Accounting Rules for Debt Covenants[*]

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November 2012

Abstract. This paper examines the properties of accounting rules designed to maximize the efficiency of accounting-based debt covenants in a setting with incomplete contracts and asset substitution. Accounting takes the role of a state-contingent decision facilitator by inducing covenant violations, and hence a transfer of decision rights from the borrowing firm to the lender, whenever the lender has less detrimental decision incentives than the firm. This representation of accounting is distinctive in two respects. First, accounting does not have to enlarge the contracting parties' information sets but is nonetheless valuable because it provides verifiable information and hence improves contract efficiency. Second, accounting is an endogenously chosen information aggregation rule rather than an information signal with exogenous properties. The optimal accounting rule in this setting is a function of cash flows and fair value estimates and implements a debt contract with a unique optimal tradeoff between decision rights and interest rates. The optimal degree of conservatism of this accounting rule, measured as the tendency to yield low accounting measures and hence more frequent covenant violations, is higher for firms with high leverage, low profitability, and high liquidation values.

[*] I would like to thank Stefan Reichelstein, Anne Beyer, Ilan Guttman, Ivan Marinovic, Maureen McNichols, Jeffrey Zwiebel, and participants at the faculty workshop and the joint accounting and finance student seminar at Stanford University for many helpful comments and suggestions.

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1. Introduction

The objective of this paper is the derivation and characterization of accounting rules that maximize the efficiency of debt contracts. In practice, debt contracts commonly contain covenants that are based on accounting measures such as book value of equity, leverage, or income. The properties of the accounting rules underlying the computation of these measures therefore determine when the borrower has violated the covenant. Covenant violations, also referred to as "technical default," tend to result in changes to the borrowing firm's operating or financing decisions, most commonly because the lender receives the right to call the debt. For example, the lender may force the firm into liquidation by insisting on immediate payment of the borrowed amount or, in exchange for waiving its right to do so, compel the firm to curtail dividends, undertake or forgo certain investments, pay a higher rate of interest, or maintain a certain level of liquidity or leverage. Covenants can also specify remedies directly, e.g. an automatic requirement for additional collateral or an increase in interest rates upon violation.

The decision consequences of covenant violations, or the absence thereof, can have detrimental effects on total firm value. In debt contracts, the asymmetric distribution of payoffs between borrower and lender induces the well-known asset substitution problem, in which the borrower has a preference for (possibly inefficiently) high risk while the lender has a preference for (possibly inefficiently) low risk. The accounting process can mediate this agency conflict and thereby limit the resulting inefficiency by summarizing information about the state of the world into an accounting measure. Based on this measure, a debt covenant can then induce decision consequences that are least affected by the contracting parties' detrimental incentives. In other words, accounting takes the role of a state-contingent rule that assigns decision rights. The derivation of its optimal functional form is the objective of the following analysis.

The model investigates how the optimal accounting rule and the agency cost of debt are determined in this setting. Specifically, the model addresses the following questions. i) How do the decision incentives of borrower and lender relate to the first-best decision rule that maximizes total firm value? How are these incentives affected by information about past and future outcomes? ii) What is the functional form of the accounting rule that implements the optimal debt covenant? Is it unique? iii) How does the optimal accounting rule vary with the borrower's lev-

erage, the profitability of the underlying investment, the opportunity cost of decision alternatives, and the variance of payoffs? The results of the analysis suggest that the optimal allocation of decision rights is unique. The accounting measure that implements this optimal allocation increases in past cash flows and decreases in the firm's estimated liquidation value. Optimal accounting rules are more conservative, in the sense that they induce more frequent covenant violations, when the borrowing firm has higher leverage, lower profitability, or a relatively high liquidation value. Finally, debt financing becomes more costly in general if the firm has high operating risk.

Prior research has studied the problem of control allocation in the presence of incentives for asset substitution but has generally focused on different aspects. Caskey and Hughes (2012) examine state-contingent control allocation based on an accounting rule in a setting with complete information and a non-contractible project selection problem. The authors consider several functional forms of the accounting rule and find that rules allocating control predominantly to the lender perform best. Sridhar and Magee (1997) analyze an incomplete contract setting in which the borrowing firm can, with some probability, overstate the contractible accounting measure underlying the debt covenant in order to avoid costly covenant violations. The authors conclude that such uncertain, opportunistic accounting manipulation can lead to underinvestment but submit that the optimal contract terms may not be unique. Gârleanu and Zwiebel (2009) study costly renegotiation of debt contracts when the borrowing firm is privately informed about the severity of the asset substitution problem before the contract is signed. Because of the resulting adverse selection problem, the optimal contract in this setting assigns control rights to the uninformed lender. Control allocation in Gârleanu and Zwiebel is unconditional and hence does not require accounting information.

An important feature of the model in this paper is that accounting does not by itself generate new information but rather serves the role of a decision facilitator by aggregating existing information and allocating control rights. A number of other accounting models in the debt contract setting have focused on the informational role of accounting instead. Gigler, Kanodia, Sapra and Venugopalan (2009) study a complete contract setting in which accounting provides information about future outcomes. Under the assumption that the liquidation value of the firm's invested assets is below the expected continuation value, the authors conclude that accounting conservatism, interpreted as the degree to which high realizations of the accounting measure are more informative than low realizations, reduces the efficiency of the contract. In Göx and Wagenhofer

(2010), accounting is an information signal about the value of the borrowing firm's collateral. The authors conclude that the optimal accounting system is designed so that low accounting signals, which indicate low collateral values, are more precise but less frequent than high accounting signals because the firm seeks to maximize the probability that the reported collateral value is above the minimum threshold required to obtain financing, even if such high reported collateral values are less precise. Burkhardt and Strausz (2009) model a firm that can engage in asset substitution after signing a debt contract but must sell its existing investment in a secondary capital market in order to do so. If the firm's accounting report is uninformative, the secondary market does not learn the value of the existing investment, and the resulting adverse selection problem prevents a large proportion of firms from selling their investments and engaging in inefficient asset substitution.

The properties of accounting-based debt covenants have also been studied extensively in empirical research. For example, Beneish and Press (1993) document that technical default is costly to violating firms because subsequent renegotiation of the contract tends to result in higher interest rates, increases in collateral, or the prohibition of certain investing or financing transactions, and may force firms to divest part of their operations in order to meet payment obligations. Dichev and Skinner (2002) observe that firms tend to make discretionary accounting choices in order to avoid debt covenant violations and that covenants are generally tight and violated frequently. The authors also note that covenants are often based on GAAP that have been adjusted for borrower-specific characteristics.

The role of accounting conservatism in debt contracting has been investigated in a number of empirical studies. Ahmed, Billings, Morton and Stanford-Harris (2002) present evidence that firms apply more conservative accounting practices when the conflict between shareholders and bondholders over dividend policy is severe. In addition, their results indicate that firms with more conservative accounting achieve higher credit ratings. Zhang (2008) finds that borrowing firms with more conservative accounting practices are more likely to violate their debt covenants after negative stock price shocks but pay lower interest rates to their lenders. Beatty, Weber and Yu (2008) investigate to what extent accounting conservatism and the restrictiveness of debt covenants, both of which increase the likelihood of technical default, are used as substitutes and find that accounting conservatism and conservative covenant modifications are both frequently

found in debt contracts and are often applied jointly, which suggests that the two are not completely substitutable in practice.

The asset substitution problem underlying the latent contractual inefficiency in this setting is specific to the payoff structure of the debt contract. Hence, the model cannot explain the optimality of debt financing relative to other forms of capital and takes the contracting parties' payoff functions as exogenously given. While not modeled explicitly, the reasons for preferring debt over other forms of financing can take various forms. For example, research in corporate finance has suggested that debt may be useful as a signaling device when managers need to convey their private knowledge about firm value to outside investors in a credibly way (Ross, 1977), or as a commitment mechanism for managers to take some optimal action in a moral hazard setting (Grossman and Hart, 1982). In both cases, debt financing is beneficial because bankruptcy is modeled as a personally costly event to managers. Further, debt may be an efficient choice if verification of outcomes is costly (Townsend, 1979), insider equity investors can consume private benefits at the expense of outsider equity investors (Jensen and Meckling, 1976), or issuers of securities prefer the unconditional payoff function of debt because it can mitigate the adverse selection resulting from information asymmetry in the capital market (Myers and Majluf, 1984).

The assumption of a fixed payoff function rules out a large class of decision mechanisms that rely on contingent transfer payments, which would require a deviation from the payoff structure of the standard debt contract.[1] For example, the asset substitution problem does not arise in connection with equity financing. The model therefore differs from studies like Aghion and Bolton (1992) in that it solves neither a complete security design problem (which would require endogenously chosen payoff functions) nor a complete capital structure problem (which would require modeling the agency cost of alternative forms of financing). Yet, solving security design or capital structure problems in generality almost invariably requires simplifying assumptions such as one-dimensional contractible information signals or reduced-form representations of agency costs. The model in this paper, by contrast, contains an endogenous agency problem with two-dimensional information signals. While limited to a specific type of financing contract, this setting can yield some insights into the intrinsic relationship between incentives, information and inefficiency that more general but simplified settings might not permit.

[1] The term 'standard debt contract' refers to a fixed interest rate or a fixed spread above the risk-free rate. In practice, some deviations from this rigid structure occur. For example, Asquith, Beatty and Weber (2005) document various benefits of performance pricing, i.e. the adjustment of spreads depending on the borrower's financial performance.

The remainder of the paper is organized as follows. Section 2 introduces the setup of the debt contracting model and the related accounting system. The optimal debt contract and its underlying accounting rules are derived in Section 3, along with a discussion of its salient properties. Section 4 contains an analysis of the relationship between the optimal contract terms, accounting conservatism and the firm's capital structure. Section 5 provides some general concluding comments and caveats about the implications of the results.

2. Model Setup

A firm has an investment opportunity with positive expected net present value.[2] The project requires a capital outlay of I at time $t = 0$ and yields uncertain cash flows of c_t during the subsequent periods $t = 1,2$ if continued to the end. Alternatively, the firm's owners can sell the enterprise after the first period for a liquidation value v. The total payoffs are therefore either $c_1 + c_2$ under continuation or $c_1 + v$ under liquidation. The cash flows are distributed on the interval $[0, \bar{c}]$ with cumulative probability distribution functions F_t and continuously differentiable density f_t. Likewise, the liquidation value is distributed with cumulative probability F and continuously differentiable density f on the same interval $[0, \bar{c}]$. Cash flows and liquidation values are independent and their distributions are common knowledge. For simplicity, the firm is assumed to undertake only this single project, so that the only business decision to be made subsequent to initiating the project is the binary choice between liquidation and continuation at $t = 1$.

For lack of internal funds, the firm must obtain the requisite capital for the project from an outside investor. Initially, the firm is assumed to finance I entirely through debt for now. This assumption will be relaxed in Section 4. The lender's payoff function is defined by a maturity value L, which the firm must repay upon conclusion of the project or at the time of liquidation. If the firm's cash flows or liquidation proceeds are insufficient to cover the maturity value L, the lender receives all available payoffs. In addition, the debt contract includes a covenant upon whose violation at $t = 1$ the lender may demand immediate payment of L and thereby force the firm into liquidation. The debt covenant is based on the firm's accounting book value of equity

[2] Throughout this text, "the firm" refers to the firm's owners and managers, who are assumed to seek to maximize the value of the firm's equity. Intra-firm agency conflicts between owners and managers are not part of this model.

$BV(\cdot)$, which can be an arbitrary function of any contractible variables. In particular, the firm is considered to be in violation of the covenant if its book value falls below some threshold value BV_0. The accounting rule $BV(\cdot)$ and the threshold value BV_0 are agreed upon as part of the contract at time $t = 0$.[3]

The lender is assumed to break even in expectation, i.e., the firm raises I in a competitive capital market.[4] Both parties are risk-neutral and the risk-free interest rate is normalized to zero. Hence, the terms of the financing contract are set so that the investor's expected payoff is I, after considering the anticipated liquidation or continuation decisions at $t = 1$ across all possible realizations of BV. In order to simplify the analysis, realized cash flows must first be used to repay the lender, and the financing contract is assumed to prohibit the firm from issuing additional debt or equity, distributing dividends, and making new investments for the duration of the project. Permitting these decisions would complicate the analysis but not alter the fundamental dynamics of the problem.[5]

The information structure of the problem is as follows. The firm's cash flows c_t are assumed to be observable and verifiable for contracting purposes after they have been realized, for example via an audit. No new information about cash flows arises over time, i.e., all available information about c_t is summarized by the prior distribution F_t. The liquidation value v, in contrast, is assumed to be observable but unverifiable even at time $t = 1$. This lack of contractibility arises in practice when the value of the firm's assets depends on factors that cannot be described in operational terms ex ante. For example, a manufacturer of board games may become aware of a shift in consumer taste toward electronic games at $t = 1$, which would adversely affect v, but this observation could not be anticipated and cast in verifiable terms in a contract at $t = 0$.

While v itself is not directly verifiable, some verifiable information correlated with v is assumed to be available in the form of an information signal s. For example, the value a firm's inventory could be estimated by observable market prices of like goods, the value of its buildings by prevalent real estate prices, or the value of its notes receivable by interest levels and historical default rates. Such inputs can be used in a valuation model to construct a contractible estimate of

[3] For simplicity, the variable BV will refer to both the accounting rule, i.e. the rules according to which book value is computed, and the firm's actual book value, i.e. the output.
[4] One could without loss of generality require that the investor obtain a positive economic profit in expectation.
[5] In the language of Christensen and Nikolaev (2011), such restrictions on financing transactions would constitute a 'capital covenant,' whereas the covenant based on BV above would be considered a 'performance covenant.'

the firm's liquidation value at time $t = 1$. The inputs are necessarily imperfect because they are constrained to come from verifiable information sources, and so s can be thought of as a 'noisy' version of v. In accounting terminology, s represents an estimate of the fair value of the firm's invested assets in the resale market.[6] It induces a posterior distribution with cumulative probability $G(v|s)$ and density $g(v|s)$ whose properties are common knowledge. In the extreme case when verifiable information sources provide no useful information about v, s is uninformative and so the conditional distribution $G(v|s)$ is equal to the prior distribution $F(v)$. In the converse case when s is a perfect estimate of v, $G(v|s)$ is degenerate, with $\Pr(v = s) = 1$.

