However, a formal proof is outstanding

Figure 1 Stability of the Service Chain



excess of a minimum level λ^1. This minimal level is clearly related to, and often equal to λ^0, the minimum level guaranteeing the existence of a Nash bargaining solution in the chain and hence the feasibility of the complete outsourcing option.

The reason defection from the service chain may be beneficial is that the chain adopts a significantly larger price than, say, the price p^* in a decentralized system so as to drive the profits of the participating retailers to their maximal level. A defecting retailer, may, when $\rho < \rho^*(N)$ exploit this by adopting a significantly lower price and thus attracting a significantly larger market share. Paradoxically, the chain can prevent defections, by inducing the participating retailers to adopt the price p^* instead of p^C, even though the resulting profits for the retailers are lower than under the coordination scheme of Theorem 4.2 Stability of the service chain under outsourcing is enhanced to the extent the supplier exploits the benefits of service pooling: when $A > 0$, the minimum demand value λ^1 decreases as ν increases, and when $A < 0$, the maximum demand value λ^2 increases. Finally, it is easily verified that any cost rate advantages for the supplier, reduce the potential for defections of individual firms from a service chain with outsourcing, see Appendix B.

Equilibrium in Two-Stage Outsourcing Game

Theorem 4.1 identifies the necessary and sufficient conditions for *universal* outsourcing, i.e., for a service chain with all N firms, to arise as a SPNE in the two-stage game. We now address the more general question, when a Nash equilibrium arises with m^i firms maintaining in house service, and the remaining $m^o = N - m^i$ firms outsourcing their service processes, for any $0 \leq m^i \leq N-1$. In other words, when is a service chain with m^o participating firms stable, in the sense that none of the firms inside the chain has an incentive to bring the service process in house, while any of the firms providing in house service incurs a *strict* profit loss by outsourcing and joining the service chain.[11] (As before, we assume the profits in the service chain are determined in accordance with the Nash bargaining solution , with V and $\{\pi_i^*\}$ as the disagreement values and η_0 and η as the supplier's and retailers' bargaining power.}

Let $\pi^i(m^0)$, $[\pi^o(m^o)]$ denote the equilibrium profits earned by a firm which chooses to keep the service in-house [to outsource], assuming that m^o firms select the outsourcing option. An SPNE with m^o firms outsourcing, exists iff

$$\pi^o(m^o) \geq \pi^i(m^o-1), \text{if } 1 < m^o \leq N, \text{and } \pi^i(m^o) > \pi^o(m^o+1), \text{if } 1 < m^o \leq N-1 \tag{16}$$

(Define $\pi^i(0) = \pi^i(1) = \pi^o(1) = \pi_i^*$ and $\pi^o(N) = \pi^o$, see (14). Theorem 4.3 below shows that any chain is viable, i.e. admits a Nash bargaining solution, provided the (benchmark) demand volume exceeds a minimum threshold, which depends on m^o, the number of participating retail firms.

The theorem shows that each of the equilibrium conditions in (16) reduces to (i) a bound for the benchmark demand value and (ii) an industry power condition involving only the number of firms with in-house and outsourced service, the competitive intensity ρ and η_0/η, the relative bargaining power of the supplier. This reduction of the equilibrium conditions is similar to the one obtained in Theorem 4.2 (a) for the special case with $m^o = N$. Analogous to their definition when all N firms outsource, let $\nu(m^o)$ denote the efficiency index achieved in a chain with m^o firms due to partial or complete pooling of the service processes, and $A(m^o) = V - m^o F - \nu(m^o)\frac{\gamma}{w}$. Also define

$$\lambda^o(m^o) = \sqrt{\frac{Abm^o}{m^o}} \Big/ \sqrt{(1-\rho)\left(\frac{2(N-2)-\rho(N-3)}{4(N-1)-4(N-2)\rho-\rho^2}\right)^2 - \left[\left(\frac{N\eta}{N\eta+\eta_0}\right)\frac{1}{4(1-\rho)} + \left(\frac{\eta_0}{N\eta+\eta_0}\right)\frac{1}{(2-\rho)^2}\right]}$$

$$\lambda^1(m^o) \equiv \max\left\{\lambda^o(m^o), \sqrt{\frac{bA(m^o)\eta}{m^o\eta+\eta_0}\frac{1}{\sqrt{Q^{m^o}(N,\rho,\eta_0/\eta}}}\right\}$$

$$\lambda^2 \equiv \sqrt{\frac{bA(m^o+1)}{m^o+1+\eta_0/\eta}\frac{1}{\sqrt{|R^{m^i}(N,\rho,\eta_0/\eta|}}}$$

$$\begin{aligned} Q^{(m^o)}(N,\rho,\eta_0/\eta) \equiv & \left[\frac{2(N-1)+(3-m^o)\rho}{2(N-1-(m^o-2)\rho)(2(N-1)-(N-m^o)\rho)-(m^o-1)(N-m^o-1)\rho^2}\right]^2 \\ & - \frac{\eta_0/\eta)}{m^o+\eta_0/\eta)}\frac{1}{(2-\rho)^2} - \frac{m^o}{m^o+\eta_0/\eta}\left[(N-1)^2-(m^o-1)\rho(N-1)\right] \\ & \cdot \left[\frac{2(N-1)+\rho}{2(N-1-(m^o-1)\rho)(2(N-1)-(N-m^o-1)\rho)-\rho^2 m^o(N-m^o)}\right]^2 \end{aligned}$$

[11] This asymmetric definition favors the outsourcing option, in case of a tie.

$$R^{(m^o)}(N,\rho,\eta_0,\eta) \equiv \left[\frac{2(N-1)+(2-m^o)\rho(N-1)}{2(N-1-(m^o-1)\rho)(2(N-1)-(N-m^o-1)\rho)-m^o(N-m^o)\rho^2}\right]^2$$
$$-\frac{\eta_0/\eta)}{m^o+1+\eta_0/\eta)}\frac{1}{(2-\rho)^2}-\frac{m^o+1}{m^o+1+\eta_0/\eta}\left[(N-1)^2-m^o\rho(N-1)\right]$$
$$\cdot\left[\frac{2(N-1)+\rho}{2(N-1-m^o\rho)(2(N-1)-(N-m^o-2)\rho)-\rho^2(m^o+1)(N-m^o-1)}\right]^2$$

Theorem 4.3*(General Stability Conditions) Let $2 \le m^o \le N-1$. (a) A service chain with m^o firms is viable, i.e. it permits a Nash bargaining solution iff $\lambda(c+\gamma,w) \ge \lambda^o(m^o)$.*

(b) Fix $2 \le m^o \le N$. Assume $A(m^o) \ge 0$. The stability condition $\pi^o(m^o) \ge \pi^i(m^o-1)$ is equivalent to

(i) $Q^{(m^o)}(N,\rho,\eta_0/\eta) \le 0$, and, (ii) $\lambda(c+\gamma,w) \ge \lambda^1(m^o)$

(c) Fix $1 \le m^o \le N-1$. Assume $A(m^o+1) \ge 0$. The stability condition $\pi^i(m^o) \ge \pi^o(m^o+1)$ is equivalent to (iii) $\lambda(c+\gamma,w) \le \lambda^o(m^o+1)$, or (iv) $R^{(m^o)}(N,\rho,\eta_0,\eta) \ge 0$, or (v) $R^{(m^o)}(N,\rho,\eta_0/\eta) < 0$ and $\lambda(c+\gamma,w) \le \lambda^2$

Theorem 4.3 specifies the stability conditions, in the most common case where $A(m^o) \ge A(m^o+1) \ge 0$ (Clearly, $A = A(N) \ge A(N-1) \ge \cdots \ge A(2)$). At the same time, if $A(m^o) < 0$, the conditions in part (b) and (c) are easily adjusted, as follows: For $2 \le m^o \le N$, it follows from (44) that the stability condition $\pi^o(m^o) \ge \pi^i(m^o-1)$ is equivalent to:

(i) $Q^{(m^o)}(N,\rho,\eta_0/\eta) \le 0$, or

(ii) $Q^{(m^o)}(N,\rho,\eta_0/\eta) > 0$, and $\lambda(c+\gamma,w) \le \lambda^2(m^o) = \sqrt{\frac{A(m^o)b}{(m^o+\eta_0/\eta)}\frac{1}{Q^{(m^o)}(N,\rho,\eta_0/\eta)}}$

Similarly, for $2 \le m^o \le N$, the stability condition $\pi^i(m^o) > \pi^o(m^o+1)$ is equivalent to

(iii) $R^{(m^o)}(N,\rho,\eta_0/\eta) \ge 0$, and $\lambda(c+\gamma,w) \ge \lambda^2(m^o) \equiv \sqrt{\frac{|A(m^o+1)|b}{(m^o+1+\eta_0/\eta)}\frac{1}{R^{(m^o)}(N,\rho,\eta_0/\eta)}}$

(Recall that if $A(m^o+1) \le A(m^o) < 0$, a service chain with m^o firms, and, a fortiori, one with m^o+1 firms, is viable under any benchmark demand volume, i.e. $\lambda^o(m^o) = \lambda^o(m^o+1) = 0$).

The following Theorem shows that a (pure) SPNE exists, for at least one value of m^o, and it provides a sufficient condition for m^o, the *size* of the outsourced service chain, to be *unique* among all Nash equilibria. (Clearly, if a Nash equilibrium exists with $1 \le m^o \le N$ firms in the chain, any of the $\binom{N^o}{m^o}$ nets of m^o firms among the industry of N, gives rise to a Nash equilibrium.) We have observed that, except when the number of firms in the industry is very small ($N \le 3$), or when the competitive intensity ρ is very large, equilibria arise with *partial* outsourcing, i.e. $1 < m^o < N$; this, even in an industry where all firms have identical characteristics.

Theorem 4.4*(a) The two stage outsourcing game has a (pure) SPNE.*

(b) Assume the difference function $\Delta(m^o) \equiv \pi^o(m^o) - \pi^i(m^o - 1)$ has at most a single root, m^{o*}: i.e. a single value which $\Delta(m^o) \geq 0$, while $\Delta(m^o+1) < 0$. Then, a SPNE exists for a single value of outsourcing firms m^o: if the difference function has a root , $m^o = m^{o*}$; Otherwise $m^o = N$. This unique value of outsourcing firms increases with the efficiency index $\nu(\cdot)$.

Thus, the more the service supply chain is able to exploit the benefits of service pooling, the larger a service chain arises in any equilibrium of the two stage outsourcing game. We conclude the section with a discussion of the very special benchmark case where $V = F = \nu = \eta_0 = 0$, i.e. (i) the supplier's minimum profit level may be reduced to zero, (ii) his relative bargaining power is negligible, (iii) the retailers do not incur any fixed costs when providing in-house service, and (iv) the supplier serves each of the participating firms in a dedicated facility. The following corollary is immediate from Theorems 4.2 and 4.3.

Corollary 4.5*Assume $V = F = \nu = \eta_0 = 0$.*

(a) Let $2 \leq m^o \leq N-1$. In the two stage game, an SPNE with m^o outsourcing firm arises iff

$$\left[\frac{2\left(1+\frac{\rho}{N+1}\right) - \frac{\rho}{N-1}(N-m^o)}{2\left(1-(N-m-1)\frac{\rho}{N-1}\right)\left(2-(m^o-1)\frac{\rho}{N-1}\right) - (N-m^o)m^o\left(\frac{\rho}{N-1}\right)^2}\right]^2 >$$

$$\left(1-(N-m^o)\frac{\rho}{N-1}\right)\left[\frac{2+\frac{\rho}{N-1}}{2\left(1-(N-m^o)\frac{\rho}{N-1}\right)\left(2-(m^o-2)\frac{\rho}{N-1}\right) - \left(\frac{\rho}{N-1}\right)^2(m^o-1)(N-m^o+1)}\right]^2, \textit{and}$$

$$\left[\frac{2\left(1+\frac{\rho}{N+1}\right) - \frac{\rho}{N-1}(N-m^o-1)}{2\left(1-(N-m^o-2)\frac{\rho}{N-1}\right)\left(2-m^o\frac{\rho}{N-1}\right) - (N-m^o-1)(m^o+1)\left(\frac{\rho}{N-1}\right)^2}\right]^2 < \quad (17)$$

$$\left(1-(N-m^o-1)\frac{\rho}{N-1}\right)\left[\frac{2+\frac{\rho}{N-1}}{2\left(1-(N-m^o-1)\frac{\rho}{N-1}\right)\left(2-(m^o-1)\frac{\rho}{N-1}\right) - \left(\frac{\rho}{N-1}\right)^2 m^o(N-m^o)}\right]^2$$

(b)A SPNE with all firms outsourcing arises iff $\rho \leq -2(N^2-4N+5) + 2\sqrt{(N^2-4N+5)^2 + (N-1)(N-3)}$

(c)A SPNE with all firms providing in-house service arises iff $0 > \frac{\eta}{2\eta+\eta_0}\left[\frac{2\lambda^2(c+\gamma,w)}{b}\left\{[(N-1)^2 - \rho(N-1)K^2 - \frac{1}{(2-\rho)^2}\right\}\right]$
where $K = \frac{2(N-1)+\rho}{2(N-1-\rho)(2(N-1)-(N-1)\rho)-\rho^2 2(N-2)}$

We have observed that for almost all values of N and ρ, the difference function $\Delta(\cdot)$ has at most a single root so that by Theorem 4.3, the equilibrium size of the service chain is unique. However, for some very high competitive intensities, the property may fail to hold, and multiple equilibrium chain sizes may arise.

Figure 2 Stability of the Service Chain

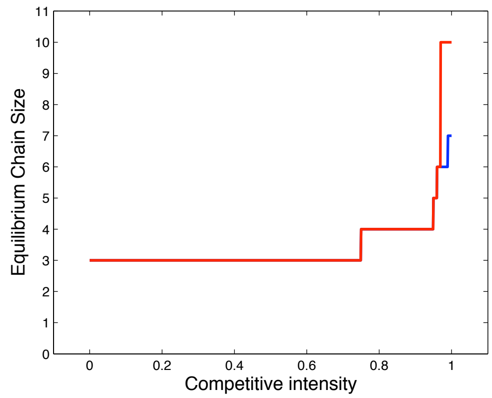


Figure 2 shows for an industry with $N = 10$ firms, how the equilibrium value(s) of m^o vary as a function of ρ; even under a low competitive intensity, $m^{o^*} = 3$; as ρ increases so does m^{o^*}. When $\rho \geq \underline{\rho}(N)$, a chain with $m^o = N = 10$ firms is stable, but so is one with $m^o = 6$ firms.

