Disclosures and Investments*

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Abstract

This paper examines the determinants and economic efficiency of corporate voluntary disclosure. The focus is on the trade-off for an individual firm when the benefits and costs of voluntary disclosure stem from the consequences of its investment decision. When a firm's investment incentive is provided by the short-term market pricing of its shares as well as long-term cash flow payoffs, investment and voluntary disclosure decisions are intertwined. First, by transmitting value-relevant information to the market, voluntary disclosure leads to a more accurate pricing which, in turn, improves investment efficiency. Second, the firm may affect the market pricing of its shares in its favor by strategically disclosing or withholding its private information. This opportunistic use of disclosure has a feedback effect on investment efficiency, which may be distorted at the margin. The presence of a separate mandatory accounting report, while not directly useful in the firm's investment decision, improves the market pricing and may discipline the voluntary disclosure because the accounting information helps disentangle the firm's disclosure incentive, which limits the opportunistic behavior and thus limits the efficiency loss. Further, this paper investigates how the efficiency of investment decisions and the propensity for providing voluntary disclosure respond to various environmental variables. Two such key variables are the quality of the mandatory accounting report and the general economic outlook of the investment opportunity. These comparative statics results are the basis of many empirical predictions generated by the model. Finally, the implication of prohibiting voluntary disclosure on efficiency is also explored, and the results show that prohibiting voluntary disclosure may be efficiency enhancing under certain circumstances.

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1 Introduction

Corporate voluntary disclosure has become an important element of capital market dynamics. It has many salient features. First, it conveys value-relevant information for market pricing.[1] Second, it typically contains information related to firm activities which may not be immediately reported in accounting reports. Third, it interacts with other information sources, such as mandatory accounting reports.[2] This paper examines the determinants and economic efficiency of corporate voluntary disclosure. The focus is on the trade-off for an individual firm when the benefits and costs of voluntary disclosure stem from the consequences of its investment decision. When a firm's investment incentive is provided by the short-term market pricing of its shares, as well as long-term cash flow payoffs, investment and voluntary disclosure decisions are intertwined.[3] First, an investment in a project brings about an option to voluntarily disclose information about the project. By transmitting value-relevant information to the market, voluntary disclosure leads to a more accurate pricing which, in turn, improves investment efficiency. Here the voluntary disclosure option plays a positive role and provides an ex ante benefit to the firm. Second, the firm may affect the market pricing of its shares in its favor by strategically disclosing or withholding its private information. This opportunistic use of voluntary disclosure has a feedback effect on the investment efficiency, which may be distorted at the margin. Here the voluntary disclosure option plays a negative role and incurs an ex ante cost to the firm. In particular, when the investment profitability is low on average, firms may be tempted to implement not-so-profitable projects and disclose in order to distinguish from those of even worse types. When the investment profitability is high on average, firms may withhold information on slightly profitable projects in order to avoid being compared with those of even better types. Such propensity to selectively disclose becomes intense and detrimental to investment efficiency when the firm knows a lot more about its investment profitability than outsiders (i.e., the information asymmetry is large). The overall efficiency is determined by both positive and negative effects of voluntary disclosure.[4]

[1] Substantial empirical evidence confirms the value relevance and information content of corporate voluntary disclosure (e.g., Patell [1976] and Penman [1980]).

[2] Chen, Defond and Park [2002] provide empirical evidence that managers voluntarily disclose balance sheets when quarterly earnings are less informative. Lennox and Park [2006] find the propensity to issue earnings forecasts is related to the informativeness of financial reports.

[3] For example, major mergers-and-acquisitions (M&As) and their announcements typically go hand-in-hand. On October 9, 2006, Google Inc. announced a $1.65 billion purchase of YouTube Inc. resulting in a rise of its share price, which The Wall Street Journal interpreted as its expansion into the lucrative online video and social networking markets. Yahoo's failure to reach a deal for Facebook highlights the slower pace of its efforts to expand. See The Wall Street Journal dated Oct 12, 2006 for more details.

[4] Rajan and Saouma [2006] study how the owner's payoffs are related to the quality of the manager's private information in a contract setting. In contrast, this paper investigates how the firm's investment

Also playing a role is a mandatory accounting report. While not directly useful in firm's investment decision, the mandatory report improves the market pricing and may discipline the firm's voluntary disclosure. This disciplining role limits the opportunistic behavior and, thus, reduces the efficiency loss. In particular, the mandatory report helps the market distinguish among firms that do not disclose and disentangle firms' disclosure incentives. Marginal improvement in the informational quality of the mandatory report mitigates firms' distorted disclosure incentives. This disciplining role of the mandatory report becomes effective when the firm's private signal is precise. However, when the private signal is coarse, a marginal improvement in the mandatory report may reduce the efficiency because it draws pricing weights away from the voluntary disclosure and, thus, induces sub-optimal investment. This demonstrates an interaction between the economic efficiency induced by the mandatory accounting report and the private signal.[5]

The model setup is consistent with findings in archival and survey research. For instance, the model studies how a firm opportunistically affects the market pricing in its favor through voluntary disclosure, which is consistent with the survey evidence from Graham, Havey and Rajgopal [2005] that firms' voluntary disclosure decisions are motivated by market prices. Archival empirical evidence also shows that corporate voluntary disclosure is motivated by short-term share price interests, such as CEO stock-based compensation (Aboody and Kasznik [2000]; Nagar, Nanda and Wysocki [2003]), insider trading (Noe [1999]; Cheng and Lo [2006]), and equity offering (Lang and Lundholm [2000]). In this paper, the accounting report is mandatory and not subject to firms' discretion, and the investment incentive is provided partially by the market price. These modeling choices are consistent with the results in Graham, Havey and Rajgopal [2005] that managers would rather take economic actions and make moderate sacrifices in long-term value, than make within-GAAP accounting choices to avoid negative market reactions in the Sarbanes-Oxley Act environment. In addition, this paper studies how disclosure decisions relate to different characteristics of voluntary disclosure and mandatory reporting as suggested by several empirical studies (e.g., Waymire [1985] and Lennox and Park [2006]). It also suggests implications on the market reaction to voluntary disclosure and mandatory reporting, which has been extensively studied in the empirical literature (e.g., Pownall, Wasley and Waymire [1993], Anilowski, Feng and Skinner [2006], and Chen, Matsumoto and Rajgopal [2006]). Beyond that, this paper shows that corporate voluntary disclosure is related to the economic outlook of investment opportunity (possibly driven by industry, business or firm life cycle), which provides the

efficiency is related to the quality of its private information (voluntary disclosure) in a market setting.

[5] Kanodia and Lee [1998] study a different setting where investments undertaken are publicly observable, and mandatory reports alleviate the perverse incentives driven by information asymmetry between firm and outsiders.

basis of new empirical hypotheses.

The insights in this paper may shed light on the efficiency issues associated with voluntary disclosure. In recent surveys of the literature, Dye [2001] and Verrecchia [2001] emphasize the importance of a theory designed to identify efficient resource allocations in a disclosure setting. Though the effects of mandatory financial reports on the efficiency of managerial behavior have been studied extensively,[6] the literature on how voluntary disclosure affects the economic efficiency has been scant.[7] By focusing on the link between investment and voluntary disclosure, this study addresses the economic relevance of disclosure as well as its determinants. In this paper, voluntary disclosure partially mitigates the under-investment in soft projects by transmitting value-relevant information to the market; but, when used opportunistically, it may induce investment inefficiency. Unlike the existing literature, this paper also points to the interactive effects between private and public information on firms' investment and disclosure decisions.

The disciplining role of accounting information is widely discussed in the literature. Much recent work has been based on the agency model in which accounting systems are viewed as a monitoring device for managers' self reporting (Gigler and Hemmer [1998], Liang [2000], Christensen and Demski [2002], and Arya, Glover, Mittendorf and Zhang [2004]). In this paper, by disciplining the firm's disclosure incentive, in a market setting (as opposed to a contract setting), an accurate accounting report may allow the firm's voluntary disclosure to be more beneficial for the economy.[8]

Specifically, this paper considers a model in which a firm is confronted with an investment decision and a voluntary disclosure option created by the investment. The firm's objective is to maximize a weighted average of the long-term cash flow payoffs and the short-term share price, net of the investment costs. The firm privately learns whether it will have an investment opportunity or not, which has a profitability correlated with the firm's ongoing

[6] Dye and Sridhar [2004] study the efficiency of accounting aggregation in the reliability and relevance of components of the report. Kanodia, Sapra and Venugopalan [2004] examine the economic trade-offs of intangible measurements. Stock and Verrecchia [2004] examine the efficiency of a manager's financial reporting system choice. Bhojraj and Libby [2005] examine the effect of mandatory disclosure frequency on managerial myopia in an experimental setting. Kanodia, Singh and Spero [2005], and Liang and Wen [2007] analyze the efficiency of accounting measurement basis.

[7] Verrecchia [1990] and Pae [1999] find a monotonically negative relationship between the efficiency and the quality of private information. In Dye [1985] and Jung and Kwon [1988], voluntary disclosure is neutral in terms of economic efficiency. Cheng, Subramanyam and Zhang [2006] find empirical evidence that firms that frequently issue quarterly earnings guidance behave myopically.

[8] In this paper, both the accounting report and the voluntary disclosure are assumed to be credible, and the analysis focuses on the disciplinary role of the accounting report on the distorted disclosure incentive, rather than its credibility. Gigler and Hemmer [1998], Liang [2000], and Arya, Glover, Mittendorf and Zhang [2004] examine the role of accounting in disciplining softer sources of information in contract settings, where disclosure cannot be credibly communicated, but a verifiable public signal creates an environment that enables the agent to credibly convey a private signal.

activities. If the investment opportunity is available, the firm observes a private signal informative of its profitability. Conditional on this private signal, the firm chooses whether to invest (and if so, how much) in the project or not. If the project is implemented, the firm acquires an additional option to make a voluntary disclosure about the project's gross return. A key assumption is that the implementation of the project is a prerequisite for voluntary disclosure. Firms without the investment opportunity do not have a credible means of disclosure as such, and are indistinguishable from firms with the opportunity that choose not to disclose.[9]

After the disclosure decision is made, a mandatory accounting report is announced, which is informative about the firm's ongoing activities. As the investment profitability is correlated with the firm's ongoing activities, the mandatory report indirectly conveys information on both the project (if implemented) and the firm's private signal (if available). Based on the mandatory report and the possible voluntary disclosure, the capital market prices the firm's shares competitively. The market's inability to distinguish among non-disclosure firms may induce distorted disclosures, in that the firm might be motivated to invest in unprofitable projects, or to withhold information on profitable projects. The direction of the distortion is determined by the ex ante expectation of the project's profitability. When the ex ante expected profitability is low, average firms in the market have less profitable projects and the market average conjecture on non-disclosure firms is also low. If the project has a slightly negative net return, but is still higher than the market average conjecture, the firm may choose to undertake the project and disclose in order to avoid under-valuation. In other words, exercising the disclosure option has a positive value ex post, which outweighs the investment loss. Alternatively, when average firms have profitable projects, the market average conjecture on non-disclosure firms is high. The firm with a slightly profitable project may choose to undertake the project but withhold information to avoid being compared to other disclosers (who disclose highly profitable projects). In other words, a voluntary disclosure is costly ex post, so the firm chooses not to exercise the disclosure option. In short, this paper identifies investment cycle (due to business/industry cycle or firm life cycle) as a major determinant of voluntary disclosure.