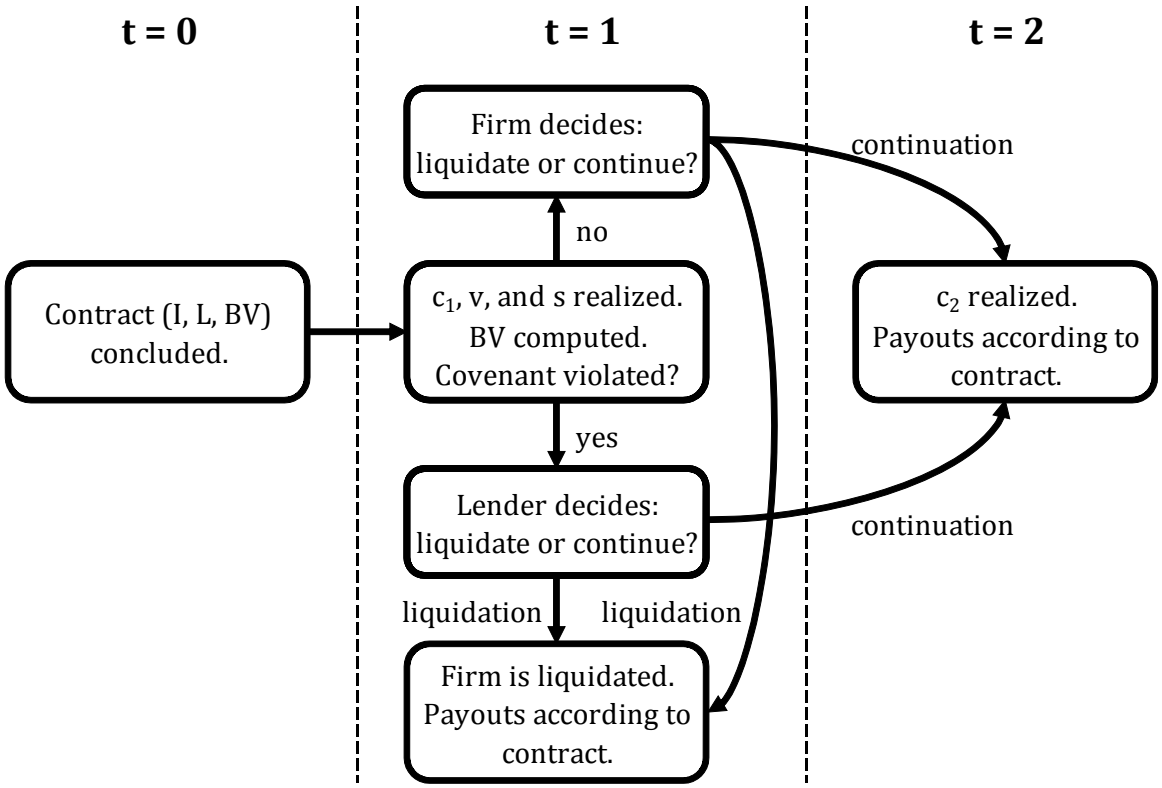

Figure 1. Timeline of the model.

The timeline of the model is summarized in Figure 1. At $t = 0$, both parties observe the distributional properties of all variables and agree on a debt contract with elements I, L and a covenant based on the firm's book value of equity BV. If BV falls below the contractually specified threshold value BV_0 at $t = 1$, the firm is considered to have violated the covenant and the lender

[6] Alternatively, s could be assumed to provide information about c_2 or about both v and c_2. The conclusions obtained in these scenarios would be consistent with those presented here.

may call the debt and thus enforce liquidation. Otherwise, decision rights remain with the firm. If liquidation occurs, proceeds in the amount of v are realized. The lender receives any amount up to L and the firm receives the remainder. If the project is continued instead, c_2 is realized in $t = 2$, and the lender receives all payoffs up to L while the firm receives the remainder.

Given the above information structure, the book value BV at $t = 1$ can be based on the contractible information variables c_1 and s.[7] For the sake of brevity, a pair (c_1, s) will be denoted by θ and the set of all contractible states θ by $\Theta \subset \mathbb{R}^2$. The cumulative probability of θ will be denoted by H and the joint density by h. The accounting rule is a continuous mapping

$$BV: \Theta \to \mathbb{R}$$

The covenant effectively partitions Θ into the 'technical default' subset

$$D \equiv \{\theta: BV < BV_0\}$$

of states in which the firm has violated the covenant and the 'no-default' subset

$$N \equiv \{\theta: BV \geq BV_0\}$$

of states in which book value is above BV_0 and hence no violation has occurred.[8] Figure 2 provides a graphical illustration of a possible partition of Θ into D and N, whereby the path of cutoff states θ^* along which $BV = BV_0$ identifies the boundary between the two subsets.

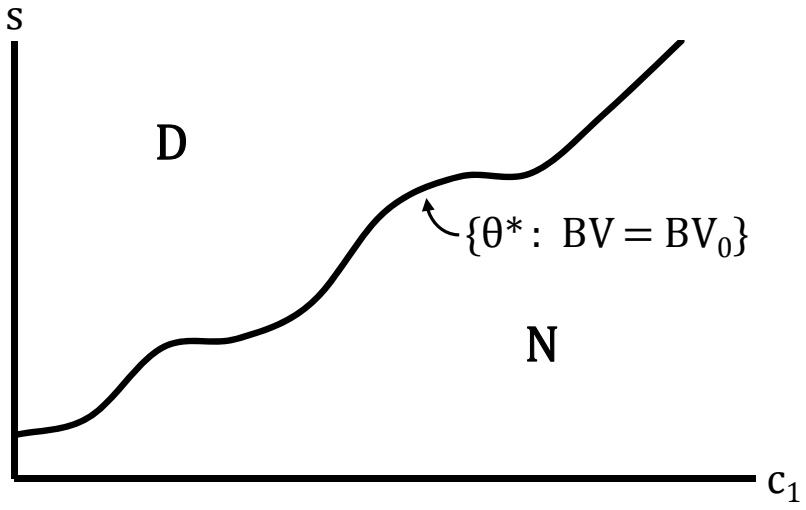

Figure 2. Control allocation between firm and lender.

[7] The accounting rule is solely based on verifiable information, i.e. discretionary accounting choices and strategic manipulation of accounting estimates are not part of this model.
[8] Assigning $\{\theta: BV = BV_0\}$ to N rather than D is without loss of generality. All results in this paper are formulated to hold in either case.

This paper is concerned with the efficiency of the debt contract and the underlying accounting rule. Efficiency is the degree to which the liquidation or continuation decisions taken under a given contract maximize total firm value, including both debt and equity. Liquidation and continuation are opportunity costs and both parties are risk-neutral, so liquidating the firm at $t = 1$ is efficient whenever

$$v > \mu_2$$

i.e., the liquidation value exceeds the expected cash flows in the second period. If the estimate s were a perfect signal of v, the optimal accounting rule BV could prescribe liquidation directly whenever the firm's book value is below BV_0, and the accounting rule to compute this book value would be

$$BV = BV_0 + \mu_2 - s \tag{1}$$

where $\mu_2 \equiv E(c_2)$. The result would be a complete contract that incurs no inefficiency. Since the underlying accounting rule is independent of the maturity value L in this case, the latter could then be chosen freely to satisfy the lender's break-even constraint. This first-best outcome under complete information will serve as the efficiency benchmark in the analysis to follow.

The option to renegotiate the contract at $t = 1$ is not considered in this paper.[9] This restriction may appear to limit the applicability of the results, and indeed the optimal accounting rule would be almost arbitrary in the symmetric information setting considered here if renegotiation were costless because any ex-ante control allocation would yield first-best efficiency ex-post. The results from a setting without renegotiation are nonetheless relevant for several reasons. First, renegotiation is generally infeasible for publicly traded debt obligations because of frictions such as free-rider problems. Public bonds are therefore rarely renegotiated (Smith and Warner, 1979). Second, renegotiation is generally not costless in reality even if it is feasible. Among other things, renegotiation costs may represent direct legal and administrative expenses, which are likely to be high when the debt capital is provided by a large number of lenders, as well as negative externalities. For example, a major customer of the firm may become aware of the renegotiation, interpret the event as a signal that the firm is in financial difficulty, and seek a new supplier that does not have a potential going concern problem. Hence, a strategic, state-contingent allocation of de-

[9] The term "renegotiation" refers to a negotiated change in contract terms. A waiver of decision rights by one party without any alteration of the contract terms is assumed to be feasible and costless in all cases.

cision rights is meaningful as long as renegotiation costs would outweigh the benefits in at least some states. Finally, a trivial renegotiation to the first-best outcome is generally not attainable when information is asymmetric. While not considered in this paper, the optimal accounting rule in a setting with asymmetric information is affected by the same incentives as those demonstrated in the following analysis, even if renegotiation is permissible and costless. Thus, this paper characterizes basic properties of an optimal accounting rule for debt contracts in a simplified scenario but its results should prove useful in extensions of the model to more realistic settings.

3. Optimal Accounting-Based Debt Covenants

The contract agreed upon at time $t = 0$ depends on the anticipated liquidation or continuation decisions at $t = 1$. It will therefore be convenient to begin the analysis with the two parties' decision preferences. The controlling party at time $t = 1$ chooses liquidation or continuation in order to maximize its payoff. The extent to which this payoff maximization deviates from the first-best rule, i.e. to liquidate whenever $v > \mu_2$, will ultimately determine the optimal allocation of control rights. The parties' decision preferences follow from their payoff functions. In particular, the firm maximizes residual claims and therefore prefers liquidation at $t = 1$ whenever

$$x_v > x_2 \tag{2}$$

where

$$x_v \equiv \max(0, c_1 + v - L)$$

is its expected payoff under liquidation and

$$x_2 \equiv E(\max(0, c_1 + \tilde{c}_2 - L))$$

is its expected payoff under continuation.[10] It should be noted that c_1 has been realized and observed at this time and can therefore be treated as a constant. The firm's liquidation payoff is zero for $v \leq L - c_1$ and changes one-for-one with v from thereon. Then ceteris paribus, there exists at most one cutoff value $v = k^f$ for which (2) holds with equality and above which the firm prefers liquidation.

Similarly, the lender maximizes debt claims and therefore prefers liquidation at time $t = 1$ whenever

[10] Random variables inside expectations are denoted by a tilde.

$$u_v > u_2 \tag{3}$$

where

$$u_v \equiv \min(c_1 + v, L)$$

and

$$u_2 \equiv E(\min(c_1 + \tilde{c}_2, L))$$

As a mirror image to the firm's case, the lender's liquidation payoff increases one-for-one with v when $v \leq L - c_1$ and remains constant once total payoffs exceed the maturity value L. Hence, the lender also has at most one indifference point $v = k^l$ at which, ceteris paribus, (3) holds with equality. The indifference points k^f and k^l will be referred to as *liquidation thresholds*.

As noted above, the decision to liquidate maximizes total firm value whenever $v > \mu_2$ because v and c_2 are opportunity costs. The efficiency of the debt contract thus depends on the distance between μ_2 and the liquidation thresholds k^f and k^l. The inequalities in (2) and (3) suggest that k^f and k^l do not generally coincide with the first-best benchmark value μ_2 because they depend on c_1 and on the debt value L, whereas μ_2 is independent of both. The following proposition establishes a critical regularity in the relationship between the liquidation thresholds and μ_2. The distance $\left|k^i - \mu_2\right|$ for $i \in \{f, l\}$ will be referred to as the *inefficiency range*. The proof of this and of all subsequent results can be found in the Appendix.

Proposition 1. *For any state θ and any maturity value L, the firm never prefers inefficient liquidation and the lender never prefers inefficient continuation. The firm's inefficiency range increases in L and decreases in c_1 while the reverse holds for the lender's inefficiency range.*

In essence, Proposition 1 is simply a reflection of the well-known asset substitution problem. Similar observations arise in other debt contract models, e.g., in the continuous time setting in Décamps and Faure-Grimaud (2002). The misalignment of preferences is illustrated in Figure 3. The lender's liquidation payoff u_v, shown in the left panel, follows the total liquidation value v up to the level of the maturity value L and remains constant thereafter. The lender's continuation payoff u_2 is bounded by the lower of L and μ_2, and hence the indifference point at the intersection of u_v and u_v is bounded from the right by the first-best indifference point $v = \mu_2$. The graph of the firm's liquidation payoff x_v, shown in the right panel, follows the total liquidation value v at a constant distance of L units below, while the distance between the firm's continuation payoff

x_2 and the total expected continuation value μ_2 is bounded above by L. Hence, the firm's indifference point at the intersection of x_v and x_2 is bounded from the left by the first-best indifference point $v = \mu_2$. The liquidation thresholds thus describe the fundamental tension in this model: both parties prefer to maximize the investment's payoff ex ante at $t = 0$ but are unable to implement the maximally efficient outcome because neither can commit to taking the first-best action at $t = 1$.