5. Identical Retailers: Waiting Time Competition

In this section we assess the benefits of outsourcing when the retailers compete by selecting their waiting time standards, under a given, exogenously specified, price level p. We follows the same agenda as outlined in the previous section

In-House Service

Corollary 2 in Allon and Federgruen (2007) establishes for general systems with possibly non-identical retailers, that a unique equilibrium vector w^* exists. In our case, this equilibrium satisfies the first order conditions $\frac{\partial \pi_i}{\partial w_i} = -a(p-c-\gamma) + \frac{\gamma}{w_i^2} = 0$, i.e. $w_1^* = \cdots = w_N^* = \sqrt{\frac{\gamma}{a(p-c-\gamma)}}$. Allon and Federgruen (2007) show, in fact, that this equilibrium is a *dominant* solution, i.e. it is optimal for each firm i to adopt this waiting time standard regardless of the choices made by its competitors. Since each firm $i's$ equilibrium profit $\pi_i^* = \lambda_i(p, w^*)(p-c-\gamma) - \frac{\gamma}{w^*}$, we obtain after substituting (6) and $w^* = \sqrt{\gamma}/\sqrt{a(p-c-\gamma)}$ that

$$\pi_i^* = (a^0 - (b - (N-1)\beta))(p-c-\gamma) - \sqrt{a\gamma(p-c-\gamma)}(2-\theta) \tag{18}$$

Universal Outsourcing

In comparing the case of in-house service with the equilibrium when all firms choose to outsource, we first need the following Lemma:

Lemma 5.1*A service chain achieves first-best performance, when all firms adopt a common waiting time standard w^c, both when the supplier serves the different firms in dedicated facilities and when it pools the service process. In the former case $w^c = \frac{1}{\sqrt{1-\theta}} w^* = \sqrt{\frac{1}{1-\theta}\frac{\gamma}{a(p-c-\gamma)}} \geq w^*$; in the latter case $w^c = \frac{1}{\sqrt{N}}\frac{1}{\sqrt{1-\theta}} w^* = \sqrt{\frac{1}{N(1-\theta)}\frac{\gamma}{a(p-c-\gamma)}} \geq w^*$ iff $\theta \geq 1 - \frac{1}{N}$. Moreover $\lambda^c \leq \lambda^*$ iff $w^c \geq w^*$*

Thus, aggregate profits in the service chain, under an optimal (common) waiting time standard w, are given by $\Pi^c(w) = (p-c-\gamma)\sum_{i=1}^N \lambda_i - \gamma\frac{N}{w}$ when dedicated facilities are used, and by $\Pi^c(w) = (p-c-\gamma)\sum_{i=1}^N \lambda_i - \frac{\gamma}{w}$, under service pooling. As in the case of price competition, we give a unified treatment to both settings, as well as intermediate ones with *partial* pooling, giving rise to profit function $\Pi^c(w) = (p-c-\gamma)\sum_{i=1}^N \lambda_i - \gamma\frac{N-\nu}{w}$, for a general efficiency index $0 \leq \nu(N) \leq N-1$ and a corresponding optimal waiting time standard $w^c = \sqrt{(1-\frac{\nu}{N})\frac{\gamma}{(1-\theta)a(p-c-\gamma)}}$. Thus, $w^c \geq w^*$ iff $\theta \geq \frac{\nu}{N}$ and w^c is a decreasing function of the efficiency index ν. In §7, we show that the service chain can achieve first best level aggregate profits under a continuum of volume based and capacity based fee pairs.

It follows from (4) that a service chain with all N firms outsourcing to the supplier is viable, i.e. permits a Nash equilibrium solution iff

$$\begin{aligned}
\Pi^c(w^c) - N\pi^* &= N(p-c-\gamma)\lambda(p,w^c) - N\pi^* \qquad (19)\\
&= N\sqrt{a\gamma(p-c-\gamma)}(2-\theta) - Na(1-\theta)(p-c-\gamma)\sqrt{\left(1-\frac{\nu}{N}\right)\frac{\gamma}{(1-\theta)a(p-c-\gamma)}}\\
&- N\sqrt{1-\frac{\nu}{N}}\sqrt{a\gamma(p-c-\gamma)(1-\theta)}\\
&= N\sqrt{a\gamma(p-c-\gamma)}\left[2-\theta-2\sqrt{\left(4-\frac{\nu}{N}\right)(1-\theta)}\right]\\
&= N\sqrt{a\gamma(p-c-\gamma)}\left[\frac{\nu(N)}{N}(1-\theta) + \left(1-\sqrt{\left(1-\frac{\nu(N)}{N}\right)(1-\theta)}\right)^2\right] \geq V - NF
\end{aligned}$$

Both the second and the third items to the right of (19) are positive, implying that the service chain is viable for N sufficiently large; it is certainly viable if $N \geq N^* \equiv \lceil \frac{V}{\sqrt{a\gamma(p-c-\gamma)}\left[1-\sqrt{1-\theta}\right]^2+F} \rceil$ ($N \geq N^* \Rightarrow N\sqrt{a\gamma(p-c-\gamma)}\left[\frac{\nu(N)}{N}(1-\theta) + \left(1-\sqrt{\left(1-\frac{\nu(N)}{N}\right)(1-\theta)}\right)^2\right] + F \geq N\left\{\sqrt{a\gamma(p-c-\gamma)}\left[1-\sqrt{1-\theta}\right]^2 + F\right\} > V$.)

Assuming the relative efficiency index $\frac{\nu(N)}{N}$ is non-decreasing in N, as is the case in the absence of service pooling ($r = 0$) and under complete service pooling ($r(N) = N-1$), the viability condition is more easily satisfied as the number of firms in the industry increases. It is also enhanced as (i) the gross profit margin $(p-c-\gamma)$ increases for a fixed capacity cost γ, or (ii) vice versa, the capacity cost rate γ increases under a given gross profits margin, or as (iii) the efficiency index increases, or (iv) the demand sensitivity to the waiting time standard increases. Finally, the condition is enhanced as the competitive intensity θ increases as a long as $\theta \geq \frac{\nu(N)}{N}$, i.e. as long as w^c, the waiting time standard adopted by the chain exceeds w^* - the waiting time standard adopted when all firms keep the service in-house. Let $B \equiv (V-NF)/N\sqrt{a\gamma(p-c-\gamma)}$. (19), the necessary and sufficient condition for the service chain with all N firms being viable, can then be rewritten as

$$2-\theta-2\sqrt{\left(1-\frac{\nu(N)}{N}\right)(1-\theta)} \geq B. \tag{20}$$

Assuming (19) holds, i.e. the chain permits a Nash bargaining solution, each firm's profit is given by $\pi_i^o = \pi_i^* + \frac{\eta}{N\eta+\eta_o}\left[\Pi^c(w^c) - N\pi^* - V + NF\right]$. Using (18) and (19), it follows that

$$\pi_i^o = (a^o - (b-(N-1)\beta)p)(p-c-\gamma) - \sqrt{a\gamma(p-c-\gamma)}\left[\frac{\eta_0}{N\eta+\eta_0}(2-\theta) + \frac{2N\eta}{N\eta+\eta_0}\sqrt{\left(1-\frac{\nu(N)}{N}\right)(1-\theta)}\right] - \frac{\eta}{N\eta+\eta_0}(V-NF) - F \tag{21}$$

Stability of the Full Service Chain

Theorem 5.2, below, characterizes when the full service chain with all N firms outsourcing is (part) of a SPNE in the following two-stage game: in stage one, all firms choose whether to outsource or not; in the stage two, all firms select their waiting time standards, with the understanding that the service chain consisting of the supplier and the outsourcing retail firms is perfectly coordinated and that its profits are distributed according to the Nash bargaining solution.

Theorem 5.2*(Stability of the full chain under waiting time competition)*

(a) The full service chain is stable, i.e., the solution in which all firms decide to outsource in the first stage of the two-stage game, and adopt the waiting time standard w^c in stage two, is a SPNE iff

$$S(N,\theta,\nu,\eta_0/\eta) \equiv 2-\theta-2\sqrt{\left(1-\frac{\nu(N)}{N}\right)(1-\theta)} - \left(1+\frac{\eta_0/\eta}{N}\right)\theta\left(\sqrt{\frac{N-1-\nu(N-1)}{N-1-(N-2)\theta}}-1\right)^+ \geq B \tag{22}$$

(b) Assume $V \geq NF$, i.e. $B > 0$, In the absence of service pooling $(\nu = 0)$, the service chain is never stable when $N \geq 5$. When $N = 4$, the chain is stable only if $\theta \geq 0.96$.

(c) Under complete service pooling, i.e. $\nu(N) = N-1$*, the service chain is stable whenever it is viable.*

We conclude that the stability condition for the full service chain is obtained from the viability condition (20) by the addition of a single non-positive term to its right hand side. Note that $S(N,\theta,\nu,\eta_0/\eta)$, the right hand side of the stability condition (22), only depends on the number of firms in the industry, the competitive intensity, the efficiency index, and the supplier's relative bargaining power. The structural form of the stability condition resembles that obtained in Theorem 4.2 for the case of price competition. Recall that, in the latter model, stability depends on the sign of the functions $Q(N,\rho,\eta_0/\eta)$ and $R(N,\rho,\eta_0/\eta)$, which, similarly, depend only on the number of firms in the industry, the competitive intensity and the supplier's relative bargaining power. As with the case of price competition, the full service chain is increasingly likely to be stable as the efficiency index ν increases. However, the impact of the efficiency index, i.e. the degree of service pooling, is even more dramatic in the waiting time competition model: under full service pooling, the chain is stable when it is viable, while in the absence of service pooling the chain is *never* stable when $N \geq 5$ (and only under extremely high competitive intensities when $N = 4$.)

Equilibrium of Two-Stage Outsourcing Game

To characterize the general equilibrium in the two-stage game, define as in Section 4:

$\pi^o(m^o) =$ the profit earned by an outsourcing firm, assuming m^o firms decide to outsource in stage one of the game
$\pi^i(m^o) =$ the profit earned by a firm that keeps the service in-house, assuming m^o firms decide to outsource in stage one

As before, an SPNE with m^o firms exists iff

$$\Delta(m^o) \equiv \pi^o(m^o) - \pi^i(m^o-1) \geq 0, \text{ while } \Delta(m^o+1) = \pi^o(m^o+1) - \pi^i(m^o) \leq 0 \qquad (23)$$

It follows from the proof of Theorem 4.4 that an SPNE is guaranteed to exist in the two-stage game and that the number of outsourcing firms m^o in an SPNE is unique if the difference function $\Delta(\cdot)$ has at most a single root. As in the price competition model, if $m^o = 1$, effectively no outsourcing occurs, so that $\pi^o(1) = \pi^i(1) = \pi^*$. Also, we define $\pi^o(m^o) = -\infty$, if a chain with m^o outsourcing firms fails to be viable. The necessary and sufficient condition for a chain's viability is analogous to (20). First, it is easily verified that all firms with in-house service employ the waiting time standard $w^i = \sqrt{\frac{\gamma}{a(p-c-\gamma)}}$, irrespective of how many firms are part of the outsourcing service chain. For those inside a chain with m^o firms, it is in equilibrium, optimal to employ one identical waiting time standard w^o, so that the demand volume of an outsourcing firm and the aggregate profits of the chain are given by

$$\begin{aligned}\lambda^o &= a^0 - (b-(N-1)\beta)p - (a-(m^o-1)\alpha)w^o + (N-m^o)\alpha w^i, \text{ and} \\ \Pi^c(m^o) &= m^o(p-c-\gamma)[a^0 - (b-(N-1)\beta)p + (N-m^o)\alpha w^i] \\ &\quad - m^o(p-c-\gamma)\left[a-(m^o-1)\alpha\right]w^o - (m^o - \nu(m^o))\frac{\gamma}{w^o}. \end{aligned} \tag{24}$$

It follows that

$$w^o(m^o) = \sqrt{\frac{\gamma(m^o-\nu(m^o))}{(p-c-\gamma)a\left(1-\frac{m^o-1}{N-1}\theta\right)}} \tag{25}$$

Substituting the expressions for w^* and w^o into (24) and (25), we show that

$$\begin{aligned}\Pi^c(m^o) - m^o\pi^* - V + m^oF &= m^o\sqrt{a\gamma(p-c-\gamma)}\left\{2 = \frac{m^o-1}{N-1}\theta - 2\sqrt{\left(1-\frac{\nu(m^o)}{m^o}\right)\left(1-\frac{(m^o-1)\theta}{N-1}\right)}\right\} - V + m^oF \\ &= m^o\sqrt{a\gamma(p-c-\gamma)}\left\{\left(1-\sqrt{1-\frac{m^o-1}{N-1}\theta}\right)^2 + 2\sqrt{1-\frac{m^o-1}{N-1}\theta}\left(1-\sqrt{1-\frac{\nu(m^o)}{m^o}}\right)\right\} - V + m^oF \end{aligned} \tag{26}$$

The expression within curled brackets is non-negative, i.e. the aggregate equilibrium profits of the service chain are always larger those obtained by the participating retail firms when all firms provide in-house service. For the service chain to be viable, this increment in aggregate profits must at least equal $(V - mF)$. Thus, with $B(m^o) \equiv (V - m^oF)/m^o\sqrt{a\gamma(p-c-\gamma)}$, a service chain with m^o firms is viable iff

$$2 - \frac{m^o-1}{N-1}\theta - 2\sqrt{\left(1-\frac{\nu(m^o)}{m^o}\right)\left(1-\frac{m^o-1}{N-1}\theta\right)} > B(m^o) \tag{27}$$

Finally, $\pi^o(m^o) = \pi^* + \frac{\eta}{m^o\eta+\eta_0}(m^o\eta + \eta_0)\left[\Pi^c(m^o) - m^o\pi^* - V + m^oF\right]$, while $\pi^i(m^o) = (p - c - \gamma)\lambda^i(m^o) - \frac{\gamma}{w^*} - F$ with $\lambda^i(m^o) = a^o - [b-(N-1)\beta]\,p - [a-(N-m^o-1)\alpha]\,w^* - m^ow^o$. Substituting (18) and (27) and the expressions for w^o and w^*, we then obtain closed form expressions for $\pi^o(m^o)$ and $\pi^i(m^o)$ and hence for the stability conditions in (23). These take the form $T^{(m^o)}(N,\theta,\nu,\eta_0/\eta) \geq B(m^o)$ and $U^{(m^o+1)}(N,\theta,\nu,\eta_0/\eta) \leq B(m^o+1)$ for specific functions $T^{(m^o)}$ and $U^{(m^o+1)}$ which only depend on the number of firms in the industry, the competitive intensity, the efficiency index, and the relative bargaining power of the supplier.

6. Identical Retailers: Simultaneous Competition

When the retailers compete simultaneously in terms of their prices and waiting times, the benefits of outsourcing are somewhat more difficult to assess. For one, a sufficient condition is required to ensure that the system has a unique equilibrium which is symmetric when all firms keep the service in-house. The same applies when all firms outsource. When discussing this last type of competition, we follow the same agenda as in the previous two sections.