The second determinant is the information asymmetry between the firm and the market. When the private signal is noisy, the firm has less information advantage over the market. The disclosure is not informative and does not significantly revise the market's perception. As a result, the coarse private information lessens the incentive to distort the disclosure

[9] The model captures two popular rationales leading to a partial voluntary disclosure. First, building on Dye [1985] and Jung and Kwon [1988], the model introduces an information endowment uncertainty created by the uncertainty of the investment opportunity. Then, building on Verrecchia [1983; 1990], the model introduces a disclosure cost, which is the endogenous inefficiency of the firm's investment decision.

choice. If the private signal is precise, the information asymmetry between the firm and the market widens and the firm's disclosure is influential to the market pricing. Ex post, the firm is motivated to take advantage of its private information and to inflate its share price by selective disclosures, which may hurt the investment efficiency ex ante. On the other hand, a precise private signal allows the firm to better distinguish good or bad projects and therefore, the firm is better able to make efficient investment decisions (because it is partially motivated by long-term firm value). So the dominant effect determines the total efficiency effect of the private signal.[10]

The mandatory accounting report also plays an important role in the firm's investment and disclosure decisions.[11] Since the mandatory report indirectly conveys information on both the project (if implemented) and the firm's private signal (if available), a favorable report from non-disclosure firms indicates a higher likelihood of a lack of an investment opportunity (and thus no disclosure opportunity). And similarly, an unfavorable report from non-disclosure firms indicates a higher likelihood that the firm purposely withholds bad news. As a result, the mandatory report helps the market better distinguish among firms that do not disclose and disentangle the firm's disclosure incentive. Thus the firm's distorted disclosure incentive is alleviated. When the quality of the mandatory report is high enough, a precise private signal improves the efficiency. However, when the private signal is coarse and the distorted disclosure incentive is trivial, coarsening the mandatory report may be efficiency enhancing because it alleviates the under-investment. The results demonstrate an interaction between the mandatory accounting report and the firm's private signal.[12]

Finally, this paper investigates how prohibiting voluntary disclosure affects the investment efficiency. Without voluntary disclosure, the distorted disclosure incentive (which represents the endogenous disclosure cost) is eliminated and no disclosure cost is incurred (because firms implement the project only if the net return is positive). However, the market is also unable to fully price in the benefit of the project, and thus provides a more muted investment incentive (leading to the traditional under-investment problem). This analysis shows that when the investment profitability is low on average and the information asymmetry is large, prohibiting voluntary disclosure is efficiency enhancing because the gain from

[10] In Verrecchia [1990] and Pae [1999], the firm is, ex ante, best off by choosing to be completely uninformed. In other words, the efficiency effect of the private signal is always negative. This model allows a positive role of firm's private signal, so the efficiency effect of the private signal could also be positive.

[11] Dye [1998], Dutta and Trueman [2002] and Suijs [2006] study the manager's disclosure strategy when he is uncertain of investor response, because he does not know the market participants' entire information set.

[12] Einhorn [2005] examines the interaction between mandatory and voluntary disclosure in a setting in which a manager makes a disclosure decision after observing both public and private signals. In this setting, the disclosure decision is made before the mandatory accounting report is announced. As such, we introduce a disciplining role of the accounting report on disclosure decisions.

eliminating disclosure cost is higher than the loss from under-investment.

The rest of the paper is organized as follows. Section 2 describes the model in details. Section 3 presents the equilibrium investment and disclosure decisions and the market pricing. Section 4 analyzes how the firm's disclosure choice responds to a private/public information structure and the prior expectation of investment profitability. Section 5 looks further into the firm's investment decisions and efficiency implications. Section 6 introduces an extension of prohibiting voluntary disclosure. Section 7 concludes the paper. The appendix contains all proofs.

2 The Model

2.1 Investment Decision

There are three dates in the model. On date 1, the firm privately learns whether it faces an investment opportunity (called a project). One can think of this project as an expansion of the firm's ongoing activities through internal growth or mergers-and-acquisitions. For example, the firm may discover a growing market demand or a new technology may allow additional innovations (such as consumer-friendly features) in its products. It is assumed that the investment opportunity occurs with probability $q \in (0, 1)$, and the availability of the project is independent with all other random variables in the setting.[13] Outsiders do not know whether the firm has an investment opportunity or not.

The firm confronted with the investment opportunity observes a private signal θ which is informative about the profitability of the project, denoted by x. The private signal θ is a garbling of x:

$$\theta = x + \varepsilon_\theta,$$

where ε_θ is normally distributed with mean zero and variance σ_θ^2, i.e., $\varepsilon_\theta \backsim N[0, \sigma_\theta^2]$, and independent with all other random variables. The variance σ_θ^2 represents the informational quality of the private signal θ. If σ_θ^2 is lower, θ conveys more information about the prospect of the project.

On date 2, the firm with the investment opportunity must choose whether to invest the project or not and how much to invest based on the realization of θ. The cash flow generated by the project z depends on both the investment level I ($I \in \mathbb{R}^+$) and the project's

[13] This assumption of independence is used for tractability and is common in the literature (e.g., Dye [1985; 1998]).

profitability parameter x[14],

$$z = 2\sqrt{I} + x.$$

The profitability of the project is closely related to the firm's ongoing activities. Let y denote the cash flow generated by the firm's ongoing activities, which is realized at the end of the game and not observable to the firm or outsiders.[15] The project's profitability x and the ongoing activities' payoff y follow the jointly normal distribution given by:

$$\begin{bmatrix} x \\ y \end{bmatrix} \sim N\left(\begin{bmatrix} \mu \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix} \right),$$

where $\rho \in (0,1)$ is the positive correlation coefficient between the project and the firm's ongoing activities. The mean of the project profitability μ could be interpreted as an indicator of the business/industry cycle, or as the firm life cycle.[16] Following the literature, the firm's investment decision is made privately.[17]

It proves convenient to frame the firm's decisions on the project as (i) a project selection choice, whether to implement the project based on the private signal (θ), and (ii) an investment level choice (I) (i.e., how much to invest in a chosen project). The socially optimal investment level is 1. The socially optimal project selection choice is to implement only if the expected profitability (x) conditional on the private signal (θ) is higher than -1.[18] When the private signal is more precise, the firm is able to make a more efficient project selection choice.

[14] All of the qualitative results remain if we generalize the production function as exhibiting decreasing returns to scale in the form of $z = \lambda \cdot I^{\alpha} + x$, where $\lambda > 0$, and $0 < \alpha < 1$.

[15] Alternatively, one can assume the ongoing cash flow is generated by an endogenous choice variable of the firm. The economic efficiency of such firm activities are also affected by the upcoming disclosure choice. One can show that all of the qualitative results remain (analysis is available upon request). A more realistic interpretation is that y summarizes all cash flows related with ongoing activities and possible future investments or growth opportunities.

[16] The mean of the cash flow from ongoing activities is immaterial to this model. For simplification, we normalize the mean of y to zero.

[17] This private investment assumption is common in the literature (e.g., Dye and Sridhar [2004; 2006], Kanodia, Singh, and Spero [2005], and Liang and Wen [2007]). It can be a scenario where an entrepreneur invests in a risky project and sells a part of the shares to the open market. One could also interpret the investment as the manager's effort cost as opposed to a monetary cost. For example, in Pae [1999], an entrepreneur provides costly effort that stochastically enhances the future cash flow.

[18] To be precise, the expected x conditional on θ is equal to $\mu + \frac{\sigma_x^2(\theta-\mu)}{\sigma_x^2+\sigma_\theta^2}$. The socially optimal project selection is to implement if $\mu + \frac{\sigma_x^2(\theta-\mu)}{\sigma_x^2+\sigma_\theta^2} > -1$.

2.2 Disclosure Choice

If a project is implemented, the firm has an option to make a voluntary disclosure, denoted by s,[19] where

$$s = 2\sqrt{I} + \theta = z + \varepsilon_\theta.$$

Note s is an unbiased estimate of z with the noise level of σ_θ^2. As a result, the informational quality of disclosure is determined by the private signal (θ). As θ becomes more precise, the disclosed signal (s) is more informative about z. One can interpret s as a forecast of future cash flows generated by the project, such as a management projection of cash flows added to the firm after an acquisition transaction. Firms who invest in a project, but choose not to disclose, are indistinguishable from firms without an investment opportunity. In other words, firms cannot credibly communicate their lack of opportunity. So the implementation of a project endows the firm with an option to credibly disclose its private signal and the investment.[20] Because the disclosure is not mandatory and is subject to the firm's discretion, the firm can make a strategic disclosure choice to influence investors' assessment of the firm's value. Let the function $d \in \{D, ND\}$ describe the disclosure rule adopted by the firm if a project is undertaken, where D stands for disclosure and ND for non-disclosure.

On date 3, the accounting system publicly reports an unbiased estimate of y, denoted by r,

$$r = y + \varepsilon_r,$$

where ε_r is normally distributed with mean zero and variance σ_r^2, i.e., $\varepsilon_r \backsim N[0, \sigma_r^2]$, and independently distributed with all other random variables. The scenario that the model describes is common. For example, accounting reports usually provide data on past transactions, and may not be timely enough to include all value-relevant information. As an alternative information source, the disclosure of the firm's assessment on soft investments generally remains voluntary and timely.[21]

[19] All of the qualitative results remain if we define voluntary disclosure as the firm's rational expected cash flows of the project given θ, which is $s = 2\sqrt{I} + \mu + \frac{\sigma_x^2(\theta-\mu)}{\sigma_x^2+\sigma_\theta^2}$.

[20] The project works as a carrier of the firm's private information. Only through a measurement of the project's future cash flow, is the firm able to credibly communicate its private information to the outsiders.

[21] In Dutta and Reichelstein [2005], the accounting system is imperfect in its ability to measure "softer" investments, like those in product development and personnel training.

2.3 Firm's Objective

Finally, the firm value is priced in a competitive capital market such that the market price, denoted by P, is determined by the expected total cash flow V, conditional on all available information, including the market's rational conjectures about the firm's disclosure rule. Denote the publicly available information set by Ω (assuming no discounting or dividend payments):

$$\begin{aligned} P &= E[V|\Omega], \\ where\ V &= \begin{cases} y, & if\ I = 0 \\ y + z, & if\ I > 0 \end{cases}, \quad \text{and} \quad \Omega = \begin{cases} \{r\}, & if\ d = ND \\ \{r, s\}, & if\ d = D \end{cases}. \end{aligned}$$

Following Stein [1989] and Liang and Wen [2007], the firm is motivated by both the long-term and the short-term interests. In particular, the firm's objective is to maximize a weighted average of the market price P and the total cash flow V (net of the investment costs I). For a realization of θ, the objective function is[22]

$$U(I(\cdot), d(\cdot)|\theta) = E[\beta P + (1-\beta)V|\theta] - I.$$

The sequence of the events is summarized below.