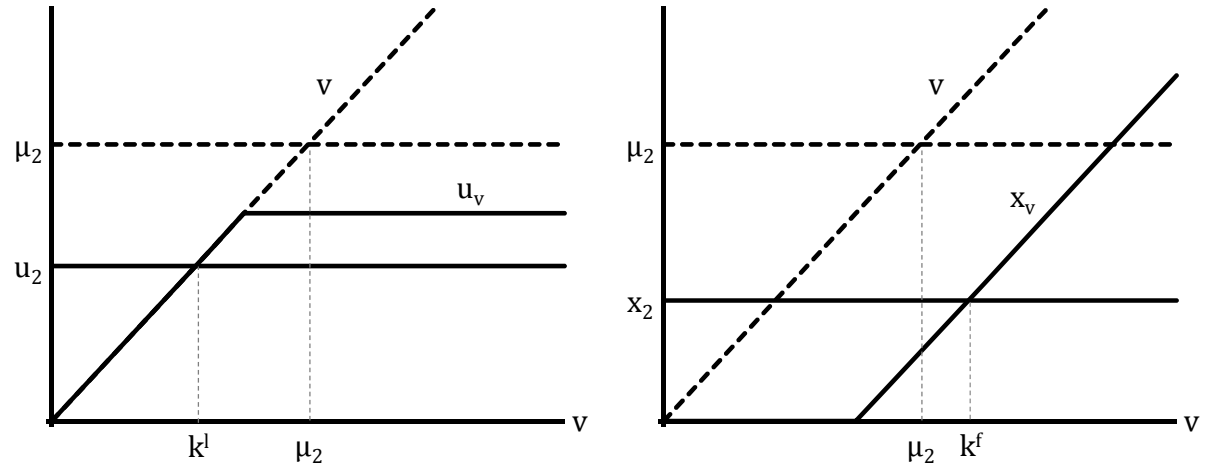

Figure 3. Liquidation thresholds relative to first-best for the lender (left) and the firm (right).[11]

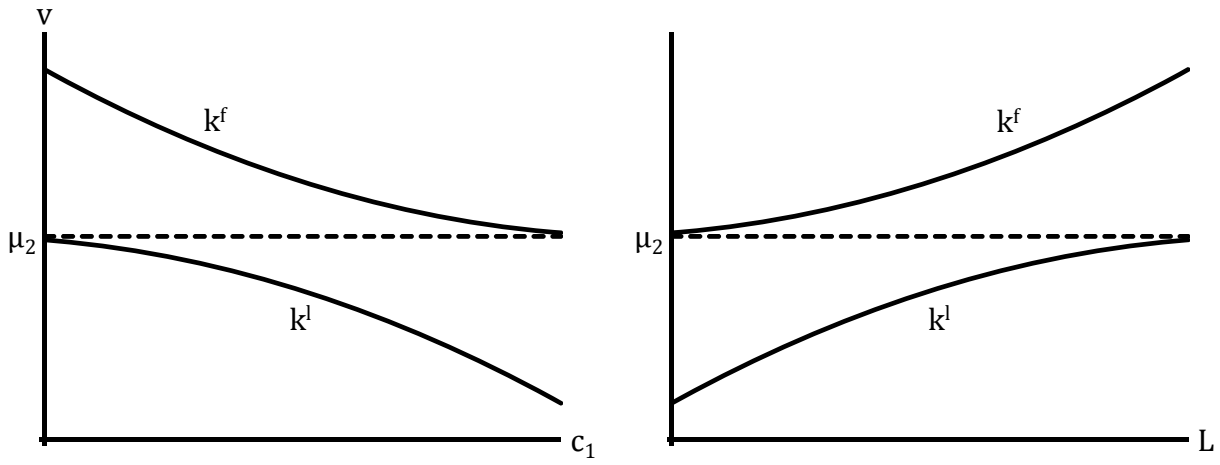

Figure 4. Liquidation preferences as a function of first-period cash flows and liquidation values.

[11] For ease of exposition, the first-period cash flow has been normalized to $c_1 = 0$ in this diagram.

If the contracting parties were to make decisions based on their liquidation thresholds, the firm would cause more inefficiency when the maturity value L is high and c_1 is low while the opposite holds for the lender. The reason is that the balance $L - c_1$ of outstanding debt is similar to the strike price of a call option on total firm value held by the firm's owners: the higher the strike price, the greater the value of potentially inefficient volatility in firm value, and hence the greater the incentive to forgo efficient but riskless liquidation. The lender's incentives naturally move in the opposite direction. This observation is an important determinant of the optimal debt covenant and holds under any assumptions about probability distributions. Figure 4 provides a graphical illustration of the liquidation thresholds as functions of c_1 and L.

Relative to the first-best cutoff point $v = \mu_2$, the expected inefficiency cost incurred if either party makes the liquidation decision according to its incentives is then

$$q^f = \int_{\mu_2}^{k^f} (v - \mu_2)\, dG(v|s)$$

in a given state $\theta = (c_1, s)$ if the decision is delegated to the firm and

$$q^l = \int_{k^l}^{\mu_2} (\mu_2 - v)\, dG(v|s)$$

if the decision is delegated to the lender.[12] Similarly, the payoff to the lender given an allocation of decision rights to party $i \in \{f, l\}$ in a given state θ is

$$u^i = \int_0^{k^i} u_2\, dG + \int_{k^i}^{\bar{c}} u_v\, dG = \int_0^{k^i} E(\min(c_1 + \tilde{c}_2, L))\, dG + \int_{k^i}^{\bar{c}} \min(c_1 + v, L)\, dG$$

The firm's payoff x^i is given by the same expression if u_2 and u_v are replaced by x_2 and x_v.

Whether inefficiency and payoffs are realized according to the liquidation thresholds k^f or k^l depends on the accounting rule BV. Specifically, the expected inefficiency at time $t = 1$ in a given state θ and under a given accounting rule BV can be written as

$$q(BV) = \mathbf{I}_D(BV) \cdot q^l + \big(1 - \mathbf{I}_D(BV)\big) \cdot q^f \tag{4}$$

where $\mathbf{I}_D$ is the technical default indicator function

$$\mathbf{I}_D(BV) = \begin{cases} 0 & \text{if} \quad BV(\theta) \geq BV_0 \\ 1 & \text{if} \quad BV(\theta) < BV_0 \end{cases}$$

Raising or lowering the default threshold BV_0 is equivalent to adding or subtracting a constant from BV, and so no loss of generality is incurred by normalizing the former to $BV_0 = 0$ hereafter.

[12] For brevity, $dG(v|s)$ will be written as dG from hereon. Likewise, $dH(\theta)$ will be abbreviated dH.

The term $q(BV)$ reflects the agency cost of the contract incurred in a particular state θ. Analogously, the firm's and lender's payoffs $u(BV)$ and $x(BV)$ in a given state for a given accounting rule are defined by replacing q in (4) with x and u, respectively. The relationship between q, u and x can be summarized by the following identity. In each state θ, the total payoff from the firm's investment must equal

$$w - q(BV) = x(BV) + u(BV) \tag{5}$$

where the left-hand side is the first-best payoff

$$w \equiv c_1 + \max(v, \mu_2) \tag{6}$$

less the agency cost q, and the right-hand side is the allocation of this payoff to the two parties.

One can interpret (5) as a type of balance sheet equation, where assets are shown on the left and liabilities and equity are shown on the right. The value of the firm's equity, conditional on θ, can therefore be written as

$$x(BV) = w - q(BV) - u(BV) \tag{7}$$

by rearranging (5). When the contract is negotiated at time $t = 0$, the state θ is yet unrealized and so the firm seeks to maximize its equity value x in expectation across all θ, subject to the lender's break-even constraint. The firm's optimization problem at $t = 0$ is therefore

$$\max_{BV(\cdot),L} \int_\Theta [w(\theta) - q(BV(\theta), \theta) - u(BV(\theta), \theta)]\, dH$$

subject to

$$\int_\Theta u(BV(\theta), \theta)\, dH \geq I$$

The firm's optimization program is a simple form of an optimal control problem with a static constraint. For ease of exposition, it will be convenient to write the objective function in Lagrange form as

$$Y(BV, L, \lambda) = \int_\Theta y(\theta, BV, L, \lambda)\, dH \tag{8}$$

where

$$y(\theta, BV, L, \lambda) = w(\theta) - q(BV(\theta), \theta) + (\lambda - 1) \cdot u(BV(\theta), \theta) - \lambda I$$

and $\lambda \geq 0$ is the Lagrange multiplier for the lender's break-even constraint. Determining when this constraint binds is the first step in setting up the necessary conditions for an optimal contract.

Lemma 1. *The lender's break-even constraint binds under any optimal contract.*

Lemma 1 rules out the possibility that the firm may ever choose to overpay the lender. One might suspect that such an overpayment could be optimal if it improves the lender's decision incentives and thereby reduces inefficiency by more than the amount of the overpayment. As equation (7) shows, the firm would reap the entire benefit of this efficiency gain. The reason why such an overpayment is never optimal from the firm's perspective is that whenever the lender's expected payoff is above the required level I, the firm could increase its equity value by either reducing the maturity value L or expanding the no-default set N.

An optimal accounting rule BV induces a violation of the debt covenant, and hence a transfer of decision rights to the lender, in all states θ in which y is higher under $\theta \in D$ than under $\theta \in N$. This condition is equivalent to requiring that

$$\Gamma(L, \lambda, \theta) \equiv q^l(\theta) - q^f(\theta) + (1 - \lambda) \cdot \Big(u^l(\theta) - u^f(\theta)\Big) = 0 \qquad (9)$$

in all critical states $\theta = \theta^*$, i.e. states in which $BV(\theta) = BV_0 = 0$. The formal derivation of Γ can be found in the Appendix under the proof of Lemma 2. Equation (9) states that in any state θ^*, the firm is indifferent between retaining and ceding control if and only if the net effect of transferring control to the lender on the firm's equity value is zero. In economic terms, Γ thus represents the net cost or benefit of technical default, which always accrues to the firm because the lender breaks even by Lemma 1. It should be noted that equation (9) imposes no restrictions on BV for $\theta \neq \theta^*$. This observation is intuitive: if the book value is not at the default threshold, a small increase or decrease in BV does not change whether the firm has violated the covenant or not and hence does not affect control rights, so the value of Y would remain unchanged.

The first-order necessary condition (9) implies that $BV = BV_0$ and $\Gamma = 0$ must hold simultaneously in an optimal contract or, equivalently, control allocation to either party is equally efficient if and only if the book value is at the technical default threshold. In all other states, Γ must either be strictly positive or negative. In particular, $\Gamma < 0$ means that the firm is better off ceding control to the lender, while $\Gamma > 0$ means that the equity value is higher when the firm retains control over the liquidation decision. Given the normalization $BV_0 = 0$, it is therefore optimal to specify the accounting rule so that $BV < 0$ in the former and $BV > 0$ in the latter case, which yields the following result.

Lemma 2. *An accounting rule $BV(\cdot)$ is optimal if and only if $BV(\theta) = z(\theta) \cdot \Gamma$ for some function $z(\theta) > 0$ at all θ.*

The solution given in Lemma 2 holds for any optimal contract, but the result does by itself not yet identify the globally optimal contract terms. The reason is that Γ is a function of the choice variable L, and a further necessary condition for an optimal contract is that $\frac{\partial Y}{\partial L} = 0$. Hence, $BV = z(\theta) \cdot \Gamma$ states the solution function only in implicit form. In order to conclude that Lemma 2 identifies the global optimum, it remains to be shown that (9) and $\frac{\partial Y}{\partial L} = 0$ can only hold simultaneously for a unique value of L and that this critical point indeed attains a maximum. The following proposition resolves this problem.

Proposition 2. *There exists a unique maturity value L and a unique partition of Θ into the default set D and the no-default set N at which Y attains its global maximum.*

The uniqueness result applies to L and to the partition $\{D, N\}$ that BV implements, but not to the accounting rule BV itself because $z(\theta)$ can take arbitrarily many functional forms as long as $z > 0$ for all θ. But even though the accounting rule that implements a given allocation of control rights is not unique, this multiplicity is inconsequential because the objective function Y takes the same values under all such accounting rules, i.e., contractual efficiency and the payoffs to both parties are the same. The requirement that D and N remain unchanged under all optimal accounting rules implies that the critical level set $\{\theta^*\} \equiv \{\theta: BV(\theta) = 0\}$ must be the same for all possible solution functions BV. In particular, the normalization $z(\theta) \equiv 1$ for all θ is without loss of generality with respect to the properties of BV on $\{\theta^*\}$.

The following heuristic provides some intuition why the optimal control allocation and maturity value must be unique. The level of L and the size of D are substitutes with respect to the lender's payoff, i.e., one can raise the expected debt payments either by increasing the interest rate on the debt or by enlarging the number of states in which the lender has control. Control rights are valuable because the two parties' decision incentives are misaligned, as shown in Proposition 1. Since the lender's break-even constraint binds by Lemma 1, one can evaluate the set of all possible solutions systematically by raising the maturity value L and reducing the de-

fault set D such that the lender's expected payoff is always equal to the investment cost I. By Proposition 1, raising L exacerbates the firm's incentives but improves the lender's, so reducing D at the same time gives more control to the party whose incentives are deteriorating. This monotonicity ensures that there exists a unique point at which the rate of increase in the agency cost caused by the firm equals the rate of decrease in the agency cost caused by the lender.

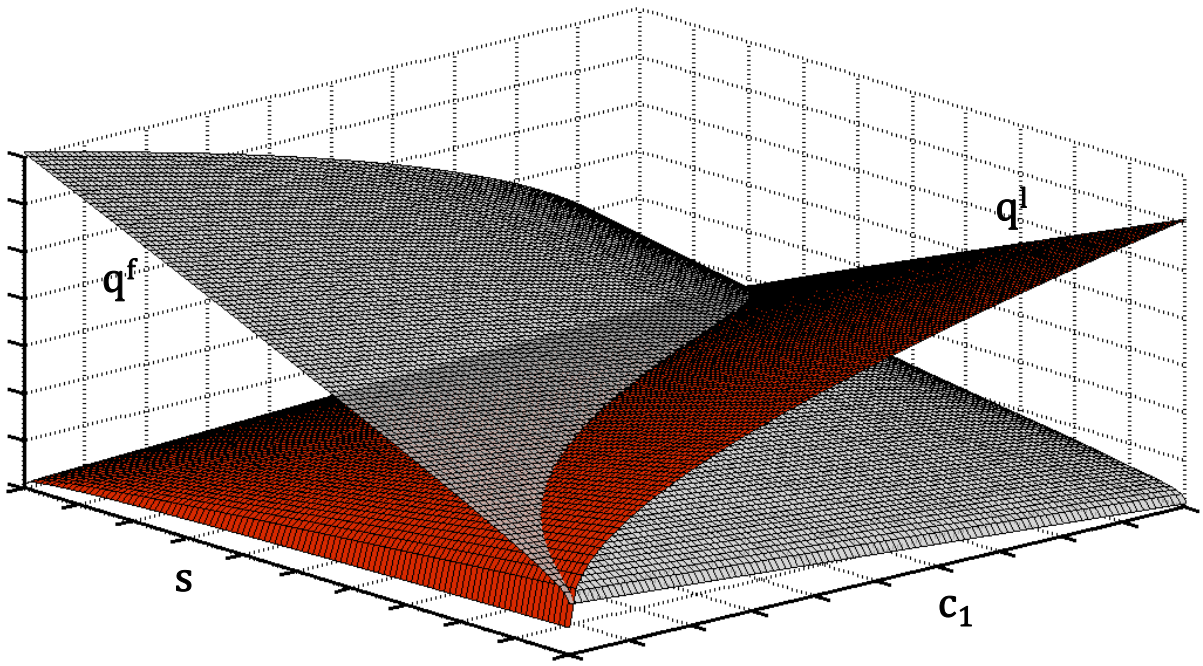

Figure 5. Agency costs by state under control allocation to either the firm or the lender.

