

Who Benefits from Fair Value Accounting?

An Equilibrium Analysis with Strategic Complementarities

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1. Introduction

The move to fair value accounting is arguably the most radical shift in accounting standards during the past decade. Under fair value accounting a firm's assets and liabilities are marked to market at each reporting date rather than maintained at their original acquisition cost (less some mechanical adjustment for depreciation). The gains and losses arising from such revaluations are reported as part of a firm's comprehensive income¹. There is widespread support among regulators and academics for fair value accounting. The only concerns that have been expressed are those stemming from the difficulty of determining fair market values in settings where markets are thin or missing. There isn't much skepticism, which is surprising because not enough is known about important questions such as: What are the equilibrium economic consequences of fair value accounting? Who benefits and why?

While the arguments supporting fair value accounting are not based on any formal analytical models that we are aware of, the intuition underlying its support seems to be the following. The current market values of a firm's assets and liabilities are much more descriptive of a firm's financial position/wealth than their historical acquisition cost. Therefore, the assessment and recording of fair values will better inform outside stakeholders who make decisions whose payoffs depend at least partially upon the firm's true wealth. Also, fair value information is obviously relevant to valuation and fair values are used as inputs into analytical models of valuation. Empirically it has been found that changes in fair values seem to be reflected in capital market assessments of debt and equity values. Thus the provision of fair value information would make markets more "efficient" and capital market valuations would be more consistent with the fundamentals of the firm. It is believed that these

¹ We are ignoring the effect of incorporating conservatism and other imperfections into fair value measurements, in order to focus solely on the *principle* of fair value reporting.

arguments are so obvious and compelling that any formal analysis is unnecessary.²

Although the financial crises of 2007-09 raised significant concern that the accounting principle of mark-to-market was aggravating and prolonging the downward economic spiral, supporters of fair value accounting argue that bank regulators, rather than the accounting numbers, were at fault.

But the above intuitive arguments supporting fair value accounting are drawn from a Robinson Crusoe economy where the firm's wealth (financial position) is treated as a state of Nature, and the interaction between decisions and wealth is entirely one – sided. In such settings more information (in the Blackwell sense) is always preferred to less. Thus, since fair value accounting ostensibly provides incremental information about a firm's wealth, Blackwell's theorem would imply that fair value accounting is strictly preferred to historical cost accounting in any decision setting where the firm's wealth is payoff relevant to decision makers. More precisely, if the wealth of the firm is a given random variable $\tilde{w}$, q is some decision to be made by a decision maker, with payoff $f(q, w)$ then, since the expectation of a maximum is always greater than the maximum of an expectation,

$$E_w \left[\text{Max}_q f(q, w) \right] > \text{Max}_q \left[E_w f(q, w) \right]$$

The difference, known as the expected value of perfect information, is always positive and the inequality continues to be true when the information is less than perfect.

However, we do not live in the sterile environment of a Robinson Crusoe economy, and assessments of a firm's wealth are not analogous to assessments of the

² See Mary Barth: "Why It's Not Fair to Blame Fair Value", (2010) *IESE Insight* 7: 48-54 (available from Harvard Business School Publishing), for a fuller articulation of such arguments.

states of Nature. A firm's wealth endogenously depends upon the actions of a vast multitude of individuals: corporate managers, and outside stakeholders such as the firm's customers, suppliers of labor and suppliers of capital. When the actions of corporate managers and outsiders interact sequentially in the determination of a firm's wealth, the actions taken by corporate managers will depend at least partially upon their anticipation of outside stakeholders' actions that are taken in the light of accounting and other sources of information. Therefore, if outside stakeholders' actions are partly guided by assessments of the firm's wealth, then information provided to help in the assessment of a firm's wealth will impact the decisions of both insiders and outsiders and will therefore change the wealth distribution that is being assessed. The purpose of this paper is to move beyond a Robinson Crusoe setting and study a realistic example of such complex interactive settings in order to gain additional insights into the economic consequences of fair value accounting.

In our analysis, the outside stakeholders who affect the firm's wealth are customers who place orders for the single good that the firm produces. Customers' demand for the good produced by the firm depends partially upon assessments of the firm's wealth because the ability of the firm to service the future needs of customers could be severely affected if the firm is financially weak. The firm's wealth is affected not just by the decisions that customers make, but also by an asset portfolio decision that is made by the firm's manager at a date earlier to the date on which customer orders are placed. This asset allocation decision consists of allocating capital between a risk free asset and a risky asset whose expected return is larger than the risk free rate of return. The firm is risk averse. Investment in the risky asset increases the firm's expected wealth, but also increases the risk that the firm must bear. Fair value

accounting for the risky asset provides incremental decision facilitating information about the firm's wealth to its customers.

The results we obtain are striking and quite contrary to popular belief. We find that, while the information provided by fair value accounting is uncertainty decreasing from the perspective of the firm's outside stakeholders, it is uncertainty *increasing* from the perspective of corporate managers. More precise information causes more variation in customer decisions. This implies that the more precise is the information provided by fair value accounting, the *greater* becomes the volatility of the firm's income and wealth from an *ex ante* perspective. This increased volatility is not cosmetic in the sense of volatility of *reported* income rather than real income. The firm's *real* income becomes more volatile. Corporate managers respond to this situation by decreasing the firm's investment in the risky asset thus additionally altering the distribution of the firm's wealth. Thus, in our setting, the wealth distribution that is being assessed by outside stakeholders is itself affected by the information that is being provided to facilitate this assessment. We find that the net result of these actions and interactions is that fair value accounting makes the firm (i.e. its shareholders) unambiguously worse off. The firm's customers are better off only in a *sequential* sense, i.e. at the time they need to make their decisions they would exhibit a positive demand for fair value accounting and the more precise is the information provided by fair value accounting the more they would benefit. But, we find that *in equilibrium*, taking into account the actions and reactions of both the manager and the outside stakeholders, these outside stakeholders could actually become worse off, especially if the information provided by fair value accounting is *too precise*.

Our results cast doubt on the desirability of fair value accounting. Plantin, Sapra and Shin (2008) and Allen and Carletti (2007) have also raised concerns about

fair value accounting and have identified some of its negative consequences. However, in this previous research the concerns originate from a lack of liquidity in the market for the firm's assets, thus creating measurement problems. Our analysis is free from measurement and liquidity issues and questions the very *principle* on which fair value accounting is based.

2. The Economic Setting

We assume customers are atomistic, so no single customer's action, by itself, has any measurable effect on the firm's wealth. In order to capture this we model a continuum of customers, uniformly distributed over the unit interval. Let:

q_i = order placed by customer i .

$Q = \int_0^1 q_i di$ = the aggregate of customer orders.

There are 3 dates, 0, 1 and 2, with date 2 being the terminal date. The firm begins at date 0 with an endowment of m units of a riskless asset. One unit of the riskless asset held until the terminal date produces one unit of wealth at the terminal date. However, the firm has the opportunity to convert some or all of its endowment into a risky illiquid asset whose expected return at date 2 is greater than that of the riskless asset. Let z be the amount that the firm chooses to invest in the risky asset at date 0 and let $z\tilde{\theta}$ be the return at date 2. *Ex post*, at date 2, the wealth of the firm is:

$$w = m - z + z\theta + Q \tag{1}$$

Thus, the firm's wealth depends partly upon a decision made by the firm's inside manager and partly upon the aggregate of decisions made by a continuum of outside stakeholders (customers).

We assume that except for informational differences, all customers are identical. These identical customers place their orders with the firm at date 1. The payoff to a customer for ordering from our incumbent firm depends partly upon a parameter η that describes how well the characteristics of the good produced by the firm match the needs of its customers, and partly upon the financial strength of the firm. The ex post payoff to a customer who places an order of size q_i is:

$$u_i = Aq_i - \frac{1}{2}q_i^2 \quad (2)$$

where $\frac{1}{2}q_i^2$ is the known cost of using the good in whatever manner the customer uses it³. The marginal benefit to a customer from purchasing its needs from the incumbent firm is $A \equiv \tau\eta + (1 - \tau)w$, where $0 < (1 - \tau) < 1$ describes the relative extent to which customers are affected by the financial strength of the firm that supplies them⁴.

Before the customers place their orders at date 1, the accounting system provides a fair value estimate of the value of the risky asset in which the firm has invested, and this estimate is incrementally informative about the terminal wealth of the firm. The fair value estimate is public information. We assume that this public information would not exist under historical cost accounting. Customers use the fair

³ The model of customers used here is a variation on the model of individual investment decisions with strategic complementarities in Angeletos and Pavan (2004).

⁴ That customers are less willing to place orders with suppliers who are perceived to be financially weak, is a well known empirical phenomenon. General Motors was faced with this predicament during the recent financial crisis. A possible reason for this phenomenon is that the benefit from today's purchase may depend on continued supplies of goods or services from the incumbent supplier in the future.

value estimate along with any other information, public or private, that is available to them to assess the firm's wealth before choosing their orders.

