Discussion of
"Welfare Accounting"
Dávila and Schaab (2023)

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Aggregating a Disaggregated Economy

- Part of a coherent research agenda that is focused on building the macroeconomy from the ground up
- Breaking up the representative agent and the representative firm:
  - heterogeneity in households
  - disaggregated production structure
  - frictions/markups/entry-exit at the firm-level
  - dispersed information
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- This paper: a decomposition of welfare assessments in an economy with **heterogeneous individuals** and **disaggregated production**

$$\mathrm{d}W/\mathrm{d}\theta = f(\text{ind. } \mathrm{MRS}, \text{agg. } \mathrm{MRS}, \mathrm{SNV}, \text{network-adjusted } \mathrm{SNV}, \ldots)$$

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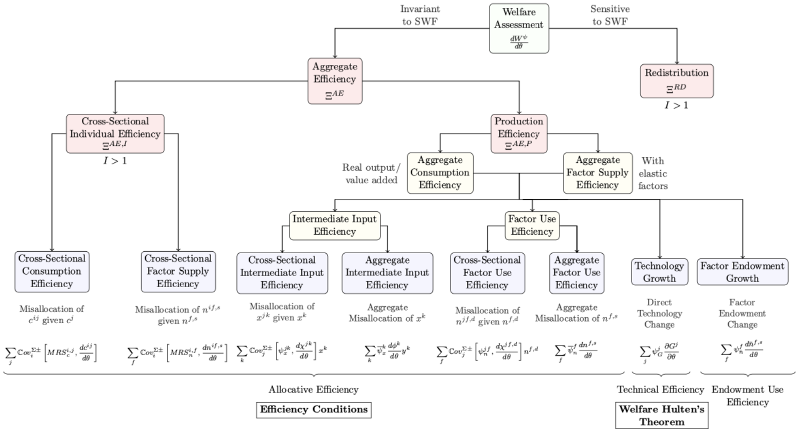
differences with the "textbook version" of Hulten's theorem:

(i) applies to **welfare** as opposed to **output**
(ii) does not require **fully inelastic factor supply**

Main Result: Welfare Decomposition



Application $\sharp 1$:

Efficiency Conditions

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(1) cross-sectional consumption efficiency

$$\mathrm{MU}_c^{ij} = \begin{cases} = \overline{\mathrm{MU}}_c^{j} & \text{if } c_{ij} > 0 \\ < \overline{\mathrm{MU}}_c^{j} & \text{if } c_{ij} = 0, \end{cases}$$

(2) cross-sectional factor supply efficiency

$$\mathrm{MRS}_n^{if} = \begin{cases} = \overline{\mathrm{MU}}_n^{f} & \text{if } n^{if} > 0 \\ > \overline{\mathrm{MU}}_n^{f} & \text{if } n^{if} = 0, \end{cases}$$

(3) cross-sectional intermediate input efficiency

(4) aggregate intermediate input efficiency

(5) cross-sectional factor use efficiency

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- Importance of corner or non-interior allocations—when goods and factors are not used in production or when they are only used in the production of a single good.
- Particularly important when production is disaggregated and when individuals are heterogeneous.

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$$\begin{aligned} \text{max} \quad & \sum_{i=1}^{m} \frac{1}{\lambda_i} u_i(c_{i1}, \ldots, c_{in}, l_i^s) \\ \text{subject to} \quad & \sum_{i=1}^{n} c_{ij} + \sum_{k=1}^{m} x_{kj} = f_j(l_j^d, x_{j1}, \ldots, x_{jn}), \quad \sum_{i=1}^{n} l_i^s = \sum_{j=1}^{m} l_j^d \\ & c_{ij}, x_{kj}, l_j^d, l_i^s \geq 0. \end{aligned}$$

Efficiency Condition: Optimality with respect to c_{ij}

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- Kuhn-Tucker conditions:

$$\frac{1}{\lambda_i} \frac{\partial u_i}{\partial c_{ij}} = \theta_j - \eta_{ij},$$

or alternatively,

$$\frac{1}{\lambda_i} \frac{\partial u_i}{\partial c_{ij}} = \mathrm{MU}_c^{ij} = \begin{cases} \overline{\mathrm{MU}}_c^j & \text{if } c_{ij} > 0 \\ < \overline{\mathrm{MU}}_c^j & \text{if } c_{ij} = 0, \end{cases}$$

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  - same exact expression as in the paper.
  - the second requirement **is nothing but complementarity slackness** under strong duality.