Figure 5 illustrates how the sets D and N are chosen optimally. The state space $\Theta = \{c_1, s\}$ is two-dimensional in this problem, and so the c_1-s-plane covers all possible contractible realizations of θ. For each point in this plane, one can plot on the z-axis the inefficiency incurred if the firm or the lender, respectively, were in control of the liquidation decision. The distance between the resulting two surfaces at any point $\theta = (c_1, s)$ is $|q^f - q^l|$. The level set $\{\theta^*\}$ identifies the boundaries between region N, in which the inefficiency given by the firm's surface is incurred, and region D, in which the inefficiency given by the lender's surface is incurred. If the choice of $\{\theta^*\}$ were unconstrained, this boundary would optimally follow the intersection of the two surfaces, but the binding of the lender's break-even constraint generally prevents this outcome. Yet, $\{\theta^*\}$ is not arbitrary. Given that each θ^* is a solution to $\Gamma = 0$, the multiplier

$$\lambda - 1 = \frac{q^f(\theta^*) - q^l(\theta^*)}{u^f(\theta^*) - u^l(\theta^*)}$$

must be constant across all θ^*. In economic terms, its value can be interpreted as the firm's cost of reducing inefficiency. The intuition for this observation arises from the requirement that an optimal contract may not permit any rearrangement of D and N such that the firm's payoff increases. In particular, if the ratio were not constant, there would exist states in D and N over which firm and lender could swap control rights such that the break-even constraint is maintained but inefficiency decreases.

It has yet to be determined whether precise statements are possible about how the input variables c_1 and s determine the properties of the optimal accounting rule BV. In order to improve tractability when answering this question, the distribution $g(v|s)$ is hereafter assumed to have a monotone likelihood ratio, as stated in Assumption 1 below. The monotone likelihood ratio property operationalizes the intuitive idea that s and v should be positively correlated and ensures that higher realizations of s shift the posterior distribution of v to the right in a consistent manner. A monotone likelihood ratio implies first-order stochastic dominance but is more restrictive because it requires a monotonic shift in the density $g(v|s)$ on any subinterval of $[0, \bar{c}]$.

Assumption 1 (Monotone likelihood ratio). *The ratio* $l(v|s) \equiv \frac{\partial g(v|s)}{\partial s} \cdot \frac{1}{g(v|s)}$ *is increasing in* v.

The perhaps most elementary question about BV is whether book value is, in its optimal configuration, an increasing or decreasing function of the firm's past cash flows c_1 and of the estimate s of the firm's liquidation value. Taken together, Proposition 1 and Assumption 1 permit an unambiguous answer to this question. Under standard accounting practices, one might expect the firm's book value to increase in both of these inputs, but as the following result shows, optimal accounting in the debt contract setting differs slightly from this conjecture.

Proposition 3. *In any critical state* θ^**, the optimal accounting rule* BV *is increasing in past cash flows* c_1 *and decreasing in the liquidation value estimate* s.

The rationale for the cash flow part of Proposition 3 follows directly from the dependence of the inefficiency ranges on c_1, as shown in Figure 4. Higher past cash flows lessen the firm's in-

centive to seek inefficiently high risk and thus decrease its inefficiency range. At the same time, the lender's incentive to seek inefficiently low risk is exacerbated. The debt contract is therefore generally more efficient if the firm retains decisions rights when realized cash flows are high, consistent with the observation that firms tend to violate covenants more often in practice if they performed poorly in the past.

That book value should be lowered when the firm's estimated liquidation value s increases may seem counterintuitive at first but follows economic logic. Liquidation is efficient when v is high. Given Assumption 1, a high realization of s then indicates that the probability that the firm will forgo efficient liquidation is larger than the probability that the lender will enforce inefficient liquidation because $k^f \geq \mu_2 \geq k^l$ for all θ and L by Proposition 1. In other words, technical default is more likely to result in an efficient decision for high s. One can thus view the firm's book value as a reflection of the continuation value, or 'value-in-use,' of its assets, net of the opportunity cost of disposing of them via liquidation. The accounting treatment of s is therefore consistent with the going concern presumption of standard financial reporting, augmented by the inclusion of opportunity costs in the computation of an asset's carrying amount.[13]

The monotonicity of BV in both c_1 and s on the level set $\{\theta^*\}$ implies that there exists a bijective mapping between the critical values c_1^* and s^*, i.e., for each c_1, there exists a unique cutoff value s^* above which the firm is in technical default, and for each s, there exists a unique cutoff value c_1^* below which the firm is in technical default. The path of $\{\theta^*\}$ is therefore increasing monotonically in the c_1-s-space, consistent with the illustration in Figure 2. This monotonicity permits the construction of a useful alternative representation of the optimal accounting rule in the form of a *separated solution*, i.e., BV can be written as a sum of two functions that only depend on either c_1 or s but not both variables jointly.

Corollary to Proposition 3. *The optimal accounting rule can be written as a separated solution*

$$BV = bv_0 + bv_c(c_1) + bv_s(s)$$

where bv_0 is a constant, $bv_c(\cdot)$ is bounded and everywhere positive and increasing, and $bv_s(\cdot)$ is bounded and everywhere negative and decreasing.

[13] Book value would naturally be increasing in s if s were instead defined to be informative about c_2 rather than v.

The separated solution identified in the preceding corollary shows similarity to the familiar accrual accounting system, in which cash flows from past transactions are aggregated and adjustments in the form of accruals are made. The constant bv_0 can be interpreted as scheduled accruals that do not depend on the realization of cash flows or other economic inputs, e.g., depreciation, amortization, or accrued interest. Further, any differences between $bv_c(c_1)$ and c_1 or between $bv_s(s)$ and s can be viewed as variable accruals computed based on realized economic inputs. For example, the difference between bv_1 and c_1 may reflect accruals related to units-of-production depreciation, warranty costs, or inventory obsolescence, and the difference between $bv_s(s)$ and s may reflect impairments.

Notwithstanding the above interpretation of BV in terms of regular financial accounting principles, the economic nature of accrual accounting in the debt contract setting differs from general-purpose financial reporting in one important respect. The objective of general-purpose financial reporting is the recognition of revenues and expenses in accordance with economic resource flows rather than cash flows, for example as reflected in the matching principle. In contrast, the accounting rule in the debt contract problem considered here produces accruals that reflect the firm's contingent losses from asset substitution. In other words, high accrued expenses do not necessarily imply a high rate of usage of the firm's invested assets during the period in question, but rather indicate a strong incentive for the firm to seek inefficiently high business risk.

Despite the different economic motivation for the recognition rules in the accounting process underlying BV, the book value BV is built on the principles of historical cost and fair value accounting familiar from regular financial reporting. For example, when s is a perfect estimate of v, one can readily verify that the separated solution becomes

$$BV = \mu_2 - s \tag{10}$$

and leads to a straightforward implementation of the first-best outcome already noted in (1) because a perfect estimate of v effectively yields a complete contract. Given the result of Proposition 1, partitioning Θ at $s = \mu_2$ then removes all agency cost. The optimal accounting rule in this scenario can be viewed as an example of fair value accounting based on a current estimate s of the firm's asset values.

In the converse extreme case when s is entirely uninformative about v, the posterior distribution of v is equal to the prior distribution for all s, and so Γ is invariant under s. The separated solution of the optimal accounting rule is then reduced to

$$BV = bv_0 + c_1 \tag{11}$$

where $bv_0 \leq 0$.[14] Given its independence of the liquidation value estimate s, this accounting rule can be viewed as a form of pure historical cost accounting because book value is solely a reflection of transactions concluded in the past. For all intermediate precision levels of s, the accounting rule BV therefore always has characteristics of both fair value and historical cost accounting.

In terms of economic fundamentals, the solution in (10) for precise s is solely calibrated toward the first-best benchmark rule $v = \mu_2$ and is therefore independent of the potential agency cost reflected in the contracting parties' inefficiency ranges $\left|k^f - \mu_2\right|$ and $|k^l - \mu_2|$. In contrast, the optimal accounting rule in (11) for uninformative s is calibrated to partition the state space at the set of points where the probability-weighted inefficiency ranges are equal, i.e., the optimal BV for uninformative s is determined by the need to balance the agency cost. The solution for intermediate precision levels of s must therefore reflect two optimality benchmarks that are in part determined independently.

The accounting rules (10) and (11) are also related to the concepts of relevance and reliability of accounting information. The cash flow value c_1 is measured without error and therefore highly reliable because it represents past transactions that have already been concluded and whose payments have been settled, but it has no bearing on the first-best decision rule because the latter requires forward-looking information about v and c_2. Nonetheless, past cash flows are pertinent to the problem indirectly through their influence on the contracting parties' incentives, as shown in Proposition 1. In contrast, the fair value estimate s is forward-looking and therefore inherently uncertain but highly relevant because it provides a direct estimate of the critical variable v needed to make the optimal liquidation decision. The relative sensitivity of BV to c_1 and s is thus a reflection of a relevance-reliability tradeoff.

A simple illustration of the relevance-reliability tradeoff arises in the special case when the sensitivities of Γ to changes in c_1 and to changes in s are in a constant proportion

$$-\frac{\partial \Gamma}{\partial c_1}\left(\frac{\partial \Gamma}{\partial s}\right)^{-1} = \gamma$$

[14] One might conjecture that adding or omitting an uninformative s from BV is a matter of indifference because both parties are risk-neutral, but the control allocation induced by this s would be uncorrelated with v and hence with the contracting parties' incentives. Therefore, s could at most not overturn a control allocation that is efficient ex ante based on knowledge of c_1, but it will do so in some cases and is hence optimally excluded from BV.

for all θ^* and some constant $\gamma > 0$. In this case, the separated solution of the optimal accounting rule, as detailed in the proof of the corollary to Proposition 3, is reduced to the linear function

$$BV = bv_0 + \frac{\gamma}{\gamma + 1} c_1 - \frac{1}{\gamma + 1} s \tag{12}$$

Large values of γ imply that s is relatively uninformative and hence has little impact on Γ, while changes in c_1 result in substantial shifts in agency costs. Then the coefficient on c_1 approaches 1 while the coefficient on s shrinks toward zero. Conversely, a low value of γ indicates that s is highly correlated with v while the agency cost implications of changes in c_1 are relatively minor. In this case, the coefficient on c_1 is reduced toward zero while the coefficient on s increases toward unity. A decrease in γ can thus be viewed as a shift from historical cost toward fair value accounting. Graphically, an increase in the information content of s can be represented as a clockwise rotation of the graph of $\{\theta^*\}$ in the c_1-s-plane in Figure 2.

These observations about γ are consistent with the simple intuition that more informative input data should have a greater impact on the accounting measure. One would therefore expect the accounting rules underlying debt covenants in industries with relatively more verifiable inputs to asset valuation to exhibit greater reliance on fair value measures, e.g., among financial services firms whose assets and liabilities are mainly in the form of securities with verifiable market prices. A highly informative s permits a debt contract with nearly first-best efficiency, so it stands to reason that these firms would rely heavily on debt financing. Conversely, firms with proprietary intangibles that are difficult to value based on verifiable information are expected to deemphasize fair value measures in their debt covenants and use debt financing to a lesser degree.[15]

4. Contract Terms, Accounting Conservatism and Capital Structure

This section investigates the tradeoffs involved in the choice of L and BV, the determinants of the optimal degree of accounting conservatism in BV, and the resulting implications for the firm's capital structure and the efficiency of the contract. Interest rates and control rights are sub-

[15] The decrease in the usefulness of accounting measures that include imprecise fair value estimates seems consistent with the empirical evidence in Demerjian (2011), who documents a decline in the use of balance sheet-based debt covenants, concurrent with standard setters' efforts to expand the application of fair value accounting to assets and liabilities whose values are inherently difficult to estimate.

stitutes with respect to payoffs, i.e., an increase in the expected debt payments can be achieved either by raising the maturity value L or by expanding the set of states D in which the lender has control. An important implication of this substitutive relationship is that the interest rate a firm pays on its debt does not imply how costly debt financing is for the firm. In particular, while empirical research has suggested that conservative accounting, which increases the likelihood of covenant violations and hence effectively provides more control rights to the lender, is beneficial because firms that apply more conservative accounting practices have been found to pay lower interest rates on their debt on average (Zhang, 2008), the substitutive relationship suggests that these firms may only have traded one mechanism of providing payoff to the lender for another. The empirical finding is therefore moot with respect to the benefits of conservative accounting.

Yet, while the lender's break-even constraint can be met by a large set of possible combinations of L and D, Proposition 2 shows that there exists a unique optimal choice. In other words, the tradeoff between the interest rate implied in L and the control allocation implemented by BV is not arbitrary because not all contracts that provide the lender with a given payoff are equally efficient. This section examines some of the fundamental economic factors that determine this tradeoff and to explore some implications for the firm's optimal choice of financial leverage. The following definition of accounting conservatism will be adopted to facilitate the discussion.

Definition. *An accounting rule BV is said to be more conservative than an accounting rule BV' if $BV(\theta) \leq BV_0$ whenever $BV'(\theta) = BV_0$ in at least some settings, and never $BV(\theta) \geq BV_0$ whenever $BV'(\theta) = BV_0$.*[16]

Intuitively, greater accounting conservatism implies an overall decrease in book value, consistent with the general notion that conservatism means a greater tendency to recognize losses than to recognize gains.[17] A more conservative accounting rule under this definition involves a general increase in expense recognition and a general decrease gain recognition. A covenant based on more conservative accounting rules is thus more easily violated and hence assigns more

[16] A 'setting' refers to a given set of distributional parameters for c_1, c_2, v and s.

[17] It is generally not possible to meet the stricter definition of a decrease in BV at all θ^* in all possible settings unless $\{\theta^*\}$ is a singleton. The latter case can only arise if the state space is one-dimensional, e.g., as in Sridhar and Magee (1997). A sufficient condition under which the stricter definition can be met even if Θ is not one-dimensional is $\frac{\partial^2 y}{\partial BV \partial \lambda} \propto \frac{\partial^2 y}{\partial BV \partial L}$ on $\{\theta^*\}$, i.e., the effects of BV on the first-order necessary conditions with respect to L and λ are proportional across θ^*.

control rights to the lender. In comparison to other models of accounting conservatism, e.g. in Gigler et al. (2009) or in Göx and Wagenhofer (2010), this definition does not require any specific changes to the informational properties of BV.