3. Customers' Ordering Decisions

Let $E_i(\tilde{A})$ be customer i 's expectation of $\tilde{A}$ conditional on the information she receives at date 1. Then, the order placed by customer i is the unique solution to:

$$\text{Max}_{q_i} E_i(\tilde{A})q_i - \frac{1}{2}q_i^2 \quad (3)$$

The first order condition to (3) yields:

$$q_i = E_i(\tilde{A}) = \tau\eta + (1-\tau)(m-z) + (1-\tau)zE_i(\tilde{\theta}) + (1-\tau)E_i(\tilde{Q}) \quad (4)$$

Since the random variable $\tilde{\theta}$ is a state of Nature, expectations of $\tilde{\theta}$ are defined by Bayes' Theorem, but expectations about the aggregate order $\tilde{Q}$ is a much more complex object. These latter expectations depend upon what customer i expects other customers to do, and therefore on customer i 's beliefs of the beliefs of other customers and i 's beliefs of other customers' beliefs of other customers beliefs, and so on. We show below that $\tilde{Q}$ and $E_i(\tilde{Q})$ can be calculated iteratively, and are described by an infinite hierarchy of higher order beliefs of $\tilde{\theta}$.

Since $Q = \int_0^1 q_i di$, it follows from the first order condition (4) that:

$$Q = \tau\eta + (1-\tau)(m-z) + (1-\tau)z \int_0^1 E_i(\theta) di + (1-\tau) \int_0^1 E_i(Q) di \quad (5)$$

We refer to $E_i(\theta)$ as the first order belief of customer i , and $\int_0^1 E_i(\theta) di$ as the average first order belief about θ in the population of customers. No customer knows what this average belief is, but each customer can form a belief of this average belief which I denote by $E_i \int_0^1 E_j(\theta) dj$. From (5),

$$E_i(Q) = \tau\eta + (1-\tau)(m-z) + (1-\tau)zE_i \int_0^1 E_j(\theta) dj + (1-\tau)E_i \int_0^1 E_j(Q) dj \quad (6)$$

In (6) the expression $E_i \int_0^1 E_j(Q) dj$ is *conceptually* well defined since it is customer i 's belief of the average belief of Q in the customer population, but we don't yet know how to calculate it. Inserting (6) into the customer's first order condition yields:

$$q_i = \tau\eta + (1-\tau)(m-z) + (1-\tau)zE_i(\theta) + (1-\tau)\tau\eta + (1-\tau)^2(m-z) + (1-\tau)^2 zE_i \int_0^1 E_j(\theta) dj + (1-\tau)^2 E_i \int_0^1 E_j(Q) dj \quad (7)$$

Integrating the expression in (7) over the customer population yields:

$$Q = \tau\eta + (1-\tau)(m-z) + (1-\tau)z \int_0^1 E_i(\theta) di + (1-\tau)\tau\eta + (1-\tau)^2(m-z) + (1-\tau)^2 z \int_0^1 E_i \int_0^1 E_j(\theta) dj di + (1-\tau)^2 \int_0^1 E_i \int_0^1 E_j(Q) dj di \quad (8)$$

In (8) the expression $\int E_i \int E_j(\theta) dj di$ is the average expectation of the average expectation of θ in the customer population. We refer to it as the average second order

expectation of θ . Now, (8) can be used to obtain an updated calculation of $E_i(Q)$ and this updated expression for $E_i(Q)$ can be inserted into the customer's first order condition (4) to yield an updated expression for q_i . Integrating this updated expression for q_i yields the following updated expression for the aggregate order quantity Q .

$$\begin{aligned}
Q = & \tau\eta + (1-\tau)\tau\eta + (1-\tau)^2\tau\eta + (1-\tau)(m-z) + (1-\tau)^2(m-z) + (1-\tau)^3(m-z) + \\
& (1-\tau)z \int_0^1 E_i(\theta) di + (1-\tau)^2 z \int_0^1 E_i \int_0^1 E_j(\theta) dj di + \\
& (1-\tau)^3 z \int_0^1 E_i \int_0^1 E_k \int_0^1 E_j(\theta) dj dk di + (1-\tau)^3 \int_0^1 E_i \int_0^1 E_k \int_0^1 E_j(Q) dj dk di
\end{aligned} \tag{9}$$

Comparing (5), (8), and (9) it is clear that repeated iteration yields:

$$\begin{aligned}
Q = & \tau\eta[1 + (1-\tau) + (1-\tau)^2 + \dots] + \\
& (1-\tau)(m-z)[1 + (1-\tau) + (1-\tau)^2 + \dots] + \\
& (1-\tau)z[\theta^{(1)} + (1-\tau)\theta^{(2)} + (1-\tau)^2\theta^{(3)} + \dots]
\end{aligned} \tag{10}$$

where, $\theta^{(t)}$, $t=1, 2, 3, \dots$ denotes the average t th. order expectation of θ . Since

$0 < (1-\tau) < 1$, each of the infinite series contained in (10) is convergent and well

defined. Carrying out the summation yields the final expression:

$$Q = \frac{\tau\eta + (1-\tau)(m-z)}{\tau} + (1-\tau)z \sum_{t=0}^{\infty} (1-\tau)^t \theta^{(t+1)} \tag{11}$$

Notice from (11) that the undefined expectations of Q have vanished and have been replaced by well defined higher order expectations of θ .

We assume that the information structure in the economy is as follows. The commonly known prior distribution of $\tilde{\theta}$ is Normal with mean μ and variance $\frac{1}{\alpha}$.

Equivalently, $\tilde{\theta} = \mu + \tilde{\xi}$, $\tilde{\xi} \sim N(0, \frac{1}{\alpha})$. We assume $\mu > 1$ so that investment in the risky asset is *a priori* desirable. Both historical cost accounting and fair value accounting reveal the amount z of investment in the risky asset but, at date 1, fair value accounting provides an additional signal that is not provided by historical cost accounting. Fair value accounting provides an estimate of the date 1 value of the risky asset. Conceptually, such an estimate is equivalent to providing a noisy signal of the final return $\tilde{\theta}$ on the risky asset. Therefore, we model fair value accounting as providing the unbiased public signal:

$$\tilde{y} = \theta + \tilde{\varepsilon}, \quad \tilde{\varepsilon} \sim N(0, \frac{1}{\beta})$$

Higher values of β represent more precise measurement of fair values, and the lowest value of β , i.e. $\beta = 0$ is equivalent to historical cost accounting. Additionally, customers may have private sources of information about the return to the risky asset. We model this as private unbiased signals, x_i , with some common precision γ :

$$x_i = \theta + \tilde{\omega}_i, \quad \tilde{\omega}_i \sim N(0, \frac{1}{\gamma})$$

A setting where the only source of information is the publicly provided fair value accounting signal is captured by specifying $\gamma = 0$, so the inclusion of private signals is without loss of generality. The existence of private signals captures the realistic idea

that in the absence of a public source of information, individual customers will have entirely idiosyncratic beliefs of the return to the risky asset. We assume that all of the noise terms, $\tilde{\xi}$, $\tilde{\varepsilon}$, and $\tilde{\omega}_i$ are independent of each other and independent of $\tilde{\theta}$.

We now proceed to derive the aggregate order quantity Q for the specific information structure described above. The first order belief of customer i is:

$$E_i(\tilde{\theta}) = \frac{\alpha\mu + \beta y + \gamma x_i}{\alpha + \beta + \gamma} \quad (12)$$

It is convenient to rewrite the above expression in a different way. Let

$$\delta \equiv \frac{\gamma}{\alpha + \beta + \gamma}, \text{ and}$$

$$P \equiv \frac{\alpha\mu + \beta y}{\alpha + \beta}$$

Then (12) is equivalent to:

$$E_i(\tilde{\theta}) = \delta x_i + (1 - \delta)P, \quad (13)$$

where P can be thought of as the public information in the economy and x_i as the private information of customer i . Then, the average first order belief of $\tilde{\theta}$ is:

$$\theta^{(1)} \equiv \int E_i(\theta) di = \int (\delta x_i + (1 - \delta)P) di = \delta \theta + (1 - \delta)P,$$

from which it follows that i 's belief of the average first order belief is:

$$\begin{aligned}
E_i \int E_j(\theta) dj &= \delta E_i(\theta) + (1-\delta)P \\
&= \delta[\delta x_i + (1-\delta)P] + (1-\delta)P \\
&= \delta^2 x_i + (1-\delta^2)P
\end{aligned}$$

Therefore the average second order belief of θ is:

$$\theta^{(2)} = \int E_i \int E_j(\theta) dj di = \delta^2 \theta + (1-\delta^2)P$$

Iterating in this way gives the average t th. order expectation of θ :

$$\theta^{(t)} = \delta^t \theta + (1-\delta^t)P \tag{14}$$

Notice that the weight on the fundamental θ decreases and the weight on the public signal P increases in successively higher order beliefs. This implies that the aggregate order received by the firm is overly sensitive to the public signal and insufficiently sensitive to the unknown fundamental θ , relative to what would be prescribed by Bayes theorem. This phenomenon is common to settings with higher order beliefs (see Morris and Shin (2002), and Angeletos and Pavan (2004)). In the specific context of fair value accounting, what this implies is that the error contained in the accounting estimate of the fair value of the risky asset, will have a disproportionate influence on how outsiders respond to the firm's asset allocation decision z .