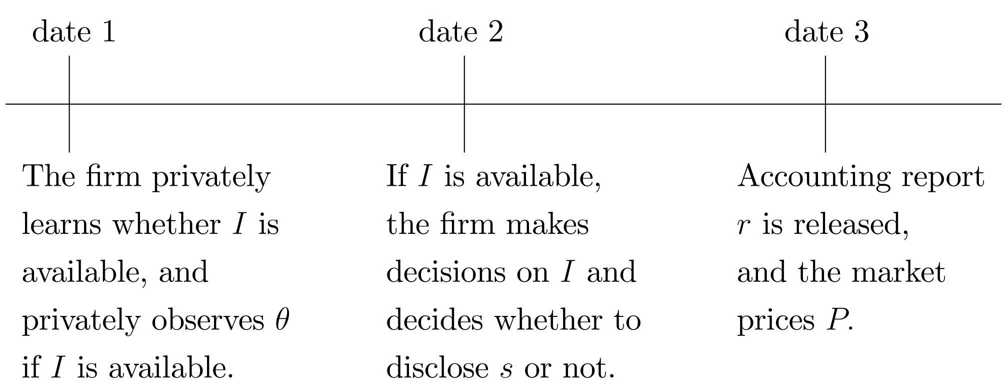


Figure 1: Time Line

3 Equilibrium Characterization

I now define the equilibrium where the investment and disclosure decisions are made in the self-interest of the firm.

[22] We assume a portion of the firm, denoted β, must be sold to the new shareholders at the end of period 3. The remaining $(1-\beta)$ portion will be held by the old shareholders. Another interpretation may be that the firm faces a probability β of takeover risk (see Stein [1989] for additional discussions).

Definition 1 *An equilibrium consists of an investment function $I(\cdot)$, a disclosure rule $d(\cdot)$, and a market pricing function $P(\cdot)$, such that:*

(i) Given $P(\cdot)$ and the firm's private signal θ, the optimal investment $I(\cdot)$ and disclosure rule $d(\cdot)$ maximize $U(I(\cdot), d(\cdot)|\theta) = E[\beta P(\cdot) + (1-\beta)V|\theta] - I$, subject to the constraint that $d = ND$ if $I = 0$;

(ii)Given $I(\cdot)$ and $d(\cdot)$, the market pricing function $P(\cdot)$ satisfies $P = E[V|\Omega, I(\cdot), d(\cdot)]$.

The definition of equilibrium is straightforward. In part (i), taking the market pricing as given and conditioning on its private signal (θ), the firm chooses the optimal investment $I(\cdot)$ and disclosure rule $d(\cdot)$ to maximize the weighted average of the market price (P) and the future total cash flow (V). In part (ii), the share price P is determined based on the capital market's assessment of the total cash flow, making a rational expectation of the firm's investment and disclosure decisions.

3.1 Equilibrium Investment and Disclosure Decisions

Let $P^N(r)$ ("N" for no disclosure) denote the equilibrium price function if no voluntary disclosure is made by the firm, and $P(r,s)$ denote the equilibrium price function if the firm makes voluntary disclosure. The following proposition presents the firm's investment and disclosure decisions, given market price functions $P^N(r)$ and $P(r,s)$.

Proposition 1 *There exists an equilibrium characterized by a threshold value θ^*, where the firm's investment and disclosure decisions are given by*

(i) equilibrium investment functions:

$$If\ \theta^* > \theta^0,\ I = \begin{cases} [1-\beta+\beta P_s'(r,s)]^2, & if\ \theta > \theta^* \text{ and the investment is available} \\ (1-\beta)^2, & if\ \theta^* > \theta > \theta^0 \text{ and the investment is available} \\ 0, & if\ \text{otherwise.} \end{cases} \quad (1)$$

$$If\ \theta^* < \theta^0,\ I = \begin{cases} [1-\beta+\beta P_s'(r,s)]^2, & if\ \theta > \theta^* \text{ and the investment is available} \\ 0, & if\ \text{otherwise.} \end{cases} \quad (2)$$

(ii) an equilibrium disclosure rule:

$$d = \begin{cases} D, & if\ \theta > \theta^* \text{ and the investment is available} \\ ND, & if\ \text{otherwise.} \end{cases} \quad (3)$$

where:

$P_s'(\cdot)$ *is the first order derivative of* $P(\cdot)$ *with respect to* s, $\theta^0 = -\dfrac{(1-\beta)^2(\sigma_x^2+\sigma_\theta^2)+\sigma_\theta^2\mu}{\sigma_x^2}$.

Proof. All proofs are placed in the appendix. ■

We make two key observations on the firm's equilibrium investment and disclosure behavior:

- *Disclosure rule*: The firm's disclosure rule is represented by a simple cut-off (threshold) value θ^*. If the firm observes a news better than θ^*, it will invest in the project and disclose such news (i.e., $d = D$ if $\theta > \theta^*$).

- *Project selection choice*: The firm's project selection choice is also represented by a cut-off value, the lower of θ^* and θ^0 (i.e., $I > 0$ if $\theta > \min(\theta^*, \theta^0)$).[23] It is possible for the firm to invest but not disclose ($\theta^0 < \theta < \theta^*$), or to invest in an unprofitable project and disclose ($\theta^* < \theta < \theta^0$).

According to equation (3), the firm chooses to disclose when its private signal is high and refrains from disclosing when its private signal is low. Though this type of disclosure strategy is similar to extant results in the literature,[24] the underlying driving force is different. In this model, the firm's disclosure decision is associated with its investment decision, because the disclosure is only possible if the project is implemented. This analysis endogenizes the disclosure cost as the possible inefficiency of the firm's investment decision.

To see this, assuming the ex ante expectation of the profitability is low (i.e., μ is low, and thus $\theta^* < \theta^0$), average firms have less profitable projects. Suppose the firm's expected net project return conditional on its private signal (θ) is slightly negative, but its cash flow prospect of the ongoing activities is higher than the market's average conjecture of non-disclosure firms. If the firm chooses not to disclose, its ongoing activities will be undervalued. If the firm chooses to disclose and distinguish itself (because of the correlation between the disclosure and the ongoing activities), there would be an efficiency loss because the firm must implement such a project. For these firms, the investment reduces the expected cash flow (negative NPV), but allows them to disclose and raise their share prices (i.e., the investment is the cost of disclosure). At a knife-edge case ($\theta = \theta^*$), the two choices lead to the identical

[23] θ^0 is the break-even level where the firm's expected net payoff from investing in the project is zero where $I = (1-\beta)^2$.

[24] In Dye [1985] and Jong & Kwon [1988], the disclosure strategy is driven by the market's inability to distinguish informed and uninformed firms. In Verrecchia [1983, 1990], Jorgensen and Kirschenheiter [2003], and Einhorn [2005], the driving force of the disclosure threshold is the exogenous cost of disclosing the private signal.

amount of losses and the firm is indifferent between disclosure and non-disclosure. For firms with profitable projects ($\theta > \theta^0$), investment and disclosure are both beneficial. For firms choosing not to disclose ($\theta < \theta^*$), either the disclosure itself is not beneficial, or the disclosure is beneficial but too costly.

One can also interpret the above results from a real option perspective.[25] Consider the investment opportunity as a real option for the firm, exercising the real option (i.e., the investment) generates another voluntary disclosure option. This embedded disclosure option makes the real investment option more valuable than a traditional NPV perspective suggests. Figure 2 illustrates the firm's disclosure strategy for different intervals of θ when μ is low.

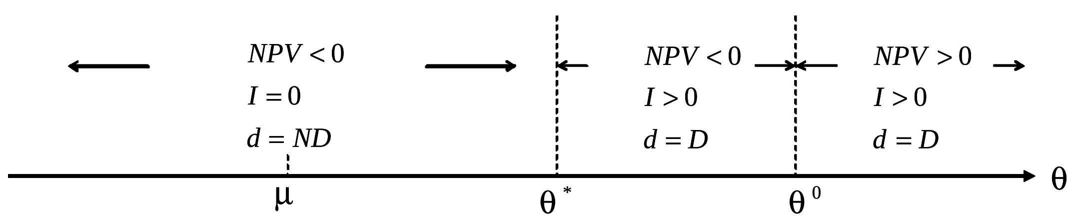


Figure 2: Disclosure strategy when μ is low

In contrast, now assume average firms have profitable projects (i.e., μ is high, and thus $\theta^* > \theta^0$). Suppose the firm's expected net project return conditional on its private signal (θ) is slightly positive, but its cash flow prospect of the ongoing activities is lower than the market's average conjecture on non-disclosure firms. If the firm chooses to disclose, the price effect could be positive because the market recognizes the implementation of the profitable project. However, the disclosure may lower the market assessment of the ongoing activities' cash flow. So choosing not to disclose is also attractive in that the firm is pooled with non-disclosure firms whose average type is higher. The disclosure decision is determined by which brings more positive effects. Figure 3 illustrates the firm's disclosure strategy for different intervals of θ when μ is high.

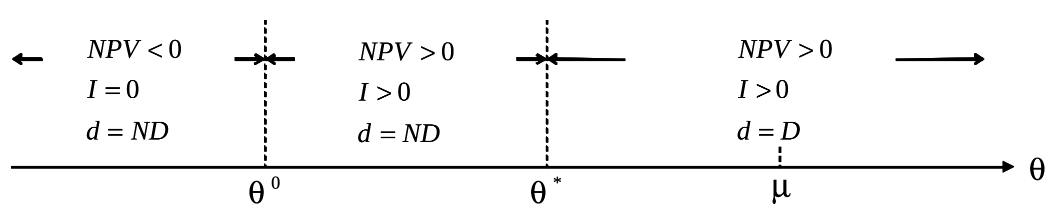


Figure 3: Disclosure strategy when μ is high

[25] Arya and Glover [2001] study a principal-agent model in which a control (incentive) problem makes the option to wait valuable when it would not have been valuable otherwise.

Even when the firm invests in a profitable project, the investment level may deviate from the socially optimal level. The first line of equations (1) and (2) indicate the investment level (I) when the firm chooses to disclose. As the disclosure conveys information about the investment, I is increasing in the market response to the disclosure ($P_s'(r,s)$). As $P_s'(r,s)$ approaches one, I becomes more efficient and approaches the socially optimal level. When the disclosure threshold (θ^*) is high (i.e. $\theta^* > \theta^0$), and the private signal is lower than θ^* but still higher than θ^0, the expected net return of the project is positive and the firm chooses to invest but not to disclose. In this case, the firm's investment decision is purely motivated by its long-term interest, which leads to under-investment.

3.2 Market Pricing

Proposition 1 characterizes the firm's investment and disclosure decisions given the market pricing functions. To complete the description of the equilibrium, Proposition 2 presents the equilibrium market pricing functions.