In-House Service vs. Universal Outsourcing

We start with the case where an outside supplier serves the customers of each firm in a dedicated facility. For $i = 1, \ldots, N$, let $\pi_i^*(p, w)$ denote the equilibrium profit for retailer i, under the price vector p and the vector of waiting time standards w, and let $\Pi^c(p, w)$ denote the optimal aggregate profit of the service chain under outsourcing.

Theorem 6.1[12]*(Comparison of price and waiting time choices, with and without outsourcing, under simultaneous competition) There exists a value $\underline{\gamma}$ such that for all $\gamma \geq \underline{\gamma}$:*

(a) Assume $2b > a + (N-1)(\alpha + \beta)$. When all firms keep the service in-house, there exists a unique equilibrium (p^*, w^*), which is symmetric, i.e. $p_1^* = \cdots = p_N^*$ and $w_1^* = \cdots = w_N^*$, where

$$p^* = \frac{a^0 - w^* a(1-\theta) + b(c+\gamma)}{b(2-\rho)}$$

$$w^* = \begin{cases} \textit{the unique root on } [0, w^{max}] \textit{ of } C^*(w) \equiv w^3 - w^2 \frac{a^0 - (c+\gamma)b(1-\rho)}{a(1-\theta)} + \frac{\gamma b(2-\rho)}{a^2(1-\theta)}, \\ \qquad\qquad if \gamma \leq \frac{a^2(1-\theta)}{b(2-\rho)} (w^{max})^2 \left[\frac{a^0 - (c+\gamma)b(1-\rho)}{a(1-\theta)} - w^{max} \right] \\ w^{max}, \qquad \textit{otherwise} \end{cases}$$

The retailers' equilibrium sales volume. $\lambda^* = \frac{b\gamma}{a(w^*)^2}$ if $w^* < w^{max}$ and $\lambda^* \leq \frac{b\gamma}{a(w^*)^2}$ if $w^* = w^{max}$.

(b) Assume $2b > a + (N-1)\alpha + 2(N-1)\beta$. When all firms outsource there exists a unique optimal solution (p^c, w^c), in the centralized problem, which is symmetric, i.e. $p_1^c = \cdots = p_N^c$ and $w_1^c = \cdots = w_N^c$, where

$$p^c = \frac{a^0 - w^c a(1-\theta)}{2b(1-\rho)} + \frac{c+\gamma}{2}$$

[12] The parameter conditions are overly restrictive. They provide guarantees under the simplest verifiable condition that the Nash equilibrium under in-house service and the optimal solution under outsourcing are symmetric

$$w^c = \begin{cases} \textit{the unique root on } [0, w^{max}] \textit{ of } C^C(w) \equiv (w^c)^3 - (w^c)^2 \frac{a^0-(c+\gamma)b(1-\rho)}{a(1-\theta)} + \frac{2\gamma b(1-\rho)}{a^2(1-\theta)^2}, \\ \qquad\qquad if \gamma \le \underline{\gamma} \equiv \frac{a^2(1-\theta)}{b(2-\rho)}(w^{max})^2 \left[\frac{a^0-(c+\gamma)b(1-\rho)}{a(1-\theta)} - w^{max}\right] \\ w^{max}, \qquad\qquad \textit{otherwise} \end{cases}$$

Let λ^c denote the retailers' optimal sales volume, in the centralized problem. Then $\lambda^c = \frac{\gamma b(1-\rho)}{a(1-\theta)(w^c)^2}$ if $w^c < w^{max}$ and $\lambda^c < \frac{\gamma b(1-\rho)}{a(1-\theta)(w^c)^2}$ if $w^c = w^{max}$

(c) Assume $2b > a + (N-1)\alpha + 2(N-1)\beta$. If $\theta \ge [<] \frac{\rho}{2-\rho}$ then $w^c \ge [\le] w^*$.

(d) Assume $2b > a + (N-1)\alpha + 2(N-1)\beta$ Assuming $w^*, w^c < w^{max}$, $\frac{w^*}{w^c} = \sqrt{1-\theta}\sqrt{\frac{p^c-c-\gamma}{p^*-c-\gamma}}$. In particular, if $p^c \le p^*$, $w^c \ge w^*$.

(e) If $\rho = \theta$, $w^c \ge w^*$ and $\lambda^c \le \lambda^*$.

If the cubic functions $C^*(w)$ and $C^c(w)$ have a root, this root can, of course, be solved in closed form, see e.g, Abramovitz and Stegun (1965). Contrary to the waiting time competition model under a *given* price level p, it is no longer certain that the firms choose a lower waiting time standard when providing in-house service, as compared to when they outsource. However, if the intensity of the waiting time competition θ is sufficiently large compared to the intensity of the price competition (as specified by the condition $\theta \ge \frac{\rho}{2-\rho}$), and, in particular when $\theta \ge \rho$ this will be the case: the larger the price competitive intensity ρ, the smaller the minimal waiting time competitive intensity $\theta = \rho/(2-\rho)$ for which outsourcing results in lower service, i.e., a higher waiting time. Part (d) shows that it is impossible for both prices *and* the waiting time standards under outsourcing to be *lower* than those selected under in-house service. The expressions for (p^c, w^c) permit us to provide a closed form expression for Π^c, the optimal aggregate profit in the service chain that arises when all N firms outsource. Assuming, again, that the supplier demands a minimum profit level V, and each of the retail firms a profit level π^* (- again obtainable in closed form, given the expression for (p^*, w^*) in Theorem 6.1 -), it is then possible to specify the conditions under which the full service chain permits a Nash bargaining solution as well as the resulting profit levels for the retail firms.

Stability of the Full Service Chain; Equilibrium in the Two Stage Outsourcing game

By a simple extension of Theorem 4.4, it is again possible to show that the two-stage outsourcing game always has a SPNE. To verify whether an SPNE exists with m^o firms outsourcing, is again, equivalent to the pair of inequalities $\Delta(m^o) \equiv \pi^o(m^o) - \pi(m^o - 1) \geq 0$ and $\Delta(m^o + 1) < 0$. [13] For any given value of m^o, these conditions can be evaluated by verifying that the second stage game has a Nash equilibrium in which all firms with in-house service adopt a common price- and waiting time pair (p^i, w^i) while those in the service chain share a pair (p^o, w^o). These two pairs can be computed by solving a system of four equations in the four variables which arise from the first order conditions for the profit functions of a firm with in-house service and that of the service chain. It is thus possible to evaluate the equilibrium profit $\pi^i(m^o)$ as well as the equilibrium aggregate profit for the service chain, and hence $\pi^o(m^o)$, by computing the Nash bargaining solution of these profits via (5).

As in the case where the firms compete along one dimension only, the full service chain fails, in general, to be immune to defections, even when *all* of the benefits of outsourcing are assigned to the retailers, i.e. $V = \eta_0 = 0$, in which case the full service chain is always viable . When $N = 2$, the chain is always immune to defections, since, after a defection the system returns to the original decentralized system. In the special where $\theta = \rho$, and $\frac{\alpha}{a} = \frac{\beta}{b}$, the demand model (6) reduces to a full price model, i.e. consumers aggregate each firm's price and waiting time standard into a *single*, so called full price measure $F_i^p = p_i + kw_i$, so that all demand volumes can be expressed as functions of the vector F^p only: $\lambda_i = a^0 - bF_i^p + \sum_{j \neq i} \beta F_j^p \; i = 1, \ldots, N$ (Here $k = \frac{a}{b}$). Cachon and Harker (2002) addressed the benefits of outsourcing for this specification, in an industry with $N = 2$ firms and $\beta = b$ i.e., $\theta = \rho = 1$. [14] In stark contrast to our results, which do not include this limiting value, but cover a much larger parameter space, the authors conclude that outsourcing is *always* beneficial to the retailers . They also claim that the service chain is always immune to defections as long as the supplier does not charge too large volume based fee. With $N = 2$, we also obtain that the full service chain is always stable, whenever viable. For $N \geq 3$, the necessary and sufficient conditions for its stability no longer reduces to a single condition in terms of the measure(s) of competitive intensity and the number of firms in the industry (When $V = F = \eta_0 = 0$, this is the case under price- or waiting time competition, see Theorem 4.2(b) and Theorem 5.2(a).) We have observed, however, that the chain is more likely to be immune when θ or ρ increases, i.e. when the competitive intensities act as deterrents for defections. Example 1 in Appendix A exhibits this phenomenon. As in the case of one-dimensional (Price or Waiting Time) Competition, the benefits of outsourcing are significantly larger, if the outside supplier is

[13] For $m^o = N$, the single inequality $\pi^o(N) \geq \pi^o(N-1)$ suffices, while $\pi = \pi^i(1) \geq \pi^0(2)$ represents the necessary and sufficient conditions for no firms outsourcing.

[14] As mentioned in the introduction, another important distinction in this paper is that the supplier specifies he retail firms' waiting time standards, along with a volume based fee.

able to benefit from lower cost rates or by pooling the service processes of the individual firms. Pursuing the latter option, the cost savings from service pooling make it more likely that the waiting time standard w^c and price p^c adopted under outsourcing are lower than their counterparts w^* and p^*, under in-house service. Following the proof of Proposition 2, the profit function $\Pi^c(p,w) = \sum_{i=1}^N \lambda_i(p_i - c - \gamma) - \gamma \frac{\sum_{i=1}^N \lambda_i}{\sum_{i=1}^N \lambda_i w_i}$. Following the proof of Theorem 1 in Allon and Federgruen (2004), it can be shown that the function is jointly concave in (p,w) if the demand values $\{\lambda_i\}$ are sufficiently large on the feasible price/waiting time space. Also, $\frac{\partial \Pi^c}{\partial w_i} = -a(p_i - c - \gamma) + \alpha \sum_{j \neq i}(p_j - c - \gamma) - \gamma \frac{(\alpha(N-1)-a)\sum_{j=1}^N(\lambda_j w_j) - (\sum_{j=1}^N \lambda_j)(\sum_{j=1}^N a w_j - w_i \lambda_i)}{(\sum_{j=1}^N \lambda_j)^2}$, and $\frac{\partial \Pi^c}{\partial p_i} = \lambda_i - b(p_i - c - \gamma) + \beta \sum_{j \neq i}(p_j - c - \gamma) - \gamma \frac{(\beta(N-1)-b)\sum_{j=1}^N(\lambda_j w_j) - (\sum_{j=1}^N \lambda_j)(\sum_{j=1}^N a w_j - w_i \lambda_i)}{(\sum_{j=1}^N \lambda_j)^2}$ Since Π^c is jointly concave. it has a symmetric maximum with $p_1^c = \cdots = p_N^c = p^c$ and $w_1^c = \cdots = w_N^c = w^c$. Substituting these identities, we obtain that (p^c, w^c) is the *unique* solution to the system of equations: $-(a-(N-1)\alpha)(p-c-\gamma) + \frac{\gamma}{Nw^2} = 0$, and $-(b-(N-1)\beta)(p-c-\gamma) + \lambda = 0$.

Eliminating p, we obtain, as in the proof of Theorem 6.1, that w^c is the *unique* root of the cubic equation $w^3 - w^2 \frac{(a^0-(c+\gamma))(b-(N-1)\beta)}{a-(N-1)\alpha} + \frac{2\gamma(b-(N-1)\beta)}{N(a-(N-1)\alpha)^2} = 0$, unless $\gamma > N\underline{\gamma}$, in which case $w^c = w^{max}$. We conclude that $w^c \leq w^*$ iff $\theta \leq 1 - \frac{2(1-\rho)}{N(2-\rho)}$. Thus, for given competitive intensity values θ and ρ, it is increasingly likely that this condition is satisfied as the number of firms in the industry increases. It is also more likely, though still not guaranteed, that the retailers benefit from outsourcing under the again unique two part pricing scheme which induces perfect coordination. Finally, it is, also, more likely that the service chain is immune to defections, assuming the retailers reap all benefits from outsourcing. To illustrate this, when adapting Example 1 to the case of pooled service, the chain is stable when $N = 4$ and $\theta = \rho = 0.57$ (while it fails to be so under dedicated service). At the same time, the chain is again prone to defections for the same values of θ and ρ when $N = 5$.

7. Efficient Outsourcing: Pricing Schemes

In most of the comparisons in §3-6, we have assumed the *best case* scenario for outsourcing, where the resulting service chain operates at maximum efficiency. In this §, we show that this can be achieved via a coordinating pricing scheme which induces the retail firms to adopt a first-best price, waiting time standard or a combination thereof (, depending on the type of competition the firms engage in).

Price Competition:

Under this type of competition, a vector of waiting time standards w^0 is exogenously given. The retail firms can be induced to adopt *any* desired price vector p^I - for example p^c – via a simple volume based fee c_i^W for each of its customers demand, a capacity based fee e_i^W for each unit of capacity requirement to service firm i's customers (in a dedicated facility), or a combination of these two fees. The profit function of firm i is then given by $\pi_i^o = \lambda_i(p, w^0)(p_i - c_i^W - e_i^W) - \frac{e_i^W \gamma}{w_i^0}$, assumed to be strictly concave in the firm's own price p_i. Thus, firm i chooses the desired price p_i^I if $\frac{\partial \lambda_i(p^I, w)}{\partial p_i}(p_i^I - c_i^W - e_i^W) + \lambda_i(p^I, w^0) = 0$ and perfect coordination is achieved for any pair of fees (c_i^W, e_i^W), with $c_i^W + e_i^W = p_i^I + \left[\frac{\partial \lambda_i(p^I, w^0)}{\partial p_i}\right]^{-1} \lambda_i(p^I, w^0) < p_i^I$ Under the demand model (6) in §3-6, this reduces to: $c^W + e_i^W = p^I - \frac{\lambda}{b}(p^I, w^0)$.

Waiting Time Competition

In this case, a vector of prices p^0 is exogenously given. To induce the firms to adopt any desired vector of waiting time standards $0 < w^I < w^{max}$ (e.g, the vector maximizing chain wide profits), it suffices again to use a combined volume based fee c_i^W and capacity based fee e_i^W. The profit function of firm i is now given by $\pi_i^o = \lambda_i(p^0, w)(p_i^0 - c_i^W - e_i^W) - e_i^W / w_i$ which is strictly concave in w_i, since the demand function is concave in w_i.[15] Thus, firm i adopts the desired waiting time standard w_i^I if $0 = \frac{\partial \pi_i^o}{\partial w_i} = \frac{\partial \lambda_i(p^0, w^I)}{\partial w_i}(p_i^0 - c_i^W - e_i^W) + \frac{e^W}{(w_i^0)^2}$. In other words, perfect coordination is achieved as long as the fees (c_i^W, e_i^W) are chosen from the line segment: $\left\{(c_i^W, e_i^W) : c_i^W + \left(1 + (w_i^I)^{-2}\left[\frac{-\partial \lambda_i(p^0, w^I)}{\partial w_i}\right]^{-1}\right) e_i^W = p_i^0; e_i^W \geq 0\right\}$.