Inserting (14) into the general expression for Q that was derived in (11), determines the value of the aggregate order Q that is specific to the information

structure under consideration. Also, from this expression for Q the individual values of $E_i(Q)$ and q_i can be calculated. These calculations yield:

Proposition 1:

In a fair value accounting regime, the equilibrium response of the firm's customers to the firm's asset allocation decision z is:

$$q_i = \frac{1}{\tau} \left\{ \tau\eta + (1-\tau)(m-z) + (1-\tau)z (\lambda x_i + (1-\lambda)P) \right\} \quad (15)$$

and,

$$Q = \frac{1}{\tau} \left\{ \tau\eta + (1-\tau)(m-z) + (1-\tau)z (\lambda\theta + (1-\lambda)P) \right\} \quad (16)$$

where,

$$\lambda \equiv \frac{\tau\delta}{1-(1-\tau)\delta}$$

Proof: See the Appendix.

Because $\left(\frac{\tau}{1-(1-\tau)\delta} \right) < 1$, $\forall \delta > 0$ and $\tau < 1$, the equilibrium weight on

x_i in (15) and on θ in (16) is strictly less than δ , which is the weight that would be used in Bayesian updating. This implies that, because of the need to assess the beliefs of others, each individual customer under-weights his private information about θ and over-weights the public information in deciding how much to order from the firm. In turn, this results in the equilibrium aggregate order quantity Q becoming less sensitive

to fluctuations in the fundamentals θ and overly sensitive to the public information provided by fair value accounting. The effect of this distortion on social welfare will be developed in a later section.

4. The Firm's Asset Allocation Decision:

We now turn to the firm's asset portfolio decision that is made at date 0. As specified earlier, the firm's terminal wealth is $w = m - z + z\theta + Q$. We assume the firm is risk averse with constant absolute risk aversion $\rho > 0$. If $\tilde{w}$ is distributed Normal, as will be the case, the firm's objective function is:

$$\text{Max}_z \left\{ E(\tilde{w}) - \frac{1}{2} \rho \text{Var}(\tilde{w}) \right\} \quad (17)$$

Because the firm's wealth is affected by the aggregate customer order, and because this aggregate is sensitive to average assessments of the firm's wealth, the firm must be mindful of how its investment z in the risky asset, together with the fair value signal, affects outsiders' assessments of its wealth. From the firm's perspective at date 0, $\tilde{Q}$, as determined in (16), is a Normally distributed random variable since it depends linearly on the Normally distributed return $\tilde{\theta}$ as well as on the Normally distributed fair value report $\tilde{y}$ that is released later at date 1.

Using the facts that $E(\tilde{\theta}) = E(\tilde{P}) = \mu$, we obtain from (16):

$$E(\tilde{Q}) = \frac{\tau\eta + (1-\tau)[m + z(\mu - 1)]}{\tau} \quad (18)$$

and,

$$E(\tilde{w}) = m + z(\mu - 1) + E(\tilde{Q}) = \frac{\tau\eta + m + z(\mu - 1)}{\tau} \quad (19)$$

The effect of the firm's risky asset investment on its expected wealth is twofold. There is a direct effect and an indirect effect. Since $\mu > 1$, the direct effect is that the expected return on the firm's investment is larger resulting in larger expected wealth. The indirect effect operates through the firm's customers. When customers perceive the firm as being more financially sound, (higher expected wealth), they are more willing to buy from the firm, so the aggregate order quantity Q is strictly increasing in z . This additionally augments the expected wealth of the firm.

We turn now to the uncertainty in the firm's wealth caused by investment in the risky asset.

$$\text{var}(\tilde{w}) = z^2 \text{var}(\tilde{\theta}) + \text{var}(\tilde{Q}) + 2z \text{cov}(\tilde{\theta}, \tilde{Q}) \quad (20)$$

The first term in (20) captures the direct effect that greater investment in the risky asset increases the volatility of the firm's wealth. The second and third terms encompass customer assessments of the firm's wealth and the decisions that are contingent on such assessments. We assess the variance of the firm's wealth term by term.

$$\text{var}(\tilde{\theta}) = \text{var}(\tilde{\xi}) = \frac{1}{\alpha}$$

From (16):

$$\begin{aligned} \text{var}(\tilde{Q}) &= \left(\frac{1-\tau}{\tau} \right)^2 z^2 \text{var}[\lambda \tilde{\theta} + (1-\lambda) \tilde{P}] \\ &= \left(\frac{1-\tau}{\tau} \right)^2 z^2 [\lambda^2 \text{var}(\tilde{\theta}) + (1-\lambda)^2 \text{var}(\tilde{P}) + 2\lambda(1-\lambda) \text{cov}(\tilde{\theta}, \tilde{P})] \end{aligned} \quad (21)$$

where,

$$\begin{aligned}
\text{var}(\tilde{P}) &= \left(\frac{\beta}{\alpha + \beta} \right)^2 \text{var}(\tilde{y}) \\
&= \left(\frac{\beta}{\alpha + \beta} \right)^2 \left(\frac{1}{\alpha} + \frac{1}{\beta} \right) = \left(\frac{\beta}{\alpha + \beta} \right) \frac{1}{\alpha}
\end{aligned}$$

and,

$$\begin{aligned}
\text{cov}(\tilde{\theta}, \tilde{P}) &= \text{cov} \left(\tilde{\theta}, \left(\frac{\beta}{\alpha + \beta} \right) \tilde{y} \right) \\
&= \left(\frac{\beta}{\alpha + \beta} \right) \text{var}(\tilde{\theta}) = \left(\frac{\beta}{\alpha + \beta} \right) \frac{1}{\alpha}
\end{aligned}$$

Inserting these last two calculations into (21) gives:

$$\text{var}(\tilde{Q}) = \left(\frac{1-\tau}{\tau} \right)^2 z^2 \frac{1}{\alpha} \left[\lambda^2 + (1-\lambda)^2 \left(\frac{\beta}{\alpha + \beta} \right) + 2\lambda(1-\lambda) \left(\frac{\beta}{\alpha + \beta} \right) \right]$$

which simplifies to:

$$\text{var}(\tilde{Q}) = \left(\frac{1-\tau}{\tau} \right)^2 z^2 \frac{1}{\alpha} \left[\lambda^2 + (1-\lambda^2) \left(\frac{\beta}{\alpha + \beta} \right) \right] \quad (22)$$

The above calculations show that the firm's investment in the risky asset not only has a direct effect on the riskiness of its wealth, but also an indirect effect by increasing the risk arising from the uncertain aggregate order quantity. This additional risk is caused by the fair value signal, as shown in Lemma 1 below:

Lemma 1:

Increases in the precision of information provided by fair value accounting increases:

- (i) *The uncertainty in the size of the aggregate order received by the firm and*
- (ii) *The marginal effect of the firm's investment in the risky asset on the uncertainty in the aggregate order.*

Proof: See the Appendix.

The result in Lemma1 is a special case of a quite general phenomenon. Information provided to a decision maker allows her to vary her decision to better fit the circumstances that exist at the time. From an *ex ante* perspective, such variability in the decision makes the world look more uncertain. The more precise is the information provided to the decision maker the more sensitive will be the decision to the information signal causing the decision to become more uncertain from an *ex ante* perspective. *Ex ante* a decision maker's action is most predictable if no new information can possibly arrive prior to making that decision. In the context of our model, no information arrival (public or private) prior to the decisions made by customers, is equivalent to $\beta = \gamma = 0$. But in this case,

$$\delta \equiv \frac{\gamma}{\alpha + \beta + \gamma} = 0, \text{ implying that } \lambda \equiv \frac{\tau\delta}{1 - (1 - \tau)\delta} = 0. \text{ Then, it is immediate from (22)}$$

that $\text{var}(\tilde{Q}) \rightarrow 0$ as $(\beta, \gamma) \rightarrow 0$.

The remaining term in the calculation of $\text{var}(\tilde{w})$ as specified in (20), is:

$$\begin{aligned} \text{cov}(\tilde{\theta}, \tilde{Q}) &= \text{cov}\left(\tilde{\theta}, \left(\frac{1-\tau}{\tau}\right)z(\lambda\theta + (1-\lambda)\tilde{P})\right) \\ &= \left(\frac{1-\tau}{\tau}\right)z[\lambda \text{var}(\tilde{\theta}) + (1-\lambda)\text{cov}(\tilde{\theta}, \tilde{P})] \end{aligned}$$

$$= \left(\frac{1-\tau}{\tau} \right) \frac{1}{\alpha} z \left[\lambda + (1-\lambda) \left(\frac{\beta}{\alpha + \beta} \right) \right] \quad (23)$$

which is also strictly increasing in β .

