Discussion of "Bonds, Currencies, and Expectational Errors"
Granziera and Sihvonen (2022)

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Empirical Motivation

- Borrowing in low interest rate currencies and investing in high interest rate currencies delivers large excess returns (Forward premium puzzle).

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  → downward-sloping term structure

- Hard to rationalize using standard structural models or no-arbitrage models.

This Paper

- A unified theory of bond and currency returns based on departure from full-information rational expectations (FIRE) benchmark

- Sticky-information model a la Mankiw and Reis (2002): at each period, only a fraction $k \in [0, 1]$ of agents adopt the correct expectation

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- Main analytical result: if the true process is AR(1), then the sticky-information model can simultaneously generate
  (1) the forward-premium puzzle
  (2) the downward-sloping term structure
  (3) under-reaction of exchange rate forecasts

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- Under rational expectations, all objects are unpredictable given time t information
- How about under misspecified expectations?

Comment 1: Currency and Bond Returns with Misspecified Expectations

- General model of subjective expectations

- Forecast revisions:

$$\mathrm{FR}_{t+1}^{(\tau)} = \mathbb{E}_{t+1}[x_{t+\tau}] - \mathbb{E}_t[x_{t+\tau}].$$

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- Forecast revisions:

$$\mathrm{FR}_{t+1}^{(\tau)} = \mathbb{E}_{t+1}[x_{t+\tau}] - \mathbb{E}_t[x_{t+\tau}].$$

- Impose two assumptions:

  (a) UIP under subjective expectations:

  $$s_t = x_t + \mathbb{E}_t[s_{t+1}]$$

  (b) subjective expectations satisfy the law of iterated expectations

Subjective Expectations: UIP + LIE

- currency excess return:

$$\mathrm{rx}_{t+1} = \sum_{\tau=1}^{\infty} \mathrm{FR}_{t+1}^{(\tau)}$$

- bond excess return differential:

$$\mathrm{rx}_{t+1}^{(k)} = \sum_{\tau=k}^{\infty} \mathrm{FR}_{t+1}^{(\tau)}$$

- relative excess return:

$$\Delta\mathrm{rx}_{t+1}^{(k)} = -\sum_{\tau=1}^{k-1} \mathrm{FR}_{t+1}^{(\tau)}$$

- Exchange rate forecast error:

$$s_{t+h} - \mathbb{E}_t[s_{t+h}] = \sum_{\tau=0}^{\infty} \sum_{k=1}^{h} \mathrm{FR}_{t+k}^{(h+\tau-k+1)}$$

Subjective Expectations: UIP + LIE

- currency excess return:

$$\text{cov}^*(\mathsf{rx}_{t+1}, x_t) = \sum_{\tau=1}^{\infty} \text{cov}^*(\text{FR}_{t+1}^{(\tau)}, x_t)$$

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Assumption: $\mathrm{cov}^*(\mathrm{FR}_{t+h}^{(\tau)}, x_t) > 0$

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Comment 1: Driving Force Behind the Results

- One can reproduce all the paper's predictions as long as subjective expectations and the true data-generating process jointly satisfy:

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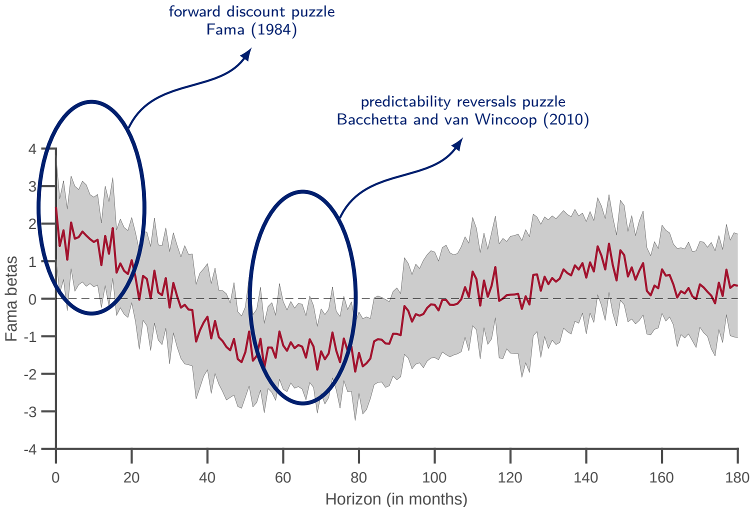
- In the model with sticky information and AR(1) data-generating process:

$$\mathrm{cov}^*(\mathrm{FR}_{t+h}^{(\tau)}, x_t) = \frac{1-\lambda^2}{1-\lambda^2(1-k)} \lambda^{\tau+h-1}(1-k)^h \left(k\mathbf{1}_{\{\tau>1\}} + \mathbf{1}_{\{\tau=1\}}\right).$$

- Positive as long as $k < 1$ (information is sticky) and $\lambda \neq 0$ (serial correlation)

Comment 2: Predictability Reversal Puzzle

$$rx_{t+h} = \alpha_h + \beta_h x_t + \epsilon_{t,t+h}$$



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- Higher interest rates today predict positive excess returns at all horizons h
- Inconsistent with the predictability reversal puzzle

- In general, too much to ask a simple model to explain all puzzles simultaneously
- But the key force that drives all the results in the model is exactly what violates the reversal puzzle.

Taking Stock

- One needs $\text{cov}^*(\text{FR}^{(\tau)}_{t+h}, x_t)$ to change signs to simultaneously explain
  (1) the forward premium puzzle
  (2) the predictability reversal puzzle
  (3) downward-sloping terms structure of term premia

- But how exactly?