In the symmetric demand model (6) used in §5, this reduces to: $\left\{(c^W, e^W) : e^W = \frac{a(w^0)^2}{1 + a(w^0)^2}(p - c^W) : c^W \leq p\right\}$. Note, to induce a retailer to accept a larger waiting time, the capacity based fee needs to be increased significantly, if the volume based fee is left unaltered; conversely, if the capacity based fee is maintained, the volume based fee needs to be chosen closer to the retail price.

Simultaneous Competition

To induce the firms to adopt any desired price vector p^I, along with any desired waiting time vector $0 < w^I < w^{max}$, a combined volume- and capacity based fee, again, suffices. However, perfect coordination now requires a *unique* pair of fee rates (c_i^W, e_i^W) which are obtained from the set of equations: $\frac{\partial \pi_i^o(p^I, w^I)}{\partial p_i} = 0$ and $\frac{\partial \pi_i^o(p^I, w^I)}{\partial w_i} = 0$. (Specific parameters restrictions may be required to ensure that the profit function is jointly concave in (p_i, w_i), see §6): $c_i^W + e_i^W = p_i^I + \left[\frac{\partial \lambda_i(p^I, w^I)}{\partial p_i}\right]^{-1} \lambda_i(p^I, w^I); e_i^W =$

[15] An alternative to the capacity based fee is to charge the retailers in proportion to *any* convexly decreasing function of the waiting time standard.

$\left[\frac{(w_i^I)^2 \frac{-\partial\lambda_i(p^I,w^I)}{\partial w_i}}{1+(w_i^I)^2 \frac{-\partial\lambda_i(p^I,w^I)}{\partial w_i}}\right][p_i^* - c_i^w] \geq 0$ Under the symmetric demand model (6) used in §6, this reduces to: $c^W \equiv p^I - (1 + a(w^J)^2)\frac{\lambda(p^I,w^I)}{b} < p^I$, and $e^W = a(w^I)^2 \frac{\lambda(p^I,w^I)}{b} > 0$.

8. Asymmetric Models: Numerical Study

Existence and characterization of equilibria via first order conditions all carry over, under mild conditions, to the general model with non-identical retailers, both under in-house service and the various outsourcing options, see Allon and Federgruen (2004, 2007). However, with non-identical retailers, it is no longer possible to obtain closed form expressions for these equilibria. In appendix A we report on a numerical study aimed to investigate which of the insights obtained in §3-6 for the symmetric model, carry over to general asymmetric industries. Here we confine ourselves to a few general insights.

Our initial set of problems involves an industry with $N = 3$ firms and demand functions of the quasi-separable form in (3). Under price competition, we showed in §3, that under outsourcing with a maximally efficient service chain, prices are higher and demand volumes lower than when the firms provide service in-house. These patterns are, in general, maintained by our asymmetric instances, but a few exceptions arise. Focusing on the special case where the retail firms are able to extract the full aggregate profits in the service chain ($V = \eta_0 = 0$), one easily verifies from the proof of Theorem 4.2, that in a symmetric model with $N = 3$ firms, the service chain is always stable, even in the absence of service pooling. With three asymmetric firms, the chain is ,again, stable in most, but not all, instances. As in the case of symmetric retailers, stability of the full chain is increasingly likely as the competition intensity ρ increases. Under pooled service, the full service chain (, with 3 asymmetric firms,) is always stable. We also report, for all instances, the equilibrium under simultaneous competition, and investigate when the full chain is stable, under this type of competition. The above observations for the case of price competition continue to apply here.

In the second part of the study, we consider price competition instances with $N = 3$ firms, whose demand functions arise from an attraction model. In this case, we conclude, for example, that the full service chain under outsourcing, with $N = 3$ firms, is *always* stable.

9. Conclusions and Extensions

In many service industries, competing firms consider the option of outsourcing an important service process to a common service provider. When some or all the firms choose to outsource, this gives rise to a service supply chain, possibly competing with the remaining firms that have chosen to keep the service in-house. We have developed analytical models to characterize the potential benefits of outsourcing. Two factors are critical determinants of these benefits: (i) the ability of the common service supplier to exploit economies of scope by complete or partial pooling of the service processes for the different firms, and (ii) the ability of the outside service supplier to operate at lower cost rates.

A first focal question has been when a service chain is *viable*, in the sense of its aggregate equilibrium profits being larger than or equal to those earned by the participating retail firms, plus a minimum profit level which guarantees the participation of the supplier. The second question addressed by the paper is when a given (viable) service chain, in a given competitive environment is *stable* in the sense that no firm in the chain has an incentive to leave the chain and bring the service in-house and no firm with in-house service to join the chain. Our models always allow for numerical comparisons of the equilibrium prices, waiting time standards, demand volumes and profit levels under in-house service and the various outsourcing options. Such numerical comparisons have been carried out in §8. For symmetric models, in which the retail firms have identical characteristics, these comparisons can be made analytically, resulting in important general insights. See the Introduction for a summery of the general conclusions we have been able to prove.

In particular, we have shown how the necessary and sufficient conditions for viability or stability of a given service chain depend on a few key characteristics, such as (i) the number of firms with in-house vs. outsourced service, (ii) the competitive intensity, (iii) the efficiency index, and (iv) the relative bargaining power of the supplier.

One important restriction of the model is the assumption that each service facility can be modeled as an M/M/1 system. For the in-house service and outsourcing with dedicated facilities options, the ability to generalize our results to more general queueing systems, depends on the ability to obtain an analytical characterization of the dependence of the service rate (= average number of customers which can be served per unit of time) with respect to a firm's demand volume and waiting time standard. Allon and Federgruen (2003) have shown that such analytical characterizations are indeed possible for a wide variety of queueing systems. More specifically, the capacity function (1) can be extended to one of the type $\mu = B_1\lambda + B_2/w + \sqrt{B_3\lambda^2 + B_4\lambda/w + B_5/w^2}$ either exactly (e.g. for M/G/1 systems, Jackson networks) or as a close approximation (e.g. GI/GI/1 or GI/GI/S) systems.

In other settings, the service facilities represent multi-server queueing systems, whose capacity is given by the *number* of servers. One of the most frequently used models to represent call- or contact centers is the M/M/s model with or without abandonments. It is well known that the required capacity in the system can be approximated closely by a function of the form $\frac{\lambda}{\mu} + \beta(w)\sqrt{\frac{\lambda}{\mu}}$, where μ is the service rate of a single server and $\beta(w)$ a constant which depends on the chosen waiting time standards, e.g. Gans et al. (2003). The square root terms in these various capacity cost functions render the analysis more complex and prevent simple closed form solutions even in symmetric models. However, the process laid out in this paper to investigate the viability and stability of any given chain can be pursued analogously and so can the full characterization of the equilibrium of the two-stage outsourcing game.

Similarly, extensions of our equilibrium results under *pooled* service require the ability to characterize the so-called achievable performance space, i.e., the region of the feasible demand volume - and waiting time vectors which can be achieved under a *given* (pooled) capacity level. Such characterizations are available for multi-class M/G/C and G/M/C systems, see for example, Federgruen and Groenevelt (1988) and Shanthikumar and Yao (1992). (For the former, the waiting time standard must be specified as the expected delay before service.) Bertsimas et al. (1994) have developed *approximate* characterizations of the achievable performance space for various multiclass queueing networks. Future work should investigate how the benefits of various outsourcing options depend on the characteristics of the queueing systems (for example, the coefficients of variation of service and interarrival times), and what priority disciplines should be used under pooled service.

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Appendix A: Benefits of outsourcing when the supplier enjoys a cost advantage

In this Appendix, we characterize the benefits of outsourcing when the outside suppler is able to operate with lower cost rates than the retailers. Let $\Delta = (c+\gamma) - (c_0 + \gamma_0) > 0$. Let p^{cC}, λ^{cC} and Pi^{cC} denote the price, demand volume, and aggregate profits in the service chain that arises under outsourcing, when the chain operates at maximum efficiency.

Theorem A.1*(Price and profit comparison between in-house and outsourcing with lower cost rates for the supplier)*
(a)$p^{cC} < p^*$ *if and only if the supplier's cost advantage* $\Delta > \frac{\rho[a^0 - w(a-(N-1)\alpha)]}{(1-\rho)(2-\rho)b} - \frac{\rho(c+\gamma)}{2-\rho}$
(b)Assume $\Delta \geq 2\sqrt{[V - NF - \frac{\gamma\nu}{w}]^+/\bar{b}}$. *There exists a Nash bargaining solution and the retail firms are better off under outsourcing, irrespective of their (benchmark) demand volume.*
(c)Assume $\Delta < 2\sqrt{[V - NF + \frac{\gamma\nu}{w}]^+/\bar{b}}$ *Thee exists a Nash bargaining solution and the retail firms are better off under outsourcing, if and only if the (threshold) demand volume* $\lambda(c+\gamma, w) \geq \frac{\Delta/2 + \sqrt{\frac{1-\rho}{(2-\rho)^2}\Delta^2 + 4\left(\frac{1}{4b(1-\rho)} - \frac{1}{b(2-\rho)^2}\right)(\frac{V}{N} - F - \frac{\gamma}{w}(\frac{\nu}{N}))}}{\frac{1}{2b(1-\rho)} - \frac{2}{b(2-\rho)^2}}$

Proof: (a) Analogous to (13), we obtain:

$$p^{cC} = \frac{a^0 - w(1-(N-1)\alpha)}{2b(1-\rho)} + \frac{c_0 + \gamma_0}{2} = \frac{a^0 - w(1-(N-1)\alpha)}{2b(1-\rho)} + \frac{c+\gamma}{2} - \frac{1}{2}.$$

From (10), $p^{cC} - p^* = \frac{a^0 - w(1-(N-1)\alpha)}{b}\left[\frac{1}{2(1-\rho)} - 12 - \rho\right] + (c+\gamma)\left[\frac{1}{2} - \frac{1}{2-\rho}\right]$, and $p^{cC} < p^*$, iff $\Delta > \frac{a^0 - w(1-(N-1)\alpha)}{2b(1-\rho)(2-rho)} - \frac{\rho(c+\gamma)}{2-\rho}$

(b) and (c) It follows from Proposition 4.1 that $\Pi^{cC} = \frac{N}{4\bar{b}}\lambda^2(c_0+\gamma_0, w) - \frac{\gamma\nu}{w}$. Note that $\lambda(c_0+\gamma_0, w) = \lambda(c+\gamma, w) + \bar{b}\Delta$, such that $\lambda^2(c_0+\gamma_0, w) = \lambda^2(c+\gamma, w) + 2\bar{b}\Delta\lambda(c+\gamma, w) + \bar{b}^2\Delta^2$. Thus,

$$\Pi^{cC} - \sum_{i=1}^{N}\pi_i^* - V + NF + (N-\nu)\gamma w = N\lambda^2(c+\gamma, w)\left[\frac{1}{b(4-4\rho)} - \frac{1}{b(2-\rho)^2}\right] + \frac{N\Delta\lambda(c+\gamma, w)}{2} + \frac{\bar{b}N\Delta^2}{4} - \Big[V - NF + (\nu - N)\frac{\gamma}{w} \tag{28}$$

Note, first, that if $\Delta \geq 2\sqrt{\left[\frac{V}{N} - F + \left(\frac{\nu}{N} - 1\right)\frac{\gamma}{w}\right]^+/\bar{b}}$, all terms to the right of (28) are non-negative, so that a Nash bargaining solution exists for *any* demand volume $\lambda(c+\gamma, w)$. At the same time, when $\Delta < 2\sqrt{\left[\frac{V}{N} - F + \left(\frac{\nu}{N} - 1\right)\frac{\gamma}{w}\right]^+/\bar{b}}$, the right hand side of (28) is a quadratic equation which has a negative and positive root. Thus, a Nash bargaining solution exists if and only if $\lambda(c+\gamma, w)$ is at least equal to the positive root. ∎

Appendix B: Examples

Example 1: Let $a^0 = 1000; a = 100, b = 50, \alpha = 0.75, \beta = 0.375$ and $c = \gamma = 1$. In an industry with $N = 4$ firms, the retail price and waiting time standard under outsourcing are $p^C = 11.6$ and $w^C = 0.0328$; when all benefits of outsourcing are assigned to the retailers, each earns $\pi_i^o = 4.305$. An individual firm can increase its profits (to 4.306) by switching to in-house service, simultaneously decreasing the price by \$0.22 and the waiting time standard by 1×10^{-3}. In this instance $\theta = \rho = 0.03$. It is only when the competitive intensities are increased to $\theta = \rho = 0.6$ that the possibility of defections from the service chain can be eliminated. At the same time, for the same value of $\theta = \rho = 0.6$ the service chain is again unstable when $N = 5$. Here $p^C = 25.95$ and $w^C = 0.032$ and the firms earn a profit of 11.458. By switching to in-house service, an individual firm can increase its profits by more than 4%, again simultaneously decreasing the price to \$17.47 and the waiting time standard to 0.025.

Appendix C: Numerical Study

As mentioned in §8, our initial set of problem instances consider an industry with $N = 3$ firms, and demand function of the quasi-separable form, as follows: (3)

$$\begin{aligned}
\lambda_1(p,w) &= \left[405 - 0.1w_1 + 0.04w_2 + 0.04w_3 - \frac{7}{1-\rho}p_1 + \frac{3.5\rho}{1-\rho}p_2 + \frac{3.5\rho}{1-\rho}p_3\right]^+ \\
\lambda_1(p,w) &= \left[405 - 0.1w_2 + 0.04w_1 + 0.04w_3 - \frac{7}{1-\rho}p_2 + \frac{3.5\rho}{1-\rho}p_1 + \frac{3.5\rho}{1-\rho}p_3\right]^+ \\
\lambda_3(p,w) &= \left[825 - 0.1w_3 + 0.04w_2 + 0.04w_1 - \frac{7}{1-\rho}p_3 + \frac{3.5\rho}{1-\rho}p_2 + \frac{3.5\rho}{1-\rho}p_1\right]^+
\end{aligned}$$

Thus, the total price sensitivity $\bar{b} = 7$ for all three firms. As to the cost parameters, $c_1 = c_2 = 10$, and $\gamma_1 = \gamma_2 = 5$, while $c_3 = 5$ and γ_3 varies. We specify $c_0 = \frac{1}{3}\sum_{i=1}^{3} c_i$ and $\gamma_0 = \frac{1}{3}\sum_{i=1}^{3} \gamma_i$, i.e., the supplier fails to face systematic cost rate advantages or disadvantages compared to the in-house service option.