Proposition 2 *The equilibrium market pricing functions take the following form:*

$$P(r,s) = 2\sqrt{I} + \mu + b_r r + b_s(s - 2\sqrt{I} - \mu), \tag{4}$$

$$P^N(r) = \begin{cases} \frac{r\sigma_y^2}{\sigma_y^2+\sigma_r^2} - \frac{qb\sigma' f(c)}{1-q+q\Phi(c)}, & if \quad \theta^* < \theta^0 \\ \\ \frac{r\sigma_y^2}{\sigma_y^2+\sigma_r^2} - \frac{q\sigma'[b_s f(c)-(b_s-b)f(g)]-q[\Phi(c)-\Phi(g)][2(1-\beta)+\mu]}{1-q+q\Phi(c)}, & if \quad \theta^* > \theta^0 \end{cases} \tag{5}$$

where

$\Phi(\cdot)$ *is the cumulative distribution function of a standard normal variable,*

$f(\cdot)$ *is the probability density function of a standard normal variable,*

$$b_r = \frac{(1-\rho^2)\,\sigma_x^2\sigma_y^2 + (\sigma_y + \rho\sigma_x)\,\sigma_y\sigma_\theta^2}{(1-\rho^2)\,\sigma_x^2\sigma_y^2 + \sigma_y^2\sigma_\theta^2 + \sigma_x^2\sigma_r^2 + \sigma_\theta^2\sigma_r^2},\quad b_s = \frac{(1-\rho^2)\,\sigma_x^2\sigma_y^2 + (\sigma_x + \rho\sigma_y)\,\sigma_x\sigma_r^2}{(1-\rho^2)\,\sigma_x^2\sigma_y^2 + \sigma_y^2\sigma_\theta^2 + \sigma_x^2\sigma_r^2 + \sigma_\theta^2\sigma_r^2},$$

$$b = \frac{\rho\sigma_x\sigma_y\sigma_r^2}{(1-\rho^2)\,\sigma_x^2\sigma_y^2 + \sigma_y^2\sigma_\theta^2 + \sigma_x^2\sigma_r^2 + \sigma_\theta^2\sigma_r^2},\quad c = \frac{\theta^* - \mu'}{\sigma'},\quad g = \frac{\theta^0 - \mu'}{\sigma'},$$

$$\mu' = \mu + \frac{r\rho\sigma_x\sigma_y}{\sigma_y^2 + \sigma_r^2},\quad \sigma' = \sqrt{\sigma_x^2 + \sigma_\theta^2 - \frac{\rho^2\sigma_x^2\sigma_y^2}{\sigma_y^2 + \sigma_r^2}}.$$

Proposition 2 describes the market's rational expectation of the total future cash flow given all public information available. When the firm discloses, indicating $I > 0$, equation

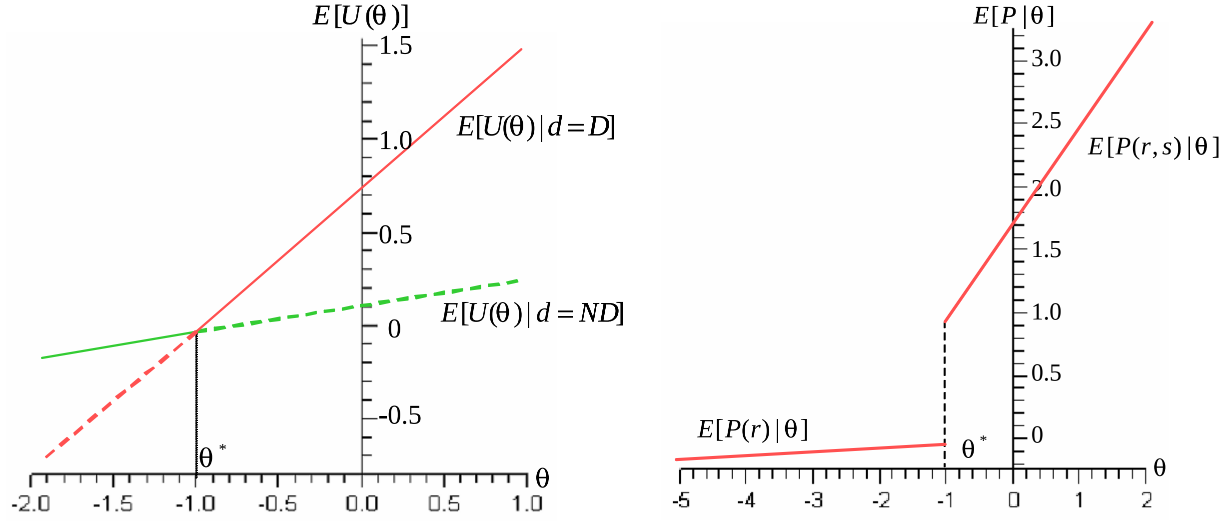
(4) shows that the market price is simply the expected total cash flow conditional on the mandatory report (r) and the disclosure (s). For any deviation of the signals, r and s, from their means, the market revises its valuation of the firm by response coefficients b_r and b_s respectively. The magnitude of the response coefficients measures the extent to which the signal r or s updates the market's belief about the total cash flow.

When no information is disclosed ($d = ND$), the market infers that either the firm does not find investment opportunities, or the firm chooses to withhold its private information. Since the market is not able to perfectly distinguish among non-disclosure firms, the price reflects an average assessment based on the mandatory report (r). In equation (5), the first item reflects the market's updated conjecture on y conditional on r, and the second item reflects the market's downward adjustment given a portion of non-disclosure firms must have observed low private signals.

To illustrate the equilibrium characterized by Proposition 1 & 2, consider the following numerical example. Let $\beta = 0.1$, $q = 0.5$, $\rho = 0.5$, $\sigma_r^2 = 10$, $\sigma_\theta^2 = \sigma_x^2 = \sigma_y^2 = 1$, and $\mu = -1$. The break-even level θ^0 is -0.62 where the firm's expected net payoff from investment is zero. The disclosure threshold θ^* is -1.015 which is lower than θ^0. Figure 4(a) illustrates the relation between the firm's expected net payoff ($E[U|\theta]$) and its private signal (θ) when the firm chooses to disclose (the steep line) and not to disclose (the less steep line). These two lines cross at θ^* where the firm is indifferent between disclosure and non-disclosure. Solid parts of these two lines indicate the firm's expected net payoff in equilibrium, while the dotted lines indicate off equilibrium choices. Figure 4(b) illustrates the relation between the firm's expected share price ($E[P|\theta]$) and its private signal (θ) given its optimal disclosure choice. The discontinuity at θ^* confirms the discussion on the disclosure choice when μ is low, that if the firm does not disclose, the firm is undervalued and if it discloses, the share price goes up, but it incurs an efficiency loss, so the firm is indifferent between disclosure and non-disclosure.

One can also interpret Figure 4(b) as the market average response to the firm's voluntary disclosure (s).[26] It suggests that the market pricing is positively related with the disclosure, and the discontinuity suggests a negative average market reaction to a non-disclosure firm which is consistent with the empirical evidence provided in Chen, Matsumoto and Rajgopal [2006] who find the average negative return around the announcement to stop earnings guidance.

[26] In equilibrium, the investment level I is constant and perfectly anticipated by the market. Since $s = 2\sqrt{I} + \theta$, Figure 4(b) can also be interpreted as the market average response to the disclosure.



(a) (b)

Figure 4: (a) Relation between the firm's expected net payoff and θ;
(b) Relation between the firm's expected market price and θ.

The following corollary summarizes the intuitive properties of the market responses and their effects on the investment decision in equilibrium.

Corollary 1 *(i) Both b_s and I (if $d = D$) decrease (increase) in σ_θ^2 (σ_r^2); and b_r decreases (increases) in σ_r^2 (σ_θ^2);*

(ii) When $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently high, b_r (b_s) increases (decreases) in ρ; when $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently low, b_r (b_s) decreases (increases) in ρ.

The noise term ε_r (ε_θ) reduces the value of the mandatory accounting report r (the voluntary disclosure s) in communicating information about the future cash flow. Any increase in the variance weakens the relationship between the cash flow and the reported value, so the capital market responds less accordingly, which explains part (i) that the market response coefficient b_r (b_s) decreases in the noise level of the reported value σ_r^2 (σ_θ^2). By Proposition 1, the investment level I (if $d = D$) is increasing in b_s, which is thus decreasing in σ_θ^2.

Here, we are particularly interested in the interactive effect between the mandatory report and the disclosure on I. The impact is driven by the interaction between the market responses to r and s. Corollary 1 shows the market response to the disclosure (b_s) increases in the noise level of the mandatory report (σ_r^2). In other words, if the mandatory report is noisier, the market transfers some pricing weight from r to s. This induces a substitution between the two information sources. The pricing weight is allocated between r and s depending on their relative noise levels. Since the firm's investment is motivated by b_s, we have the results that I (if $d = D$) increases in σ_r^2. Thus the information substitution also has real economic effects.

The correlation parameter ρ defines the extent to which the market infers the project's (the ongoing activities') profit from the mandatory (voluntary) report. As ρ increases, the interaction effect is more intense. When the information asymmetry is small, indicating that s is much noisier than r, more pricing weight is transferred from s to r when ρ increases (i.e., b_r (b_s) increases (decreases) in ρ). Alternatively, when the information asymmetry is large, more pricing weight is transferred from r to s when ρ increases (i.e., b_r (b_s) decreases (increases) in ρ).

4 Disclosure Strategy Analysis

Having characterized the equilibrium, I now analyze how the firm's disclosure choice responds to various parameters of the model. In particular, the analysis focuses on the impact of different features of the information structure (the firm's private signal and the mandatory accounting report) and the prior expectation of the investment profitability.

4.1 Effect of Information Structure

We begin with the case in which the informational quality of the private signal (θ) is relatively low compared with the public report (r). That is, the information asymmetry between the firm and outsiders is small. The following corollary presents the sufficient conditions under which the firm's disclosure threshold approaches the level where the project yields zero expected net return.

Corollary 2 *When $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently high, or ρ is sufficiently low, or β is sufficiently low, the disclosure threshold θ^* approaches $-\frac{\left[1-\beta^2(1-b_s)^2\right](\sigma_x^2+\sigma_\theta^2)+\sigma_\theta^2\mu}{\sigma_x^2}$ (i.e., $E(z-I|\theta^*, d=D)$ approaches 0).*

From the discussion in the last section, the firm has an incentive to affect the market's perception in its favor by undertaking a less profitable project ($\theta^* < \theta < \theta^0$), or by withholding information of a profitable project ($\theta^0 < \theta < \theta^*$). When the private signal is relatively noisy, the firm can barely affect the market's perception through selective disclosure, since it becomes easier for the market to distinguish among non-disclosure firms and translate the firm's disclosure choice based on the accurate mandatory report. In this sense, the noisy private signal makes the opportunistic use of voluntary disclosure less effective, which reduces the distortion. The firm chooses to undertake the project and discloses only if θ indicates the project yields a positive expected net return. At the same time, the benefit of voluntary disclosure is also diminished.[27]

Next, we consider the case in which the informational quality of the private signal is high relative to the mandatory report. That is, the information asymmetry between the firm and outsiders is large. The firm tends to utilize its information advantage to affect the market's perception in its favor. Intuitively, such a tendency becomes more intense as the information asymmetry widens. The following proposition analyzes how the disclosure threshold responds to the noise level of the mandatory report when the informational asymmetry is large.[28]

Proposition 3 *When $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently low, then:*

(i) if μ is sufficiently low, the disclosure threshold θ^* strictly decreases in the noise level of the mandatory report σ_r^2;

(ii) if μ is sufficiently high, the disclosure threshold θ^* strictly increases in the noise level of the mandatory report σ_r^2.

