Discussion of
"Consensus Expectations and Conventions"
Golub and Morris (2015)

Alireza Tahbaz-Salehi

Columbia Business School

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This Discussion

(1) My own (non-technical) understanding of consensus expectations.

(2) A decomposition result

(3) “Souped-up” consensus expectations

Consensus Expectations: Definition

- First-order expectations:

$$X^i(1) = \mathbb{E}^i[Y|t^i]$$

- Iterated average expectations:
  (*i*'s expectation of the "average" expectation in the society)

$$X^i(k+1) = \mathbb{E}\left[\sum_{j=1}^{n} \gamma^{ij} X^j(k) \big| t^i\right]$$

- Consensus expectations:
  (the average expectation of the average expectation of .... everybody)

$$c^* = \lim_{k \to \infty} X^i(k)$$

Example: "Hot Potato Game"

- A security traded among n different traders
- Trader i runs into trader j with probability γ^{ij} and "dumps" the asset on her.
- Traders uncertain about the fundamental value of the asset θ and the valuations of others.
- With probability $1-\rho$ the world ends you're stuck with the hot potato
- The asset price reflects
  - asset's fundamental value θ;
  - i's expectation of the average valuation of her counterparties;
  - i's expectation of the average expectation of her counterparties' average expectations of their own counterparties;
- Consensus expectations: equilibrium asset price in the highly speculative game as $\rho \to 1$.

Consensus Expectations

- Main result: CE is a deterministic object and does not depend on agent index i

$$c^* = \sum_{i=1}^{n} \sum_{k=1}^{K^i} p_k^i \mathbb{E}^i \left[\theta \middle| t_k^i \right]$$

- Consensus expectations is a convex combination of agents' first-order expectations.
- Extremely simple characterization.

- Caution: weights p_k^i are extremely complicated objects and depend on
  - the extent of network externalities: γ^{ij}
  - agents' expectations: $\pi^i(t_l^j | t_k^i)$

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Consensus Expectations

$$c^* = \sum_{i=1}^{n} \sum_{k=1}^{K^i} p_k^i \mathbb{E}^i \left[\theta \middle| t_k^i\right]$$

- Define $B_{kl}^{ij} = \pi^i(t_l^j | t_k^i)$

$$B = \begin{bmatrix} \gamma^{11} B^{11} & \cdots & \gamma^{1n} B^{1n} \\ \vdots & \ddots & \vdots \\ \gamma^{n1} B^{n1} & \cdots & \gamma^{nn} B^{nn} \end{bmatrix}$$

- weights in consensus expectations:

$$p' B = p'$$

- p_k^i: information & interaction centrality
- captures not only whether people care about you, but also what they think about you.

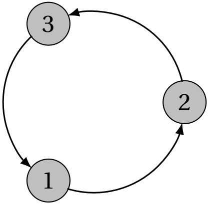
Whose Expectations Matter?

$$B = \begin{bmatrix} \gamma^{11} B^{11} & \cdots & \gamma^{1n} B^{1n} \\ \vdots & \ddots & \vdots \\ \gamma^{n1} B^{n1} & \cdots & \gamma^{nn} B^{nn} \end{bmatrix}$$

- If trader i never faces trader j it doesn't matter what he thinks of her!
- What matters is what j's *direct* counterparties think of her!
- Even though i is certain that j would eventually obtain the security.

An Example from the Paper: Cyclic Optimism

- Two possible states G and B with returns 1 and 0, *ex ante* equally likely.
- Each trader can be of two types (g or b)
- $\mathbb{P}(\theta = G | t_i = g) = p$
- $\mathbb{P}(\theta = G | t_i = b) = 1 - p$



- $\mathbb{P}(t_{i+1} = g | t_i = g) \approx 1 \quad \mathbb{P}(t_{i+1} = g | t_i = b) = \frac{1}{2}$
- $\mathbb{P}(t_{i-1} = g | t_i = g) = \frac{1}{2} \quad \mathbb{P}(t_{i+1} = g | t_i = b) \approx 0$

equilibrium price $= p$

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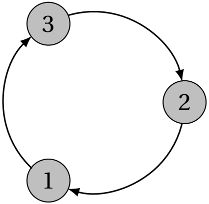

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Network and Information Interaction

$$B = \begin{bmatrix} \gamma^{11} B^{11} & \cdots & \gamma^{1n} B^{1n} \\ \vdots & \ddots & \vdots \\ \gamma^{n1} B^{n1} & \cdots & \gamma^{nn} B^{nn} \end{bmatrix}$$

- Consensus expectations is determined by the eigenvector of B.
- Expectations and network interactions are not necessarily "separable".

Comment: A Decomposition Result

- Suppose agents have common type sets: $T_i = T$
- Agents hold "symmetric expectations":

$$B^{ij}_{kl} = \pi^i(t^j_l | t^i_k) = \widehat{\pi}(t_l | t_k) = \widehat{B}_{kl}$$

- In this case $B = \Gamma \otimes \widehat{B}$

Theorem

If $\gamma^{ii} = 0$ and agents hold symmetric expectations, then

$$p^i_k = \textit{network centrality}_i \cdot \widehat{\pi}(t_k).$$

- The interaction network and beliefs no longer interact.
- Complete information and CPA-T would be special cases.
- The real bite of the results is when agents hold asymmetric expectations.

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- Consensus Expectations:

$$X^i(k+1) = \mathbb{E}\left[\sum_{j=1}^{n} \gamma^{ij} X^j(k) \middle| t^i\right]$$

- But what if agents care about a potentially different average expectation at different levels?

$$\widehat{X}^i(k+1) = \mathbb{E}\left[\sum_{j=1}^{n} \gamma^{ij}(k) \widehat{X}^j(k) \middle| t^i\right]$$

Theorem

Under suitable connectivity assumptions on $\{\Gamma(k)\}_{k=1}^{\infty}$ and beliefs, "Souped-up" consensus expectations

$$\lim_{k\to\infty} \widehat{X}^i(k)$$

is always a deterministic object and independent of the index of the player i.

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Summary

- A very interesting paper, formalizing and characterizing a new concept
- Many applications (coordination games, relaxing the common prior assumption, equilibrium robustness)
- Meta-Theorem 1: network interactions and incomplete information interact with one another.
- Meta-Theorem 2: at some level, network interactions and incomplete information are the same object (see Stephen's other paper).
- (Almost) all infinite regress of average expectations lead to a consensus expectation!