We start with 4 instances, corresponding with 4 values of γ_3 ($\gamma_3 = 5, 10, 15, 20$), a price competition intensity $\rho = 0.3$ and a common waiting time standard $w = 1$. This quartet is followed by two others, one with $\rho = 0.3$ replaced by $\rho = 0.9$, and one with w replaced by $w = 0.01$. These instances may, for example represent an industry with a well established domestic provider (firm 3) facing competition by two more recent and identical foreign entrants who are able to operate at lower capacity costs, but higher transaction costs per customer. (The fact that firm 3 is the more established or recognized provider is reflected by the larger intercept of its demand function.) Since the total marginal cost per customer is given by $c_i + \gamma_i$, firm 3 enjoys a cost advantage over its competitors when $\gamma_3 = 5$, while firms 1 and 2 have the advantage when $\gamma_3 \geq 10$. (It follows from (8) that if two firms have identical *total* marginal cost rates $c_i + \gamma_i$, the one with the lower γ value enjoys a cost advantage.)

In Table 1, we compare for all 12 instances the equilibrium under(i) in-house service, (ii) outsourcing to a supplier using a dedicated facility for each of the firms, and (iii) outsourcing to a supplier who pools the service processes. Under the in-house service option (i), we report each firm's equilibrium price, demand volume and profit level. Under the outsourcing options (ii) and (iii), we similarly report the equilibrium prices and demand volumes. As to the firms' profit levels under outsourcing (in (ii) and (iii)), we present the outsourcing option in the most favorable light, i.e., we assume that the service chain is perfectly coordinated while *all* of its profits are earned by the retail firms. In other words, we consider a special case of our model where $V = \eta_0 = 0$. (We also assume $F_i = 0$). In the symmetric case, this uniquely characterizes each firm's profit level, but with asymmetric firms this is no longer the case. First, the full service chain permits a Nash bargaining solution for any vector of bargaining power indices η if and only if it is *viable*, i.e. its aggregate profits are larger than those earned under in-house service. Since the supplier's cost rates are chosen to be the average of those of the three retail firms, this condition does not always apply, see below. Second, assuming the chain's aggregate profits are larger than those earned under in-house service, the specific profit allocation depends on the firms' vector of bargaining indices η. When dedicated facilities are used, it seems reasonable to assume that η_i, the bargaining power of each retail firm i in the service chain is proportional to the revenues that are obtained from its customers and, net of the cost of the dedicated facility the supplier operates on its behalf. We therefore report *this* Nash bargaining solution and profit allocation, along with the firms' price and demand volumes, whenever the service chain is viable (and none otherwise). In the case of service pooling, there is no "obvious" choice for the bargaining power indices η.

The brand recognition firm 3 enjoys, allows it, under in-house service, to position itself in the market with a higher price than its competitors, irrespective of the value of γ_3 or the competitive intensity ρ. When $\rho = 0.3$, firm 3's price differential is approximately \$19. When $\rho = 0.9$, the much more intense price competition compels firm 3 to reduce the price difference to less than \$1.50. (Note, that all prices reduce sharply when ρ moves from 0.3 to 0.9; while variable profit margins are above 100% and sometimes 200% when $\rho = 0.3$, they shrink to 20% for firms 1 and 2 in case $\gamma_3 = 5$ and to approximately 20% for firm 3, in case $\gamma_3 = 20$.) When $\gamma_3 = 0.3$, firms 3's market share is above 50% (except when $\gamma_3 = 5$, where it is slightly below 50%), its large price differential notwithstanding. When $\rho = 0.9$, the intensive price competition causes firm 3's market share to shrink to the 42-45% range. Comparing corresponding instances, one notes that firm 3's profit reduces approximately by a factor of four, and those of its competitors by approximately tenfold, when going from $\rho = 0.3$ to $\rho = 0.9$. Finally, under $\rho = 0.3$, as γ_3 and hence the marginal cost rate per customer of firm 3 decreases in increments of \$5, approximately 60% of the cost saving (or \$2.94) is passed on to the customer, forcing firms 1 and 2 to reduce their prices by an almost identical amount. In contrast, when $\rho = 0.9$, 90% of the same cost rate reductions is passed on to the customer.

In the symmetric model of §4, we proved that, under outsourcing, prices are higher and demand volumes lower than when the firms perform service in-house. This pattern is maintained by our asymmetric instances, with the exception of two instances with $\gamma_3 = 20$ and $\rho = 0.3$, where, under outsourcing, firm 3 faces a slight *increase* in its demand volume, along with a slight price increase. (Note, however, that this demand increase is the result of the supplier operating at a lower capacity cost rate $\gamma_0 = 10$ compared with $\gamma_3 = 20$).

When the supplier uses dedicated facilities, the full service chain fails, in some cases, to be viable, i.e. aggregate profits are lower then those under in-house service. This phenomenon occurs only because the supplier operates with higher cost rates than some of the participating firms. At the same time, when $\rho = 0.9$, the service chain is always viable. Here, the benefits of outsourcing clearly stem from the ability to circumvent the cut throat competition, by creating a perfectly coordinated service chain. For the first eight instances, only small differences arise between the

Table 1 Price Competition

Scenario		In-house			Dedicated			Pooled
	Firm	Price	Demand	Profit	Price	demand	profits	Profit
$\gamma3 = 20, w = 1, \rho = 0.3$	1,2	40.25	152.52	3846.60	41.95	138.57	4297.20	
	3	59.79	347.87	12081.00	60.10	349.83	12531.60	
	total						21126.00	21141.00
$\gamma3 = 15, w = 1, \rho = 0.3$	1,2	37.31	173.11	3857.40	41.40	142.42		
	3	56.85	368.46	13561.00	59.55	353.68		
	total						21141.00	22201.00
$\gamma3 = 10, w = 1, \rho = 0.3$	1,2	34.37	193.70	3747.00	40.30	150.12	3966.33	
	3	53.90	389.05	15126.00	58.45	361.38	15345.33	
	total						23278.00	23290.00
$\gamma3 = 5, w = 1, \rho = 0.3$	1,2	31.43	214.29	3515.50	39.75	153.97	3712.50	
	3	50.96	409.63	16775.00	57.90	365.23	16972.00	
	total						24397.00	24406.00
$\gamma3 = 20, w = 1, \rho = 0.9$	1,2	28.99	279.21	3900.80	47.45	142.14	6625.93	
	3	31.44	450.63	2881.00	49.65	338.83	5606.13	
	total						18858.00	18874.00
$\gamma3 = 15, w = 1, \rho = 0.9$	1,2	24.44	311.03	2932.10	46.35	149.84	6513.63	
	3	26.89	482.45	3310.20	48.55	346.53	6891.73	
	total						19919.00	19933.00
$\gamma3 = 10, w = 1, \rho = 0.9$	1,2	19.90	342.85	1674.20	45.80	153.69	6305.33	
	3	22.35	514.27	3768.20	48.00	350.38	8399.33	
	total						21010.00	21023.00
$\gamma3 = 5, w = 1, \rho = 0.9$	1,2	15.35	374.66	127.01	44.70	161.39	6000.27	
	3	17.80	546.09	4255.20	46.90	358.08	10128.46	
	total						22129.00	22139.00
$\gamma3 = 20, w = .01, \rho = 0.3$	1,2	40.22	152.22	3839.00	41.95	138.57		
	3	59.73	347.33	12064.00	60.10	349.85		
	total						18157.00	20153.00
$\gamma3 = 15, w = .01, \rho = 0.3$	1,2	37.28	172.80	3850.10	41.40	142.42		
	3	56.79	367.92	13536.00	59.55	353.70		
	total						19713.00	21377.00
$\gamma3 = 10, w = .01, \rho = 0.3$	1,2	34.34	193.39	3740.00	40.30	150.12	3315.00	
	3	53.85	388.51	15094.00	58.45	361.40	14669.00	
	total						21299.00	22631.00
$\gamma3 = 5, w = .01, \rho = 0.3$	1,2	31.40	213.98	3508.80	39.75	153.97	3228.60	
	3	50.91	409.10	16736.00	57.90	365.25	16455.80	
	total						22913.00	23912.00

outsourcing option with dedicated facilities versus pooled service. This is due to the cost of the service based capacity being very small compared to the other cost components, in these instances. (The cost savings that arise from service pooling are confined to the cost of the service based capacity.) Table 2 exhibits whether the full service chain that arises under outsourcing with dedicated facilities, is immune to defections. Given the ambiguity about how profits should be allocated among non-identical firms, we define a chain to be stable if *some* profit allocation exists which deters defections. Clearly, this is the case if and only if the aggregate profits in the chain are bigger than or equal to the sum of the profits each firm can obtain when defecting. In terms of the equilibrium profits, we therefore only report the profit of the defecting firm. It follows from Theorem 4.3 that in symmetric models, with $V = F = \eta_0 = 0$, instances with up to 3 firms, stability always prevails, but that the chain fails to be stable when the number of firms is larger and the competitive intensity is below a threshold $\rho^*(N)$, which is increasing in N. Table 2 shows that, with 3 asymmetric firms, the chain is stable in all but two instances, where ρ is low and the dominant firm 3's capacity cost rate under in-house service is considerably lower than that of the supplier. Clearly, the full chain is ever more likely to be stable , when its service processes are pooled.

Table 3 exhibits and discusses, for all eight combinations of ρ and γ_3, the equilibria which arise under *simultaneous*

Table 2 Price Competition, defections

Scenario	3rd firm defects				1st firm defects			
	Firm	Price	demand	profit	Firm	Price	demand	profit
$\gamma 3=20, w=1, \rho=0.3$	1,2	38.1	171.29		1	35.35	196.305	3989.8
	3	60.1	338.28	11854	2	39.2	152.04	
					3	57.35	363.315	
$\gamma 3=15, w=1, \rho=0.3$	1,2	37	176.515		1	35.35	194.655	3956.2
	3	57.35	362.48	13524	2	38.65	156.715	
					3	56.8	367.99	
$\gamma 3=10, w=1, \rho=0.3$	1,2	35.9	181.74		1	35.35	191.355	3889.1
	3	54.6	386.68	15303	2	37.55	166.065	
					3	55.7	377.34	
$\gamma 3=5, w=1, \rho=0.3$	1,2	34.8	186.965		1	35.35	189.705	3855
	3	51.85	410.88	17190	2	37	170.74	
					3	55.15	382.015	
$\gamma 3=20, w=1, \rho=0.9$	1,2	27.1	336.565		1	22.7	488.63	3757
	3	30.95	365.78	2156	2	25.45	209.515	
					3	27.65	406.215	
$\gamma 3=15, w=1, \rho=0.9$	1,2	24.9	317.315		1	22.15	457.83	3268.5
	3	27.65	458.18	3490.1	2	24.35	234.54	
					3	26.55	431.24	
$\gamma 3=10, w=1, \rho=0.9$	1,2	22.7	298.065		1	22.15	388.53	2773
	3	24.35	550.58	5137.9	2	23.25	276.89	
					3	25.45	473.59	
$\gamma 3=5, w=1, \rho=0.9$	1,2	20.5	261.49		1	21.6	357.73	2356
	3	20.5	681.48	7150.5	2	22.15	301.915	
					3	24.35	498.615	
$\gamma 3=20, w=.01, \rho=0.3$	1,2	38.1	171.2999		1	196.3248	35.35	3495.2
	3	60.1	338.2998	9874.3	2	152.0499	39.2	
					3	363.3249	57.35	
$\gamma 3=15, w=.01, \rho=0.3$	1,2	37	176.5249		1	35.35	194.6748	3461.6
	3	57.35	362.4998	12039	2	38.65	156.7249	
					3	56.8	367.9999	
$\gamma 3=10, w=.01, \rho=0.3$	1,2	35.9	181.7499		1	35.35	191.3748	3394.5
	3	54.6	386.6998	14313	2	37.55	166.0749	
					3	55.7	377.3499	
$\gamma 3=5, w=.01, \rho=0.3$	1,2	34.8	186.9749		1	35.35	189.7248	3360.9
	3	51.85	410.8998	16696	2	37	170.7499	
					3	55.15	382.0249	

competition, both when the firms provide in-house service and when they outsource to a supplier using dedicated facilities.

Commenting briefly on the results in Table 3, under in-house service, firm 3 positions itself as the firm with the highest service level, when $\gamma_3 = 5$, i.e., when it operates with lower costs than its competitors. When $\gamma_3 = 10$ and all firms exhibit identical marginal costs, they offer virtually identical waiting time standards. Finally, firm 3's waiting time is higher when $\gamma_3 \geq 15$, i.e. when firm 3 is at a cost disadvantage. As in the case of price competition, firm 3 exploits its larger brand recognition to charge a higher price than its competitors, albeit that the price differential is considerably smaller under high values of ρ. Under simultaneous competition, the prices charged by firms 1 and 2 are much less sensitive to the cost value γ_3 and the corresponding price of firm 3 than under price competition above.

Comparing the equilibria under in-house service and outsourcing, we note that the coordinated service chain is consistently able to charge higher prices and offer higher waiting time expectations. As in the case of price competition, the benefits of outsourcing are considerably higher when $\rho = 0.9$. Under outsourcing, firm 3 adopts a considerably lower price and service level, when $\rho = 0.9$ compared to $\rho = 0.3$, while firms 1 and 2 adopt a higher price and lower service.