As demonstrated by Proposition 1, the firm with a lower private signal generally tends to refrain from disclosure to benefit from pooling. The mandatory report, to some extent, conveys information about the private signal θ (via v_1 and v_2). Indirectly, the mandatory report removes some benefits of pooling. A precise mandatory report makes it harder for the firm with a lower private signal to pool with those without an investment opportunity. This disciplining role discourages (distorted) disclosures when μ is low, and encourages more (beneficial) disclosures when μ is high.

More specifically, assuming μ is low, average firms in the market have less profitable projects. The benefit from disclosure motivates firms to make distorted disclosure decision. When the noise level of the mandatory report (σ_r^2) decreases and the report conveys more value-relevant information, the marginal benefit of the distorted disclosure is lessened. Thus the firm becomes more reluctant to undertake a less profitable project and disclose. The disclosure threshold θ^* shifts upward, and the project selection choice becomes more efficient. In this case, the mandatory accounting information is a substitute for the firm's disclosure.

[27] The same results exist when either ρ or β is sufficiently low. When ρ is sufficiently low, θ is uninformative to the ongoing activities, and the market conjecture on the ongoing activities is irrelevant to s. Thus the distorted incentive is eliminated. Similarly, when β is sufficiently low, the firm's payoff is more consistent with the underlying firm value, which also dampens the distorted disclosure incentive.

[28] Here, we do not analyze the effect of the private signal's noise level (σ_θ^2) on the disclosure threshold (θ^*) because those results are not comparable. Imagine if σ_θ^2 is changed, the firm adjusts its expected profitability accordingly. Then the results on the change of the threshold are less illuminating, since it could be either driven by the firm's adjustment on expected profitability, or by the change of information structure.

Better accounting information discourages detrimental voluntary disclosures. This substitution result is consistent with the empirical evidence from Chen, Defond and Park [2002] that when quarterly earnings are less informative, managers are more likely to voluntarily disclose information (which is related to balance sheets) to supplement information in earnings.

On the other hand, when μ is high and the mandatory report conveys more information about θ, the benefit of pooling with non-disclosure firms is diminished. More firms choose to disclose and the disclosure threshold (θ^*) shifts downward. In this case, the mandatory accounting information is a complement for the firm's disclosure. Better accounting information encourages more beneficial voluntary disclosures.

4.2 Effect of Investment Prospects

In this subsection, we examine the effect of investment prospects (represented by the ex ante expectation of the investment profitability μ) on the disclosure threshold (θ^*). Since neither the mandatory accounting report nor the firm's private signal perfectly reveals the firm value, the prior expectation (μ), to some extent, influences both the firm's and the market's perception. The following proposition summarizes how the prior expectation (μ) affects the firm's disclosure strategy.

Proposition 4 *If $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently low, the firm's disclosure threshold θ^* strictly increases in μ; If $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently high, the firm's disclosure threshold θ^* strictly decreases in μ.*

To interpret the results in Proposition 4, recall that the firm's disclosure threshold (θ^*) is determined by the cost (or benefit) of undertaking the project and making disclosure, and the cost (or benefit) of pooling with non-disclosure firms. When the prior expectation (μ) changes, both sides of the trade-off are affected.

How the investment prospect (μ) relates to the firm's disclosure trade-off depends on the relative informational quality of the private signal to the mandatory report. For instance, if $\frac{\sigma_\theta^2}{\sigma_r^2}$ is low, the firm's perception of the cost (or benefit) of disclosure is not affected by μ, while the market's conjecture heavily relies on μ. As μ is influential to the market perception, the pooling effect is enlarged. When μ increases, the cost (or benefit) of pooling with non-disclosure firms is reduced (increased) substantially, and the firm's perception of the cost (or benefit) of disclosure does not change much. So the more firms choose not to disclose, and the disclosure threshold shifts upward. Alternatively, when $\frac{\sigma_\theta^2}{\sigma_r^2}$ is high, the market price is relatively more accurate in evaluating firm value. The pooling effect is diminished, and the firm's payoff is more consistent with the underlying firm value. When μ increases, the firm's perception of the profitability is increased substantially, so more firms choose to disclose, and the disclosure threshold shifts downward. This analysis suggests an empirical prediction that when the information asymmetry is small, more firms voluntarily disclose if the investment prospect is high on average.

Table 1 summarizes the basic results of the firm's disclosure strategy analysis.

Table 1: Summary of the firm's disclosure strategy

	Effect of σ_r^2	Effect of μ
$\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently high	Ambiguous	$\frac{\partial\theta^*}{\partial\mu} < 0$
$\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently low	$\frac{\partial\theta^*}{\partial\sigma_r^2} < 0$ if μ is low $\frac{\partial\theta^*}{\partial\sigma_r^2} > 0$ if μ is high	$\frac{\partial\theta^*}{\partial\mu} > 0$

5 Investment Decisions and Efficiency Implications

To analyze the investment efficiency of the equilibrium characterized by Proposition 1 and 2, we need to consider the firm's investment decisions from the following two perspectives:

- *The project selection choice:* Since the firm's disclosure opportunity is decided by the project selection choice, the efficiency of selection choice is closely related to the firm's disclosure strategy. When the disclosure threshold is low ($\theta^* < \theta^0$), the selection choice is directly driven by the disclosure choice. When the threshold is high ($\theta^* > \theta^0$), the selection threshold is θ^0 where the firm's expected net payoff from the investment is zero.

- *The investment level (I):* If the firm discloses, I increases in the market response to the disclosure. If the firm does not disclose, and no information on I released directly to the market, I is severely under-invested.

We study the efficiency of the firm's investment decision by analyzing the effects of both the private signal and the mandatory accounting report, and the interaction between these two information sources.

5.1 Efficiency Effects of Private Information

Several studies examine the effects of private information on the firm's ex ante payoff. Verrecchia [1990] and Pae [1999] reach the same conclusion that the firm's ex ante payoff is monotonically decreasing in the quality of the private signal. That is, the firm is ex ante best off by choosing to be completely uninformed. The argument is that when the firm is more informed, it is more willing to disclose, which incurs more associated costs. A similar force is present in this setting. If the firm's private signal is precise, it is tempted to selectively make disclosure to maximize the market's perception on the firm's value. When the ex ante investment prospect (μ) is low, the firm would like to undertake a less profitable project and disclose for distinction; when μ is high, the firm would like to withhold private information for pooling. Such a propensity for selectively making disclosure or withholding information might be detrimental to the efficiency in both situations.

However, this setting allows for a positive role of private information. When the firm obtains better knowledge of underlying profitability, it is better able to distinguish good from bad projects, therefore, is better able to make efficient investment decisions. As the firm is partially motivated by its long-term value, better information endowment could also improve efficiency. Thus the positive and negative roles of the private information jointly determine whether private information improves efficiency or not. The economy may be better-off, ex ante, by having a privately informed firm. Proposition 5 provides a comparison between the efficiency effects of a precise private signal versus a coarse private signal.

Proposition 5 *Let V_H (V_L) denote the ex ante expected net return of the project when σ_θ^2 is sufficiently high (low), then:*
(i) when μ is sufficiently low, $V_H > V_L$;
(ii) when β is sufficiently low, $V_H < V_L$;
(iii) when σ_r^2 is sufficiently low, $V_H < V_L$.

Proposition 5 implies that an informative private signal can increase or decrease the efficiency. When μ is low, an informative private signal hurts efficiency. To explain this, recall that by Proposition 4, when σ_θ^2 is sufficiently low, the firm's disclosure threshold increases in μ. That is, a lower μ leads to a lower disclosure threshold, which implies that the firm tends to invest in a less profitable project and disclose, since the market puts more pressure on those without disclosure. As μ decreases, the firm incurs a higher cost for disclosure, which leads to a deterioration in project selection choice. Thus, the negative effects of private information are aggravated.

For a more informative private signal to improve efficiency, the firm must take advantage of its private information in an appropriate way. Recall that Corollary 2 shows that a lower β dampens the firm's distorted disclosure incentive, since the firm's payoff is more consistent with the underlying firm value. Meanwhile, a lower β also induces less distorted investment level decisions. As the negative role of the private signal is diminished, a more informative private signal enhances the efficiency because the firm becomes better able to make efficient investment decisions.

Part (iii) of Proposition 5 demonstrates that when the informational quality of the mandatory accounting report is high, the firm is ex ante better off by having a precise private signal. Recall in Proposition 3, the mandatory report enables the market to imperfectly distinguish among non-disclosure firms, so it plays a disciplining role on the voluntary disclosure. With a precise mandatory report, the firm's incentive to selectively disclose or withhold private information is easier for the market to disentangle. In other words, the market pricing is more aligned to the underlying firm value, and the firm is motivated to use its private signal to choose better projects and improve efficiency.

5.2 Efficiency Effects of Mandatory Accounting Report

In the preceding discussion, we showed that the informational quality of the mandatory accounting report influences the firm's investment and disclosure decisions through a disciplining effect. To gain further insight into the efficiency effects of the mandatory report, we examine how the noise level of the mandatory report affects the investment efficiency when the information asymmetry is large or small. The results appear in the following proposition.

Proposition 6 *(i) When $\frac{\sigma_\theta^2}{\sigma_r^2}$ and μ is sufficiently high, a marginal increase in σ_r^2 improves the investment efficiency; (ii) When $\sigma_\theta^2 < \rho\sigma_1\sigma_2$ and σ_r^2 is sufficiently high, a marginal decrease in σ_r^2 improves the investment efficiency.*

Part (i) of Proposition 6 implies that when the information asymmetry is small, coarsening the mandatory report may enhance the efficiency. To understand this, recall that the market reacts more to the mandatory report (b_r) than to the disclosure (b_s) when $\frac{\sigma_\theta^2}{\sigma_r^2}$ is high. This leads to a severe under-investment decision. A marginal increase in the noise level of the mandatory report alleviates the under-investment, because the market would shift some valuation weight to the voluntary disclosure. Though coarsening the mandatory report weakens the disciplining role, when μ is high, the negative effect from weaker disciplining is small and is outweighed by the improvement in the investment level decision. Opposite results occur when the information asymmetry is large. If $\sigma_\theta^2 < \rho\sigma_1\sigma_2$ and σ_r^2 is sufficiently high, b_s is much higher than b_r, which induces over-investment decision. In this case, marginal decrease in the noise level of the mandatory report (σ_r^2) improves the investment efficiency from both respects. It not only alleviates the over-investment, but also improves the project selection choice by a better disciplining role.

Overall, Proposition 6 demonstrates that the efficiency effect of the mandatory report is contingent on the information environment. When the private signal is coarse, a precise mandatory report induces lower efficiency, since it provides under-investment incentive despite its better disciplining role. Alternatively, when the private signal is precise, a precise mandatory report induces higher efficiency for both better investment level decision and better project selection choice. The results demonstrate an interaction between the economic efficiency induced by the mandatory accounting report and by the private signal.