Inserting (22) and (23) into (20) gives:

$$\begin{aligned} \text{var}(\tilde{w}) = & z^2 \frac{1}{\alpha} + z^2 \frac{1}{\alpha} \left(\frac{1-\tau}{\tau} \right)^2 \left[\lambda^2 + (1-\lambda^2) \frac{\beta}{\alpha + \beta} \right] + \\ & 2z^2 \frac{1}{\alpha} \left(\frac{1-\tau}{\tau} \right) \left[\lambda + (1-\lambda) \frac{\beta}{\alpha + \beta} \right] \end{aligned} \quad (24)$$

Proposition 2:

Keeping fixed the firm's investment in the risky asset, the more precise is the fair value information provided to outside stakeholders to assist in assessing the firm's wealth the more uncertain the wealth of the firm becomes from an ex ante perspective.

Proof: See the Appendix.

Proposition 2 is shocking, since one would normally think that the effect of information is to decrease uncertainty. But, it is important to ask from whose perspective are we assessing the uncertainty in the environment. It is certainly true that information provided at date 1 decreases the uncertainty faced by economic agents who make decisions at date 1. But, what happens to the uncertainty faced by economic agents who must move earlier, before the information is provided? If the decision made by later economic agents is

sensitive to the information provided to them, the earlier economic agents must perceive the decisions made by the later economic agents as random variables and therefore the information actually *increases* the uncertainty they face. Proposition 2 is also a stark example of how misleading accounting disclosure studies could be when the variable of interest is assigned an exogenously specified distribution. In such studies information *always* reduces uncertainty, since statistically a conditional variance is smaller than an unconditional variance. In our study too, if the wealth of the firm is an exogenously given random variable then information about wealth can only decrease the uncertainty in wealth. But such a scenario is an over-simplification of the real world. Realistically, a firm's wealth depends not just upon the state of Nature, but also upon decisions made by both insiders and outsiders. If the disclosure of information alters the decisions of outsiders and if these decisions affect the distribution of the firm's wealth then it is not necessarily true that information is uncertainty reducing even on an *ex post* basis.

We can now characterize the firm's date 0 investment in the risky asset. Inserting (19) and (24) into the firm's objective function, as described in (17), and differentiating with respect to z gives the first order condition:

$$z = \frac{\mu - 1}{\tau \rho \frac{1}{\alpha} \left[1 + \left(\frac{1 - \tau}{\tau} \right)^2 \left(\lambda^2 + (1 - \lambda^2) \frac{\beta}{\alpha + \beta} \right) + 2 \left(\frac{1 - \tau}{\tau} \right) \left(\lambda + (1 - \lambda) \frac{\beta}{\alpha + \beta} \right) \right]} \quad (25)$$

Equation (25) indicates that the firm is not passive to the provision of fair value information. Anticipating that its asset portfolio will be revalued at interim dates the firm shifts its investments away from risky assets to riskless assets. This can be seen by examining how variations in the precision β of the fair value information affects z as described in (25). From (25) it is clear that the effect of β on z is through the two factors

$\lambda^2 + (1 - \lambda^2) \left(\frac{\beta}{\alpha + \beta} \right)$ and $\lambda + (1 - \lambda) \left(\frac{\beta}{\alpha + \beta} \right)$ contained in the denominator of (25). In

the proofs of Lemma 1 and Proposition 2, we established that both factors are strictly increasing in β . Therefore, the denominator in (25) is strictly increasing in

β , implying that $\frac{\partial z}{\partial \beta} < 0$. We have established:

Proposition 3

Increases in the precision of the fair value signal provided to outside stake holders causes the firm's investment in the risky asset to decline.

The result described in Proposition 3 is due to the fact that the precision of the fair value information increases not only the prior uncertainty in the aggregate order Q but also increases the marginal effect of z on this uncertainty. The firm decreases its holdings of risky assets in order to decrease the uncertainty in customer orders.

Proposition 3 is a testable result. It predicts that fair value reporting of a firm's assets will significantly change the firm's portfolio of asset holdings. Such a prediction is inconsistent with studies of fair value accounting in exchange economies where it is implicitly assumed that the accountant measures and reports on an objective reality, a reality that is invariant to accounting measurements.

5. Welfare Analysis

Having characterized the equilibrium decisions of both insiders and outsiders, we now turn to the main question of interest: In equilibrium, who benefits from fair value accounting? This question can be answered by examining variations in the precision of the

fair value information. If the welfare of both parties (the firm and its customers) is uniformly declining in β , then fair value accounting unambiguously decreases social welfare. If the equilibrium payoff to the firm is declining in β , but the welfare of the firm's customers is increasing in β then there is a conflict of interest, and so on.

Welfare from the Firm's Perspective:

The firm's welfare is simply the maximized value of its objective function at the equilibrium value of $\tilde{Q}$. Therefore, the effect of fair value accounting on the firm's welfare is described by:

$$\frac{\partial}{\partial \beta} \left\{ \text{Max}_z \left(E(\tilde{w}) - \frac{1}{2} \rho \text{var}(\tilde{w}) \right) \right\}$$

where $E(\tilde{w})$ is as described in (19) and $\text{var}(\tilde{w})$ is as described in (24). Using the envelope theorem, this derivative is:

$$-\frac{1}{2} \rho \left[z^2 \frac{1}{\alpha} \left(\frac{1-\tau}{\tau} \right)^2 \frac{\partial}{\partial \beta} \left\{ \lambda^2 + (1-\lambda^2) \frac{\beta}{\alpha + \beta} \right\} + 2z^2 \frac{1}{\alpha} \left(\frac{1-\tau}{\tau} \right) \frac{\partial}{\partial \beta} \left\{ \lambda + (1-\lambda) \frac{\beta}{\alpha + \beta} \right\} \right]$$

We have previously established in the proofs of Lemma 1 and Proposition 2 that both

$\lambda^2 + (1-\lambda^2) \left(\frac{\beta}{\alpha + \beta} \right)$ and $\lambda + (1-\lambda) \left(\frac{\beta}{\alpha + \beta} \right)$ are strictly increasing in β . Therefore

the firm's welfare is strictly decreasing in β , which establishes the result:

Proposition 4

In equilibrium, the firm's welfare is strictly decreasing in the precision of the fair value signal.

Proposition 4 implies that, given a choice, firms would strictly prefer historical cost to fair value accounting. This result is consistent with the actual lobbying behavior of firms who opposed the fair value accounting standard, protesting that mark-to-market would significantly increase the volatility of their reported income. However, FASB dismissed this line of argument arguing that mark-to-market does not *create* volatility; it only makes the volatility that is already present more transparent to outside stake holders. Our analysis indicates that FASB's argument has merit only when the actions taken by a firm's stakeholders in response to accounting information has no impact at all on the wealth of the firm. We feel that such a setting is unrealistic. When the actions taken by outsiders in the light of fair value information affects the wealth of the firm (i.e., when accounting information has *real effects*), fair value accounting does create additional volatility in the firm's true income, and this increased volatility does have negative economic consequences.

Welfare from Customers' Perspective:

We now examine the effect of fair value accounting on the social welfare of the firm's customers. We define the *ex post* social welfare Ω of the customer population as the aggregate of the *ex post* payoffs of individual customers, i.e.,

$$\begin{aligned}\Omega &\equiv \int u_i di = A \int q_i di - \frac{1}{2} \int q_i^2 di \\ &= AQ - \frac{1}{2} \left[\int (q_i - Q)^2 di + Q^2 \right]\end{aligned}$$

Inserting the expression for A gives:

$$\Omega = [\tau\eta + (1-\tau)(m-z+z\theta) + (1-\tau)Q]Q - \frac{1}{2}Q^2 - \frac{1}{2}\int (q_i - Q)^2 di ,$$

or, equivalently,

$$\Omega = [\tau\eta + (1-\tau)(m-z+z\theta)]Q - \frac{1}{2}(2\tau-1)Q^2 - \frac{1}{2}\int (q_i - Q)^2 di \quad (26)$$

The expression $\int (q_i - Q)^2 di$ in (26) indicates that social welfare is enhanced if individual customer purchases are coordinated so that each customer orders exactly the same amount, i.e. if $q_i = Q, \forall i$. This social benefit to coordination is due to the convexity in the cost function of individual customers. If the only source of information to individual customers was the publicly provided fair value information, then this perfect coordination would naturally occur. However, the presence of private information prevents such perfect coordination.