Table 3 Simultaneous Competition

Scenario		In-house Price	In-house Waiting Time	In-house demand	In-house profit	Dedicated Price	Dedicated Waiting Time	Dedicated demand	Dedicated profit
$\gamma 3=20, \rho=0.3$	1,2	34.788	1.5896	197.8801	3913	42	10	137.504	4510.33
	3	58.9724	2.4263	339.724	11533	60	2.08	351.592	12130.33
	total								21151.00
$\gamma 3=15, \rho=0.3$	1,2	34.582	1.5979	195.8201	3831	42	10	137.504	4251.33
	3	56.4435	2.0288	364.4354	13274	60	2.08	351.592	13694.33
	total								22197.00
$\gamma 3=10, \rho=0.3$	1,2	34.3759	1.6064	193.7595	3751	40	10	151.504	3968.33
	3	53.9148	1.603	389.1482	15137	58	2.08	365.592	15354.33
	total								23291.00
$\gamma 3=5, \rho=0.3$	1,2	34.1697	1.615	191.697	3672	40	10	151.471	
	3	51.3865	1.0991	413.8645	17124	58	1.42	365.658	
	total								24407.00
$\gamma 3=20, \rho=0.9$	1,2	21.5737	2.7579	460.1624	3023.2	46.6667	4.72	162.3521	7067.50
	3	28.0988	8.0337	216.9172	669.7	49.3333	6.04	311.4403	4714.00
	total								18849.00
$\gamma 3=15, \rho=0.9$	1,2	20.7408	2.9518	401.6838	2303.3	46	4.06	146.0914	6774.70
	3	25.225	5.358	365.7469	1908.2	48	6.7	362.6548	6379.60
	total								19929.00
$\gamma 3=10, \rho=0.9$	1,2	19.9032	3.1933	343.2239	1681.3	46	3.4	146.098	6307.63
	3	22.3505	3.6884	343.413	3779.4	48	6.04	362.668	8405.73
	total								21021
$\gamma 3=5, \rho=0.9$	1,2	19.0647	3.5073	284.527	1155.1	44	3.4	160.032	5664.83
	3	19.4739	2.2973	663.1725	6280.6	46	4.72	376.8	10790.33
	total								22120

Table 4 Simultaneous Competition, defections

Scenario	3rd firm defects Firm	Price	Waiting time	demand	profit	1st firm defects Firm	Price	Waiting time	demand	profit
$\gamma 3=20, w=1, \rho=0.3$	1,2	42	10	137.6515		1	35.4263	1.5646	204.2634	4169.2
	3	60.0881	2.3875	350.8806	12303	2	42	10	127.3009	
						3	60	2.08	341.4097	
$\gamma 3=15, w=1, \rho=0.3$	1,2	39	10	158.7104		1	34.9762	1.5821	199.7625	3987.3
	3	57.14	2.0097	371.3995	13786	2	39	10	152.1267	
						3	57	2.08	366.2355	
$\gamma 3=10, w=1, \rho=0.3$	1,2	39	10	154.9425		1	34.9762	1.5821	199.7625	3987.3
	3	54.6421	1.5883	396.4206	15709	2	39	10	152.1267	
						3	57	2.08	366.2355	
$\gamma 3=5, w=1, \rho=0.3$	1,2	39	10	151.1713		1	34.9743	1.5822	199.7427	3986.6
	3	52.1446	1.0892	421.4455	17757	2	39	10	152.0841	
						3	57	1.09	366.3315	
$\gamma 3=20, w=1, \rho=0.9$	1,2	30	5.05	254.606		1	24.5708	2.2857	2.2857	6409.9
	3	31.8933	5.3864	482.5327	3322.5	2	30	7.03	7.03	
						3	33	4.06	4.06	
$\gamma 3=15, w=1, \rho=0.9$	1,2	30	4.06	175.8594		1	24.57	2.2858	669.8981	6408.7
	3	29.3938	3.996	657.5626	6173.2	2	30	5.05	118.1862	
						3	33	3.07	233.9634	
$\gamma 3=10, w=1, \rho=0.9$	1,2	30	4.06	97.0792		1	23.8943	2.371	622.6043	5535.5
	3	26.8945	2.8995	832.6174	9900.1	2	30	3.07	2.6063	
						3	30	3.07	422.6063	
$\gamma 3=5, w=1, \rho=0.9$	1,2	30	3.07	18.3		1	23.8943	2.371	622.6043	5535.5
	3	24.3947	1.8637	1007.6	14502	2	30	3.07	2.6063	
						3	30	3.07	422.6063	

Table 4 exhibits the equilibria when either firm 3 or one of its two competitors defects from the service chain under outsourcing (, with dedicated facilities). This allows us to verify whether the chain is immune to defections. As in the case of price competition, the chain is sometimes unstable.

We conclude this section with a set of instances with demand functions from an attraction model.

(B) ***Demand functions given by an attraction model***

Here, there is a given potential number of customers M in thr market. Each firm's market share is determined by the so called attractiveness value z_i, itself a general function of the firm's price p_i and waiting time standard w_i, i.e.

Table 5 Attraction demand model

Scenario		In-house			Dedicated		
		Price	demand	profit	Price	demand	profit
$\gamma 3 = 20$	1,2	75.74	2371.1	120310	87.3333	2058.1	142005.69
	3	93.63	2858.2	181870	100.6667	2860.4	235505.90
	total						519517.28
$\gamma 3 = 15$	1,2	75.71	2354.7	119400	86.8	2079.1	145819.74
	3	91.61	2947.6	196340	100.4	2861.2	239588.56
	total						531228.05
$\gamma 3 = 10$	1,2	75.68	2338.6	118520	86	2106.8	149576.13
	3	89.61	3034.8	211250	99.6	2881.5	243768.23
	total						542920.49
$\gamma 3 = 5$	1,2	75.65	2322.9	117650	86	2106.8	153096.156
	3	87.62	3120.3	226590	99.6	2881.5	248582.005
	total						554774.317

Table 6 Attraction demand model, defections

Scenario	3rd firm defects				1st firm defects			
	Firm	Price	demand	profit	Firm	Price	demand	profit
$\gamma 3 = 20$	1,2	79.6	2148	131590	1	74	2611.8	**154090**
	3	94	2995.6	**206680**	2	82	2003.5	168910
					3	95.6	2721.5	168910
$\gamma 3 = 15$	1,2	78.8	2166.7	134610	1	74	2596.5	**153190**
	3	92.4	3048.1	**220670**	2	81.2	2035.6	173080
					3	94.8	2749.3	173080
$\gamma 3 = 10$	1,2	78	2178.6	137250	1	74	2589	**152750**
	3	90	3133.6	**235010**	2	81.2	2029.6	177180
					3	94	2784.9	177180
$\gamma 3 = 5$	1,2	77.2	2196.5	140280	1	74	2581.5	**152300**
	3	88.4	3183.1	**249550**	2	80.4	2067.2	181310
					3	94	2776.8	181310

$z_i = z_i(p_i, w_i)$. For a given positive constant z_0, the demand rates of the firms are thus given by the system of equations

$$\lambda_i = M \frac{z_i(p_i, w_i)}{\sum_{i=1}^{N} z_j(p_j, w_j) + z_0}, \ i = 1, \ldots, N \tag{29}$$

Without loss of generality, we assume

$$\frac{\partial z_i}{\partial p_i} \leq 0 \ , \ \frac{\partial z_i}{\partial w_i} \leq 0. \tag{30}$$

See Bell et al. (1980) and Leeflang et al. (2000) for an axiomatic foundation of the class of attraction models and various specifications of the attraction functions. The MultiNomial Logit specification arises with the choice $z_i = e^{a_i(w_i) - b_i p_i}$.

Under the attraction model (29), a firm maintains a *positive* market share irrespective of how extreme and uncompetitive its price and service level choices are.

In our numerical study, we assume again an industry with $N = 3$ firms operating with the same cost rates as before, $M = 15000$, $z_1(p, w) = z_2(p, w) = 1800 - 20p + 15\log(10/w)$ and $z_3(p, w) = 220 - 20p + 15\log(10/w)$. As with the previous instances, firm 3's larger brand recognition manifests itself in the larger intercept of its attraction function. In Table 5 we compare, for each of the four γ_3 values, the equilibrium price under in-house service with that arising under outsourcing *without* service pooling, assuming all three firms adopt the same waiting time standard $w = 1$. Profit allocations within the service chain are again determined as the Nash bargaining solution with the bargaining power induces determined in the same way as under the linear demand scenario. Table 6 describes the equilibrium when either firm 3 or one of its two (identical) competitors defects from the service chain. As with the previous instances, firm 3 is able to exploit its superior brand recognition to charge a significantly higher price than its competitors, while continuing to be the largest provider in the market. These observations hold, both when service is provided in-house and when it is outsourced and irrespective of firm 3's capacity cost rate γ_3. Under in-house service, firm 3's price is rather sensitive to the value of γ_3, with roughly 40% of the cost savings due to a reduction of γ_3, being passed on to the customer. In contrast, under outsourcing, the prices are quite insensitive to the capacity cost rate. In this case, outsourcing enables an increase of aggregate profits by more than 20%. The chain is always immune to defections.

Appendix D: Proofs

Proof of Theorem 4.2 Consider a setting where all N firms are part of a common chain which operates at maximum efficiency. Assume now that one of the firms, without loss of generality firm N, decides to leave the chain and to service its customers in-house. After the "defection", the industry operates as a two player game: the chain comprised of firms $1,\dots,N-1$ is the first player, deciding on a vector $(p_1,\dots,p_{N-1})$ and firm N is the second player, selecting a single price $p_N = p^i$ for its *in-house* service. For all $i = 1,\dots,N-1$, $\lambda_i = a_i^0 + \beta p^i + \alpha w - w(a-(N-2)\alpha) - bp_i + \beta\sum_{j\neq i}^{N-1} p_j$. Thus, for any price level p^i, the best response problem faced by the chain is equivalent to the centralized problem in a system with $N-1$ firms and no outside competitors, and an intercept for the firms' demand function given by $\widehat{a}^0 = a^0 + \beta p^i + \alpha w$. As shown above, it is optimal in the centralized system to select identical prices for all $N-1$ firms. Thus, with p^o denoting the common price among all firms $1,\dots,N-1$, for their *outsourced* service, we obtain from (12) that

$$p^o = \frac{a^0 - w(a-(N-1)\alpha)}{2(b-(N-2)\beta)} + \frac{c+\gamma}{2} + \frac{p^i\beta}{2(b-(N-2)\beta)}. \tag{31}$$

Firm N's profit $\pi_N = \lambda^i(p^i - c - \gamma) - \frac{\gamma}{N}$ with $\lambda^i = a^0 - w(a-(N-1)\alpha) - bp^i + (N-1)\beta p^o$, so

$$p^i = \frac{a^0 - w(a-(N-1)\alpha)}{2b} + \frac{c+\gamma}{2} + \frac{p^o\beta(N-1)}{2b}. \tag{32}$$

Substituting (31) into (32) we obtain $p^i\left[\frac{4b(b-(N-2)\beta)-\beta^2(N-1)}{4b(b-(N-2)\beta)}\right] = \frac{a^0-w(a-(N-1)\alpha)}{2b}\left[\frac{2(b-(N-2)\beta)+\beta(N-1)}{2(b-(N-2)\beta)}\right] + \frac{c+\gamma}{2}\left[\frac{2b+\beta(N-1)}{2b}\right]$ and $p^i = (a^0 - w(a-(N-1)\alpha))\frac{2(b-(N-2)\beta)+\beta(N-1)}{4b(b-(N-2)\beta)-\beta^2(N-1)} + (c+\gamma)\frac{(2b+(N-1)\beta)(b-(N-2)\beta)}{4b(b-(N-2)\beta)-\beta^2(N-1)}$. By (9)

$$\begin{aligned}
\pi_N &= b(p^i - c - \gamma)^2 - \frac{\gamma}{w} - F \\
&= b\left[(a^0 - w(a-(N-1)\alpha))\frac{2(b-(N-2)\beta)+\beta(N-1)}{4b(b-(N-2)\beta)-\beta^2(N-1)} + (c+\gamma)\left[\frac{(2b+(N-1)\beta)(b-(N-2)\beta)}{4b(b-(N-2)\beta)-\beta^2(N-1)} - 1\right]\right]^2 - \frac{\gamma}{w} - F \\
&= b\left[\frac{2b-\beta(N-3)}{4b(b-(N-2)\beta)-\beta^2(N-1)}\right]^2\left[a^0 - w(a-(N-1)\alpha) - (c+\gamma)(b-(N-1)\beta)\right]^2 - \frac{\gamma}{w} - F \\
&= b\left[\frac{2b-\beta(N-3)}{4b(b-(N-2)\beta)-\beta^2(N-1)}\right]^2 \lambda(c+\gamma,w)^2 - \frac{\gamma}{w}.
\end{aligned} \tag{33}$$

Thus, using (14), the full chain is stable iff

$$\lambda^2(c+\gamma,w)\ b\left[\frac{2b-\beta(N-3)}{4b(b-(N-2)\beta)-\beta^2(N-1)}\right]^2$$

$$\leq \quad \frac{\lambda^2(c+\gamma,w)}{b}\left[\left(\frac{N\eta}{N\eta+\eta_0}\right)\frac{1}{4(1-\rho)}+\frac{\eta_0}{N\eta+\eta_0}\frac{1}{(2-\rho)^2}\right]-\frac{\eta}{N\eta+\eta_0}\left[V-NF-\frac{\gamma\nu}{w})\right] \Longleftrightarrow$$

$$\lambda^2(c+\gamma,w)\left[b\left(\frac{2b-\beta(N-3)}{4b(b-(N-2)\beta)-\beta^2(N-1)}\right)^2-\left(\left(\frac{N\eta}{N\eta+\eta_0}\right)\frac{1}{4(1-\rho)}+\frac{\eta_0}{N\eta+\eta_0}\frac{1}{(2-\rho)^2}\right)\right] \leq -\frac{\eta}{N\eta+\eta_0}A \Longleftrightarrow$$

$$\lambda^2(c+\gamma,w)Q(\rho,N,\eta_0/\eta) \leq \frac{-A\eta b}{N\eta+\eta_0} \tag{34}$$

Thus, if $A>0$, stability requires $Q(\rho,N,\eta_0/\eta)\leq 0$ and $\lambda(c+\gamma,w)\geq \sqrt{\frac{Ab}{N}}/\sqrt{\frac{1}{4(1-\rho}-\frac{1}{(2-\rho)^2}+(1+\frac{\eta_0}{\eta})\left[\frac{1}{(2-\rho)^2}-\left(\frac{2(N-1)-\rho(N-3)}{4(N-1)-4(N-2)\rho-\rho\ldots}\right.\right.}$ In addition, stability of course, requires that the full service chain permits a Nash bargaining solution, i.e., $\lambda(c+\gamma,w)\geq\sqrt{\frac{Ab}{N}}/\sqrt{\frac{1}{4(1-\rho}-\frac{1}{(2-\rho)^2}}$, see (13) . It then easily follows that the stability condition is equivalent to $Q(\rho,N,\eta_0/\eta)\leq 0$ and $\lambda(c+\gamma,w)\geq\lambda^1$, thus proving (a). Similarly, if $A<0$, (34) is clearly satisfied if $Q(\rho,N,\eta_0/\eta)\leq 0$, and if $Q(\rho,N,\eta_0/\eta)>0$, it is satisfied iff in addition $\lambda(c+\gamma,w)\leq\lambda^2$, proving part (b) of the theorem. (Note that, when $A<0$, the service chain permits a Nash bargaining solution under any benchmark demand volume, i.e. $\lambda^0=0$, see (13).)

part(c): Since the last term to the right of (34) is positive, $Q(\rho,N,\eta_0/\eta)\leq 0$ only if $\left[\frac{2(N-1)-\rho(N-3)}{4(N-1)-4(N-2)\rho-\rho^2}\right]^2-\frac{1}{4(1-\rho)}\leq 0 \Longleftrightarrow \left[\frac{2(N-1)-\rho(N-3)}{4(N-1)-4(N-2)\rho-\rho^2}\right]^2\leq\frac{1}{4(1-\rho)} \Longleftrightarrow \underline{Q}(\rho,N)\equiv 4(N-1)(N-3)-4\rho(N^2-4N+5)-\rho^2\leq 0$. For N=3, the inequality is satisfied for all $\rho\geq 0$. For $N\geq 4$, the quadratic expression in ρ is non-positive for ρ larger than or equal to its positive root. Clearly $\underline{\rho}(N)\geq 0$, since $(N-1)(N-3)\geq 0$ for $N\geq 3$. Also, $\underline{\rho}(N)\leq 1\Leftrightarrow 2\sqrt{(N^2-4N+5)^2+(N-1)(N-3)}\leq 1+2(N^2-4N+5)\Leftrightarrow 4(N-1)(N-3)<1+4(N^2-4N+5)$, which holds for all N. Moreover, since $\underline{\rho}(N)\leq 1$,

$$\underline{Q}(\underline{\rho}(N),N+1)=4(2N-3)(1-\underline{\rho}(N))+Q(\underline{\rho}(N),N)\geq 0. \tag{35}$$

