

Discussion of “Collateralized Debt Networks with Lender Default”  
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# Financial Networks

- Growing literature on how financial linkages...
  - (i) function as a mechanism for propagation and amplification of shocks
  - (ii) generate systemic risk from micro shocks
- For the most part, the literature makes two simplifying assumptions
  - ▶ unsecured lending
  - ▶ exogenously-specified network of relationships
- But in reality...
  - ▶ interbank lending is mostly collateralized
  - ▶ banks choose their partners and terms of contracts

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# This Paper: A Model of Collateralized Debt Markets

- Allows for collateralized lending and borrowing
- Quantities and prices (including that of the collateral) are endogenously determined
- Two methodological contributions:
  - ▶ a framework for propagation of shocks in a collateralized lending network
  - ▶ a model of financial network formation  
(though sidesteps some of the intricate issues of network formation by assuming lenders/borrowers are competitive)
- Main results:
  - ▶ trade-off between counterparty risk and leverage
  - ▶ under-diversification in equilibrium
  - ▶ an application to loss coverage by a CCP

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## Propagation of Shocks over Collateralized Debt Networks

- An exogenous network of pairwise collateralized debt contracts

$y_{ij}$  : borrowed cash per unit of collateral

$c_{ij}$  : amount of posted collateral

- Financial network represented by a pair of matrices  $(Y, C)$

- Payment from  $j$  to  $i$ , per unit of posted collateral:

$$x_{ij} = \min\{y_{ij}, p\},$$

where  $p$  is the equilibrium price of collateral (non-recourse)

# Propagation of Shocks over Collateralized Debt Networks

- Nominal wealth of agent  $j$ :

$$m_j = e_j - \epsilon_j + h_j p + \sum_k c_{jk} \min\{p, y_{jk}\} - \sum_k c_{kj} \min\{p, y_{kj}\} - \sum_{k:m_k < 0} \zeta(c_{kj}) [p - y_{kj}]^+$$

- failure of lender makes the borrower incur a cost to recover her collateral
  - ▶ counterparty risk channel
- nominal wealth depends on the equilibrium price of the asset
  - ▶ collateral price channel of contagion

## Payment Equilibrium

- Collection of nominal wealth  $(m_1^*, \dots, m_n^*)$  and asset price  $p^*$  such that:

- (i) nominal wealths are mutually consistent with pairwise contracts:

$$m_j^* = e_j - \epsilon_j + h_j p^* + \sum_k c_{jk} \min\{p^*, y_{jk}\} - \sum_k c_{kj} \min\{p^*, y_{kj}\} - \sum_{k: m_k^* < 0} \zeta(c_{kj}) [p^* - y_{kj}]^+$$

- (ii) asset market clears:

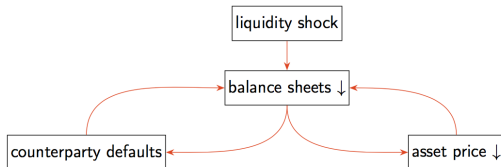
$$\begin{cases} \sum_{i=1}^n \max\{m_i^*, 0\} = p^* \sum_{i=1}^n h_i & \text{if } p^* < s \\ \sum_{i=1}^n \max\{m_i^*, 0\} \geq p^* \sum_{i=1}^n h_i & \text{if } p^* = s, \end{cases}$$

where  $s$  is the asset's payoff.



## Comment 1: Monotone Comparative Statics?

- Paper discusses the interaction of fire sale and counterparty risk channels:



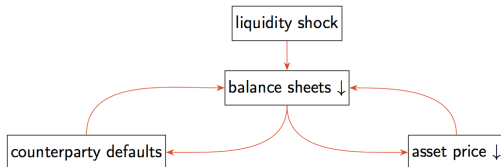
- But this is an argument about “best responses” and not **equilibrium**.
- Should not be hard to prove a formal result for equilibrium using monotone comparative statics arguments:

### Proposition

$p^*$ ,  $(m_1^*, \dots, m_n^*)$ , and aggregate welfare, are decreasing in the shock  $\epsilon_j$ .

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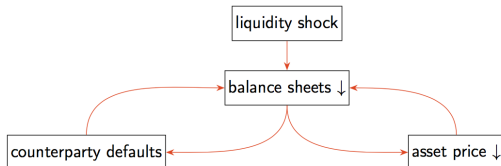
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## Comment 2: Network Comparative Statics?

- There is a literature that studies the role of network structure for systemic risk, but mostly focused on unsecured lending.
- Current framework can be used to investigate the robustness of those results.
  
- Can one say anything about how changes in  $C$  and  $Y$  impact equilibrium objects?
  
- For example, suppose  $y_{ij} \geq \tilde{y}_{ij}$  for all  $i \neq j$ . What can one say about  $p^*$  and  $\tilde{p}^*$ ?
- How about uniformly higher haircuts? More diversified patterns of lending?

## Network Formation Stage

- Where do  $C$  and  $Y$  come from?

Assume agents disagree about asset payoff  $s \rightarrow$  gains from trade.

- Each agent takes pairwise interest rates and the price of the asset (today and tomorrow) as given and chooses the contracts:

$$\begin{array}{ll} \max_{\{c_{jk}, c_{kj}, y_{jk}, y_{kj}\}} & \mathbb{E}_j \left[ \max \left\{ m_j \frac{s}{p_1}, 0 \right\} \right] \\ \text{subject to} & \text{budget constraint} \\ & \sum_k c_{kj} = \sum_k c_{jk} + h_j. \end{array}$$

- Interest rates and prices determined such that asset market and contract markets clear.

## Network Equilibrium Results

- (1) If there is a cost to recovering collateral, borrowers have an incentive to borrow from more than one lender  
→ **counterpart risk vs. leverage trade-off**
  
- (2) Yet, they do not internalize the full benefit of diversification on others  
→ **under-diversification externality.**

## Comment 3: Framing?

- Both results are novel, and yet, they have counterparts in the earlier literature and are well known (for non-secured lending).
- Would be great to use the powerful framework (and characterization) to obtain novel comparative statics and answer policy-relevant questions.
- Already some of the ingredients are there:
  - ▶ impact of distress on lending volume, velocity of collateral
  - ▶ CCP
- Would be great to push this further