In order to facilitate interpretation, it is useful to first calculate customer welfare if the information that customers receive perfectly reveals the value of θ to all of them. This corresponds to the case where $\beta \rightarrow \infty$. In this case:

$$q_i = A = \tau\eta + (1-\tau)(m-z+z\theta) + (1-\tau)Q, \forall i$$

implying:

$$q_i = Q = \frac{1}{\tau}[\tau\eta + (1-\tau)(m-z+z\theta)], \forall i$$

Therefore, from (26):

$$\begin{aligned} \Omega(z | \text{perfect information}) &= \frac{1}{\tau}[\tau\eta + (1-\tau)(m-z+z\theta)]^2 - \frac{1}{2}(2\tau-1)\frac{1}{\tau^2}[\tau\eta + (1-\tau)(m-z+z\theta)]^2 \\ &= \frac{1}{2\tau^2}[\tau\eta + (1-\tau)(m-z+z\theta)]^2 \end{aligned}$$

Since welfare must be assessed ex ante, the welfare of the customer group given perfect information is the expectation of the above expression with respect to θ :

$$\begin{aligned}
 E(\Omega | z, \text{ perfect information}) &= \frac{1}{2\tau^2} \{E[\tau\eta + (1-\tau)(m-z+z\theta)]\}^2 + \\
 &\quad \frac{1}{2\tau^2} \text{var}(\tau\eta + (1-\tau)(m-z+z\theta)) \\
 &= \frac{1}{2\tau^2} [\tau\eta + (1-\tau)(m-z+z\mu)]^2 + \frac{1}{2\tau^2} (1-\tau)^2 z^2 \frac{1}{\alpha} \quad (27)
 \end{aligned}$$

The first term in (27) is the expected welfare of customers in a regime where no fair value information is provided (i.e. in a historical cost regime), and the second term is the gain in expected welfare due to perfect information. This gain is not due to a decrease in risk, as is the case in pure exchange economies with risk aversion. Here, the gain is due to the fact that information allows customers to better fit their *real* decisions to the true wealth of the firm. Because of the quadratic nature of aggregate payoffs to customers, the amount of the gain is described by the extent of uncertainty reduction caused by the information.

Now, consider the case of noisy public and private information. Then, working with q_i and Q as described in (15) and (16) and Ω as described in (26), we obtain:

Proposition 5:

$$\begin{aligned}
 E(\Omega | z, \text{ noisy public and private information}) &= \frac{1}{2\tau^2} [\tau\eta + (1-\tau)(m-z+z\mu)]^2 \\
 &\quad + \frac{1}{2\tau^2} (1-\tau)^2 z^2 \left[\frac{1}{\alpha} - \frac{1}{(\alpha + \beta + \tau\gamma)} - \frac{\tau\gamma(1-\tau)}{(\alpha + \beta + \tau\gamma)^2} \right] \quad (28)
 \end{aligned}$$

Proof: See the Appendix

As before, the first term in (28) is expected customer welfare in a historical cost regime, while the second term is the change in customer welfare brought about by the provision of noisy fair value information. Not surprisingly, the change in customer welfare caused by fair value information is positive even though the information is noisy and even though private information results in coordination losses. The change is positive because:

$$\begin{aligned} \frac{1}{\alpha} - \frac{1}{\alpha + \beta + \tau\gamma} - \frac{\tau\gamma(1-\tau)}{(\alpha + \beta + \tau\gamma)^2} &> \frac{1}{\alpha} - \frac{1}{\alpha + \beta + \tau\gamma} - \frac{\tau\gamma(1-\tau)}{\alpha(\alpha + \beta + \tau\gamma)} \\ &= \frac{\alpha + \beta + \tau\gamma - \alpha - \tau\gamma + \tau^2\gamma}{\alpha(\alpha + \beta + \tau\gamma)} = \left(\frac{\beta + \tau^2\gamma}{\alpha + \beta + \tau\gamma} \right) \frac{1}{\alpha} > 0 \end{aligned}$$

The overweighting of public information caused by higher order beliefs makes the gain from fair value accounting smaller than it would otherwise be. To see this, compare (28) to (27).

When the public information is infinitely precise, as in (27), private information becomes redundant so there is no overweighting of public information. In this case the gain from fair

value accounting is proportional to $\text{var}(\theta) = \frac{1}{\alpha}$ since this is the amount of uncertainty

eliminated by the information that is being provided. With noisy public and private information the residual uncertainty after providing information is

$$\text{var}(\theta | x_i, y) = \frac{1}{\alpha + \beta + \gamma}, \text{ so that the uncertainty eliminated by the information provided}$$

is $\frac{1}{\alpha} - \frac{1}{\alpha + \beta + \gamma}$. But, the benefit from fair value information is proportional to:

$$\frac{1}{\alpha} - \left(\frac{1}{\alpha + \beta + \tau\gamma} + \frac{\tau\gamma(1-\tau)}{(\alpha + \beta + \tau\gamma)^2} \right) < \frac{1}{\alpha} - \frac{1}{\alpha + \beta + \gamma} \text{ because,}$$

$$\frac{1}{\alpha + \beta + \gamma} - \frac{1}{\alpha + \beta + \tau\gamma} - \frac{\tau\gamma(1-\tau)}{(\alpha + \beta + \tau\gamma)^2} = \frac{(\alpha + \beta + \tau\gamma)^2 - (\alpha + \beta + \tau\gamma)(\alpha + \beta + \gamma) - \tau\gamma(1-\tau)(\alpha + \beta + \gamma)}{(\alpha + \beta + \gamma)(\alpha + \beta + \tau\gamma)^2} < 0$$

The full benefit from uncertainty reduction is not obtained precisely because public information is over-weighted and private information is under-weighted.

The following result is obvious by visual inspection of (28):

Proposition 6:

Customer welfare is strictly increasing in the precision of fair value information, if the firm's asset portfolio is viewed as fixed and sunk .

Proposition 6 succinctly captures the usual argument given in support of fair value accounting: fair value information is relevant to stakeholders and facilitates better decisions. It is also consistent with the common intuition, based on Blackwell's theorem, that more information is preferred to less. But Blackwell's theorem is concerned with information about states of Nature that are invariant to the provision of information. Similarly, at the time that our customers (outside stakeholders) make their decisions, the firm's asset portfolio is sunk and the size of the aggregate order Q is beyond the control of any individual customer. Therefore, from the perspective of an individual customer, at date 1, the wealth of the firm is an exogenous random variable just like the state of Nature. So, at the time that customers need to make their decisions (at date 1), each individual customer would demand the information provided by fair value accounting and would want the information to be as precise as possible.

But, from the perspective of a regulator who mandates corporate disclosure policy, it is fallacious to view the firm's asset portfolio as fixed and given. A mandate to fair value a

firm's assets could very well result in the firm choosing a different asset portfolio. This endogeneity to the firm's choice of assets has been largely overlooked in the literature and is missing from debates about the merits of fair value accounting. In Proposition 3 we established that the amount z that the firm chooses to invest in the risky asset *declines* with the precision of the fair value information that is later provided to outside stakeholders. So the welfare result described in Proposition 6, while true in a partial equilibrium sense, may no longer be true when the decline in z is taken into account. Given that $\mu > 1$, customer welfare is strictly increasing in z as can be seen from visual inspection of (28). Therefore, an increase in the precision of the fair value signal generates two *opposing* effects: better decisions which increases customer welfare, but lower investment in the risky asset which decreases customer welfare. Whether or not customers are better off, in an overall sense, depends on which of these two effects dominate. Below, we investigate the net effect on customer welfare.

The aggregate *ex ante* welfare of customers, as derived in (28) consists of two terms. The first term in (28) depends on β only through z and since z is declining in β the first term in (28) is unambiguously declining in β . The second term in (28) captures the two opposing effects described earlier. The direct dependence on β captures the decision facilitating benefit of fair value information, and the indirect dependence through z captures the expected wealth decreasing effect of fair value information. To gauge the net effect, insert the equilibrium value of z , as derived in (25) into the second term of (28). After considerable algebraic simplification, and using the facts that

$$\lambda + (1 - \lambda) \left(\frac{\beta}{\alpha + \beta} \right) = \frac{\beta + \tau\gamma}{\alpha + \beta + \tau\gamma} \text{ as shown in the proof of Proposition 2, and}$$

$$\lambda^2 + (1 - \lambda^2) \left(\frac{\beta}{\alpha + \beta} \right) = \frac{(\beta + \tau\gamma)(\alpha + \beta + \tau\gamma) - \alpha\tau\gamma}{(\alpha + \beta + \tau\gamma)^2} \text{ as shown in the proof of Lemma 1,}$$

this substitution for z into the second term of (28) yields:

$$\begin{aligned} & \frac{1}{2\tau^2} (1 - \tau)^2 z^2 \left[\frac{1}{\alpha} - \frac{1}{(\alpha + \beta + \tau\gamma)} - \frac{\tau\gamma(1 - \tau)}{(\alpha + \beta + \tau\gamma)^2} \right] = \\ & \left(\frac{(1 - \tau)^2}{2\tau^2} \right) \left(\frac{(\mu - 1)^2}{\tau^2 \rho^2} \right) \left[\frac{(\alpha + \beta + \tau\gamma)(\beta + \tau\gamma) - \alpha\tau\gamma(1 - \tau)}{(\alpha + \beta + \tau\gamma)^2 \frac{1}{\alpha} \left[1 + \left(\frac{1 - \tau^2}{\tau^2} \right) \left(\frac{\beta + \tau\gamma}{\alpha + \beta + \tau\gamma} \right) - \left(\frac{1 - \tau}{\tau} \right)^2 \frac{\alpha\tau\gamma}{(\alpha + \beta + \tau\gamma)^2} \right]^2} \right] \end{aligned} \quad (29)$$

We wish to study the behavior of (29) with respect to variations in β . Unfortunately the effect of β on (29) is ambiguous, so the overall effect of the precision of fair value information on the aggregate welfare of customers is parameter specific when there is both public and private fair value information. However, considerable insights are obtained by examining some simpler settings.