Equation (35) implies that $\underline{\rho}(N+1)$, the positive root of the quadratic equation $Q(\underline{\rho},N+1)=0$, is larger than $\underline{\rho}(N)$. Finally, the positive root of the quadratic equation can be written as $\frac{4(N-1)(N-3)}{\sqrt{4(N^2-4N+5)^2+4(N-1)(N-3)}+2(N^2-4N+5)}$, from which $\lim_{N\uparrow\infty}\underline{\rho}(N)=1$ follows

■

Proof of Theorem 4.3

Assume $m^o\equiv N-m^i$ firms outsource, without loss of generality, firms $\{m^i+1,\ldots,N\}$. For given prices $\{p^i_1,\ldots,p^i_{m^i}\}$ of the firms outside the chain, the aggregate profit of the chain is given by $\Pi^c=\sum_{i=m^i+1}^N\left[\lambda_i(p_i-c-\gamma)-\frac{\gamma}{w}\right]$, where $\lambda_i=a^0-[a-(N-1)\alpha]\,w-bp_i+\beta\sum_{j=m^i,j\neq i}^N p_j+\beta\sum_{j=1}^{m^i}p^i_j$. It is easily verified that the chain's aggregate profit function is a concave quadratic function, such that the optimal prices for the participating firms satisfy the first order conditions

$$0=\frac{\partial\Pi^c}{\partial p_i}=\lambda_i-b(p_i-c-\gamma)+\beta\sum_{j=m^*+1,j\neq i}^N(p_j-c-\gamma) \tag{36}$$

$$= a^0 - [a - (N-1)\alpha]\, w - bp_i + \beta \sum_{j=1}^{m^i} p_j^i + \beta \sum_{j=m^i, j\neq i}^{N} p_j + \beta \sum_{j=1}^{m^i} p_j$$
$$- \; b(p_i - c - \gamma) + \beta \sum_{j=m^*+1, j\neq i}^{N} (p_j - c - \gamma), i = m^i + 1, \ldots, N$$

This represents a linear system of $(N - m^i)$ equations in as many unknown prices $\{p_{m^i+1}, \ldots, p_N\}$. By writing the system in matrix form, one easily verifies that this system of equations has a unique solution which is symmetric, i.e. there exists a price p^o, such that $p_j = p^o$ for all $j = m^i + 1, \ldots, N$

$$p^o = \frac{a^0 - w(a - (N-1)\alpha)}{2[b - (N - m^i + 1)\beta]} + \frac{c + \gamma}{2} + \frac{\beta \sum_{j=1, j\neq i}^{m^i} p_j^i}{2(b - (N - m^i + 1)\beta)} \tag{37}$$

The firms $i = 1, \ldots, m^i$ performing in-house service, have, under a given (common) price p^o for the outsourcing firms, a profit function $\pi_i = \lambda_i(p_i - c - \gamma) - \frac{\gamma}{w}$, with $\lambda_i = a^0 - [a - (N-1)\alpha]\, w - bp_i + \beta \sum_{j=1, j\neq i}^{m^i} p_j + \beta(N - m^i)p^o$. For a given price p^o for the outsourcing firms, the remaining m^i firms engage in a m^i-person game in which the Nash equilibrium satisfies the first order conditions:

$$0 = \frac{\partial \pi_i}{\partial p_i} = \lambda_i - b(p_i - c - \gamma) = [a^0 - w(a - (N-1)\alpha)] + (N - m^i)\beta p^o - 2bp_i + \beta \sum_{j=1, j\neq i}^{m^i} p_j + b(c + \gamma), i = 1, \ldots, m^i. \tag{38}$$

Once again, this system of linear equation has a unique solution, which is symmetric. In other words, for any price p^o selected by the outsourcing firms, the remaining firms choose a common price p^i, i.e. $p_j = p^i, j = 1, \ldots, m^i$, where

$$p^i = \frac{a^0 - w(a - (N-1)\alpha) + (N - m)\beta p^o + b(c + \gamma)}{2b - (m^i - 1)\beta}. \tag{39}$$

An overall Nash equilibrium in the $m^i + 1$ person game, consisting of the service chain and m^i firms with in house service, uses a price pair $\{p^i, p^o\}$ which satisfies (33) and (39). This system of two equations has the following solution

$$\begin{aligned}
p^i &= (a^0 - w(a - (N-1)\alpha)) \frac{2(b + \beta) - \beta(N - m^i)}{2(b - (N - m^i - 1)\beta)(2b - (m^i - 1)\beta) - (N - m^i)m^i\beta^2} \\
&+ (c + \gamma) \frac{(b - (N - m^i - 1)\beta)(2b + (N - m^i)\beta)}{2(b - (N - m^i - 1)\beta)(2b - (m^i - 1)\beta) - (N - m^i)m^i\beta^2} \\
p^o &= (a^0 - w(a - (N-1)\alpha)) \frac{2b + \beta}{2(b - (N - m^i - 1)\beta)(2b - (m^i - 1)\beta) - (N - m^i)m^i\beta^2} \\
&+ (c + \gamma) \frac{(b - (N - m^i - 1)\beta)(2b - (m^i - 1)\beta) + b\beta m^i}{2(b - (N - m^i - 1)\beta)(2b - (m^i - 1)\beta) - (N - m^i)m^i\beta^2}
\end{aligned}$$

By (38), $\lambda_i = b(p^i - c - \gamma)$ for all $i = 1, \ldots, m^i$, and

$$\pi^i(m^o) = \lambda_i(p^i - c - \gamma) - \frac{\gamma}{w} - F = b(p^i - c - \gamma)^2 - \frac{\gamma}{w} - F \tag{40}$$

After some algebra, we obtain that,

$$p^i - c - \gamma = \frac{[a^0 - w(a - (N-1)\alpha][2(b + \beta) - \beta(N - m^i)]}{2(b - (N - m^i - 1)\beta)(2b - (m^i - 1)\beta) - (N - m^i)m^i\beta^2}$$

$$
\begin{aligned}
&+\ (c+\gamma)\left\{\frac{(b-(N-m^i-1)\beta)(2b+(N-m^i)\beta)}{2(b-(N-m^i-1)\beta)(2b-(m^i-1)\beta)-(N-m^i)m^i\beta^2}-1\right\}\\
&=\ \frac{2(b+\beta)-\beta(N-m^i)}{2(b-(N-m^i-1)\beta)(2b-(m^i-1)\beta)-(N-m^i)m^i\beta^2}\left\{a^0-w(a-(N-1)\alpha)-(b-(N-1)\beta)(c+\gamma)\right\}\\
&=\ \frac{2(b+\beta)-\beta(N-m^i)}{2(b-(N-m^i-1)\beta)(2b-(m^i-1)\beta)-(N-m^i)m^i\beta^2}\lambda(c+\gamma,w)
\end{aligned}
$$

Thus by (22) ,

$$
\pi^i(m^o)\ =\ b\lambda^2(c+\gamma,w)\frac{[2(b+\beta)-\beta m^o]^2}{[2(b-(m^o-1)\beta)(2b-(N-m^o-1)\beta)-m^o(N-m^o)\beta^2]^2}-\frac{\gamma}{w}-F \tag{41}
$$

$$
=\ \frac{\lambda^2(c+\gamma,w)}{b}\left[\frac{2(N-1)+(2-m^o)\rho}{2(N-1)-(m^o-1)\rho)(2(N-1)-(N-m^o-1)\rho)-m^o(N-m^o)\rho^2}\right]-\frac{\gamma}{w}-F \tag{42}
$$

Similarly, one verifies that

$$
\begin{aligned}
p^o-c-\gamma\ &=\ \frac{(2b+\beta)\left[a^0-(a-(N-1)\alpha)w-(c+\gamma)(b-(N-1)\beta)\right]}{2(b-(m^o-1)\beta)(2b-(N-m^o-1)\beta-m^o(N-m^o)\beta^2}\\
&=\ \frac{(2b+\beta)\lambda(c+\gamma,w)}{2(b-(m^o-1)\beta)(2b-(N-m^o-1)\beta-m^o(N-m^o)\beta^2}
\end{aligned}
$$

If follows from (18) that $\lambda_i=[b-(N-m^i+1)\beta](p^o-c-\gamma)$, for all $i=m^i+1,\ldots,N$. Thus

$$
\begin{aligned}
\Pi^c(m^o)\ =&\ (N-m^i)[b-(N-m^i-1)\beta]\left[\frac{2b+\beta}{2(b-(N-m^i-1)\beta)(2b-(m^i-1)\beta)-\beta^2m^i(N-m^i)}\right]^2\lambda^2(c+\gamma,w)\\
&-\ \frac{m^o\gamma}{w}+\frac{\nu(m^o)\gamma}{w}
\end{aligned} \tag{43}
$$

A Nash bargaining solution exists iff $\Pi^c(m^o)-m^o\overline{\pi}^*\geq V-m^oF\Leftrightarrow$

$$
\begin{aligned}
&\left\{\left[(N-1)^2-(m^o-1)\rho(N-1)\right]\left[\frac{2(N-1)+\rho}{2(N-1-(m^o-1)\rho)(2(N-1)-(N-m^o-1)\rho)-\rho^2m^o(N-m^o)}\right]^2-\frac{1}{(2-\rho)^2}\right\}\\
&\frac{\lambda^2(c+\gamma,w)m^o}{b}\geq V-m^oF-\frac{\nu(m^o)\gamma}{w}=A(m^o)
\end{aligned} \tag{44}
$$

Note that in the absence of any fixed capacity or operating costs, the left hand side of (44) represents the increment in aggregate profits the service chain generates beyond those the participating firms earn if all firms in the industry keep the service in house. Theorem 4 in Zhou and Hawe (2004) establishes that this increment is non-negative. It follows that the service chain is viable, i.e., permits a Nash bargaining solution iff the benchmark demand volume $\lambda(c+\gamma,w)\geq\lambda^o(m^o)$. Finally, it follows from (5) that

$$
\begin{aligned}
\pi^o(m^o)\ =&\ \pi^*+\frac{\eta}{m^o\eta+\eta_0}\left[\frac{m^o\lambda^2(c+\gamma,w)}{b}\left\{[(N-1)^2-(m^o-1)\rho(N-1)K^2-\frac{1}{(2-\rho)^2}\right\}-V+m^oF+\frac{\nu(m^o)\gamma}{w}\right]\\
=&\ \frac{\lambda^2(c+\gamma,w)}{b}\left[\frac{\eta_0}{m^o\eta+\eta_0}\frac{1}{(2-\rho)^2}+\frac{m^o\eta}{m^o\eta+\eta_0}[(N-1)^2-(m^o-1)\rho(N-1)]K^2\right]\\
&+\ \frac{\eta}{m^o\eta+\eta_0}\left[-V+m^oF+\frac{\nu(m^o)\gamma}{w}\right]-\frac{\gamma}{w}-F.
\end{aligned} \tag{45}
$$

where $K=\frac{2(N-1)+\rho}{2(N-1-(m^o-1)\rho)(2(N-1)-(N-m^o-1)\rho)-\rho^2m^o(N-m^o)}$.

(b) It follows from part (a) that the stability condition $\pi^o(m^o) \geq \pi^o(m^o-1) \Leftrightarrow$

$$\frac{\lambda^2(c+\gamma,w)}{b}\left[\frac{\eta_0}{m^o\eta+\eta_0}\frac{1}{(2-\rho)^2}+\frac{m^o\eta}{m^o\eta+\eta_0}[(N-1)^2-(m^o-1)\rho(N-1)]\left[\frac{2(N-1)+\rho}{2(N-1-(m^o-1)\rho)(2(N-1)-(N-m^o-1)\rho)-\rho^2 m^o(N-m^o)}\right]^2\right] - \frac{\lambda^2(c+\gamma,w)}{b}\left[\frac{2(N-1)+\rho(3-m^o)}{2(N-1-(m^o-2)\rho)(2(N-1)-(N-m^o)\rho)-\rho^2(m^o-1)(N-m^o+1)}\right]^2 \geq \frac{\eta A(m^o)}{m^o\eta+\eta_0} \tag{46}$$

This inequality is satasfied if $Q^{(m^o)}(N,\rho,\eta_0/\eta) \leq 0$ and $\lambda(c+\gamma,w) \geq \lambda^1(m^o)$. (The inequality $\lambda(c+\gamma,w) \geq \lambda^o(m^o)$ is required to ensure that a service chain with m^o firms is viable, i.e, permits a Nash bargaining solution.)

(c) The stability condition $\pi^i(m^o) \geq \pi^o(m^o+1) \Leftrightarrow$ either $\lambda(c+\gamma,w) \leq \lambda^o(m^o+1)$, in which case a chain with m^o+1 firms fails to be viable, or $\frac{\lambda^2}{b}R^{m^o}(N,\rho,\eta_0/\eta) \geq -\frac{A(m^o+1)}{m^o+1+\eta_0/\eta)}$. Since $A(m^o+1) \geq 0$, this inequality is satisfied iff (iii) $R^{(m^o)}(N,\rho,\eta_0/\eta) \geq 0$ or (iv) $R^{(m^o)}(N,\rho,\eta_0/\eta) < 0$ and $\lambda(c+\gamma,w) \leq \lambda^1$. ■

Proof of Theorem 4.4

*(a)*Note $\pi^o(1)=\pi^i(0)$. Let $m^* = \max\{1 \leq m \leq N : \pi^o(m) \geq \pi^i(m-1)\}$. If $m^*=N$, a SPNE exists with $m^o=N$ firms since all firms are part of the service chain and none want to switch to in-house service. If $m^*<N$, $\pi^o(m^*) \geq \pi^i(m^*+1)$, while $\pi^o(m^*+1) < \pi^i(m^*)$ so that a chain with m^* firms is in equilibrium: all firms inside (outside) the chain are better off continuing to outsource (provide in-house service).

*(b)*If the difference function $\Delta(\cdot)$ has a root m^{o*}, it follows from the proof of part (a) that every SPNE has $m^o=m^{o*}$ firms outsourcing. If the difference function has no root, SPNE exist in which all N firms outsource. Moreover, for all $m^o<N$, firms outside the chain have an incentive to join, so that no other SPNE exist.