The first object of interest in the analysis is the investment cost parameter I. The firm so far has been assumed to raise I entirely in the form of debt, but firms in practice finance their capital needs by a variety of combinations of debt and equity. The following discussion will therefore adopt the representation

$$I = I^X + I^U \tag{13}$$

of the firm's capital structure immediately following the conclusion of the debt contract, where I^X is the amount of paid-in equity capital and I^U is the principal amount of the debt. The analysis so far has assumed that $I^U = I$ and $I^X = 0$, but replacing I by I^U in (8) would not alter the structure of the problem because an equity investment of $I^X > 0$ does not give rise to any additional constraints. The debt financing amount I^U can thus serve as a measure of financial leverage, which yields the following result.

Proposition 4. *For firms with higher leverage, optimal debt covenants are based on more conservative accounting rules.*

Ceteris paribus, an increase in the amount of outstanding debt means a greater likelihood that the firm will ultimately default and the lender receives less than the full maturity value L. In other words, the debt is more at risk when I^U increases and hence a higher maturity value L is needed to compensate the lender. As Figure 4 illustrates, the lender's inefficiency range shrinks as a result, so that contract efficiency improves if the lender receives additional decision rights. Hence, the optimal debt covenant for more highly levered firms is based on more conservative accounting rules and is thus more easily violated.

The result in Proposition 4 is stated in terms of financial leverage and hence implicitly assumes that an increase in I^U is offset by a decrease in I^X while the total investment cost I is held constant. Yet, one can readily observe that an increase in I^U might alternatively have been the result of an increase in I that the firm decided to finance, at least partly, by raising additional debt. In contrast to the leverage interpretation, this scenario implies a decrease in the firm's prof-

itability rather than a mere shift in capital structure. To illustrate this claim, one can note that the return on the firm's investment is

$$ROI \equiv \frac{W - Q}{I} = \frac{E_\theta(w - q)}{I}$$

where $W \equiv E_\theta(w)$ is the expected first-best firm value, $Q \equiv E_\theta(q)$ is the expected inefficiency, w is the first-best firm value for a given θ as defined in (6), and q is the inefficiency as defined in (4). When I increases, the return rate declines by

$$\frac{dROI}{dI} = -\frac{W - Q + \frac{dQ}{dI}I}{I^2} < -\frac{W - Q - I}{I^2} < 0$$

where the first inequality follows from Lemma 1 and the final inequality from the requirement that the investment has positive net present value.[18] Mathematically, this alternative interpretation of an increase in I^U does not alter the problem. Proposition 4 thus has a natural corollary.

Corollary to Proposition 4. *For firms with lower profitability, optimal debt covenants are based on more conservative accounting rules.*

In summary, I^U has a dual interpretation as either a measure of profitability or as a measure of financial leverage, but the effect of a change in I^U on the optimal contract terms is the same in both cases. The reason is that in both settings, the firm's payment obligation to the lender, and thus its incentive to make decisions that yield inefficiently high risk, have increased. In response, an efficient contract shifts decision rights to the lender, who now has relatively less detrimental incentives. This observation suggests that conservatism in accounting, as defined in this context, is neither unconditionally beneficial nor unconditionally disadvantageous. Rather, its optimal degree is determined by the economic situation of the firm and its capital structure. A direct empirical implication is that firms in competitive industries with low profit margins should tend to follow more conservative accounting practices under their debt covenants.

Given the possible leverage interpretation of I^U, it may seem natural to make I^U and I^X choice variables of the firm, but as noted in the introduction, an endogenous capital structure would require a full model of capital costs across all types of financing options. Nonetheless, the

[18] If $\frac{dQ}{dI} < -1$, the firm would voluntarily overpay the lender because the resulting reduction in inefficiency would exceed the amount of the overpayment, but Lemma 1 shows that this situation can never arise in an optimal contract.

model can make a limited contribution to solving the firm's capital structure problem in the following sense. In the frictionless setting of Modigliani and Miller (1958), the firm should not be able to improve its return on equity, net of financing costs, by changing its capital structure, and so the firm's return less its cost of equity financing would be the same regardless of I^U. Then if one were to introduce the agency cost from asset substitution as the sole friction, ceteris paribus, the firm's owners would choose I^U to maximize their net return

$$Y - I^X = Y + I^U - I \tag{14}$$

where the equality follows from $I = I^X + I^U$.

Given the lender's binding break-even constraint, maximizing (14) is equivalent to maximizing total firm value. The corresponding first-order condition would require that

$$\frac{dY}{dI^U} + 1 = -\lambda + 1$$

is zero at an optimal interior value of I^U, where the equality follows from the envelope theorem. As noted in the Introduction, the corner solution $I^U = 0$ would attain the first-best outcome because the agency conflict is eliminated under pure equity financing. But if I^U were bounded away from zero for exogenous reasons and λ evolved in a non-monotonic manner, one might conjecture that this stylized capital structure problem might also have a local interior maximum under some contract for which $\lambda = 1$. The following result demonstrates that this is not the case.

Proposition 5. *Firm value is convex in leverage.*

The convexity of total firm value in I^U implies that a maximization of firm value or a minimization of inefficiency would yield a corner solution in this simplified setting. Then even if the trivial optimum $I^U = 0$ is infeasible or undesirable for exogenous reasons, one would still expect firms, ceteris paribus, either to minimize their debt as much as possible or to seek the highest possible amount of debt.[19] The reason why intermediate levels of leverage are inefficient is that they require costly compromises between conflicting decision incentives. The more decision rights and payoff claims are vested with only one party, the less costly the financing choice be-

[19] For example, a corner solution need not coincide with the extreme value $I^U = 0$ if the firm only has limited access to equity capital, in which case I^U is bounded below. On the opposite extreme, I^U may be bounded above because the firm can only make a credible commitment to pay out some maximum amount to an outside financier, e.g., because of the opportunity to consume private benefits at the financier's expense.

comes. Taken together, Propositions 4 and 5 are consistent with the finding by Rajan and Zingales (1995) that firms with higher profitability tend to have lower leverage. As shown in Proposition 4 and its corollary, profitability and leverage have additively inverse effects on contract efficiency, and hence the opposing directional alignment observed in practice may in part be intended to reduce agency costs. One should, however, bear in mind that the convexity result is obtained in a setting in which asset substitution is the only financing cost that varies with leverage, while firms' leverage choices in practice are likely guided by a number of other factors.

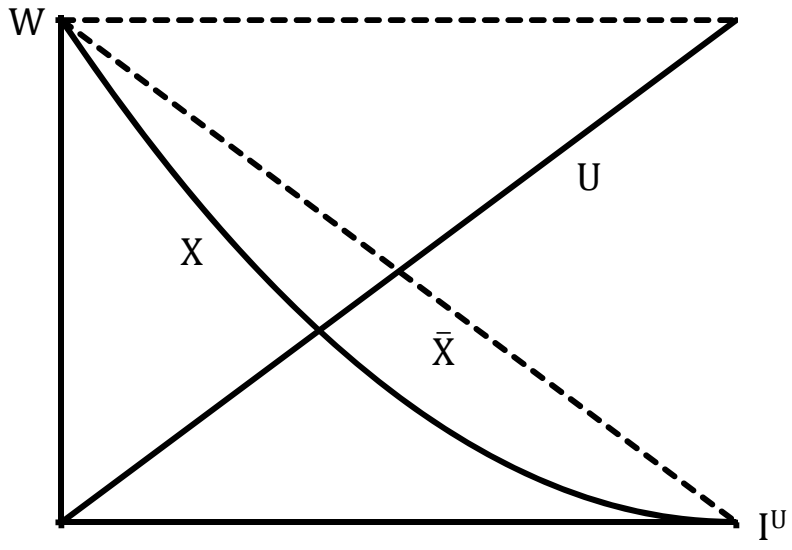

Figure 6. Debt and equity values as a function of leverage.

Figure 6 provides a graphical illustration of the values of debt and equity as functions of I^U. Total firm value is bounded above by the first-best level W, as defined above. The lender's payoff $U \equiv E_\theta(u)$ increases one-for-one with I^U because the break-even constraint $U \geq I^U$ binds under any optimal contract by Lemma 1. Hence, the value of the firm's equity $X \equiv E_\theta(x)$ is bounded above by $\bar{X} \equiv W - I^U$. One can readily observe from the convexity of X that even if the firm's choice of I^U were constrained to interior values for exogenous reasons, the value of I^U that minimizes inefficiency would always coincide with one of the endpoints of the feasible in-

terval.[20] Since the slope of X is $-\lambda$ by the envelope theorem, the convexity result also implies that λ serves as an index of leverage and profitability. Firms with higher λ are more profitable and less leveraged, with the middle case $\lambda = 1$ as a convenient benchmark.[21] References to λ in Propositions 6 and 7 below thus correspond directly to the economic fundamentals of the firm.

Proposition 4 and its corollary still leave the question whether profitability and leverage explain tradeoffs between accounting conservatism and interest rate levels. The answer is negative. As shown in the proof of Proposition 5 in the Appendix, an increase in I^U must coincide with an increase in the maturity value L, which generally also yields a higher interest rate, as implied in the ratio $\frac{L}{I^U}$.[22] Thus, increases in financial leverage or decreases in profitability do not imply that firms trade off more conservative accounting rules for lower interest rates.

The reason I^U cannot explain tradeoffs is that changing I^U also shifts the total expected payoff that must be transferred to the lender via adjustments to L and BV. This observation implies that a meaningful conclusion about the tradeoff between control rights and interest rates can only be drawn if the break-even constraint and hence the expected debt payment are held constant. Even more importantly, I^U and I^X are endogenous in reality, and so a change in any model parameter that affects the lender's payoff does not only have a direct effect on the optimal choice of L and BV but is also likely to prompt the firm to adjust its amount of debt financing in response, which would trigger a secondary adjustment to L and BV. An important empirical implication is that testing the relationship between accounting conservatism and interest rates may not yield meaningful results unless the role of the firm's capital structure and profitability as covariates of the optimal contract terms is reflected in the research design.

A genuine tradeoff consideration, on the other hand, arises when the relative weights of cash flows and liquidation values in the contracting parties' payoff functions shift but the overall profitability and leverage are held constant. In accounting terms, one can interpret a greater liquidation value as a high proportion of 'tangible' assets on the firm's balance sheet, such as plant,

[20] Naturally, the corner solution $I^U = W$ is only feasible in case of a zero-NPV investment, in which case the lender would receive all payoffs in all states.

[21] When $\lambda = 1$, the optimal partitioning line in the c_1-s-space follows the intersection of the two surfaces in Figure 5.

[22] In a first-best scenario, the break-even constraint $U = I^U$ implies that $\frac{dL}{dI^U} = \frac{1}{1-\Pr(L)}$, where $\Pr(L)$ is the probability that total payoffs are less than L. The interest rate then increases monotonically by $\frac{d}{dI^U}\left(\frac{L}{I^U}\right) = \frac{1}{I^U} \cdot \left(\frac{dL}{dI^U} - \frac{L}{I^U}\right) > 0$, where the inequality follows from the observation that $(1 - \Pr(L)) \cdot L < I^U$. The second-best setting analyzed in this paper is similar, but the interest rate need not increase monotonically at all levels of I^U.

property and equipment, which are likely to have substantial value to outside buyers if the firm were liquidated. Liquidation and continuation are opportunity costs with different degrees of risk, and so given the asymmetric risk preferences established in Proposition 1, one may suspect that favorable or unfavorable changes in either one should have predictable effects on the optimal allocation of decision rights even in the absence of differences in profitability or capital structure. Since v and c_2 are random variables when the contract is written at $t = 0$, this analysis requires a characterization of such changes in terms of distributions, as given in the following definition.

Definition. *A parameter η provides an ordering of the firm's liquidation value if the likelihood ratio $m(v|s) \equiv \frac{\partial g(v|s)}{\partial \eta} \cdot \frac{1}{g(v|s)}$ is increasing in v and decreasing in s for all v, s and η.*

Similar to Assumption 1, the definition of η utilizes the monotone likelihood ratio property to operationalize the notion of an increase in v. Firms with higher η thus have higher expected liquidation values, while the distribution of the continuation value c_2 is held fixed.[23] Since both s and η now induce monotone likelihood ratios, their interaction must be specified in order to maintain tractability. The likelihood ratio of η is therefore assumed to decrease in s.[24] Finally, one should observe that the contracting parties' decision incentives are invariant under η because v is realized before the decision whether to liquidate the firm or not is made.[25]

An increase in η has two distinct effects. First, η makes liquidation the efficient decision under the first-best rule in more states θ and thus increases the weight of v as a source of payoff relative to c_2. This will be referred to as the *composition* effect of η. Second, the two parties' payoffs X and U, and hence total firm value, increase with η because liquidation proceeds have increased while cash flows, investment costs and incentives are all unchanged. This will be referred to as the *level* effect of η. Figure 6 provides a graphical intuition for the level effect. The increased liquidation value increases the first-best firm value W, and hence both the lender's payoff function U and the firm's actual and first-best payoff functions X and $\bar{X}$ are scaled up ac-

[23] The same conclusions can be obtained if the properties of v are held constant and η is specified with respect to the distribution of c_2 instead.
[24] One can verify graphically that a likelihood ratio m that is constant or increasing in s is difficult to sustain: if s increases, $g(v|s)$ is shifted to the right, and hence the likelihood ratio in η must generally be negative over a longer interval of v.
[25] This observation does not apply when η is defined with respect to the continuation value c_2, but subject to a technical assumption on $g(v|s)$, the conclusions are the same.

cordingly. Then if I^U is held fixed, its *relative* position on the x-axis has shifted left, which yields both a lower optimal degree of accounting conservatism by Proposition 4 and, generally, a lower rate of interest. In other words, the level effect of η is equivalent to a decrease in I^U in terms of its implications for BV and L, and is therefore simply a variant of Proposition 4.