Consider the case where there are no private sources of information and the only source of information is the public fair value signal. Algebraically, this is equivalent to letting the precision of private information $\gamma \rightarrow 0$. Inserting $\gamma = 0$ in (25) and (29) yields:

$$z = \frac{\mu - 1}{\tau\rho \frac{1}{\alpha} \left[1 + \left(\frac{1 - \tau^2}{\tau^2} \right) \left(\frac{\beta}{\alpha + \beta} \right) \right]}, \text{ and}$$

$$\begin{aligned} \frac{1}{2\tau^2}(1-\tau)^2 z^2 \left[\frac{1}{\alpha} - \frac{1}{(\alpha + \beta + \tau\gamma)} - \frac{\tau\gamma(1-\tau)}{(\alpha + \beta + \tau\gamma)^2} \right] &= \frac{(1-\tau)^2 z^2 \left(\frac{\beta}{\alpha + \beta} \right) \frac{1}{\alpha}}{2\tau^2} \\ &= \frac{(1-\tau)^2}{2\tau^2} \frac{(\mu-1)^2}{\tau^2 \rho^2} \left[\frac{\frac{\beta}{\alpha + \beta}}{\left[\frac{1}{\alpha} \left[1 + \left(\frac{1-\tau^2}{\tau^2} \right) \left(\frac{\beta}{\alpha + \beta} \right) \right]^2 \right]} \right] \end{aligned}$$

Let:

$$L(\beta) \equiv \frac{\frac{1}{\alpha} \left[1 + \left(\frac{1-\tau^2}{\tau^2} \right) \left(\frac{\beta}{\alpha + \beta} \right) \right]^2}{\left(\frac{\beta}{\alpha + \beta} \right)}$$

If $L(\beta)$ is increasing in β then both the first and second terms of aggregate customer welfare are strictly decreasing in the precision of the fair value signal, so customers would be worse off from receiving fair value information. But, if $L(\beta)$ is decreasing in β then the effect of the precision of information on customer welfare is ambiguous since the first term of aggregate customer welfare is decreasing in β while the second term is increasing in β .

Simplifying the expression for $L(\beta)$ gives:

$$L(\beta) = \frac{1}{\alpha} \left[\left(\frac{\alpha + \beta}{\beta} \right) + 2 \left(\frac{1-\tau^2}{\tau^2} \right) + \left(\frac{1-\tau^2}{\tau^2} \right)^2 \left(\frac{\beta}{\alpha + \beta} \right) \right]$$

Differentiating gives:

$$\frac{\partial L}{\partial \beta} = -\frac{1}{\beta^2} + \left(\frac{1-\tau^2}{\tau^2} \right)^2 \frac{1}{(\alpha + \beta)^2}$$

Therefore,

$$\frac{\partial L}{\partial \beta} > 0 \text{ if and only if } \frac{1-\tau^2}{\tau^2} > \frac{\alpha + \beta}{\beta}$$

or, equivalently $\frac{\partial L}{\partial \beta} > 0$ if and only if :

$$\frac{\alpha}{\beta} < \frac{1}{\tau^2} - 2 \quad (30)$$

The quantity $\frac{1}{\tau^2} - 2$ must be positive for (30) to be satisfied, i.e. $\tau < 0.707$ is necessary

for $\frac{\partial L}{\partial \beta} > 0$. Recall that $(1-\tau)$ is the weight that customers put on the firm's wealth in

assessing the marginal value of placing their orders with the incumbent firm. So (30)

indicates that if the weight that customers put on the firm's wealth is at least 30% then there

is an *upper bound* to the precision of fair value information beyond which customer welfare

is guaranteed to decrease. Additionally, (30) yields the following very non-intuitive result:

Proposition 7:

The greater is the relevance of firm wealth to customer decisions (i.e. the greater is the value of $(1-\tau)$) the less precise should be the information provided by fair value accounting.

Proposition 7 would make no sense at all if the wealth maximizing decisions of corporate managers were independent of the actions of the firm's outside stakeholders. It

begins to make sense only if we take into account the *real* effects of accounting disclosure. In our setting this real effect occurs in the following way. Greater precision in the information provided by fair value accounting induces greater variability in the actions of outside stakeholders, which causes the firm's wealth to become more uncertain, which induces the firm to become more cautious in its investment strategy which, in turn, damages the welfare of the firm's outside stakeholders.

6. Concluding Remarks

The results that we have obtained contradict popular wisdom to such an extent that it behoves us to ask why outside stakeholders did not lobby against the move to fair value accounting, and why they seem to demand even more precision in fair value estimates. These empirical facts look less mysterious if one takes into account the sequential nature of decisions made by corporate management and outside stakeholders. At the time that outside stakeholders need to make their choices, the actions of corporate managers are sunk, so to these outside stakeholders the firm's wealth *feels* very much like an exogenous random variable. Sequential rationality dictates that they will demand the most accurate information possible about the firm's wealth so that they can minimize the probability of their own decision errors. This fact is reflected in Proposition 6.

It is tempting for regulators to adopt the sequential perspective of firms' outside stakeholders. If these users of fair value information demand it, regulators are likely to require that the information be supplied. Yet the higher wisdom requires an understanding of how the entire equilibrium changes in response to the disclosure mandate of regulators. In the equilibrium that we have described, and also (we think) in the real world, many of the actions taken by corporate managers are significantly influenced by the anticipation of future actions by outside stakeholders and, in turn, these outside stakeholders are affected by the

earlier decisions made by corporate managers. In such an interactive world, our results cast doubt on the wisdom of mandating fair value accounting.

Appendix

Proof of Proposition 1:

First, we calculate the value of Q from (11). From (14) it follows that:

$$\begin{aligned}
 \sum_{t=0}^{\infty} (1-\tau)^t \theta^{(t+1)} &= \sum_{t=0}^{\infty} (1-\tau)^t [\delta^{t+1} \theta + (1-\delta^{t+1})P] \\
 &= \delta \theta \left[\sum_{t=0}^{\infty} (1-\tau)^t \delta^t \right] + P \sum_{t=0}^{\infty} (1-\tau)^t - \delta P \left[\sum_{t=0}^{\infty} (1-\tau)^t \delta^t \right] \\
 &= \frac{\delta \theta}{1-(1-\tau)\delta} + \frac{P}{\tau} - \frac{\delta P}{1-(1-\tau)\delta} \\
 &= \frac{1}{\tau} \left[\left(\frac{\tau \delta}{1-(1-\tau)\delta} \right) \theta + \left(1 - \frac{\tau \delta}{1-(1-\tau)\delta} \right) P \right]
 \end{aligned}$$

Inserting this expression into (11) yields the expression described in (16).