■

Proof of Lemma 5.1

If firms are served in dedicated facilities, aggregate profits are given by $\Pi^C(w)=(p-c-\gamma)\sum_{i=1}^N \lambda_i - \gamma\sum_{i=1}^N \frac{1}{w_i}$, a jointly concave function of w, whose optimum is symmetric and satisfies the first order conditions $\frac{\partial \Pi^C}{\partial w_i}=(a-(N-1)\alpha)(p-c-\gamma)-\frac{\gamma}{w_i^2}=0$. Hence , the expressions for w^C. Under service pooling $\Pi^C(w)=(p-c-\gamma)\sum_{i=1}^N\lambda_i-\gamma\mu^*$, with μ^* given by (2). We first show that it is optimal to use identical waiting time standards for all N firms. Note first that under an optimal vector of waiting times, the maximum in (2) is achieved for $S=\{1,\ldots,N\}$. Allon and Federgruen (2004) show that a *largest* set S^* exists which achieves the maximum. Assuming to the contrary that $S^* \neq \{1,\ldots,N\}$: the waiting times $\{w_i^C : i \notin S^*\}$ could be reduced without increasing the required capacity while increasing the first term in the profit function under pooling; this contradicts the optimality of w^C. $\left(\frac{\partial(\sum_{l=1}^N \lambda_l)}{\partial w_i}=-a+(N-1)\alpha<0\right)$. Thus

$$\Pi^C(w)=(p-c-\gamma)\sum_{i=1}^N \lambda_i - \gamma\frac{\sum_{i=1}^N \lambda_i}{\sum_{i=1}^N \lambda_i w_i} \tag{47}$$

and

$$\begin{aligned}\max_w \Pi^c(w) &= \max_{w\geq 0, W>0}\left\{\sum_{i=1}^N (p-c-\gamma)\lambda_i - \gamma\frac{\sum_{i=1}^N \lambda_i}{W} \mid \sum_{i=1}^N \lambda_i w_i \geq W\right\} \\ &= \max_{W\geq 0}\min_{\delta\geq 0}\max_{w\geq 0}\left\{\sum_{i=1}^N (p-c-\gamma)\lambda_i - \gamma\frac{\sum_{i=1}^N \lambda_i}{W} + \delta\left(\sum_{i=1}^N \lambda_i w_i - W\right)\right\}\end{aligned} \tag{48}$$

where the second equality follows from the strong duality theorem of convex programming, as $\sum_{i=1}^{N}\lambda_i w_i$ is a strictly concave quadratic function of w. Note that the inner maximum (for any given $W, \delta \geq 0$) is achieved by a vector with *identical* components since the objective within curled brackets is a concave (quadratic) function of w, which is invariant to permutations of the vector $(w_1,\ldots,w_N)$. Thus, if a vector $w^{(1)}$ with non-identical components obtains the maximum, so would all of its permutations $\left\{w^{(l)}, l=1,\ldots,N!\right\}$, but their average $\widehat{w}=\frac{1}{N!}\sum_{i=1}^{N!}w^{(l)}$ with identical components achieves a higher objective value than each of these optima. We conclude that it is optimal to use the same waiting times standard w, which by (47), must therefore optimize the function $N(p-c-\gamma)\left[a^0-(b-(N-1)\beta)p-(a-(N-1)\alpha)w\right]-\frac{\gamma}{w}$. The expression for w^c follows readily. ■

Proof of Theorem 5.2

*(a)*If one of the firms, without loss of generality, firm N, leaves the chain, the industry operates as a duopoly: the remaining chain of firms $1,\ldots,N-1$, is the first player who decides on the vector $(w_1,\ldots,w_{N-1})$, and firm N is the second player choosing the single "in-house" waiting time standard w^i. It is easily verified that irrespective of firm N's waiting time choice w^i, it is optimal for the remaining firms in the chain to make identical waiting time choices $w^o=\sqrt{\left(1-\frac{\nu(N-1)}{N-1}\right)\frac{\gamma}{(a-(N-1)\alpha)(p-c-\gamma)}}$, while $w^i=w^*$ continues to be the dominant choice for firm N. Thus $\lambda^i=(a^0-p(b-(N-1)\beta))-aw^i+(N-1)\alpha w^o$ represents the demand volume of the defecting firm N. Substituting w^i and w^o we obtain firm N's profit π^i when providing in-house service $\pi^i=(a^0-p(b-(N-1)\beta))(p-c-\gamma)-2\sqrt{\gamma a(p-c-\gamma)}+(N-1)\alpha\sqrt{\left(1-\frac{\nu(N-1)}{N-1}\right)\frac{\gamma(p-c-\gamma)}{a-(N-2)\alpha}}-F$. Thus, firm N does not benefit from defection if and only if

$$
\begin{aligned}
0 \geq \pi^i-\pi^o = \sqrt{a\gamma(p-c-\gamma)}&\left[-2+\frac{\eta_0}{N\eta+\eta_0}(2-\theta)+\theta\sqrt{\left(1-\frac{\nu(N-1)}{N-1}\right)\frac{1}{1-\frac{N-2}{N-1}\theta}}+2\frac{N\eta}{N\eta+\eta_0}\sqrt{\left(1-\frac{\nu(N)}{N}\right)(1-\theta)}\right]\\
+\ \frac{\eta}{N\eta+\eta_0}(V-NF) &\Leftrightarrow\\
\left(\frac{N\eta}{N\eta+\eta_0}2+\frac{\eta_0}{N\eta+\eta_0}\theta\right)-\theta\sqrt{\frac{N-1-\nu(N-1)}{N-1-(N-2)\theta}}&-2\frac{N\eta}{N\eta+\eta_0}\sqrt{\left(1-\frac{\nu(N)}{N}\right)(1-\theta)}\geq\frac{BN\eta}{N\eta+\eta_0}\Leftrightarrow\\
\left(2+\frac{\eta_0/\eta}{N}\theta\right)-\left(1+\frac{\eta_0/\eta}{N}\right)\theta\sqrt{\frac{N-1-\nu(N-1)}{N-1-(N-2)\theta}}&-2\sqrt{\left(1-\frac{\nu(N)}{N}\right)}(1-\theta)\geq B\Leftrightarrow\\
2-\theta-2\sqrt{\left(1-\frac{\nu(N)}{N}\right)(1-\theta)}+\left(1+\frac{\eta_0/\eta}{N}\right)\theta&\left(1-\sqrt{\frac{N-1-\nu(N-1)}{N-1-(N-2)\theta}}\right)\geq B
\end{aligned}
\tag{49}
$$

In addition, (20) is required to ensure the full service chain is viable. Note that (20)) and (49) can be combined into (22).

*(b)*When $\nu=0$, the stability condition (22) reduces to

$$
2-\theta-2\sqrt{1-\theta}-\left(1+\frac{\eta_0/\eta}{N}\right)\theta\left(\sqrt{\frac{1}{1-\theta\frac{N-2}{N-1}}}-1\right)\geq B \tag{50}
$$

Note that

$$
2-\theta-2\sqrt{1-\theta}-\left(1+\frac{\eta_0/\eta}{N}\right)\theta\left(\sqrt{\frac{1}{1-\theta\frac{N-2}{N-1}}}-1\right)\leq
$$

$$2-\theta-2\sqrt{1-\theta}-\theta\left(\sqrt{\frac{1}{1-\theta\frac{N-2}{N-1}}}-1\right) = 2(1-\sqrt{1-\theta})-\theta\left(\sqrt{\frac{1}{1-\theta\frac{N-2}{N-1}}}\right) \tag{51}$$

The right hand side of (51) is negative when $N \geq 5$, so that the stability condition is violated. (To verify this claim note that

$$2(1-\sqrt{1-\theta}) \leq \theta\left(\sqrt{\frac{1}{1-\theta\frac{N-2}{N-1}}}\right) \Leftrightarrow \frac{2\theta}{1+\sqrt{1-\theta}} \leq \frac{\theta}{\sqrt{1-\theta\frac{N-2}{N-1}}} \Leftrightarrow 2\sqrt{1-\theta\frac{N-2}{N-1}} \leq 1+\sqrt{1-\theta}. \tag{52}$$

When $N \geq 5$, $2\sqrt{1-\theta\frac{N-2}{N-1}} \leq 2\sqrt{1-\frac{3\theta}{4}} \leq 1+\sqrt{1-\theta}$ since the last inequality is equivalent to $\theta+\sqrt{1-\theta} > 1$ for all $0<\theta<1$ as can be validated by squaring both sides and grouping terms.

Similarly, when $N=4$, $2\sqrt{1-\theta\frac{N-2}{N-1}}2\sqrt{1-\frac{2\theta}{3}} \leq 1+\sqrt{1-\theta}$ for all $\theta \leq 0.96$, since this inequality is equivalent to a quadratic inequality which on the interval $[0,1)$ holds whenever $\theta \leq 0.96$, so that the stability condition (50) is verified for all $\theta \leq 0.96$.

*(c)*Under complete service pooling, the stability condition (22) combined with the viability condition (20).

■

Proof of Theorem 6.1 Assume $\gamma \geq a(w^{max})^3 \max\{\frac{a}{4b}, \frac{1+\theta}{2}\}$.

*(a)*Let $\pi_i = \lambda_i(p_i-c-\gamma)-\frac{\gamma}{w_i}$. Note,

$$\frac{\partial \pi_i}{\partial p_i} = -b(p_i-c-\gamma)+\lambda_i; \quad \frac{\partial \pi_i}{\partial w_i} = -a(p_i-c-\gamma)+\frac{\gamma}{w_i^2}. \tag{53}$$

π_i is jointly concave in the pair (p_i, w_i) on $[c+\gamma, p^{max}] \times [0, w^{max}]$, since its Hessian, on this rectangle, has diagonal elements $(-2b)$ and $-\frac{2\gamma}{w_i^3}$, and a determinant $\frac{4b\gamma}{w_i^3} - a^2 > 0$, since $\gamma \geq (w^{max})^3 \frac{a^2}{4b}$. It follows that a Nash equilibrium exists. Moreover, the Nash equilibrium is *unique* since the Jacobian of the system of equations $\left\{\frac{\partial \pi_i}{\partial p_i}=0; \frac{\partial \pi_i}{\partial w_i}=0, i=1,\ldots,N\right\}$ is negative definite. (The Jacobian $J = \begin{pmatrix} -2b & \beta & \cdots & \beta & -a & \alpha & \cdots & \alpha \\ & \ddots & & & & \ddots & & \\ \beta & \cdots & \beta & -2b & \alpha & \cdots & \alpha & -a \\ -a & \alpha & \cdots & \alpha & \frac{-2\gamma}{w_i^3} & 0 & \cdots & 0 \\ & \ddots & & & & \ddots & & 0 \\ \alpha & \cdots & \alpha & -a & 0 & \cdots & 0 & \frac{-2\gamma}{w_i^3} \end{pmatrix}$ is dominant diagonal, since $2b > a+(N-1)(\alpha+\beta)$ and $\frac{2\gamma}{w_i^3} \geq a+(N-1)\alpha$. Since its diagonal elements are negative, J is negative definite.) Thus any pair (p,w), which satisfies the first order conditions:

$$\frac{\partial \pi_i}{\partial p_i}=0; \frac{\partial \pi_i}{\partial w_i}=0, \textit{if } w_i<w^{max} \text{ and } \frac{\partial \pi_i}{\partial w_i} \geq 0, \text{ if } w_i=w^{max}, i=1\ldots,N \tag{54}$$

is the *unique* Nash equilibrium. We show that (54) is indeed satisfied by a symmetric vector, i.e. $p_1=\cdots=p_N=p$ and $w_1=\cdots=w_N=w$. Using (53) this implies:

$$p = \frac{a^0-w(a-(N-1)\alpha)+b(c+\gamma)}{2b-(N-1)\beta} \tag{55}$$

Substituting this identity into the expression for $\frac{\partial \pi_i}{\partial w_i}$ in (53), we obtain after some algebra with $C = \frac{2b-(N-1)\beta}{a[a-(N-1)\alpha]}$:

$$C \frac{\partial \pi_i}{\partial w_i} w_i^2 = w^3 - w^2 \frac{a^0 - (c+\gamma)(b-(N-1)\beta)}{a-(N-1)\alpha} + \frac{\gamma(2b-(N-1)\beta)}{a(a-(N-1)\alpha)} \tag{56}$$

Clearly, $\frac{\partial \pi_i}{\partial w_i} > 0$ when evaluated at $w = 0$ (and $p = \frac{a^0 - b(c+\gamma)}{2b-(N-1)\beta}$). Thus, if $\frac{\partial \pi_i}{\partial w_i} > 0$ when evaluated at $w = w^{max}$, the pair $[\frac{a^0 - w^{max}(a-(N-1)\alpha)+b(c+\gamma)}{2b-(N-1)\beta}, w^{max}]$ is a Nash equilibrium. Otherwise, (56) has a unique root on $[0, w^{max}]$, since $\frac{\partial \pi_i}{\partial w_i}$ is monotone and the characterization of the unique equilibrium (p^*, w^*) follows. Finally, using (53) and (54), we obtain $-\frac{a\lambda^*}{b} + \frac{\gamma}{(w^*)^2} = [\geq]0$, if $w^* < [=]w^{max}$.

*(b)*The proof is analogous to that of part (a), after establishing that the profit function $\Pi(p, w) = \sum_{i=1}^{N} \left[\lambda_i(p_i - c - \gamma) - \frac{\gamma}{w_i}\right]$ is jointly concave in the vector (p, w). This follows from its Hessian being negative dominant diagonal under the stated conditions.

(c)$C^c(w)$ and $C^i(w)$ differ by +a constant. If $\theta \geq [>] \frac{\rho}{2-\rho}$, $C^c(w) \geq C^i(w) \geq [>0]0$ for all $w \leq w^*$, so that $w^C \geq [>]w^*$. Similarly, if $\theta < \frac{\rho}{2-\rho}$, $C^i(w) > C^c(w) \geq 0$, for all $w \leq w^C$, so that $w^* > w^C$.

*(d)*Since (p^*, w^*) is a Nash equilibrium under Simultaneous Competition, w^* is an equilibrium in the Waiting Time competition model, when the price level is fixed at $p = p^*$. Thus, if $w^* < w^{max}, w^* = \sqrt{\frac{\gamma}{a(p^*-c-\gamma)}}$, as shown in §4. Similarly, w^C is the optimal waiting time standard in the centralized problem when the price level is fixed at p^C, so that $w^C = \sqrt{\frac{\gamma}{(a-(N-1)\alpha)(p_i^C - c-\gamma)}}$, if $w^* < w^{max}$. Part (d) follows.

(e)$w^C \geq w^*$ follows from part (c). We distinguish between two cases: (i) $w^* < w^{max}$; (ii) $w^* = w^{max}$. In case (i), $\lambda^* = \frac{b\gamma}{a(w^*)^2} \geq \frac{b\gamma}{a(w_i^C)^2} \geq \lambda^C$, by parts (a) and (b). In case (ii), $w^C = w^*$, and p^* and p^C are, respectively, the equilibrium price and the optimal price level in a centralized system, where the waiting time standard is fixed at $w = w^{max} = w^C$. $\lambda^* \geq \lambda^C$ now follows from Proposition 1(b).

■