Discussion of
"Product Differentiation and Oligopoly: a Network Approach"
Bruno Pellegrino (2023)

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A Network Model of Oligopoly

- Generalized Hedonic-Linear (GHL) Demand: consumers have additively separable preferences over attributes:

$$u(x_1, \ldots, x_m) = \sum_{k=1}^{m} (b_k x_k - \frac{1}{2} x_k^2) - L$$

  - examples: antibodies, organisms, purification, yeast, enzymes, ...

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- n firms producing differentiated products, which can be represented on the attribute space:

$$\text{good } i\text{'s representation:} \qquad a_i = \begin{bmatrix} a_{i1} & a_{i2} & \ldots & a_{im} \end{bmatrix}'$$

  - representation of the product characteristics space: $\mathbf{A}$

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- Firms compete à la Cournot
- Cosine similarity as a natural measure of how similar two products are

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- **Main force**: firms that produce more similar products compete more intensely
- **Implication**: firms with high product market centralities...
  - set lower markups
  - have a smaller (weighted) market share

- **Empirical Finding**: a significant portion (90%) of the rise in markups can be attributed to changes in product market centrality.

A Network Model of Oligopoly

- Truly impressive paper
- Lots and lots of generalizations:
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  - input-output linkages
  - competitive fringe of firms
  - etc.

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- **This discussion**: narrow focus on the theory

Comment 1: What Is χ?

- Product market centrality of firm i as

$$\chi_i = 1 - 2\sum_{j=1}^{n} (\mathbf{I} + \mathbf{A}'\mathbf{A})^{-1}_{ij} \left(\frac{(\mathbf{A}'b - c)_j}{(\mathbf{A}'b - c)_i} \right)$$

  A measure of how intensely a firm competes with others

- Characterize equilibrium quantities, markups, consumer surplus, profits, market share, etc. in terms of product market centrality

$$q = \frac{1}{2}\mathrm{diag}(\mathbf{A}'b - c)(\mathbf{1} - \chi)$$

$$\mu = \mathbf{1} + \frac{1}{2}\mathrm{diag}^{-1}(c)\mathrm{diag}(\mathbf{A}'b - c)(\mathbf{1} - \chi).$$

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- A solid case that the product market centrality χ_i is economically relevant
  - markups:

$$\mu_i = \chi_i + (1 - \chi_i)\bar{\mu}_i$$

  - weighted market share:

$$\mathcal{M}_i = \frac{q_i}{q_i + \sum_{j \neq i} \sigma_{ij} q_j} = \frac{1 - \chi_i}{1 + \chi_i}$$

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- But beyond these, the paper doesn't explore what χ_i is or how it behaves, even though it is the central statistic in the model.

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- No matter the environment and the market structure, I can always find a χ_i as follows and call it "centrality":

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- But this would only be useful as a measure if one understands how this object depends on product characteristics.

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- There is an expression in the paper in terms of model primitives, but understanding what the object really captures requires comparative statics analysis.

$$\chi = \mathbf{1} - 2\mathrm{diag}^{-1}(\mathbf{A}'b - c)(\mathbf{I} + \mathbf{A}'\mathbf{A})^{-1}(\mathbf{A}'b - c).$$

Comment 1: Comparative Statics

- Consider the following change in the product space.
- Intuitively: goods are more become more similar as γ grows

$$\mathbf{B} \propto (1-\gamma)\mathbf{A} + \gamma \mathbf{1}\mathbf{1}'/\sqrt{n} \quad , \quad \mathbf{A} = \begin{bmatrix} 0.0641 & 0.7271 & 0.2212 \\ 0.9365 & 0.3822 & 0.9015 \\ 0.3448 & 0.5703 & 0.3719 \end{bmatrix}$$

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- Product market centrality has the flavor of "how far firm i is from every other rival j in the space of product characteristics," but it's not exactly that.

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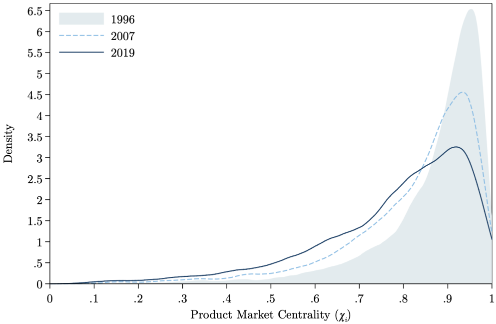

- Clear from the analysis that low centrality firms have higher markups
- But what do we learn about their product characteristics?
  - is it really because of they have more differentiated products?
  - maybe! maybe not!
  - would be great if the paper can pin this down.