Now, from (16) it follows that:

$$E_i(Q) = \frac{1}{\tau} \left\{ \tau \eta + (1-\tau)(m-z) + (1-\tau)z \left[\left(\frac{\tau \delta}{1-(1-\tau)\delta} \right) E_i(\theta) + \left(1 - \frac{\tau \delta}{1-(1-\tau)\delta} \right) P \right] \right\} \quad (A1)$$

Substituting (A1) into (4) and using $E_i(\theta) = \delta x_i + (1-\delta)P$ gives:

$$\begin{aligned}
 q_i &= [\tau \eta + (1-\tau)(m-z)] \left[1 + \left(\frac{1-\tau}{\tau} \right) \right] + (1-\tau)z [\delta x_i + (1-\delta)P] + \\
 &\quad (1-\tau)z \left(\frac{1-\tau}{\tau} \right) \left[\left(\frac{\tau \delta}{1-(1-\tau)\delta} \right) \{\delta x_i + (1-\delta)P\} + \left(1 - \frac{\tau \delta}{1-(1-\tau)\delta} \right) P \right] \quad (A2)
 \end{aligned}$$

Collect the terms in (A2) that depend on x_i and the terms that depend on P . The term

that depends on x_i is:

$$(1-\tau)z\delta x_i \left[1 + \left(\frac{1-\tau}{\tau} \right) \left(\frac{\tau\delta}{1-(1-\tau)\delta} \right) \right] = \frac{(1-\tau)z\delta x_i}{1-(1-\tau)\delta}$$

which is convenient to write as:

$$= \left(\frac{1-\tau}{\tau} \right) \left(\frac{\tau\delta}{1-(1-\tau)\delta} \right) z x_i \quad (\text{A3})$$

Also in (A2) the terms that depend on P are:

$$\begin{aligned} & (1-\tau)zP \left[(1-\delta) \left\{ 1 + \left(\frac{1-\tau}{\tau} \right) \left(\frac{\tau\delta}{1-(1-\tau)\delta} \right) \right\} + \left(\frac{1-\tau}{\tau} \right) \left(1 - \frac{\tau\delta}{1-(1-\tau)\delta} \right) \right] \\ &= (1-\tau)zP \left[\left(\frac{1-\delta}{1-(1-\tau)\delta} \right) + \left(\frac{1-\tau}{\tau} \right) \left(\frac{1-\delta}{1-(1-\tau)\delta} \right) \right] \\ &= \left(\frac{1-\tau}{\tau} \right) \left(\frac{1-\delta}{1-(1-\tau)\delta} \right) zP \\ &= \left(\frac{1-\tau}{\tau} \right) \left(1 - \frac{\tau\delta}{1-(1-\tau)\delta} \right) zP \quad (\text{A4}) \end{aligned}$$

Inserting (A3) and (A4) into (A2) and simplifying gives:

$$q_i = \frac{1}{\tau} \left\{ \tau\eta + (1-\tau)(m-z) + (1-\tau)z \left[\left(\frac{\tau\delta}{1-(1-\tau)\delta} \right) x_i + \left(1 - \frac{\tau\delta}{1-(1-\tau)\delta} \right) P \right] \right\}$$

as claimed in Proposition 1.

Proof of Lemma 1:

Both parts of the Lemma are true if the factor $\lambda^2 + (1 - \lambda^2) \left(\frac{\beta}{\alpha + \beta} \right)$ is strictly

increasing in β . Using $\lambda \equiv \frac{\tau\delta}{1 - (1 - \tau)\delta}$ and $\delta \equiv \frac{\gamma}{\alpha + \beta + \gamma}$ gives $\lambda = \frac{\tau\gamma}{\alpha + \beta + \tau\gamma}$.

Therefore the factor:

$$\begin{aligned} \lambda^2 + (1 - \lambda^2) \left(\frac{\beta}{\alpha + \beta} \right) &= \left(\frac{\tau\gamma}{\alpha + \beta + \tau\gamma} \right)^2 + \left(1 - \frac{\tau^2\gamma^2}{(\alpha + \beta + \tau\gamma)^2} \right) \left(\frac{\beta}{\alpha + \beta} \right) \\ &= \frac{1}{(\alpha + \beta + \tau\gamma)^2} \left[\tau^2\gamma^2 + [(\alpha + \beta + \tau\gamma)^2 - \tau^2\gamma^2] \left(\frac{\beta}{\alpha + \beta} \right) \right] \\ &= \frac{\tau^2\gamma^2 + \beta(\alpha + \beta) + 2\beta\tau\gamma}{(\alpha + \beta + \tau\gamma)^2} \end{aligned}$$

Therefore,

$$\begin{aligned} \text{sign} \frac{\partial}{\partial \beta} \left\{ \lambda^2 + (1 - \lambda^2) \left(\frac{\beta}{\alpha + \beta} \right) \right\} &= \\ \text{sign} \left\{ (\alpha + \beta + \tau\gamma)^2 (\alpha + 2\beta + 2\tau\gamma) - 2(\alpha + \beta + \tau\gamma)(\tau^2\gamma^2 + \beta(\alpha + \beta) + 2\beta\tau\gamma) \right\} &= \\ \text{sign} \left\{ (\alpha + \beta + \tau\gamma)[(\alpha + \beta + \tau\gamma)(\alpha + 2\beta + 2\tau\gamma) - 2(\tau^2\gamma^2 + \beta(\alpha + \beta) + 2\beta\tau\gamma)] \right\} &= \\ \text{sign} \left\{ \begin{aligned} &(\alpha + \beta + \tau\gamma)[\alpha(\alpha + \beta + \tau\gamma) + 2\beta(\alpha + \beta) + 2\beta\tau\gamma + \\ &2\tau\gamma(\alpha + \beta) + 2\tau^2\gamma^2 - 2\tau^2\gamma^2 - 2\beta(\alpha + \beta) - 4\beta\tau\gamma] \end{aligned} \right\} &= \\ \text{sign} \left\{ (\alpha + \beta + \tau\gamma)[\alpha(\alpha + \beta + \tau\gamma) + 2\alpha\tau\gamma] \right\} &> 0 \end{aligned}$$

Q.E.D.

Proof of Proposition 2:

We have previously argued that from the perspective of date 0,

$var(\tilde{w}) = z^2 var(\tilde{\theta}) + var(\tilde{Q}) + 2z cov(\tilde{\theta}, \tilde{Q})$. In Proposition 2 we are holding z fixed and $var(\tilde{\theta})$ is a prior variance that is unaffected by the precision of accounting disclosure. We have shown in Lemma 1 that $var(\tilde{Q})$ is strictly increasing in the precision of public disclosure. Therefore, it suffices to establish that $cov(\tilde{\theta}, \tilde{Q})$ is also strictly increasing in the precision of public disclosure. But, from (20), $cov(\tilde{\theta}, \tilde{Q})$ is strictly increasing in β if the factor $\lambda + (1 - \lambda) \left(\frac{\beta}{\alpha + \beta} \right)$ is strictly increasing in β .

Inserting $\lambda = \frac{\tau\gamma}{\alpha + \beta + \tau\gamma}$ gives,

$$\begin{aligned} \lambda + (1 - \lambda) \left(\frac{\beta}{\alpha + \beta} \right) &= \frac{\tau\gamma}{\alpha + \beta + \tau\gamma} + \left(1 - \frac{\tau\gamma}{\alpha + \beta + \tau\gamma} \right) \left(\frac{\beta}{\alpha + \beta} \right) \\ &= \frac{\beta + \tau\gamma}{\alpha + \beta + \tau\gamma} \end{aligned}$$

which is strictly increasing in β .

Q.E.D.

Proof of Proposition 5:

As derived in (16):

$$Q = \frac{1}{\tau} \left\{ \tau\eta + (1 - \tau)(m - z) + (1 - \tau)z (\lambda\theta + (1 - \lambda)P) \right\}$$

The term $\lambda\theta + (1 - \lambda)P$ can be written as:

$$\begin{aligned}
& \lambda\theta + (1-\lambda) \left(\frac{\alpha(\theta - \theta + \mu) + \beta(\theta + \varepsilon)}{\alpha + \beta} \right) \\
&= \theta - (1-\lambda) \left(\frac{\alpha}{\alpha + \beta} \right) (\theta - \mu) + (1-\lambda) \left(\frac{\beta}{\alpha + \beta} \right) \varepsilon \\
&= \theta - \left(\frac{\alpha}{\alpha + \beta + \tau\gamma} \right) (\theta - \mu) + \left(\frac{\beta}{\alpha + \beta + \tau\gamma} \right) \varepsilon
\end{aligned}$$

where we have used $(1-\lambda) = \frac{\alpha + \beta}{\alpha + \beta + \tau\gamma}$ as derived in the proof of Lemma1. Therefore,

$$Q = \frac{1}{\tau} \left[\tau\eta + (1-\tau)(m - z + z\theta) + (1-\tau)z \left\{ - \left(\frac{\alpha}{\alpha + \beta + \tau\gamma} \right) (\theta - \mu) + \left(\frac{\beta}{\alpha + \beta + \tau\gamma} \right) \varepsilon \right\} \right]$$

Therefore,

$$\begin{aligned}
[\tau\eta + (1-\tau)(m - z + z\theta)]Q &= \frac{1}{\tau} [\tau\eta + (1-\tau)(m - z + z\theta)]^2 + \\
&\quad \frac{1}{\tau} [\tau\eta + (1-\tau)(m - z + z\theta)] \left[(1-\tau)z \left\{ - \frac{\alpha}{\alpha + \beta + \tau\gamma} (\theta - \mu) + \frac{\beta}{\alpha + \beta + \tau\gamma} \varepsilon \right\} \right]
\end{aligned}$$

and,

$$\begin{aligned}
Q^2 &= \frac{1}{\tau^2} [\tau\eta + (1-\tau)(m - z + z\theta)]^2 \\
&\quad + \frac{2}{\tau^2} [\tau\eta + (1-\tau)(m - z + z\theta)] \left[(1-\tau)z \left\{ - \frac{\alpha}{\alpha + \beta + \tau\gamma} (\theta - \mu) + \frac{\beta}{\alpha + \beta + \tau\gamma} \varepsilon \right\} \right] \\
&\quad + \left(\frac{1-\tau}{\tau} \right)^2 z^2 \left\{ - \frac{\alpha}{\alpha + \beta + \tau\gamma} (\theta - \mu) + \frac{\beta}{\alpha + \beta + \tau\gamma} \varepsilon \right\}^2
\end{aligned}$$