The preceding discussion implies that only the composition effect can yield insights into the tradeoff between accounting conservatism and interest rates because the level effect generally decreases both of them simultaneously. Hence, two firms with different η are, ceteris paribus, not comparable with respect to the tradeoff in accounting conservatism and interest rates unless one controls for the level effect. This observation again emphasizes that profitability and leverage are important covariates of the optimal debt contract terms. The following result therefore examines a firm with a *relatively* high liquidation value, defined as a higher level of both η and I^U so that the level effect is neutralized and only the composition effect remains.[26]

Proposition 6. *For λ sufficiently close to* 1*, optimal debt contracts for firms with relatively high liquidation values are based on lower interest rates and more conservative accounting rules.*

The intuition behind Proposition 6 follows from Proposition 1. If liquidation is the efficient decision in more states of the world, decision rights should be vested more often with the lender, whose decision incentives are aligned with the first-best rule when liquidation is optimal. A second, less apparent but important cause is that lowering L becomes more beneficial with higher η because the lender's inefficient liquidation decisions now occur in states with a lower ex-ante probability. The reason Proposition 6 cannot provide an unambiguous prediction if λ is too large or too close to zero, i.e., leverage or profitability are too large or too small, is that η can increase the marginal benefit of raising the maturity value L in these cases, which may make higher L and less conservative accounting rules optimal.[27] Therefore, the predictions of Proposition 6 may not be observable in firms around the tails of the leverage and profitability distributions, which correspond to very high or very low values of λ and, graphically, the corner regions in Figure 6.

[26] Formally, isolating the composition effect is equivalent to examining an increase in η with a joint increase in I^U that offsets the increase $\frac{\partial U}{\partial \eta} > 0$ in the lender's expected payoff.

[27] For very high λ, the firm has efficient incentives in most states and bankruptcy is highly unlikely. Raising L would incur little additional inefficiency in this case but create substantial slack in the break-even constraint, so that additional control allocation to the firm can increase efficiency. Raising η increases this marginal benefit. A symmetric argument applies when λ is close to zero.

A natural corollary of Proposition 6 is the impact of a relatively high liquidation value on efficiency. One may conjecture that firms with more valuable liquidation options enter into more efficient debt contracts because they can pledge collateral of higher expected value and pay a lower rate of interest. The envelope theorem implies that the composition effect of an increase in η on contract efficiency is $-\frac{\partial Q}{\partial \eta}$.[28] Expanding shows that a greater weight of the liquidation value in the overall payoff function changes contract efficiency by

$$-\frac{\partial Q}{\partial \eta} = \int_N \int_{\mu_2}^{k^f} (v - \mu_2) \cdot m(v|s)\, dG\, dH + \int_D \int_{k^l}^{\mu_2} (\mu_2 - v) \cdot m(v|s)\, dG\, dH \qquad (15)$$

where m denotes the likelihood ratio as given in the definition of η.

Inspection of (15) reveals no particular directional bias. For small I^U, the set N is large but the firm's inefficiency range $\left|k^f - \mu_2\right|$ is small while the technical default set D is small but the lender's inefficiency range $|k^l - \mu_2|$ is large. The converse applies to large I^U. Further, the monotonic increase in m suggests that the first term in (15) is generally positive and the second term is generally negative. Taken together, these observations do not permit the conclusion that (15) is systematically different from zero and hence the efficiency of debt contracts in this model benefits from high liquidation values, even though Proposition 6 predicts such firms to pay a lower rate of interest on their debt. This conclusion again emphasizes that the interest rate level is by itself not indicative of contractual efficiency.

In practice, firms with high values of η hold assets with a high degree of fungibility, which would therefore be valuable to a broad set of potential outside buyers in liquidation and be well-suited as loan collateral. Such firms likely tend to operate in relatively competitive industries that require extensive investment in tangible assets such as plant, property and equipment. Proposition 6 is thus consistent with the notion that collateralized debt carries a lower interest rate, but as noted above, the lower interest rate and the attendant conservative accounting rule do not imply greater contract efficiency. In contrast, firms with low η hold assets that probably have little or no value in the hands of a different owner. Firm-specific intangibles such as human capital or a

[28] Formally, the total effect of η on efficiency is

$$\frac{dY}{d\eta} = W_\eta - Q_\eta + (1 - \lambda) \cdot U_\eta$$

where subscripted η denote partial derivatives. The terms W_η and U_η are level effects, and so the composition effect of η is given by $-Q_\eta \equiv -\frac{\partial Q}{\partial \eta}$.

well-functioning organization are likely to fall into this category. One would therefore expect the latter firms to be more prevalent in industries characterized by a high intensity of research and development activity, product innovation, and intellectual capital.

Proposition 6 effectively examines the implications of a shift in the mean of the liquidation value relative to expected cash flows. A natural complement to this result is an analysis of changes in the variance of payoffs. The following discussion will focus on the firm's *operating risk*, defined as the variance of the its second-period cash flows c_2. As in the case of η, it is important to avoid the confounding effects of changes the first-best firm value W. Higher operating risk is therefore modeled as a mean-preserving spread of the distribution of c_2.

Definition. *A parameter σ provides an ordering of the firm's operating risk if*

$$\int_0^c \frac{\partial F_2(c_2)}{\partial \sigma} dc_2 \geq 0$$

for any $c \in [0, \bar{c}]$, with equality when $c = \bar{c}$.

Mean preservation implies that σ has no bearing on the first-best decision rule because $\frac{\partial \mu_2}{\partial \sigma} = 0$ by construction, and hence the first-best firm value W is the same for all σ. Yet unlike η, operating risk affects the contracting parties' decision incentives. In particular, the firm adjusts its liquidation threshold upward by

$$\frac{\partial k^f}{\partial \sigma} = \frac{\partial x_2}{\partial \sigma} = -\int_{L-c_1}^{\bar{c}} \frac{\partial F_2}{\partial \sigma} dc_2 \geq 0 \tag{16}$$

while the lender adjusts its liquidation threshold downward by

$$\frac{\partial k^l}{\partial \sigma} = \frac{\partial u_2}{\partial \sigma} = -\int_0^{L-c_1} \frac{\partial F_2}{\partial \sigma} dc_2 \leq 0 \tag{17}$$

in response to an increase in σ.[29] This observation is consistent with the rationale that the firm benefits from inefficiently high risk to begin with, so that an increase in σ should reinforce its incentives further. On the other hand, higher σ also reinforces the lender's incentive to avoid risk. Taken together, (16) and (17) and the constant first-best cutoff value μ_2 suggest that efficiency

[29] Incentives would be unaffected if σ were instead defined with respect to the prior distribution of v because both parties observe the realization of v before making their decisions. One could, however, assume the contracting parties to observe imperfect information about v at $t = 1$ and define σ with respect to the variance of v conditional on this imperfect information. The results would then be consistent with those from the above construction of σ.

decreases with σ because $k^l \leq \mu_2 \leq k^f$ by Proposition 1, and hence both parties' liquidation thresholds move away from the efficient benchmark. The next result formalizes this reasoning.

Proposition 7. *For $\lambda \geq 1$, higher operating risk always reduces the efficiency of debt contracts.*

Proposition 7 follows from the observation that the incentive problem becomes more severe with higher values of σ, regardless whether the firm or the lender receives control rights. Higher σ increase the risk differential between the riskless liquidation option and the risky continuation option and thereby intensify the agency conflict. Ceteris paribus, operating risk is therefore likely to prompt firms to set their financing choices closer to the more efficient corner values of I^U. The need for a nuanced statement with respect to λ arises as follows. A higher variance of cash flows not only exacerbates incentives, and hence increases the inefficiency cost Q as a first-order effect, but also transfers wealth from the lender to the firm at the same time. The lender's expected payoff U therefore decreases in σ, but since the break-even constraint $U = I^U$ must hold, this wealth transfer requires adjustments to L and BV that compensate the lender for the expected loss. These adjustments involve higher L and more conservative BV in view of Proposition 4 and thus effectively move the contract to the right on the x-axis in Figure 6. Due to the convexity of firm value in I^U, this relative rightward shift decreases firm value when I^U is low (and hence λ is high) and increases it when I^U is high (and hence λ is low). Therefore, the efficiency loss is mitigated when $\lambda < 1$ and an unambiguous analytical statement can only be made for $\lambda \geq 1$.

As noted previously, λ can be viewed as an index of leverage or profitability. Proposition 7 therefore not only suggests that debt is generally more costly for firms with high operating risk, but that this cost increase is most pronounced in highly profitable firms. These firms would therefore benefit more from reducing I^U than marginally profitable firms would benefit from increasing I^U further.[30] If this effect is economically significant in reality, a high degree of equity financing should be more prevalent in volatile industries and among firms with high-risk business models, especially if risk and return are positively correlated in practice.

Proposition 7 implicitly assumes the variance of the liquidation value v, conditional on the contracting parties' information at $t = 1$, to remain constant. If one were to give the firm and the

[30] In addition, I^U is bounded above by I, so increasing debt financing further may not even be feasible.

lender imperfect information about v at time $t = 1$ and consider an increase in the variance of the parties' estimate of v given this imperfect information instead, the resulting incentive effects would take the opposite signs of (16) and (17), but the direction of the wealth transfer and hence the effect of λ would remain the same. Therefore, while the agency cost Q increases with σ but would decrease with the variance of v, the impact of an increase in either variance on contract efficiency is always less favorable for firms with high λ. In other words, a high degree of equity financing would be relatively more efficient for profitable high-risk firms even if risk were defined with respect to the liquidation value v.

The relationship between σ and the optimal accounting rule BV depends on a number of factors. First, the symmetry of the implications of σ with respect to the parties' decision incentives and the absence of a change in the first-best decision rule imply that BV should, on average, be unaffected by the impact of σ on the agency cost Q. Second, the wealth transfer from the lender to the firm necessitates a more conservative accounting rule to compensate the lender, as noted above. Third, the preceding discussion also suggests that firms might adjust their financial leverage in response to changes in σ: highly profitable firms would benefit from adjusting I^U, and thus their financial leverage, downward if σ increased, which by itself would imply a reduction in accounting conservatism in view of Proposition 4. Conversely, firms with low profit margins may benefit from raising I^U further. Hence, one would expect the optimal accounting rule to become more conservative when σ increases in firms with low profitability, while no clear directional prediction emerges for highly profitable firms. The possible correlation between operating risk and profitability in practice may further complicate the prediction.

5. Conclusion

Accounting measures frequently serve as a basis for debt covenants. This paper examines how the underlying accounting rules are designed optimally to minimize inefficiency in a setting with incomplete contracts and asset substitution. Accounting can serve two broad purposes in this context. First, it can provide information to the contracting parties and thus reduce the cost of decision errors. Second, it can act as a decision mechanism in the presence of an incentive misalignment problem, e.g. by triggering technical default events under a covenant. The focus of

this paper is on the second role. Accounting is modeled as an information aggregation function that implements an efficient allocation of decision rights between borrower and lender.

This setup permits several observations about the properties of the optimal accounting rule. Both past transaction data in the form of realized cash flows and forward-looking data in the form of fair value estimates are valuable in the debt contract problem. The former affects the contracting parties' decision incentives and hence the severity of the asset substitution problem, whereas the latter provides information about the first-best decision. The relative usefulness of these two types of information determines whether the optimal accounting rule has a historical cost focus or a fair value focus. Furthermore, accounting conservatism, defined as the tendency of the accounting rule to yield values below the technical default threshold, is neither unambiguously beneficial nor detrimental. In particular, the empirically observed negative correlation between measures of conservatism and interest rate levels may only reflect the substitutive nature of the two with respect to the lender's payoff.

The properties of the optimal accounting rule give rise to a number of empirical predictions. Debt financing is expected to be more prevalent in industries in which reliable estimates of the value of the borrowers' assets are available, and these debt contracts should rely more on fair value measures. Firms with high leverage or low profit margins are expected to follow more conservative accounting practices than highly profitable firms and firms predominantly financed by equity. Moreover, debt contracts are least efficient at medium levels of leverage, at which compromises between the conflicting incentives of the firm and the lender are most costly. Ceteris paribus, firms are therefore expected to align their degree of debt financing inversely to their profitability, consistent with the observation in Rajan and Zingales (1995). A high liquidation value relative to the value of the firm as a going concern increases the optimal degree of accounting conservatism and decreases the optimal interest rate level. Finally, high volatility of operating cash flows increases the cost of debt, particularly for highly profitable firms, and is therefore expected to correlate with a high degree of equity financing.

These results are subject to several qualifications. First, the predictions concerning the firms' capital structure are only partial in the sense that the model does not include the cost of alternative financing sources, specifically equity. Empirical results may be difficult to obtain if debt and equity costs are too highly correlated. In addition, debt financing is costly relative to equity in this model, and hence there must exist benefits outside the model that compel firms to choose

positive leverage ratios to begin with. Their correlation with the forces in the model may again complicate empirical tests. Similarly, cash flows, liquidation values, the precision of fair value information, profitability, operating risk and other variables in the model are assumed to be independent, but the correlation structure is unlikely to be so simple in reality. Lastly, the firm is assumed to borrow from a single lender, but in practice, debt financing is often provided by multiple lenders, whose claims may be ranked by seniority. It may be important to consider that the severity of the incentive misalignment likely depends on the seniority of the lender's claim, whereby unsecured lender's incentives are more closely aligned with those of equityholders.

A further commentary concerns the extent to which the properties of the optimal accounting rule derived in this paper are useful attributes of accounting in general. In practice, accounting serves multiple purposes, and firms generally seem to rely on a basic, general accounting system and make purpose-specific adjustments to the output information, e.g. for external financial reporting, performance evaluation, or contracts. The model in this paper examines the role of accounting for a single, specific purpose and therefore cannot identify a clear boundary between contract-specific accounting adjustments and general-purpose accounting rules. Providing insight into which aspects of the mapping from the state space to the allocation of decision rights are optimally part of general-purpose accounting rules and which aspects should be specific to debt contracts would require a more general model in which accounting has multiple roles.

Finally, the discrimination between accounting rules and covenant provisions in this model is to some extent ambiguous. The debt covenant is assumed to take the simple form of a fixed cut-off value of the accounting measure BV while all computational mechanisms are absorbed into the accounting rule, but one could alternatively specify a floating threshold whose calculation assumes some of the properties of BV without altering the outcome. An example from practice may illustrate this point: an income escalator clause, i.e., a prescription that less than one-hundred percent of the borrower's positive income in a given period is credited to owner's equity as computed under the covenant, can be viewed either as a covenant provision superimposed on the accounting rule or as an accounting rule by itself that prescribes the accrual of an expense related to agency costs. Another manifestation of this ambiguity is the possible substitution between accounting conservatism and covenant tightness. One should therefore bear in mind that some of the predicted properties of the optimal accounting rule may manifest themselves in the form of covenant provisions rather than accounting numbers from in public financial reports.