6 Voluntary Disclosure Is Prohibited

In this section, we consider the case in which voluntary disclosure is prohibited. That is, a firm who undertakes a project is not allowed to communicate with the market about its private information. The firm may find it difficult to communicate its private information about the investment, since either disclosure regulation has restrictions on certain kinds of sensitive information, or the disclosure itself cannot be credibly communicated. This analysis may be valuable in that it helps us to understand how prohibiting voluntary disclosure influences the firm's investment decision, and its subsequent effect on economic efficiency.

6.1 Equilibrium with Disclosure Prohibited

Suppose that voluntary disclosure is not available at date 2. The firm who is confronted with an investment opportunity is not allowed to issue voluntary disclosure regardless of its investment decision. There is no information about the investment is available to the market and all value relevant information publicly available is the mandatory accounting report r, i.e., $\Omega = \{r\}$. We now analyze the equilibrium behavior of the firm and the market pricing function $P^B(r)$ ("B" for prohibited) when voluntary disclosure is prohibited.

Proposition 7 *If voluntary disclosure is prohibited, there exists an equilibrium where the firm's investment decision and the market pricing are given by*

(i) equilibrium investment function:

$$I = \begin{cases} (1-\beta)^2, & if \quad \theta > \theta^0 \text{ and the investment is available} \\ 0, & if \quad otherwise. \end{cases} \tag{6}$$

(ii) equilibrium market pricing function:

$$P^B(r) = \frac{r\sigma_y^2}{\sigma_y^2 + \sigma_r^2} + q\Phi(-g)\left[2\sqrt{I} + \mu' + (b_s - b)\frac{\sigma' f(-g)}{\Phi(-g)}\right]. \tag{7}$$

Without voluntary disclosure, the market price is irrelevant to the investment decision since the mandatory report only captures information on the ongoing activities. As a result, the firm only yields a marginal return to the investment cost from long-term interests, which leads to severe under-investment decision. The firm chooses to undertake the project only if the expected net payoff is positive ($\theta > \theta^0$).

In addition, the mandatory report, to some extent, reveals information about the private signal θ (via v_1 and v_2) which is related to the firm's investment decision. When the mandatory report is favorable, the market updated conjecture on the private signal (θ) is high, which indicates a higher likelihood of undertaking a profitable project. When the mandatory report is unfavorable, a lower conjecture on θ indicates an unprofitable project and a lower likelihood of undertaking a project. In the equilibrium market pricing function (7), the first item reflects the market updated conjecture on y conditional on the mandatory report (r), and the second item reflects the market's upward adjustment on z, since the firm may undertake a profitable project.

6.2 The Efficiency Implications of Prohibiting Voluntary Disclosure

Standard intuition suggests that prohibiting voluntary disclosure would result in inferior decisions for the related reasons: (i) the share price is less accurate since the market has less value-relevant information; (ii) the firm, anticipating this less accurate pricing, will severely under-invest in the project, which leads to less efficiency. It is the purpose of this subsection to show that, notwithstanding these intuitive arguments indicating the inferiority of prohibiting voluntary disclosure, it can perform strictly better than allowing voluntary disclosure in certain circumstances. The following proposition analyzes the efficiency implications of prohibiting voluntary disclosure.

Proposition 8 *If μ and $\frac{\sigma_\theta^2}{\sigma_r^2}$ are sufficiently low (high), prohibiting voluntary disclosure increases (decreases) the investment efficiency.*

According to Proposition 8, when the investment prospect is low and the information asymmetry is large, prohibiting voluntary disclosure is efficiency enhancing. This counter-intuitive result is driven by the firm's distorted disclosure incentive when voluntary disclosure is allowed. As we have illustrated above, with a precise private signal, the firm's disclosure threshold (θ^*) differs from the break-even level (θ^0). Especially, a lower μ leads to a lower disclosure threshold, which implies that the firm tends to invest in a less profitable project and disclose since the market puts more pressure on those without disclosure. In other words, making more disclosures leads to a deterioration of investment decision. When voluntary disclosure is prohibited, the firm's distorted disclosure incentive is eliminated and no endogenous disclosure cost is incurred, which leads to more efficient investment decisions.

When the investment prospect is high, the equilibrium disclosure threshold (θ^*) is higher than the break-even level (θ^0). The firm's project selection choice is not subject to its disclosure decision, and no project with a negative net return is undertaken. In this case, prohibiting voluntary disclosure leads to less efficiency, because less accurate market pricing induces under-investment.[29]

Corollary 3 *If voluntary disclosure is prohibited, a more informative private signal (lower σ_θ^2) is always efficiency enhancing.*

In contrast to Proposition 5, Corollary 3 shows that if voluntary disclosure is prohibited, a more informative private signal always leads to more efficiency. That is, the firm is always better off by choosing to be more informed. To explain this, recall the positive role (better knowledge of profitability) and the negative role (distorted disclosure incentive) of private information when disclosure is allowed. If voluntary disclosure is prohibited, the negative role of private information is completely eliminated, while the positive role is still in effect. Though the firm does not utilize its private information to the fullest extent, the marginal efficiency improvement of private information is still strictly positive.

7 Conclusion

This paper examines the determinants and economic efficiency of corporate voluntary disclosure, where the economic efficiency stems from the consequences of firm investment decisions. Voluntary disclosure plays two roles (positive and negative) with respect to firm investment efficiency. By transmitting value-relevant information to the market, voluntary disclosure leads to a more accurate pricing and improves investment efficiency, which is the positive role. The negative role is that firms use voluntary disclosure opportunistically to affect the market pricing in its favor, which can be detrimental to investment efficiency. This paper shows that the efficiency implication of voluntary disclosure is not obvious and determined by the dominant one of these two roles. In addition, the mandatory accounting report also plays an important role on firm investment and disclosure decisions. This study finds that the mandatory accounting report helps the market distinguish among non-disclosure firms and disentangle the disclosure incentive, and thus plays a disciplining role of voluntary disclosure. The results demonstrate an interaction between the efficiency effects of the mandatory accounting report and of the private signal.

[29] Again, one can consider voluntary disclosure as the firm's option. The value of this option is related to the economic situation and the information environment.

This paper also examines the firm investment efficiency when voluntary disclosure is prohibited. By comparing the economic efficiency induced by prohibiting or allowing voluntary disclosure, this study may provide policy makers with useful knowledge for designing disclosure regulations in light of their overall impact on the firm's investment efficiency.

Future works may benefit from including more disclosure regulation implications with regard to the efficiency of discretionary disclosure arrangements. The exploration of interaction between disclosure and firm's additional operating and financial choices is also left for future research.

Appendix

Proof. (of Proposition 1&2) We begin with the market pricing functions assuming the disclosure threshold at θ^*. Given the jointly normal distribution,

$$\begin{pmatrix} x+y \\ y+\varepsilon_r \\ \theta \end{pmatrix} \sim N\left(\begin{pmatrix} \mu \\ 0 \\ \mu \end{pmatrix}, \begin{bmatrix} \sigma_x^2+\sigma_y^2+2\rho\sigma_x\sigma_y & \sigma_y^2+\rho\sigma_x\sigma_y & \sigma_x^2+\rho\sigma_x\sigma_y \\ \sigma_y^2+\rho\sigma_x\sigma_y & \sigma_y^2+\sigma_r^2 & \rho\sigma_x\sigma_y \\ \sigma_x^2+\rho\sigma_x\sigma_y & \rho\sigma_x\sigma_y & \sigma_x^2+\sigma_\theta^2 \end{bmatrix}\right).$$

Following the properties of normal distribution, we have the market pricing with disclosure s is,

$$\begin{aligned} P(r,s) &= E[y+2\sqrt{I}+x|r,s] \\ &= 2\sqrt{I}+\mu+b_r r+b_s(s-2\sqrt{I}-\mu) \qquad (8) \\ where\ b_r &= \frac{(1-\rho^2)\,\sigma_x^2\sigma_y^2+(\sigma_y+\rho\sigma_x)\,\sigma_y\sigma_\theta^2}{(1-\rho^2)\,\sigma_x^2\sigma_y^2+\sigma_y^2\sigma_\theta^2+\sigma_x^2\sigma_r^2+\sigma_\theta^2\sigma_r^2},\ b_s=\frac{(1-\rho^2)\,\sigma_x^2\sigma_y^2+(\sigma_x+\rho\sigma_y)\,\sigma_x\sigma_r^2}{(1-\rho^2)\,\sigma_x^2\sigma_y^2+\sigma_y^2\sigma_\theta^2+\sigma_x^2\sigma_r^2+\sigma_\theta^2\sigma_r^2}. \end{aligned}$$

Firms without disclosure (assuming $\theta^* < \theta^0$) are composed by firms with a low private signal who choose to withhold information and firms without an investment opportunity. For the first group of firms, the market pricing is

$$\begin{aligned} P^N(r,\theta &< \theta^*)=E[y|r,\theta<\theta^*] \\ &= b_r' r+b\left(E[\theta|r,\theta<\theta^*]-\mu\right) \qquad (9) \\ where\ b_r' &= \frac{(1-\rho^2)\,\sigma_x^2\sigma_y^2+\sigma_y^2\sigma_\theta^2}{(1-\rho^2)\,\sigma_x^2\sigma_y^2+\sigma_y^2\sigma_\theta^2+\sigma_x^2\sigma_r^2+\sigma_\theta^2\sigma_r^2}, b=\frac{\rho\sigma_x\sigma_y\sigma_r^2}{(1-\rho^2)\,\sigma_x^2\sigma_y^2+\sigma_y^2\sigma_\theta^2+\sigma_x^2\sigma_r^2+\sigma_\theta^2\sigma_r^2} \end{aligned}$$

For the second group of firms, the market pricing is

$$P^N(r)=E[y|r]=\frac{r\sigma_y^2}{\sigma_y^2+\sigma_r^2}. \qquad (10)$$

To find $E[\theta|r,\theta<\theta^*]$ in (9), the updated distribution of θ conditional on r is

$$\begin{aligned} \theta|r &\backsim N[u',(\sigma')^2], \qquad (11)\\ where\ \mu' &= \mu+\frac{r\rho\sigma_x\sigma_y}{\sigma_y^2+\sigma_r^2},\ \sigma'=\sqrt{\sigma_x^2+\sigma_\theta^2-\frac{\rho^2\sigma_x^2\sigma_y^2}{\sigma_y^2+\sigma_r^2}}. \end{aligned}$$

Then we have

$$E[\theta|r,\theta<\theta^*]=u'-\sigma'\frac{f\left(c\right)}{\Phi\left(c\right)}, \qquad (12)$$

where $c=\frac{\theta^*-\mu'}{\sigma'}$, $f(\cdot)$ and $\Phi\left(\cdot\right)$ are the probability density function and the cumulative distribution function for a standard normal variable. Substituting (12) into (9) and simplifying, we have

$$P^N(r,\theta<\theta^*)=\frac{r\sigma_y^2}{\sigma_y^2+\sigma_r^2}-\frac{b\sigma' f(c)}{\Phi(c)}. \qquad (13)$$

Since the market could not distinguish among firms that do not disclose, the market overall pricing on non-disclosure firms is a weighted average of (10) and (13), which is

$$P^N(r)=\frac{r\sigma_y^2}{\sigma_y^2+\sigma_r^2}-\frac{qb\sigma' f(c)}{1-q+q\Phi(c)}.$$

Using the same method, we obtain the market pricing of non-disclosure firms when $\theta^*>\theta^0$.