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\begin{aligned}
\max \quad & u(c_1, \ldots, c_j, \ldots, c_n, l^s) \\
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- If $x_{jk} > 0$, then KT condition becomes:

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$$\frac{\partial u}{\partial c_j} \frac{\partial f_j}{\partial x_{jk}} - \frac{\partial u}{\partial c_k} = \eta_k - \eta_j \frac{\partial f_j}{\partial x_{jk}}$$

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$$\frac{\partial u}{\partial c_j} \frac{\partial f_j}{\partial x_{jk}} - \frac{\partial u}{\partial c_k} = \eta_k - \eta_j \frac{\partial f_j}{\partial x_{jk}} = \begin{cases} = 0 & \text{if neither } j \text{ nor } k \text{ are pure intermediates} \\ \neq 0 & \text{otherwise} \end{cases}$$

Application $\sharp$1: Big Picture

- Efficiency conditions in the paper coincide with the optimality conditions of the planner's problem (as they should!)...

  ... and the KT conditions naturally take care of all non-negativity constraints **via complementarity slackness** conditions.

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- Put differently: the objects in the paper (MRS, social net valuations, network-adjusted social net valuations, etc.) are either identical to or regroupings of Lagrange multipliers in the planner's problem.

Application $\sharp 2$:

“Welfare Hulten Theorem”

Hulten's Theorem

- In an **efficient economy** with **inelastic factor supplies** and a **representative household**, the first-order impact of a shock is equal to an industry's Domar weight (sales as a fraction of output):

$$\frac{\mathrm{d}\log \mathsf{GDP}}{\mathrm{d}\log z_j} = \frac{p_j y_j}{\mathsf{GDP}}$$

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- Where does it come from? **applying the envelope theorem to the planner's problem**

$$\begin{aligned} \mathrm{GDP} = \max \quad & C(c_1, \dots, c_m) \\ \text{s.t.} \quad & c_j + \sum_{j=1}^{m} x_{ji} = z_j f_j(l_j^d, x_{j1}, \dots, x_{jn}), \quad \sum_{j=1}^{m} l_j^d = L \end{aligned}$$

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- Also shows why it requires inelastic labor supply: with elastic labor supply

$$\text{efficiency} \neq \text{maximum output}$$

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$$W = \max \quad \sum_{i=1}^{n} \frac{1}{\lambda_i} u_i(c_{ij}, l_i^s)$$

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- One line proof:

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- One line proof:

$$
\frac{\mathrm{d}W}{\mathrm{d}z_j} = \frac{\mu_j y_j}{z_j} \Rightarrow \frac{W}{\log z_j} = p_j y_j \Rightarrow \frac{1}{\sum_i \mathbf{p}_i \mathbf{c}_i} \frac{\mathrm{d}W}{\mathrm{d}\log z_j} = \text{Domar}_j
$$

- Once again, not clear if one needs the decomposition.

Summary

- This paper:
  - impressive, diligent work to understand the sources of welfare gain/loss
  - part of a larger agenda to build the economy from the bottom up (disaggregated production, heterogeneous agents, etc.)
  - two applications to showcase the applicability of the result

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  - to show where the real value-added of the results are, it would be nice to use an application that needs the decomposition machinery
- What can I—or even better, a more applied person—use these results for?
  - welfare impact of a particular shock?
  - comparison of the relevance of various channels?