Discussion of
"Productivity and Misallocation in General Equilibrium"
by David Baqaee and Emmanuel Farhi

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Hulten (1978)

- In an efficient economy, the macro impact of a shock to industry i is determined by that industry's Domar weight

$$\frac{d \log \mathrm{GDP}}{d \log A_i} = \frac{p_i y_i}{\mathrm{GDP}}$$

  up to a first-order approximation.

- Extremely powerful result because
  (i) it is agnostic with respect to the microeconomic structure of the economy.
  (ii) can be calculated using observables.

- Two key qualifiers:
  - Efficiency is necessary: Hulten is a consequence of envelope theorem.
  - First-order approximations are not useful in non-linear economies.

This Paper

- A generalization of Hulten's Theorem to inefficient economies.
  how micro productivity and markup shocks shape aggregate outcomes

- A parametric model to relate various terms to structural parameters (micro elasticities, returns to scale, input-output linkages, etc.)

- Key take-aways:
  - Micro shocks impact macro outcomes via two channels:
    (1) a pure technology effect and (2) a reallocation effect
  - The latter can be measured via changes in factor income shares
  - Unlike efficient economies, all micro intricacies become important.

Key Idea

- Allocation matrix: $\mathcal{X}(A, \mu)$, where $\mathcal{X}_{ij} = x_{ij} / y_i$ be the allocation of inputs across various firms.
- GDP $= \mathcal{Y}(A, \mathcal{X})$

- Chain rule:

$$\frac{d \log \text{GDP}}{d \log A} = \underbrace{\frac{\partial \log \mathcal{Y}}{\partial \log A}}_{\Delta \text{ technology}} + \underbrace{\frac{\partial \log \mathcal{Y}}{\partial \mathcal{X}} \frac{d \mathcal{X}}{d \log A}}_{\Delta \text{ allocative efficiency}}$$

- If the initial allocation is efficient, then by the envelope theorem, the second term is equal to zero $\rightarrow$ Hulten's Theorem (almost).
- In an inefficient economy, one needs to understand how the second term responds to shocks.

Framework

- n goods, each produced by competitive producers using intermediate goods as well as F factors that are inelastically supplied.

- Producers: constant-returns cost functions

$$\frac{1}{A_i}\mathbf{C}_i\Big(p_1,\ldots,p_n,w_1,\ldots,w_F\Big)y_i$$

- Markups:

$$p_i = \frac{\mu_i \mathbf{C}_i}{A_i}$$

- Final demand

$$\begin{aligned} Y = \max \quad & \mathcal{D}(c_1,\ldots,c_n) \\ \text{s.t.} \quad & \sum_{i=1}^{n} p_i c_i = \sum_{f=1}^{F} w_f L_f + \sum_{i=1}^{n} \pi_i \end{aligned}$$

Standard Objects

- Input-output matrix:

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}$$

- Final expenditure shares:

$$b_i = \frac{p_i c_i}{\text{GDP}}$$

- Domar weights:

$$\lambda_i = \frac{p_i y_i}{\text{GDP}}$$

- Leontief inverse:

$$\Psi = (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \ldots$$

- Market-clearing:

$$\lambda_i = \sum_{k=1}^{n} b_k \Psi_{ki}$$

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Markup-Adjusted Objects

- Markup adjusted ("cost-based") input-output matrix:

$$\tilde{\Omega}_{ij} = \mu_i \Omega_{ij}$$

- Markup adjusted ("cost-based") Leontief inverse:

$$\tilde{\Psi} = (I - \tilde{\Omega})^{-1}$$

- "Cost-based" vs. "revenue-based" Domar weights:

$$\lambda_i = \sum_{k=1}^{n} b_k \Psi_{ki}$$

$$\tilde{\lambda}_i = \sum_{k=1}^{n} b_k \tilde{\Psi}_{ki}$$

Main Result

- Key result of the paper: apply envelope theorem to each producer and combine with chain rule:

$$d \log Y = \frac{\partial \log \mathcal{Y}}{\partial \log A} d \log A + \frac{\partial \log \mathcal{Y}}{\partial \mathcal{X}} d\mathcal{X}$$

Theorem

Pure technology effect:

$$\frac{d \log \mathcal{Y}}{d \log A} d \log A = \sum_{k=1}^{n} \tilde{\lambda}_k \, d \log A_k$$

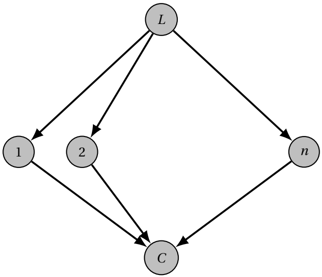
Resource reallocation:

$$\frac{\partial \log \mathcal{Y}}{\partial \mathcal{X}} d\mathcal{X} = - \sum_{k=1}^{n} \tilde{\lambda}_k \, d \log \mu_k - \sum_{f=1}^{F} \tilde{\Lambda}_f \, d \log \Lambda_f .$$

- Reduces to Hulten's theorem when $\mu_k = 1$ for all k.