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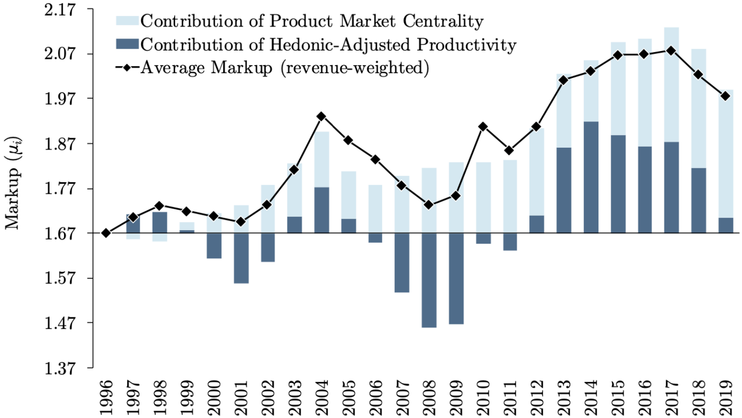
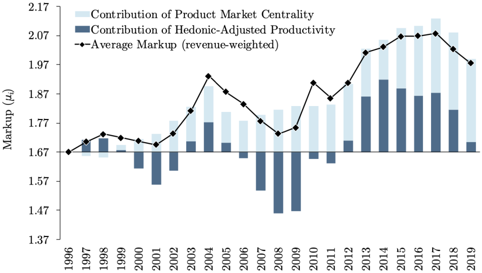


Comment 2: Markup Growth Decomposition

- Markups in the model can be expressed in terms of product market centrality and hedonic-adjust productivity

$$\mu_i = \chi_i + \frac{1}{2}(1 - \chi_i)(1 + \omega_i)$$

- Use this result to decompose the rise of markups to either increased productivity or reduction in centrality



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- But one cannot move these two objects independently:

$$\chi_i = 1 - 2\sum_{j=1}^{n} (\mathbf{I} + \mathbf{A}'\mathbf{A})^{-1}_{ij} \left(\frac{b_j - c_j}{b_i - c_i} \right)$$

$$\omega_i = b_i / c_i$$

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- For example, when all firms have identical marginal costs:

$$\chi_i = 1 - 2\sum_{j=1}^{n}(\mathbf{I} + \mathbf{A}'\mathbf{A})^{-1}_{ij}\left(\frac{\omega_j - 1}{\omega_i - 1}\right)$$

- So, an increase in the productivity of firm i also increases its centrality.
- Having a hard time thinking about this decomposition

Minor Comment 3: Complementarities?

- The paper argues that the model can handle goods that are gross complements, even though the utility function is submodular:

$$\frac{\partial^2 u}{\partial q_i \partial q_j} = -(\mathbf{A}'\mathbf{A})_{ij} \leq 0 \qquad \text{for all } i \neq j$$

$$\frac{\partial q_i}{\partial p_j} = -(\mathbf{A}'\mathbf{A})^{-1}_{ij} \lesseqgtr 0$$

- In fact, the paper finds evidence for gross complementarities in the data: "General Motors's output is gross complement vis-a-vis energy and consumer finance companies."

Comment 3: Complementarities?

$$\mathbf{A}'\mathbf{A} = \begin{bmatrix} 1 & 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & 1 & 1/\sqrt{3} \\ 0 & 1/\sqrt{3} & 1 \end{bmatrix} \quad , \quad (\mathbf{A}'\mathbf{A})^{-1} = \begin{bmatrix} 2 & -\sqrt{3} & 1 \\ -\sqrt{3} & 3 & -\sqrt{3} \\ 1 & -\sqrt{3} & 2 \end{bmatrix}$$

- Goods 1 and 3 are gross complements:

$$\frac{\partial q_1}{\partial p_3} = -1$$

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- Useful to understand what happens here
- Suppose p_3 increases
  - direct effect: increases in demand for 2 (because 2 and 3 are substitutes)
  - indirect effect: the increase consumption of good 2 reduces demand for 1 (because 1 and 2 are substitutes)
  - total effect: 1 and 3 are act as complements.

Comment 3: Complementarities?

- Back to the example:

- Automobile and fuel are complements because I have no use for gas if I don't have a car, independently of the presence of any third good (submodular preferences)

- In the model, automobile and fuel are complements only because when the price of cars go up, I switch to a third good (bicycles?) that is a substitute to both of them.

Summary

- Really impressive and ambitious paper
- It can benefit from exploring in more detail what the objects are really capturing