Discussion of "Endogenous Production Networks" by Daron Acemoglu and Pablo Azar

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Production Networks

- Growing literature on how input-output linkages can
  (i) function as a mechanism for propagation & amplification of shocks.
  (ii) translate micro shocks into aggregate fluctuations.

- Even though some papers allow for entry and exit, non-Cobb-Douglas technologies (and hence endogenous input-output matrices) etc., the literature takes the production functions to be exogenous.

- Reasonable assumption in the short run. Less so in the longer run.
  - evolution of industries over time in response to shocks
  - long-run growth

- This requires a model of network formation!

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- This requires a model of network formation!

Network Formation

- In many instances, network formation models turn out to be either "too complex" or "too simple" (though see Oberfield (2017)).
  - multiple equilibria
  - no strong guarantees for existence
  - strategic considerations $\rightarrow$ intractability
  - endogenous network inherits many of the properties of the exogenous parameter specifications.

- The paper overcomes many of these challenges by taking a competitive approach towards network formation.
- Firms in each industry take all prices as given not only (i) when making input decisions, but also (ii) when choosing their set of suppliers.
- Natural assumption for production networks at the industry level.

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Model Outline

- Instead of a single production technology, let firms in each industry have access to a discrete menu of different constant returns-to-scale production technologies, *indexed by the set of suppliers.*

- Each technology (set of suppliers) also has an associated productivity:

$$y_i = F_i(\ell_i, x_{ij}, S_i, A_i(S_i)),$$

  where $S_i \subseteq \{1, \ldots, n\}$.

- Demand side: a representative household maximizes her utility $u(c_1, \ldots, c_n)$ while providing one unit of labor inelastically to the firms.

Solution Concept

- An equilibrium is a collection $(P^*, C^*, S^*, \ell^*, x^*, y^*)$, such that

(i) Consumer maximization

(ii) Firm maximization

$$\ell_i^*, x_{ij}^* \in \arg\min_{\ell_i, x_{ij}} \ell_i + \sum_{j \in S_i^*} P_j^* x_{ij}$$

$$\text{s.t. } F_i(\ell_i, x_{ij}, S_i^*, A_i(S_i^*)) = 1$$

$$S_i^* \in \arg\min_{S_i} K_i(S_i, A_i(S_i), P^*)$$

(iii) Market clearing:

$$c_i + \sum_{j=1}^{n} x_{ji} = \ell_i$$

(iv) Markets are contestable: there is no vector of prices $\tilde{P} < P^*$ that satisfies (i)–(iii).

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Alternative (and Less Elegant) Formulation

- Firms in industry i have access to a "super-technology" of the form:

$$y_i = \mathbf{F}^i(\ell_{ik}, x_{ijk}, \mathbf{A}_i) = \sum_{S_i \subseteq \{1,\dots,n\}} F_i(\ell_i, x_{ij}, S_i, A_i(S_i)).$$

- All technologies are perfect substitutes.
- Firm i will end up using only the cost-minimizing technology (because of constant returns-to-scale).
- No longer equivalent when the sub-technologies (F_i's) are not constant returns-to-scale.

Main Results

- Existence: always!
  Non-standard argument due to discrete choice.

- Uniqueness: generically in the space of realized shocks.
  There is multiplicity if a firm is indifferent between two technologies!

- Comparative statics: under certain simple assumptions, higher productivity leads to denser networks, whereas higher taxes/distortions leads to a sparser input-output network.

- Exponential Growth: Endogenous networks lead to the exponential growth when there would have been no growth with exogenous networks.

Endogenous Networks as the Origin of Economic Growth

- A dynamic variant of the model, where one new industry arrives at every period.
- Consumer preferences and all available technologies are Cobb-Douglas:

$$y_i(S_i) = A_i(S_i) \left(\ell_i^{1-\sum_{j \in S_i} \alpha_{ij}} \right) \prod_{j \in S_i} x_{ij}^{\alpha_{ij}}$$

$$u(c_1, \ldots, c_t) = \sum_{i=1}^{t} \beta_i \log(c_i)$$

- Adding one extra input reduces the exponent corresponding to labor, while keeping the relative importance of all other goods the same.
- Log productivity shocks are drawn i.i.d. from an exponential or Gumble distribution.