Example 1: Horizontal Economy + Productivity Shocks

- Elasticity of substitution between various goods $= \theta$.



- "Pure" technology effect $= \lambda_k$
  - Holding the allocation of labor to each firm constant, a productivity shock to firm k increases its output.
  - This in turn increases aggregate output.

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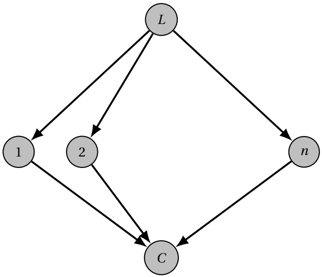

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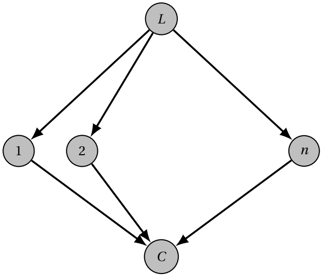
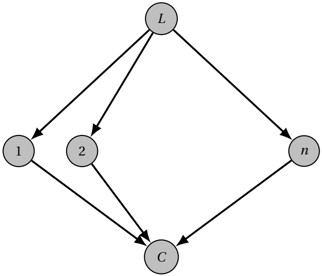


- Reallocation effect $= -\lambda_k(\theta - 1)\left(\frac{\mu_k^{-1}}{\sum_i \lambda_i \mu_i^{-1}} - 1\right)$
  - Shock to firm k reduces its price and increases its demand when $\theta > 1$ via a substitution effect. Therefore, workers are reallocated to k.
  - When k's markup is larger than average, the firm was too small from a social perspective $\rightarrow$ A positive shock to k improves the allocative efficiency.

- No change in allocative efficiency when $\theta = 1$, as factor shares do not move.

Example 1: Horizontal Economy + Markup Shocks

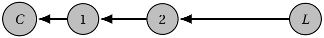
- Reallocation effect $= \lambda_k \theta \left(\frac{\mu_k^{-1}}{\sum_i \lambda_i \mu_i^{-1}} - 1 \right)$



- $\theta = 0$: There are no reallocation effects, as the HH consumes a fixed quantity.

- $\theta > 0$: A markup shock increases k's price, reduces its demand, and reallocates workers to other firms.
  Allocate efficiency would increase (decrease) depending on whether k's markup was smaller (larger) than average markup.

Example 2: Vertical Economy



- Allocative efficiency:
    - No room for misallocation.
    - Productivity or markup shocks have no effect on allocative efficiency.

- Pure technology effect: $\frac{d \log Y}{d \log A_k} = \tilde{\lambda}_k = 1$
    - Different from Hulten's prediction due to double marginalization: $\tilde{\lambda}_k > \lambda_k = \prod_{i=1}^{k-1} \mu_i^{-1}$.

So What?

- Besides comparative statistics, the non-parametric decomposition can be used for to measure *changes* in the economy's allocative efficiency

$$d\log Y - \tilde{\Lambda}' d\log L = \underbrace{\tilde{\lambda}' d\log A}_{\Delta \text{ Technology}} \underbrace{-\tilde{\lambda}' d\log \mu + \tilde{\Lambda}' d\log \Lambda}_{\Delta \text{ allocative efficiency}}$$

as well as growth accounting.

- The parametric model can be used for measuring the *level* of allocative efficiency (among other applications).

Comment: Inefficient Economies and the Origin of Distortions

- The paper provides a general framework to handle distortions.
- The framework can be generalized to endogenous distortions and productivities by applying the chain rule one more time.