Provided the market pricing functions ($P(r,s)$ and $P^N(r)$), and the disclosure threshold θ^*, when the private signal is higher than θ^*, the firm's investment maximization program becomes:

$$Choose\ I(\theta>\theta^*)\ to\ \max\ E\left[-I+\beta P\left(r,s\right)+(1-\beta)(z+y)|\theta\right].$$

The first-order condition with respect to I is,

$$\begin{aligned} 0 &= -1+\frac{(1-\beta+\beta b_s)}{\sqrt{I}},\\ I &= (1-\beta+\beta b_s)^2. \end{aligned}$$

When the private signal is lower than θ^*, the firm's investment maximization program becomes:

$$Choose\ I(\theta<\theta^*)\ to\ \max\ E\left[-I+\beta P^N\left(r\right)+(1-\beta)(z+y)|\theta\right].$$

By taking the first-order condition with respect to I, we have $I=(1-\beta)^2$. The firm chooses to invest only if the expected return is higher than zero, which is $E\left[-I+(1-\beta)z|\theta,I=(1-\beta)^2\right]>0$. Then we find the break-even level where the firm's expected net payoff from the investment is zero, at $\theta^0 = -\frac{(1-\beta)^2(\sigma_x^2+\sigma_\theta^2)+\sigma_\theta^2\mu}{\sigma_x^2}$.

To find the disclosure threshold and show the existence of θ^*, let $U(\theta, d = D)$ and $U(\theta, d = ND)$ be the firm's expected net payoff with the private signal θ when the firm chooses to disclose and not to disclose respectively. Using the above results, we have (assuming $\theta^* < \theta^0$)

$$\begin{aligned} U(\theta, d = D) &= \left[1 - \beta^2 (1 - b_s)^2\right] + \mu + (\theta - \mu) \frac{\sigma_x^2 + \rho\sigma_x\sigma_y}{\sigma_x^2 + \sigma_\theta^2}, \\ U(\theta, d = ND) &= (\theta - \mu) \frac{\rho\sigma_x\sigma_y}{\sigma_x^2 + \sigma_\theta^2} \left(1 - \frac{\beta\sigma_r^2}{\sigma_y^2 + \sigma_r^2}\right) - \beta b\sigma' \cdot h(q, \theta, c), \\ where\ h(q, \theta, c) &= E[\frac{qf(c)}{1 - q + q\Phi(c)}|\theta]. \end{aligned}$$

At the disclosure threshold θ^*, the firm is indifferent between disclosing and not disclosing, which is $U(\theta^*, d = D) = U(\theta^*, d = ND)$. We obtain the following equation, which has to hold in equilibrium

$$\left[1 - \beta^2 (1 - b_s)^2\right] + \mu + (\theta^* - \mu) \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\theta^2} = -(\theta^* - \mu) \frac{\rho\sigma_x\sigma_y}{\sigma_x^2 + \sigma_\theta^2} \frac{\beta\sigma_r^2}{\sigma_y^2 + \sigma_r^2} - \beta b\sigma' \cdot h(q, \theta^*, c). \quad (14)$$

As the range of $h(q, \theta, c)$ is bounded, the left hand side (LHS) of (14) is strictly increasing in θ and the first item in the right hand side (RHS) is strictly decreasing in θ, and all of the functions are continuous, there must be at least one root of θ^* letting the equation hold. For any $\theta > \theta^*$, the LHS is higher than the RHS and the firm chooses to disclose. For any $\theta < \theta^*$, the LHS is lower than the RHS and the firm chooses not to disclose. Using the same method, we show the existence of disclosure threshold when $\theta^* > \theta^0$. ■

Proof. (of Corollary 1) For part (i), it is easy to find $\frac{\partial b_r}{\partial \sigma_r^2} < 0$ ($\frac{\partial b_s}{\partial \sigma_\theta^2} < 0$) since the noise variance σ_r^2 (σ_θ^2) only appears in the denominator. For the same reason, $\frac{\partial I}{\partial \sigma_\theta^2}$ is also negative. From (8), we have $\frac{\partial b_s}{\partial \sigma_r^2} = \frac{\rho\sigma_x^2\sigma_y^2\left[(1-\rho^2)\sigma_x\sigma_y + \rho\sigma_\theta^2 + \frac{\sigma_y}{\sigma_x}\sigma_\theta^2\right]}{\left[(1-\rho^2)\sigma_x^2\sigma_y^2 + \sigma_y^2\sigma_\theta^2 + \sigma_x^2\sigma_r^2 + \sigma_\theta^2\sigma_r^2\right]^2} > 0$. Using the same method, we have $\frac{\partial b_r}{\partial \sigma_\theta^2} > 0$. For part (ii),

$$\frac{\partial b_r}{\partial \rho} = \sigma_x\sigma_y \frac{\sigma_\theta^2\left[(1 + \rho^2)\sigma_x^2\sigma_y^2 + \sigma_y^2\sigma_\theta^2 + \sigma_x^2\sigma_r^2 + \sigma_\theta^2\sigma_r^2\right] - 2\rho\sigma_r^2\left(\sigma_y\sigma_x^3 + \sigma_x\sigma_y\sigma_\theta^2\right)}{\left[(1 - \rho^2)\sigma_x^2\sigma_y^2 + \sigma_y^2\sigma_\theta^2 + \sigma_x^2\sigma_r^2 + \sigma_\theta^2\sigma_r^2\right]^2}.$$

When $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently high, the first item in the numerator is higher than the second item which leads to positive derivative $\frac{\partial b_r}{\partial \rho}$. Similarly, when $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently low, $\frac{\partial b_r}{\partial \rho} < 0$. And similarly

$$\frac{\partial b_s}{\partial \rho} = \sigma_x \sigma_y \frac{\sigma_r^2 \left[\left(1+\rho^2\right) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_\theta^2 + \sigma_x^2 \sigma_r^2 + \sigma_\theta^2 \sigma_r^2 \right] - 2\rho \sigma_\theta^2 \left(\sigma_y^3 \sigma_x + \sigma_x \sigma_y \sigma_v^2 \right)}{\left[\left(1-\rho^2\right) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_\theta^2 + \sigma_x^2 \sigma_r^2 + \sigma_\theta^2 \sigma_r^2 \right]^2}.$$

Using the some way, we have $\frac{\partial b_s}{\partial \rho}$ is negative (positive) when $\frac{\sigma_\theta^2}{\sigma_r^2}$ is high (low), ■

Proof. (of Corollary 2) From Proposition 1 & 2, the equation (14) must hold in equilibrium. Rewrite (14) in the following form

$$\begin{aligned} & \left[1-\beta^2\left(1-b_s\right)^2\right] + \mu + \left(\theta^* - \mu\right) \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\theta^2} \qquad (15) \\ = \quad & -\beta\rho \left[\frac{\sigma_x \sigma_y}{\sigma_x^2 + \frac{\sigma_\theta^2}{\sigma_r^2} \sigma_y^2 + \sigma_r^2} \frac{\theta^* - \mu}{} + \frac{\sigma_x \sigma_y \sigma' h(q, \theta^*, c)}{\left(1-\rho^2\right) \sigma_y^2 \frac{\sigma_x^2}{\sigma_r^2} + \sigma_y^2 \frac{\sigma_\theta^2}{\sigma_r^2} + \sigma_x^2 + \sigma_\theta^2} \right]. \end{aligned}$$

When ρ or β approaches zero, the RHS of (15) approaches to zero. When $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently high, both items in the bracket on the RHS go to zero. Then in all the cases, we have

$$\left[1-\beta^2\left(1-b_s\right)^2\right] + \mu + \left(\theta^* - \mu\right) \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\theta^2} = 0,$$

which is θ^* approaches $-\frac{\left[1-\beta^2(1-b_s)^2\right]\left(\sigma_x^2+\sigma_\theta^2\right)+\sigma_\theta^2 \mu}{\sigma_x^2}$. ■

Proof. (Proposition 3 & 4) For Proposition 3, we rewrite the equation (14) into

$$\left[1-\beta^2\left(1-b_s\right)^2\right] + \mu + \frac{\theta^* - \mu}{\sigma_x^2 + \sigma_\theta^2} \left(\sigma_x^2 + \frac{\rho \sigma_x \sigma_y \beta \sigma_r^2}{\sigma_y^2 + \sigma_r^2} \right) = -\beta b \sigma' \cdot h(q, \theta^*, c). \qquad (16)$$

Since the RHS of (16) is bounded, when μ is sufficiently low, $(\theta^* - \mu)$ must be positive to sustain the balance. Intuitively, as μ is lower, $(\theta^* - \mu)$ is getting higher. Taking the first derivative of both sides of equation (16) with respect to σ_r^2, we have

$$\text{LHS} \qquad \frac{\left(\theta^* - \mu\right) \rho \sigma_x \sigma_y \beta \sigma_y^2}{\left(\sigma_x^2 + \sigma_\theta^2\right) \left(\sigma_y^2 + \sigma_r^2\right)^2}, \qquad (17)$$

$$\text{RHS} \quad -\beta \sigma' h(\cdot) \frac{\rho \sigma_x \sigma_y \left[\left(1-\rho^2\right) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_\theta^2 \right]}{\left[\left(1-\rho^2\right) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_\theta^2 + \sigma_x^2 \sigma_r^2 + \sigma_\theta^2 \sigma_r^2 \right]^2} + \frac{\beta b \rho \sigma_x \sigma_y}{\left(\sigma_y^2 + \sigma_r^2\right)^2} h_c'(\cdot). \qquad (18)$$

The LHS of (16) represents the firm's expected net payoff by choosing to disclose at θ^*, and the RHS represents the expected net payoff by choosing not to disclose at θ^*. Following the above arguments, (17) is positive ($(\theta^* - \mu)$ is positive) when μ is sufficiently low, and getting higher as μ decreases. The range of (18) is bounded given both $h(\cdot)$ and $h_c'(\cdot)$ are bounded[30]. When $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently low, the absolute value of (18) is approaching zero. When μ is sufficiently low, (17) must be higher than (18), which implies at θ^*, the firm chooses to disclose and the disclosure threshold shifts downward. Use the same method, we have the disclosure threshold θ^* shifts upward in σ_r^2, if μ is sufficiently high.