Appendix

Proof of Proposition 1. The firm's payoff is zero under liquidation if $v \leq L - c_1$ but positive in expectation under continuation, so the firm never prefers liquidation for $v \leq L - c_1$. For $v > L - c_1$, the firm's indifference condition $x_v = x_2$ becomes

$$v = F_2(L - c_1) \cdot (L - c_1) + \int_{L-c_1}^{\bar{c}} c_2 \, dF_2 \geq \mu_2$$

with equality only when $c_1 = L$. The solution value $v = k^f$ changes with c_1 by

$$\frac{\partial k^f}{\partial c_1} = -F_2(L - c_1) \leq 0$$

and reaches its minimum when $c_1 = L$, in which case $k^f = \mu_2$. Hence, the firm never prefers inefficient liquidation and its inefficiency range $|k^f - \mu_2|$ decreases monotonically in c_1. Further, $\frac{\partial k^f}{\partial L} = -\frac{\partial k^f}{\partial c_1} \geq 0$ because k^f only depends on c_1 and L through the joint term $L - c_1$.

Conversely, the lender receives the maximum payoff of L for any $v \geq L - c_1$ and therefore always prefers liquidation for $v \geq L - c_1$. For $v < L - c_1$, the lender's indifference point is

$$v = \int_0^{L-c_1} c_2 \, dF_2 + \big(1 - F_2(L - c_1)\big) \cdot (L - c_1) \leq \mu_2$$

and so the solution $v = k^l$ changes with c_1 by

$$\frac{\partial k^l}{\partial c_1} = F_2(L - c_1) - 1 \leq 0$$

Then the lender's liquidation threshold reaches its maximum for $v \geq L - c_1$ when $k^l = \mu_2$, i.e., the lender never prefers inefficient continuation and its inefficiency range $|\mu_2 - k^l|$ decreases monotonically in c_1. As in the firm's case, $\frac{\partial k^l}{\partial L} = -\frac{\partial k^l}{\partial c_1} \geq 0$.

Proof of Lemma 1. In any given state $\theta \in N$, the firm maximizes its payoff by solving

$$\max_{a_i} x(a_i | \theta \in N)$$

where $x(\cdot\,|\theta \in N)$ is the firm's payoff conditional on θ and the absence of default, and $a_i \in \{a_1, a_2\}$ is the action the firm chooses at $t = 1$, where a_1 corresponds to liquidation and a_2 corresponds to continuation. Similarly, the lender solves

$$\max_{a_i} u(a_i|\theta \in D)$$

in any state $\theta \in D$. Then for a given state θ, it must be that

$$x(a_i|\theta \in N) \geq x(a_i|\theta \in D)$$

with equality only when

$$\arg\max_{a_i} x(a_i|\theta \in N) \supseteq \arg\max_{a_i} u(a_i|\theta \in D)$$

It follows that the firm's total expected payoff is

$$X(\widetilde{N}) \geq X(N)$$

for any partition $\{D, N\}$ and $\{\widetilde{D}, \widetilde{N}\}$ with $\widetilde{N} \supseteq N$, i.e., the firm's payoff increases with its control rights. A symmetric argument applies to the lender.

Then for any partition $\{D, N\}$ such that $U > I$, the firm could choose some $\{\widetilde{D}, \widetilde{N}\}$ with $\widetilde{N} \supseteq N$ so that its payoff increases without violating the lender's break-even constraint, and so $\{D, N\}$ could not be part of an optimal contract. If $U > I$ and $D = \emptyset$, i.e., the firm cannot expand its space of control, it can lower the maturity value L until $U = I$ because states in which payoffs depend on k^l (and hence adverse incentive effects could reduce X) are reached with probability zero when $D = \emptyset$. Hence, $U = I$ under any optimal contract.

Proof of Lemma 2. Under an optimal accounting rule, the first variation

$$\frac{d}{d\varepsilon}\int_\Theta y\big(BV + \varepsilon j(\theta)\big)\, dH\bigg|_{\varepsilon=0} = \int_\Theta j(\theta) \cdot \Gamma(L, \lambda, \theta) \cdot \delta(BV)\, dH \tag{A1}$$

must be zero for any continuous test function $j(\cdot)$, where $\Gamma(\cdot)$ is given by (9) and $\delta(\cdot)$ is the Dirac delta distribution. By the fundamental lemma of calculus of variations, (A1) is zero if and only if the Euler-Lagrange equation

$$y_B \equiv \frac{\partial y}{\partial BV} = \Gamma(L, \lambda, \theta) \cdot \delta(BV) = 0 \tag{A2}$$

holds for all θ. Since the delta distribution is zero at any $BV \neq 0$ but assigns point mass when $BV = 0$, (A2) holds if and only if $\Gamma = 0$ whenever $BV = 0$. The accounting rule $BV = z(\theta) \cdot \Gamma$ is therefore sufficient to set (A1) to zero. It remains to be shown that this solution maximizes Y. Indeed, $\Gamma > 0$ implies that the firm's equity value is higher if $\theta \in N$, while $\Gamma < 0$ implies the opposite, which corresponds to the control allocation that $BV = z(\theta) \cdot \Gamma$ implements as long as $z(\theta) > 0$ for all θ. In order to establish necessity, it suffices to note that an accounting rule $\widetilde{BV}$

cannot be written in the form $BV = z(\theta) \cdot \Gamma$ only if $\text{sgn}\left(\widetilde{BV}\right) \neq \text{sgn}(BV)$ for at least some θ. However, then the control allocation under $\widetilde{BV}$ is not optimal for any such θ, and hence $\widetilde{BV}$ cannot be an optimal accounting rule.

Proof of Proposition 2. A sufficient condition for the contract to achieve a local maximum is

$$\int_{\Theta} j^2(\theta) \cdot \det(\mathbf{M})\, dH > 0 \tag{A3}$$

where

$$\mathbf{M} = \begin{bmatrix} 0 & U_L & u_B \\ U_L & Y_{LL} & y_{BL} \\ u_B & y_{BL} & y_{BB} \end{bmatrix} \tag{A4}$$

is the bordered Hessian matrix at a given θ and $j(\theta)$ is an arbitrary, continuous test function. Then (A3) holds if and only if

$$\det(\mathbf{M}) = -U_L^2 y_{BB} + 2u_B U_L y_{BL} - u_B^2 Y_{LL} \geq 0$$

for all θ, with strict inequality for at least some θ. The elements of **M** are defined as follows.

As argued in the proof of Lemma 1, the lender's payoff decreases whenever a state θ is transferred from D to N. Hence, the change in the lender's payoff from increasing book value

$$u_B \equiv \frac{\partial u}{\partial BV} = \left(u_f - u_l\right) \cdot \delta(BV)$$

must be non-positive for all θ. Conversely, the effect of raising the maturity value L given by

$$U_L \equiv \frac{\partial U}{\partial L} = \int_{\Theta} \frac{\partial u}{\partial L}\, dH$$

must be positive in view of the following argument. Let L^* denote the optimal maturity value for a given level of I. For $I = 0$, $D = \emptyset$ and $L^* = 0$ because $U = I$ by Lemma 1, and so

$$U_L|_{L=0} = \int_{\Theta} \left(\int_0^{k^f} \frac{\partial u_2}{\partial L}\, dG + \int_{k^f}^{\bar{c}} \frac{\partial u_v}{\partial L}\, dG \right) dH > 0$$

If one were to raise L and dynamically adjust D and I such that $L = L^*$ everywhere, the resulting path would cover all possible optima. If $U_L < 0$ for any $L_1 > 0$ on this path, continuity implies that $U_L = 0$ for some optimal $L_0 < L_1$ because $U_L|_{L=0} > 0$. Then $Y_L = 0$ at L_0 if and only if $Q_L = 0$, which implies $X_L = 0$ because

$$X_L + U_L = -Q_L$$

in view of (5). Hence, L_0 must maximize both X and U, which implies that both would decrease if L were increased beyond L_0. Therefore, $L^* = L_1$ cannot be part of an optimal contract and so $U_L > 0$ at any L^*.

The second partial derivative $Y_{LL} \equiv \frac{\partial^2 Y}{\partial L^2}$ must be negative. Otherwise, the firm could raise L and obtain a higher equity value without violating the lender's break-even constraint because $U_L > 0$ as noted above, but the break-even constraint must bind by Lemma 1. Hence, no maturity value L such that $Y_{LL} > 0$ can meet the necessary condition $Y_L = 0$. The second-order condition with respect to the accounting rule is

$$y_{BB} \equiv \frac{\partial^2 y}{\partial BV^2} = \Gamma \cdot \frac{d\delta(BV)}{dBV} = \Gamma \cdot \delta'(z(\theta) \cdot \Gamma) = -\frac{1}{z(\theta)} \cdot \delta(z(\theta) \cdot \Gamma)$$

when evaluated at the optimum $BV = z(\theta) \cdot \Gamma$, and so $y_{BB} < 0$ for all θ^* because $z > 0$ by Lemma 2.[31] Finally, the cross-partial derivative with respect to BV and L is

$$y_{BL} \equiv \frac{\partial^2 y}{\partial BV \partial L} = \frac{\partial \Gamma}{\partial L} \cdot \delta(BV) = -\frac{\partial \Gamma}{\partial c_1} \cdot \delta(BV)$$

where the second equality follows from the observation that Γ only depends on L and c_1 through the joint term $L - c_1$. The delta distribution is zero for $BV \neq 0$, and hence it suffices to examine y_{BL} for $\theta = \theta^*$. When $\lambda = 1$, the u terms are eliminated and one obtains

$$-\frac{\partial \Gamma}{\partial c_1} = (k^f - \mu_2) \cdot g(k^f|s) \cdot \frac{\partial k^f}{\partial c_1} + (\mu_2 - k^l) \cdot g(k^l|s) \cdot \frac{\partial k^l}{\partial c_1} \qquad \text{(A5)}$$

By Proposition 1, both liquidation thresholds decrease in c_1 and $k^f \geq \mu_2 \geq k^l$, which yields $-\frac{\partial \Gamma}{\partial c_1} < 0$. Equation (A5) thus implies that the marginal benefit of giving control rights to the lender for a given s is monotonically decreasing in c_1, and hence the critical value c_1^* at which $\Gamma = 0$ is unique when $\lambda = 1$. If c_1^* were not unique for some $\lambda \neq 1$, there would have to exist some interval on the support of c_1 whose states are in D and whose neighboring interval to the left is in N. But then the two parties could exchange control rights over states from these two intervals such that the break-even constraint $U = I$ is maintained. This control transfer is feasible because $u_B \leq 0$ for all θ as noted above and would decrease inefficiency. Hence, an accounting rule that yields multiple c_1^* for a given s cannot meet condition (A2), and so $y_{BL} < 0$ for any θ^*.

[31] Equivalently, one could define $\frac{d\Gamma}{dBV} = \nabla\Gamma \cdot \iota$ as the directional derivative of Γ, where ∇ is the gradient with respect to $\theta = (c_1, s)$, and ι is any vector in Θ that corresponds to a unit increase in BV. Graphically, ι thus points from default to no-default states.

The above observations imply that $\det(\mathbf{M})$ must be non-negative for all θ and strictly positive for all θ^*. Hence, any accounting rule and maturity value that satisfy the first-order condition attain a maximum. Multiple local maxima would require the existence of saddle points or minima, and so the maximum must be unique.

Proof of Proposition 3. Let s^* and c_1^* denote the critical values at which $BV = BV_0 = 0$. The path of $\{\theta^*\}$ is one-to-one if the Euler-Lagrange equation

$$z(\theta) \cdot \Gamma = 0$$

holds for a unique c_1^* given s and for a unique s^* given c_1, where $BV = z(\theta) \cdot \Gamma$ is the optimal accounting rule from Lemma 2. Since $\Gamma = 0$ at all θ^* and $z > 0$ for all θ by Lemma 2, the type of stationary point at θ^* for a given c_1 is identified by the sign of

$$\frac{\partial \Gamma}{\partial s} = \int_{k^l}^{k^f} (x_2 - x_v) \cdot l(v|s)\, dG + \lambda \int_{k^l}^{k^f} (u_2 - u_v) \cdot l(v|s)\, dG$$

which is identical to Γ except for the likelihood ratio factor

$$l(v|s) \equiv \frac{\partial g(v|s)}{\partial s} \cdot \frac{1}{g(v|s)}$$

By Assumption 1, l is increasing in v. Further, the parenthetical terms in both integrands are increasing on $[k^l, k^f]$, as shown in the proof of Proposition 1. Since $\Gamma = 0$ for any θ^*, the previous observations imply that $\frac{\partial \Gamma}{\partial s} < 0$. Control is therefore optimally transferred from the firm to the lender at any s^*. The existence of multiple s^* for a given c_1 would require that $\frac{\partial \Gamma}{\partial s} > 0$ for at least one s^*, and hence the critical value must be unique. The uniqueness of the critical value c_1^* follows from $\frac{\partial \Gamma}{\partial c_1} > 0$ at any θ^*, as shown in the proof of Proposition 2 above.

Proof of Corollary to Proposition 3. Let $\Gamma_c \equiv \frac{\partial \Gamma}{\partial c_1}$ and $\Gamma_s \equiv \frac{\partial \Gamma}{\partial s}$. The level set

$$\{\theta^*\} = \{\theta : BV(\theta) = 0\} = \{\theta : \Gamma = 0\}$$

is characterized by the directional derivative

$$\nabla BV \cdot \iota = \Gamma_c \cdot dc_1^* + \Gamma_s \cdot ds^* = 0 \qquad \text{(A6)}$$

where ∇ denotes the gradient of BV with respect to θ, and ι is a vector in the c_1-s-space tangent to the path of $\{\theta^*\}$. Equation (A6) must hold for all θ^* and states the well-known result that the

gradient of BV must be orthogonal to the level set $\{\theta^*\}$. Dividing (A6) through by $\Gamma_c - \Gamma_s$ and integrating yields the separated solution

$$BV = bv_0 + bv_c(c_1) + bv_s(s)$$

where bv_0 is a constant,

$$bv_c(c_1) = \int_0^{c_1} \frac{\Gamma_c\big(t, s^*(t)\big)}{\Gamma_c\big(t, s^*(t)\big) - \Gamma_s\big(t, s^*(t)\big)}\, dt$$

is positive and increasing for all c_1 because $\Gamma_c > 0$ and $\Gamma_s < 0$ at all θ^* by Proposition 3, and

$$bv_s(s) = \int_0^{s} \frac{\Gamma_s\big(t, c_1^*(t)\big)}{\Gamma_c\big(t, c_1^*(t)\big) - \Gamma_s\big(t, c_1^*(t)\big)}\, dt$$

is negative and decreasing for all s.[32] The cutoff values s^* and c_1^* are uniquely identified for given c_1 and s, respectively, by Proposition 3. By construction, $|bv_i| \in [0,1]$ for $i = c, s$ and all θ, and hence both terms are bounded.

