Discussion of
"Financial Networks and Contagion"
Elliott, Golub, and Jackson (2013)

Alireza Tahbaz-Salehi
Columbia Business School

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Model Overview

- A model of interconnected organizations with claims on
  (i) some fundamental assets
  (ii) each other.

- Amplification mechanism: Discontinuous loss in productive value if an institution's market value falls below a certain threshold.

- Key question: how the nature of such interdependencies affect the stability of the system as a whole?

- Results: "more interconnectivity" has a non-monotonic effect.

Model

- n institutions/organizations
- m assets
- p_k: price of asset k
- D_{ik}: share of asset k held by institution i

- Interconnectivity: cross-holding of shares
  - C_{ij}: fraction of institution j owned by organization i.
  - $\widehat{C}_{ii}$: fraction held by i's outside shareholders.

$$\widehat{C}_{ii} = 1 - \sum_{j \neq i} C_{ij}$$

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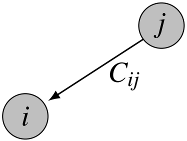
The Financial Network

- The cross-holdings define a network:



Integration and Diversification

- Integration: C' more integrated than C if

$$\widehat{C}'_{ii} \leq \widehat{C}_{ii} \qquad \forall\, i$$

- captures the total level of exposure of organizations to each other.

- Diversification: C' more diversified than C if

$$C'_{ij} \leq C_{ij} \quad \forall\, i,j \text{ such that } C_{ij} > 0$$
$$C'_{ij} > C_{ij} \quad \text{for some } i,j \text{ such that } C_{ij} = 0.$$

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Book Values

- Book value of organization i:

$$V_i = \sum_{j \neq i} C_{ij} V_j + \sum_k D_{ik} p_k$$

$$V = (I - C)^{-1} D p$$

- However,

$$\sum_i V_i > \sum_k p_k.$$

- In fact, if $\widehat{C}_{ii} = \hat{c} < 1$ for all i:

$$\sum_i V_i = \frac{1}{\hat{c}} \sum_k p_k.$$

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Market Values

- Market values: the equity value of the organization held by its *outside* investors.

$$v_i = \widehat{C}_{ii} \cdot V_i.$$

- or in vector form:

$$v = \widehat{C}(I - C)^{-1} Dp.$$

Contagion

- Contagion: A drop in the value of the an asset held by j can lead to the fall in value of i even if it does not directly hold the asset.

- Absent any amplification mechanism, however, the losses are simply reallocated across the network:

$$\frac{\partial}{\partial p_k} \sum_i v_i = 1 \qquad \text{for all networks}$$

- All network structures are alike.

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Amplification Mechanism

- Failure costs:

$$v = \widehat{C}(I - C)^{-1}\left(Dp - \beta(p)\mathbf{1}_{\{v_i < \underline{v}_i\}}\right).$$

- Discontinuous loss in productive value if an institution's market value falls below a certain threshold $\underline{v}_i$.
  - ∗ inefficient use of the assets
  - ∗ discontinuous jumps in the cost of capital
  - ∗ bankruptcy and legal costs

- Value of the organizations: fixed point $(v_1, \ldots, v_n)$.

Analysis: The Random Network Model

- Fix a matrix $\pi = [\pi_{ij}]$
- π_{ij}: the fraction of nodes that have in-degree i and out-degree j.
- $\mathcal{G}(\pi, n)$: the set of all networks on n nodes with distribution π.
- Draw a network $G \in \mathcal{G}(\pi, n)$ uniformly at random.

$$C_{ij} = \frac{cG_{ij}}{d_{j,\text{out}}}$$

$$\widehat{C}_{ii} = 1 - c$$

- c = the extent of integration.
- d = expected out-degree of the vertex at the end of a random edge

Limit Contagion

- Regularity assumptions:
  - $v_i = \underline{v}$ for all i
  - $D = I$: each bank holds a single asset.
  - $p = (1, 1, \ldots, 1)$
  - shock: $p_i \to 0$ uniformly at random.
  - $\beta_j = 1$: the asset value of a failing firm is completely wiped out.
- Fix π, and consider a sequence of networks growing in size n
- Limit contagion:
  if a non-vanishing fraction of organizations fail as $n \to \infty$.

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Main Result: Integration

Proposition

If $c(1-c) < \alpha$, *then there is no limit contagion.*

- Integration's effect is non-monotonic:
  - low: little exposure to others, failures do not trigger cascades
  - high: difficult to get the first failure (drop in own assets does not trigger failure)

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Main Result: Diversification

Proposition

Suppose integration is neither too low nor too high.

(a) $d < 1$: *no limit contagion.*

(b) $1 < d_{\text{max}} < \alpha c(1 - c)$: *limit contagion.*

(c) $d_{\text{min}} > \alpha c(1 - c)$: *no limit contagion.*

- Diversification's effect is non-monotonic:
  - low: fragmented network; no widespread contagion
  - high: little exposure to any single org; failures do not spread

Comments: Non-Random Graphs?

- Most analytical results for asymptotically large random graphs:
- Even though the intuitions are clear from the current results, still valuable to solve the problem for deterministic, small structures (even if structures are simple).
- Possible to obtain results for the “first threshold of failure”?

Comments: Micro-Foundations

- A model of value interdependencies $v_i = f_i(v_{-i})$.
- Current interpretation:
  - ∗ firms have (debt?) obligations to one another.
  - ∗ if i's value does not cover its obligations, firm j gets $C_{ij}V_i$.
  - ∗ outside owners get $v_i = \widehat{C}_{ii}V_i$.
  - ∗ If $v_i < \underline{v}$, the owner stops the operations of the firm.
- Discontinuities triggered by value and not the cash flow.
- Margin requirements? Collateral/balance sheet constraints?
- Fleshing out the micro-foundations in more detail.

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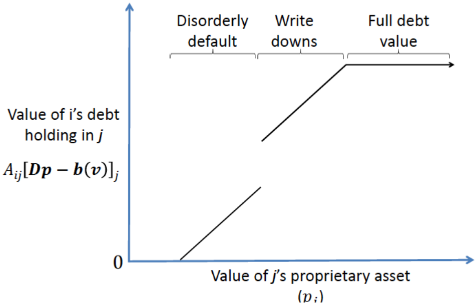
Comments: Comparative Statics

- Linear dependencies: banks cannot meet the face value of their obligations.



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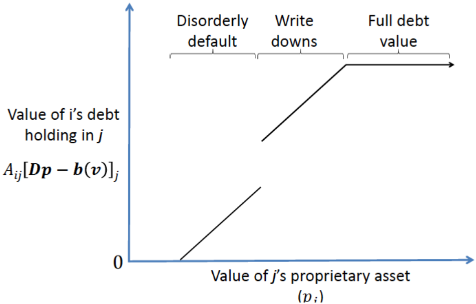

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Summary

- Very interesting and relevant paper, with clean insights (even though the problem may first look intractable).
- Fleshing out the micro-foundations in more detail
- There is still value in analyzing simple, non-random, "finite" size networks.
- Given the interpretation, comparative statics that focus on the non-distressed regime also valuable.