- But inefficiencies/distortions are only meaningful with respect to an efficiency benchmark.
- The right notion of efficiency cannot be decoupled from
  (a) the origin of the inefficiencies/wedges
  (b) the policy instruments available

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Measuring Allocative Efficiency

- Typical measure for the *level* of allocative efficiency: distance to the frontier Restuccia and Rogerson (2008) and Hsieh and Klenow (2009)

- The efficiency benchmark used in the literature is the undistorted economy with no wedges
  $\rightarrow$ Allocative inefficiency $=$ increase in output if all wedges were eliminated.

- This is regardless of where the wedges come from
  (monopoly markups, taxes, financial frictions, etc.)

Measuring Allocative Efficiency: Information Frictions

- Suppose firms make production decisions under asymmetric information about the fundamentals (Angeletos, Iovino, and La'O, 2009)

- Asymmetry of information induces a wedge with respect to the complete information benchmark.

- Yet, the economy can still be constrained efficient $\rightarrow$ a planner who cannot transfer private information across firms would face the same exact wedges.

- This would be measured as misallocation, even though the economy is (constrained) efficient: there is no policy instrument that can improve upon the allocation.

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Changes in Allocative Efficiency

- Similar idea can be applied to *changes* in allocative efficiency.

$$\begin{aligned} \log Y = \max_{\mathcal{X}} \quad & \log \mathcal{Y}(A, \mathcal{X}) \\ \text{s.t.} \quad & \log g(A, \mathcal{X}) \geq 0 \end{aligned}$$

- If the equilibrium is constrained efficient, we can reuse the envelope theorem, making the allocative efficiency terms second order again:

$$\frac{d \log Y}{d \log A} = \frac{\partial \log \mathcal{Y}}{\partial \log A} + \eta \frac{\partial \log g}{\partial \log A}$$

- Measuring either the (i) level or (ii) changes in allocative efficiency may require taking the origins of the wedges more seriously.

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Gains from Eliminating Markups

- The model can be used to estimate the gains from eliminating markups.
- Second-order approximation:

$$\log(\text{GDP}^*) - \log(\text{GDP}) \approx \frac{1}{2} \sum_{i=1}^{n} \frac{d \log Y}{d \log \mu_i} \left(\frac{1 - \mu_i}{\mu_i} \right).$$

- Using markup data and the calibrated model:

	2014/15	1997
Gutierrez & Philippon	20%	3%
Lerner Index	17%	5%
De Loecker & Eeckhout	35%	21%

- Two main observations:
  (a) Substantial increase in gains from eliminating markups.
  (b) Two order of magnitudes larger than Harberger's estimate (0.1%)

Comment: Gains from Eliminating Markups

$$\log(\text{GDP}^*) - \log(\text{GDP}) \approx \frac{1}{2}\sum_{i=1}^{n} \frac{d\log Y}{d\log \mu_i}\left(\frac{1-\mu_i}{\mu_i}\right)$$

- Unlike the decomposition of the Solow residual, measuring the gains from eliminating markups requires the structural model:

$$\frac{d\log Y}{d\log \mu_i} = -\tilde{\lambda}_k - \sum_j (\theta_j - 1)\mu_j^{-1}\lambda_j \text{Cov}\left(\tilde{\Psi}_{(k)}, \Psi_L/\Lambda_L\right) - \lambda_k \frac{\Psi_{kL}}{\Lambda_L}$$

- But the approximation above is exact only if the elasticities are close to 1 (Bigio and La'O, 2016).
- The estimation process seems to be internally inconsistent:
  - (i) Uses structural elasticities when calculating $d\log Y / d\log \mu_i$.
  - (ii) Approximates the gains as if all production functions are Cobb-Douglas

Comment: Gains from Eliminating Markups

- Calibrations for 2014/15:

	Benchmark	CD Counterfactual
GP	20%	4%
LI	17%	4%
DE	35%	7%

- Not sure how to interpret the large gap between the two calibrations.
- What if the much larger numbers of the benchmark (where the elasticities are $\epsilon = 0$, $\zeta = 8$) are driven by larger approximation errors?

- Since estimating the $d\log Y/d\log\mu_i$ requires the structural model anyways, why not estimate the gains from reducing markups also structurally?

Summary

- A generalization of Hulten's theorem to inefficient economies
- Applications:
  - comparative statics
  - growth accounting
  - measuring allocative efficiency
  - macro impacts of micro shocks
  - etc.

- The paper makes a strong argument for taking the microeconomic nature of the economy (input-output linkages, micro elasticities, returns to scale) seriously.
- Thinking about misallocation may also require taking the nature and origins of the "wedges" more seriously.