Proof of Proposition 4. By Proposition 2, the optimal accounting rule BV is identified by its first-order condition $y_B = 0$. Then BV is still optimal after an increase in I if and only if

$$\frac{dy_B}{dBV} = y_{BB}BV_I + y_{BL}L_I + u_B\lambda_I = y_{BB}BV_I + y_{BL} \cdot \left(L_I + \frac{u_B}{y_{BL}}\lambda_I\right) = 0 \qquad \text{(A7)}$$

for all θ^*, where $BV_I \equiv \frac{\partial BV}{\partial I}$, $L_I \equiv \frac{\partial L}{\partial I}$ and $\lambda_I \equiv \frac{\partial \lambda}{\partial I}$ denote the contract term adjustments and all other terms are defined as in the proof of Proposition 2. Similarly, the adjustments must solve

$$\frac{dY_L}{dI} = Y_{LL}L_I + \int_\Theta y_{BL}BV_I\, dH + U_L\lambda_I = 0 \qquad \text{(A8)}$$

and

$$\frac{dU}{dI} = U_L L_I + \int_\Theta u_B BV_I\, dH = 1 \qquad \text{(A9)}$$

If $BV_I \geq 0$ for all θ^*, (A9) implies that $L_I > 0$, which in turn yields $\lambda_I > 0$ by (A8) because $Y_{LL} < 0$ and $U_L > 0$ under any optimal contract by Proposition 2. But then (A7) cannot hold for any θ^* because $y_{BB} < 0$, $y_{BL} < 0$ and $u_B < 0$ for all θ^* by Proposition 2, and hence $BV_I \geq 0$ for

[32] If s^* is undefined for some c_1 because either the lender or the firm optimally receives control for all s, one can set Γ_s to zero so that the integrand of bv_c becomes $\frac{\Gamma_c}{\Gamma_s - \Gamma_c} = 1$ for these c_1. This normalization is without loss of generality because the constant term bv_0 can be adjusted accordingly. An analogous argument applies to values of s for which c_1^* is undefined in bv_s. Graphically, these two cases correspond to regions beyond the points at which the path of $\{\theta^*\}$ shown in Figure 2 intersects the c_1-axis or the s-axis, respectively.

all θ^* is impossible. Conversely, in order to demonstrate that $BV_I \leq 0$ for all θ^* is feasible, one can consider the scenario $y_{BL} \propto u_B$ on $\{\theta^*\}$. Then BV_I must be of the same sign for all θ^* because

$$L_I + \frac{u_B}{y_{BL}}\lambda_I = C$$

for some constant C at all θ^* in (A7). The case $C < 0$ implies $BV_I \geq 0$, which has been ruled out as a solution, and hence $BV_I \leq 0$ for all θ^* in this scenario. Therefore, the conservatism of BV increases in I.

Proof of Proposition 5. Since $\frac{dY}{dI} = 1 - \lambda$ by the envelope theorem, firm value is convex in I if $\frac{\partial \lambda}{\partial I} < 0$. Given the uniqueness of the optimal contract terms, the contract term adjustments $\lambda_I \equiv \frac{\partial \lambda}{\partial I}$, $L_I \equiv \frac{\partial L}{\partial I}$ and $BV_I \equiv \frac{\partial BV}{\partial I}$ in response to an increase in I are optimal if and only if

$$\int_\Theta \mathbf{M} \cdot \begin{bmatrix} \lambda_I \\ L_I \\ BV_I \end{bmatrix} dH = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{A10}$$

where **M** is as defined in Proposition 2. Then in each state θ^*, the integrand of (A10) is

$$\mathbf{M} \cdot \begin{bmatrix} \lambda_I \\ L_I \\ BV_I \end{bmatrix} = \begin{bmatrix} 1 + \varepsilon_1 \\ \varepsilon_2 \\ 0 \end{bmatrix} \tag{A11}$$

for some ε_i, where $E_\theta(\varepsilon_i) = 0$ for $i = 1,2$. The third element on the right-hand side of (A11) is always zero because the first-order condition $y_B = 0$ with respect to BV must hold for every θ. Solving (A11) yields

$$\lambda_I \equiv (1 + \varepsilon_1) \cdot (Y_{LL} y_{BB} - y_{BL}^2) + \varepsilon_2 \cdot (u_B y_{BL} - U_L y_{BB})$$

and

$$L_I \equiv (1 + \varepsilon_1) \cdot (u_B y_{BL} - U_L y_{BB}) - \varepsilon_2 u_B^2$$

for all θ^*, where the definitions and signs of all terms on the right-hand sides are as given in the proof of Proposition 2. Since $E_\theta(\varepsilon_2) = 0$, continuity implies that $\varepsilon_2 = 0$ for some θ. Then if $L_I > 0$, it must be that $\varepsilon_1 > -1$ at this θ, in which case λ_I is negative if

$$Y_{LL} y_{BB} - y_{BL}^2 < 0 \tag{A12}$$

It remains to the shown that both $L_I > 0$ and (A12) hold for all I. First, $L = 0$ when $I = 0$ but $L > 0$ when $I > 0$, and so by continuity, $L_I < 0$ can only hold if $L_I = 0$ for some I. The mul-

tiplier of ε_1 in L_I is always nonzero, and so $L_I = 0$ can only hold if $\varepsilon_1 = -1$ when $\varepsilon_2 = 0$, which in turn implies $\lambda_I = 0$. Then (A7) can only hold if $BV_I = 0$ for all θ^*, but then all contract terms remain unchanged while I has increased, in which case the lender's break-even constraint is no longer met. Hence, $L_I \leq 0$ cannot hold for any level of I.

In order to establish (A12), one can note that its left-hand side is equal to the determinant of the Hessian matrix in the unconstrained optimization program

$$\max_{BV,L}\big(w - q(BV) + (\lambda - 1) \cdot u(BV)\big) \tag{A13}$$

over L and BV, which differs from the original program (8) in that λ has been fixed, Θ is reduced to a single state and u is unconstrained. The unique maximum of this program is always a corner solution with either $L = 0$ and $BV > 0$ when $\lambda \leq 1$, or $L = 2\bar{c}$ and $BV < 0$ when $\lambda \geq 1$ because i) $q = 0$ in both cases, ii) w does not depend on the contract terms, iii) u attains its minimum or maximum when all payoffs and control rights are allocated to one party only, and iv) the interior stationary point given by L and BV must be unique by Proposition 2, and so local interior maxima cannot exist. Further, $Y_{LL} < 0$ and $y_{BB} < 0$ imply that L and BV cannot yield a minimum either, and therefore (A13) must be at a saddle point, which implies that (A12) holds.

Proof of Proposition 6. The level effect of η is the increase $\frac{\partial U}{\partial \eta}$ in the lender's payoff. Since Proposition 6 only refers to the composition effect of η, $\frac{\partial U}{\partial \eta}$ is assumed to be offset by an increase in I^U and does therefore not enter (A16) below. Given the uniqueness of the optimal contract, contract term adjustments in response to an increase in η are optimal if and only if

$$\frac{dy_B}{d\eta} = y_{B\eta} + y_{BB}BV_\eta + y_{BL}L_\eta + u_B\lambda_\eta = 0 \tag{A14}$$

for all θ^*, where $BV_\eta \equiv \frac{\partial BV}{\partial \eta}$, $L_\eta \equiv \frac{\partial L}{\partial \eta}$ and $\lambda_\eta \equiv \frac{\partial \lambda}{\partial \eta}$. Further, these adjustments must solve

$$\frac{dY_L}{d\eta} = Y_{L\eta} + Y_{LL}L_\eta + \int_\Theta y_{BL}BV_\eta \, dH + U_L\lambda_\eta = 0 \tag{A15}$$

and

$$\frac{dU}{d\eta} = U_L L_\eta + \int_\Theta u_B BV_\eta \, dH = 0 \tag{A16}$$

The definitions and signs of all terms not involving η follow from the proof of Proposition 2.

First, assume that $y_{B\eta} < 0$ for all θ^* and $Y_{L\eta} < 0$. Then if $BV_\eta \geq 0$ for all θ^*, (A16) can only hold if $L_\eta > 0$, which implies that $\lambda_\eta > 0$ in order for (A15) to hold. But then (A14) cannot hold for any θ^*, and so $BV_\eta \geq 0$ for all θ^* is impossible. Next, consider the scenario $u_B \propto y_{BL} \propto y_{B\eta}$ on $\{\theta^*\}$. Then

$$\frac{y_{B\eta}}{y_{BL}} + L_\eta + \frac{u_B}{y_{BL}}\lambda_\eta = C$$

for some constant C at all θ^* in (A14). The case $C < 0$ implies $BV_\eta \geq 0$ for all θ^*, which has been ruled out, and so it must be that $C > 0$ and hence $BV_\eta \leq 0$ for all θ^*.

The above arguments imply that the conservatism of BV increases in η if it can be shown that $y_{B\eta} < 0$ and $Y_{L\eta} < 0$. Since y_B is given by (A2), the effect of η on $y_{B\eta}$ takes the same sign as

$$\frac{\partial \Gamma}{\partial \eta} = \int_{k^l}^{k^f} \left(x_2 - x_v + \lambda \cdot (u_2 - u_v)\right) \cdot m(v|s)\, dG \tag{A17}$$

where

$$m(v|s) \equiv \frac{\partial g(v|s)}{\partial \eta} \cdot \frac{1}{g(v|s)}$$

as in the definition of η. The term is equivalent to Γ except for the weighting factor m. Then (A17) is negative for all θ^* because i) m is increasing in v by Assumption 1, ii) $x_2 - x_v$ and $u_2 - u_v$ are decreasing in v, and iii) $\Gamma = 0$ by the first-order condition. Then $y_{B\eta} \leq 0$ for all θ, with strict inequality for θ^*. The cross-partial derivative with respect to L and η is

$$\begin{aligned} Y_{L\eta} \equiv & \int_D (\mu_2 - k^l) \cdot \frac{\partial k^l}{\partial L} \cdot g(k^l|s) \cdot m(k^l|s)\, dH \\ & + \lambda \int_N (\mu_2 - k^f) \cdot \frac{\partial k^f}{\partial L} \cdot g(k^f|s) \cdot m(k^f|s)\, dH \\ & + (\lambda - 1) \int_N \left(\int_0^{k^f} \frac{\partial u_2}{\partial \eta} \cdot m(v|s)\, dG + \int_{k^f}^{\bar{c}} \frac{\partial u_v}{\partial \eta} \cdot m(v|s)\, dG \right) dH \\ & + (\lambda - 1) \int_D \left(\int_0^{k^l} \frac{\partial u_2}{\partial \eta} \cdot m(v|s)\, dG + \int_{k^l}^{\bar{c}} \frac{\partial u_v}{\partial \eta} \cdot m(v|s)\, dG \right) dH \end{aligned} \tag{A18}$$

For $\lambda = 1$, the last two terms are eliminated. Then (A18) must be negative because i) the integrand of the first term is always positive and the integrand of the second term is always negative in view of $k^l \leq \mu_2 \leq k^f$, ii) any s in N is lower than any s in D for a given c_1 by Proposition 3, and iii) m is increasing in v and decreasing in s, and so the inequality

$$m(k^f|s) \geq m(k^l|s) \tag{A19}$$

must hold both for all s given any c_1 and for all c_1 given any s. Then since $Y_L = 0$ must hold at the optimum and $Y_{L\eta}$ is equivalent to Y_L except for the weighting terms (A19), it must be that $Y_{L\eta} < 0$ for any λ sufficiently close to 1.

Proof of Proposition 7. By the envelope theorem, the firm's equity value changes with σ by

$$\frac{dY}{d\sigma} = \int_\Theta \left(-\frac{\partial q(BV)}{\partial \sigma} + (\lambda - 1) \cdot \frac{\partial u(BV)}{\partial \sigma} \right) dH$$

where

$$\frac{\partial q(BV)}{\partial \sigma} = \mathbf{I}_D(BV) \cdot (k^l - \mu_2) \cdot g(k^l|s) \cdot \frac{\partial k^l}{\partial \sigma} + \left(1 - \mathbf{I}_D(BV)\right) \cdot (k^f - \mu_2) \cdot g(k^f|s) \cdot \frac{\partial k^f}{\partial \sigma}$$

must be positive regardless of control allocation because $\frac{\partial k^l}{\partial \sigma} \leq 0$ and $\frac{\partial k^f}{\partial \sigma} \geq 0$ as shown in (16) and (17) and $k^l \leq \mu_2 \leq k^f$ by Proposition 1. The change in the lender's payoff is

$$\frac{\partial u(BV)}{\partial \sigma} = \mathbf{I}_D(BV) \cdot \int_0^{k^l} \frac{\partial u_2}{\partial \sigma} dG + \left(1 - \mathbf{I}_D(BV)\right) \cdot \left((\mu_2 - k^f) \cdot g(k^f|s) \cdot \frac{\partial k^f}{\partial \sigma} + \int_0^{k^f} \frac{\partial u_2}{\partial \sigma} dG \right)$$

which must be negative because

$$\frac{\partial u_2}{\partial \sigma} = -\int_0^{L-c_1} \frac{\partial F_2}{\partial \sigma} dc_2 \leq 0$$

Since the lender's break-even constraint binds for all σ by Lemma 1, $\frac{dY}{d\sigma}$ must also equal the change in total firm value, and so an increase in σ results in a decrease in both equity value and efficiency through $\frac{\partial q(BV)}{\partial \sigma}$. The decrease is larger for $\lambda > 1$ but mitigated for $\lambda < 1$ because $\frac{\partial u(BV)}{\partial \sigma} < 0$.

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