- Result: With exogenous production networks, the rate of growth is zero, whereas with endogenous networks, the economy grows exponentially fast.

Comment: What Drives Exponential Growth?

- In the model, industry i can choose any combination of various inputs:

$$S_i(t) \in \mathcal{S}_i(t) = 2^{\{1,\ldots,t\}}$$

- As a result, each industry has access to exponentially many different technologies to choose from: $|\mathcal{S}_i(t)| = 2^t$.

- What if we introduce network rigidities in the set of technologies that one can choose from by shrinking the choice set? $\mathcal{S}_i(t) \subsetneq 2^{\{1,\ldots,t\}}$

- In the paper, $|\mathcal{S}_i(t)|$ is either 2 or s^t for all industries at all times.

- There is a big gap between the two worlds!

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Network Rigidities and Exponential Growth?

- Consider two "intermediate" scenarios:

(1) Firms in industry 1 face no frictions whatsoever, whereas all other industries have face perfectly rigid production technologies:

$$|S_i(t)| = \begin{cases} 2^t & \text{if } i = 1 \\ 1 & \text{if } i \neq 1 \end{cases}$$

(2) There is some friction in all industries, bringing the number of available technologies just below exponential:

$$|S_i(t)| = 2^{t^{1-\epsilon}}$$

for some $\epsilon > 0$ arbitrarily small.

Network Rigidities and Exponential Growth?

Proposition

Suppose industry 1 faces no frictions, whereas all other industries have fully rigid production networks. Then,

$$\liminf_{t\to\infty} \frac{1}{t} \log(\mathrm{GDP}_t) > 0.$$

Proposition

Suppose $|\mathcal{S}_i(t)| = 2^{t^{1-\epsilon}}$ *for all industries* i*, where* $\epsilon > 0$*. Then,*

$$\lim_{t\to\infty} \frac{1}{t} \log(\mathrm{GDP}_t) = 0$$

for all $\epsilon > 0$.

- Exponential growth is driven by exponential choice somewhere in the economy. Conversely, frictions that bring the choice set down to sub-exponential kill exponential growth.

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How about Taxes and Distortions?

- Suppose industry i faces an effective price of $P_j(1+\tau_{ij})$ when purchasing goods from industry j, where $\tau_{ij} \geq 0$.

$$\log(\mathrm{GDP}_t^{\mathrm{dist}}) = \sum_{i,j=1}^{t} \beta_i \mathcal{L}_{ij}(S)\epsilon_j(S_j) - \sum_{i,j,k=1}^{t} \beta_i \mathcal{L}_{ij}(S)\alpha_{jk}(S_j)\log(1+\tau_{jk})$$

- Taxes and distortions not only lead to the destruction of real value, but also impact the firms' input choices.

Proposition

Suppose $\tau_{ij} \leq \overline{\tau}$ for all pairs of industries i and j. Then,

$$\lim_{t\to\infty} \frac{1}{t}\log(\mathrm{GDP}_t^{\mathrm{dist}}) = \lim_{t\to\infty} \frac{1}{t}\log(\mathrm{GDP}_t^{*}).$$

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Frictions and Exponential Growth?

- In the model, network rigidities (ruling out an input or an input combination entirely) have a fundamentally different impact on the growth rate as taxing inputs.
- Network rigidities limit i's opportunity for searching for more and more input combinations.
- In contract, in the presence of taxes/distortions, firm i can still draw as many productivity shocks as in the undistorted economy.

Summary

- Useful and portable framework to endogenize the nature of production.
- Overcomes many of the issues that are common in the network formation literature (existence, uniqueness, clean non-trivial structure).
  Disclaimer: the discussant is very biased!
- Nice illustration of how endogenizing the nature of production can shape long-run growth
  - requires a strong position on the rate of expansion of feasible technologies
  - seems that new draws of the shocks are doing some heavy-lifting.