For proposition 4, taking the first derivative of both sides of equation (16) with respect to μ, we have

$$\text{LHS} \qquad 1 - \frac{\sigma_x^2 + \frac{\rho\sigma_x\sigma_y\beta\sigma_r^2}{\sigma_y^2+\sigma_r^2}}{\sigma_x^2 + \sigma_\theta^2}, \tag{19}$$

$$\text{RHS} \qquad \frac{\beta\rho\sigma_x\sigma_y\sigma_r^2}{(1-\rho^2)\,\sigma_x^2\sigma_y^2 + \sigma_y^2\sigma_\theta^2 + \sigma_x^2\sigma_r^2 + \sigma_\theta^2\sigma_r^2} h_c'(\cdot). \tag{20}$$

When $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently high, (19) is positive and approaches one, and (20) is approaching zero since $h_c'(\cdot)$ is bounded. So we have (19) is higher than (20) when $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently high. Following the above arguments, the firm chooses to disclose more and the disclosure threshold shifts downward. Similarly when $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently low, (19) is negative and lower than (20), the firm chooses to disclose less and the disclosure threshold shifts upward. ■

Proof. (Proposition 5) When σ_θ^2 is sufficiently high, b_s approaches zero, and the ex ante expected net return approaches

$$V_H = \begin{cases} 0, & if \quad \mu < (1-\beta^2) \\ q\left[(1-\beta^2)+\mu\right], & if \quad \mu > (1-\beta^2) \end{cases}. \tag{21}$$

Similarly, when σ_θ^2 is sufficiently low, the ex ante expected net return approaches

$$V_L = \frac{q}{\sigma_x}\int_{\theta^*}^{+\infty}\left[1-\beta^2(1-b_s)^2+\theta\right] f\left(\frac{\theta-\mu}{\sigma_x}\right) d\theta. \tag{22}$$

To prove part (i), rewrite (22) into

$$V_L = q\Phi\left(\frac{\mu-\theta^*}{\sigma_x}\right)\left[1-\beta^2(1-b_s)^2+\mu\right] + q\sigma_x f\left(\frac{\theta^*-\mu}{\sigma_x}\right). \tag{23}$$

As the second item in (23) is positive and bounded. When μ is sufficiency low, the first item in (23) turns negative. So we have V_L is lower than V_H. To prove part (ii), from Corollary 2, when β is sufficiently low and σ_θ^2 approaches zero, in (22), θ^* approaches $-\left[1-\beta^2(1-b_s)^2\right]$.

[30] Since both $f(\cdot)$ and $\Phi(\cdot)$ are functions with global upper and lower bounds, as long as q is not infinitely close to one, $h(\cdot)$ must be also bounded. $h_c'(\cdot)$ is also a global bounded function. More specifically, once $q < 0.999$, the value of $h_c'(\cdot)$ is bounded between -1 and 2.5.

That is, $1-\beta^2(1-b_s)^2+\theta$ is positive for any $\theta>\theta^*$. So V_L approaches

$$\frac{q}{\sigma_x}\int_0^{+\infty} tf\left(\frac{t-1-\mu}{\sigma_x}\right)dt = q(1+\mu)+\frac{q}{\sigma_x}\int_{-\infty}^{0} -tf\left(\frac{t-1-\mu}{\sigma_x}\right)dt. \tag{24}$$

Since the last item in (24) is positive, V_L is higher than V_H. Similarly, to prove part (iii), recall (14), when σ_r^2 is sufficiently low, the RHS of (14) approaches zero, so the disclosure threshold goes to $-\left[1-\beta^2(1-b_s)^2\right]$. When σ_θ^2 is low, b_s approaches one and the disclosure threshold goes to -1. So, V_L approaches the expression (24). Thus, we have V_L is higher than V_H. ■

Proof. (Proposition 6) When μ is sufficiently high, the disclosure threshold θ^* is higher than the break-even level θ^0. So the investment function is

$$I=\begin{cases}[1-\beta+\beta b_s]^2, & if\ \ \theta>\theta^* \text{ and the investment is available}\\ (1-\beta)^2, & if\ \ \theta^*>\theta>\theta^0 \text{ and the investment is available}\\ 0 & if\ \ \text{otherwise.}\end{cases}$$

Then the expected net return of the project ($E_\theta[z-I]$) is

$$\Phi(-e)\,\beta^2(2-b_s)\,b_s+\frac{1}{\sqrt{\sigma_x^2+\sigma_\theta^2}}\int_{\theta^0}^{+\infty}\left[\left(1-\beta^2\right)+\theta\right]f\left(\frac{\theta-\mu}{\sqrt{\sigma_x^2+\sigma_\theta^2}}\right)d\theta,\ where\ e=\frac{\theta^*-\mu}{\sqrt{\sigma_x^2+\sigma_\theta^2}}.$$

Taking the first derivative of $E_\theta[z-I]$ with respect to σ_r^2, we have

$$\frac{\partial E_\theta[z-I]}{\partial\sigma_r^2}=\Phi(-e)\,\beta^2(2-2b_s)\frac{\partial b_s}{\partial\sigma_r^2}+\frac{\partial[\Phi(-e)]}{\partial\sigma_r^2}\beta^2(2-b_s)\,b_s.$$

When $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently high, b_s approaches zero, which indicates the second item is zero. From Corollary 1, $\frac{\partial b_s}{\partial\sigma_r^2}=\frac{\rho\sigma_x^2\sigma_y^2\left[\left(1-\rho^2\right)\sigma_x\sigma_y+\rho\sigma_\theta^2+\frac{\sigma_y}{\sigma_x}\sigma_\theta^2\right]}{\left[(1-\rho^2)\sigma_x^2\sigma_y^2+\sigma_y^2\sigma_\theta^2+\sigma_x^2\sigma_r^2+\sigma_\theta^2\sigma_r^2\right]^2}$ and approaches $\frac{\rho\sigma_x^2\sigma_y^2\left[\rho+\frac{\sigma_y}{\sigma_x}\right]}{\left[\sigma_y^2+\sigma_r^2\right]^2}$ when $\frac{\sigma_\theta^2}{\sigma_r^2}$ is high. So b_s is strictly increasing in σ_r^2, and the first item is positive. Then we have $\frac{\partial E_\theta[z-I]}{\partial\sigma_r^2}$ is positive.

Similarly, to prove claim (ii), the investment I is equal to $[1-\beta+\beta b_s]^2$, when $\theta>\theta^*$, where b_s approaches $\frac{\sigma_x^2+\rho\sigma_x\sigma_y}{\sigma_x^2+\sigma_\theta^2}$. When $\sigma_\theta^2<\rho\sigma_x\sigma_y$, b_s is higher than one, which implies I is over-invested. From Corollary 1, $\frac{\partial I}{\partial\sigma_r^2}$ is positive, so the expected net return of the project is decreasing in σ_r^2, when $\theta>\theta^*$. To find the effect of σ_r^2 on the disclosure threshold, recall the equation (16) which holds in equilibrium. Taking the first derivative of both sides of equation (16) with respect to σ_r^2, we have (17) and (18). When σ_r^2 is sufficiently high, both (17) and (18) approaches zero, which indicates the marginal effect of σ_r^2 on the disclosure threshold approaches zero. So the overall expected net return of the project is decreasing in σ_r^2. ■

Proof. (Proposition 7) Following the properties of normal distribution, we have the market expectation on the ongoing activities' cash flow is,

$$E[y|r] = \frac{r\sigma_y^2}{\sigma_y^2 + \sigma_r^2}. \tag{25}$$

Due to the possibility of undertaking the project, the expected gross return on the project is

$$E[2\sqrt{I} + x|r, \theta > \theta^0] = 2\sqrt{I} + \mu + (b_r - b_r')\, r + (b_s - b)\left(E[\theta|r, \theta > \theta^0] - \mu\right), \tag{26}$$

where b_r, b_r', b_s and b are given by (8) and (9). To find $E[\theta|r, \theta > \theta^0]$ in (26), invoke the updated distribution of θ conditional on r in (11) is $\theta|r \backsim N[u', (\sigma')^2]$. Then we have

$$E[\theta|r, \theta > \theta^0] = u' + \sigma' \frac{f(-g)}{\Phi(-g)}, \tag{27}$$

where $g = \frac{\theta^0 - \mu'}{\sigma'}$. Substituting (27) into (26) and simplifying, we have

$$E[2\sqrt{I} + x|r, \theta > \theta^0] = 2\sqrt{I} + \mu' + (b_s - b)\frac{\sigma' f(-g)}{\Phi(-g)}. \tag{28}$$

Since the market could not distinguish whether the project is implemented or not without disclosure, the market price is a weighted average of (25) and (28), which is

$$P^B(r) = \frac{r\sigma_y^2}{\sigma_y^2 + \sigma_r^2} + q\Phi(-g)\left[2\sqrt{I} + \mu' + (b_s - b)\frac{\sigma' f(-g)}{\Phi(-g)}\right].$$

Provided the market pricing function ($P^B(r)$), and voluntary disclosure is prohibited, the firm's investment maximization program becomes

$$\textit{Choose } I \textit{ to } \max\ E\left[-I + \beta P^B(r) + (1-\beta)(z+y)|\theta\right].$$

By taking the first-order condition with respect to I, we have $I = (1-\beta)^2$. The firm chooses to invest in the project only when the expected return is higher than zero, which is $\theta > \theta^0$.

■

Proof. (Proposition 8 & Corollary 3) When voluntary disclosure is prohibited, the ex ante expected net return is

$$V_B = \frac{q}{\sqrt{\sigma_x^2+\sigma_\theta^2}}\int_{\theta^0}^{+\infty}\left[\left(1-\beta^2\right)+\mu+\frac{\sigma_x^2\left(\theta-\mu\right)}{\sigma_x^2+\sigma_\theta^2}\right] f\left(\frac{\theta-\mu}{\sqrt{\sigma_x^2+\sigma_\theta^2}}\right) d\theta \tag{29}$$

$$= q\int_{-\frac{\left(\left(1-\beta\right)^2+\mu\right)\sqrt{\sigma_x^2+\sigma_\theta^2}}{\sigma_x^2}}^{+\infty}\left[\left(1-\beta^2\right)+\mu+\frac{\sigma_x^2\cdot t}{\sqrt{\sigma_x^2+\sigma_\theta^2}}\right] f\left(t\right) dt.$$

From (29), it is evident that V_B is decreasing in σ_θ^2. When voluntary disclosure is allowed and μ is low, the ex ante expected net return is

$$E_\theta\left[z-I\right] = q\int_{\frac{\theta^*-\mu}{\sqrt{\sigma_x^2+\sigma_\theta^2}}}^{+\infty}\left[1-\beta^2\left(1-b_s\right)^2+\mu+\frac{\sigma_x^2\cdot t}{\sqrt{\sigma_x^2+\sigma_\theta^2}}\right] f\left(t\right) dt. \tag{30}$$

From Proposition 4, when μ and $\frac{\sigma_\theta^2}{\sigma_r^2}$ is sufficiently low, θ^* becomes negative. So we have $E_\theta\left[z-I\right]$ is lower than V_B. Similarly, when μ is high, $E_\theta\left[z-I\right]-V_B$ is

$$E_\theta\left[z-I\right]-V_B = q\Phi\left(-e\right)\beta^2 b_s\left(2-b_s\right). \tag{31}$$

When $\frac{\sigma_\theta^2}{\sigma_r^2}$ is high, b_s is lower than two, which indicates $b_s\left(2-b_s\right)>0$, so we have $E_\theta\left[z-I\right]$ is higher than V_B. ■

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