Also, from (15) and (16),

$$\int (q_i - Q)^2 di = \frac{1}{\tau^2} (1-\tau)^2 z^2 \left(\frac{\tau\gamma}{\alpha + \beta + \tau\gamma} \right)^2 \int (x_i - \theta)^2 di$$

Therefore, from (26):

$$\begin{aligned}
\Omega &= [\tau\eta + (1-\tau)(m-z+z\theta)]Q - \frac{1}{2}(2\tau-1)Q^2 - \frac{1}{2}\int (q_i - Q)^2 di \\
&= \frac{1}{2\tau^2}[\tau\eta + (1-\tau)(m-z+z\theta)]^2 + \\
&\quad \left(\frac{1-\tau}{\tau^2}\right)[\tau\eta + (1-\tau)(m-z+z\theta)] \left[(1-\tau)z \left\{ -\frac{\alpha}{\alpha+\beta+\tau\gamma}(\theta-\mu) + \frac{\beta}{\alpha+\beta+\tau\gamma}\varepsilon \right\} \right] \\
&\quad - \frac{1}{2\tau^2}(2\tau-1)(1-\tau)^2 z^2 \left[-\frac{\alpha}{\alpha+\beta+\tau\gamma}(\theta-\mu) + \frac{\beta}{\alpha+\beta+\tau\gamma}\varepsilon \right]^2 \\
&\quad - \frac{1}{2\tau^2}(1-\tau)^2 z^2 \left(\frac{\tau\gamma}{\alpha+\beta+\tau\gamma} \right)^2 \int (x_i - \theta)^2 di
\end{aligned}$$

Taking an expectation over the random variables θ and ε and using

$$E[(\theta - \mu)^2] = \text{var}(\theta) = \frac{1}{\alpha} \text{ and } E[(x_i - \theta)^2] = \text{var}(x_i) = \frac{1}{\gamma}, \text{ gives:}$$

$$\begin{aligned}
E(\Omega | z) &= \frac{1}{2\tau^2}[\tau\eta + (1-\tau)(m-z+z\mu)]^2 + \frac{1}{2\tau^2}(1-\tau)^2 z^2 \frac{1}{\alpha} \\
&\quad - \left(\frac{1-\tau}{\tau^2}\right)(1-\tau)^2 z^2 \frac{\alpha}{\alpha+\beta+\tau\gamma} E[(\theta)(\theta-\mu)] \\
&\quad - \left(\frac{2\tau-1}{2\tau^2}\right)(1-\tau)^2 z^2 \left[\left(\frac{\alpha}{\alpha+\beta+\tau\gamma}\right)^2 \frac{1}{\alpha} + \left(\frac{\beta}{\alpha+\beta+\tau\gamma}\right)^2 \frac{1}{\beta} \right] \\
&\quad - \frac{1}{2\tau^2}(1-\tau)^2 z^2 \frac{\tau^2\gamma}{(\alpha+\beta+\tau\gamma)^2}
\end{aligned}$$

Collecting terms and using $E[(\theta)(\theta - \mu)] = E(\theta^2) - \mu^2 = \text{var}(\theta) = \frac{1}{\alpha}$ gives,

$$\begin{aligned}
 E(\Omega | z) &= \frac{1}{2\tau^2} [\tau\eta + (1-\tau)(m-z+z\mu)]^2 \\
 &\quad + \frac{1}{2\tau^2} (1-\tau)^2 z^2 \left[\frac{1}{\alpha} - \frac{2(1-\tau)}{(\alpha + \beta + \tau\gamma)} - (2\tau-1) \frac{\alpha + \beta}{(\alpha + \beta + \tau\gamma)^2} - \frac{\tau^2 \gamma}{(\alpha + \beta + \tau\gamma)^2} \right] \\
 &= \frac{1}{2\tau^2} [\tau\eta + (1-\tau)(m-z+z\mu)]^2 \\
 &\quad + \frac{1}{2\tau^2} (1-\tau)^2 z^2 \left[\frac{1}{\alpha} - \frac{1}{(\alpha + \beta + \tau\gamma)^2} (2-2\tau)(\alpha + \beta + \tau\gamma) + (2\tau-1)(\alpha + \beta) + \tau^2 \gamma \right] \\
 &= \frac{1}{2\tau^2} [\tau\eta + (1-\tau)(m-z+z\mu)]^2 \\
 &\quad + \frac{1}{2\tau^2} (1-\tau)^2 z^2 \left[\frac{1}{\alpha} - \frac{1}{(\alpha + \beta + \tau\gamma)} - \frac{\tau\gamma(1-\tau)}{(\alpha + \beta + \tau\gamma)^2} \right]
 \end{aligned}$$

Q.E.D.

Proof of Proposition 6

Substituting $\frac{\alpha\mu + \beta y}{\alpha + \beta} = \frac{\alpha\mu + \beta(\mu + \xi + \varepsilon)}{\alpha + \beta} = \mu + \left(\frac{\beta}{\alpha + \beta} \right) (\xi + \varepsilon)$ in (26), gives:

$$Q = \frac{1}{\tau} [\tau\eta + (1-\tau)(m-z) + (1-\tau)z\mu] + \frac{1}{\tau} [(1-\tau)z \left(\frac{\beta}{\alpha + \beta} \right) (\xi + \varepsilon)]$$

Substituting this expression for Q into (32) and using $\theta = \mu + \xi$ gives:

$$\begin{aligned}
\Omega(\xi, \varepsilon, z) = & \frac{1}{\tau} [\tau\eta + (1-\tau)(m-z) + (1-\tau)z\mu]^2 + \\
& \frac{1}{\tau} [\tau\eta + (1-\tau)(m-z) + (1-\tau)z\mu] \left[(1-\tau)z \left(\frac{\beta}{\alpha + \beta} \right) (\xi + \varepsilon) + (1-\tau)z\xi \right] + \\
& \frac{1}{\tau} (1-\tau)^2 z^2 \left(\frac{\beta}{\alpha + \beta} \right) \xi (\xi + \varepsilon) - \frac{1}{2} (2\tau - 1) Q^2
\end{aligned}$$

Therefore,

$$\begin{aligned}
E_{y,\theta} \{ \Omega(\theta, y; z) \} = E_{\xi,\varepsilon} \{ \Omega(\xi, \varepsilon; z) \} = \\
\frac{1}{\tau} (\tau\eta + (1-\tau)(m-z) + (1-\tau)z\mu)^2 + \\
\frac{1}{\tau} (1-\tau)^2 z^2 \left(\frac{\beta}{\alpha + \beta} \right) \frac{1}{\alpha} - \frac{1}{2} (2\tau - 1) E_{\xi,\varepsilon} (Q^2)
\end{aligned} \tag{A5}$$

But,

$$\begin{aligned}
E_{\xi,\varepsilon} (Q^2) = & \frac{1}{\tau^2} (\tau\eta + (1-\tau)(m-z) + (1-\tau)z\mu)^2 + \\
& \frac{1}{\tau^2} (1-\tau)^2 z^2 \left(\frac{\beta}{\alpha + \beta} \right)^2 \left(\frac{1}{\alpha} + \frac{1}{\beta} \right)
\end{aligned}$$

Substituting this expression into (A5) gives,

$$\begin{aligned}
E_{y,\theta} \{ \Omega(\theta, y; z) \} = & (\tau\eta + (1-\tau)(m-z) + (1-\tau)z\mu)^2 \left(\frac{1}{\tau} - \frac{2\tau - 1}{2\tau^2} \right) \\
& + \frac{1}{\tau} (1-\tau)^2 z^2 \left(\frac{\beta}{\alpha + \beta} \right) \frac{1}{\alpha} - \left(\frac{2\tau - 1}{2\tau^2} \right) (1-\tau)^2 z^2 \left(\frac{\beta}{\alpha + \beta} \right) \frac{1}{\alpha}
\end{aligned}$$

which implies:

$$\begin{aligned}
E_{y,\theta} \{ \Omega(\theta, y; z) \} &= \frac{1}{2\tau^2} \left(\tau\eta + (1-\tau)(m-z) + (1-\tau)z\mu \right)^2 \\
&\quad + \frac{1}{2\tau^2} (1-\tau)^2 z^2 \left(\frac{\beta}{\alpha + \beta} \right) \frac{1}{\alpha}
\end{aligned}$$

which is strictly increasing in β .

Q.E.D.

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