Information and Cost of Capital with Uncertainty about Types

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Abstract

In this paper I provide an extension on the rational expectations equilibrium proposed by Easley and O’Hara (2004). I relax the assumption that traders know their own types, but maintain the informed/uniformed type dichotomy. In this setting, uncertainty results in a higher required return and the effect of information asymmetry on cost of equity increases with the degree of uncertainty. I also reexamine empirically the relation between information attributes and cost of capital using non-parametric test, and find results consistent with Easley and O’Hara (2004).

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1 Introduction

The correct assessment of the cost of capital permeates the most important corporate decisions. It is precisely this measure that determines the true profitability of the firm and its hurdle rate for investment. When the market underestimates the value of a stock due to information asymmetry, equity financing becomes more expensive. The relationship between information and cost of capital has been object of a growing body of accounting literature and several disclosure policies. The Regulation Fair Disclosure ("Reg FD") by the U.S. Securities and Exchange Commission ("SEC"), effective on October 23, 2000, for instance, intended to stop the practice of "selective disclosure," in which firms provided information only to certain selected analysts and institutional investors prior to disclosing it publicly. Supporters of this policy have argued that this kind of disclosure was unfair, propitiating the privileged group an opportunity to make a profit or avoid a loss at the expenses of those who were uninformed. Critics, however, have claimed that information transfer between companies and investors would deteriorate and eventually would raise firms’ cost of capital. Gomes et al. (2006) find that the adoption of Reg FD resulted, in fact, in a welfare loss for small firms, facing thus, a higher cost of capital. This consequence of Reg FD and the different views regarding this policy already suggests that the effect of information in financial markets involves much debate and controversy.

In the accounting literature it is still unsettled the discussion of how information attributes affect the cost of capital, if so. Fama (1991), argues that information risk is diversifiable and should have no effect on the cost of equity. In contrast, Diamond and Verrecchia (1991), proposing a model in an imperfect competition setting in which investors are differentially informed, show that revealing public information in order to reduce information asymmetry among investors can increase liquidity
and reduce a firm’s cost of capital. In the same direction, Easley and O’Hara (2004) (henceforth, EO) develop a model in which firms with greater public and less private information have lower cost of capital.

Empirically, Verdi (2005) has documented that the relation between information asymmetry and cost of equity is sensitive to the selected proxy for cost of equity, concluding that this emphasize the lack of consistency in the relation between proxies for information asymmetry and cost of equity. Botosan and Plumlee (2006) (henceforth, BP), empirically test the EO model providing evidence that information attributes such as composition, dissemination and precision of affect the cost of equity capital. The conclusions drawn by BP are subject to misspecification choice, which is going to be discussed ahead.

One aspect about EO is that they assume that investors have knowledge of their types (informed or uninformed). This paper contributes to the literature in two aspects: first, it theoretically investigates the consequences of uncertainty about types in the relation between information and cost of capital, expanding EO model. The modification allows for the possibility of investors being unaware of their respective types due to not knowing the total number of signals existent in the market. The above proposal is addressed conjecturing two assumptions: one that consists in allowing uninformed traders to behave as if they are informed (naive hypothesis) not taking information content in price into account, and the other that allows informed traders, because of career concerns, consider price when trading (career concerns hypothesis). It is discussed that in the presence of uncertainty about types, higher returns are required by investors.

Second, even with potential limitations regarding measurement errors in proxies, it is empirically estimated EO model in its original form against Modified EO. The results provide evidence for uncertainty about types, in particular under the career
concerns hypothesis. Finally, aware of the possibility of misleading inference due to misspecification choice (linearity / non-linearity), it is reexamined the relationship between information attributes and cost of capital estimated BP using distribution-free test. The results are consistent with EO model and BP.

The paper is organized as follows: Section 2 presents the related literature, section 3 introduces the modified EO model, which allows for uncertainty about types and the hypotheses about the relation between information attributes and cost of capital are developed. Section 4 describes the proxy variables for the empirical tests, and section 5, presents the empirical results. Finally, section 6 concludes the paper.

2 Related literature

2.1 Analytical research

One stream of the literature, mainly composed by finance researchers, defends that the effect of information asymmetry can be diversified, and thus cannot affect the cost of equity (e.g. Fama, 1991.). Leland (1992) finds that even with the increase in the information asymmetry from the existence of the insider, the cost of equity will be reduced, suggesting the idea that it is not necessary that an increase in the degree of information asymmetry results in an increase in the cost of equity. Merton (1987), in a model with two types of investors where lack of information refers to not knowing about the existence of some firms, finds that in the case where private information from informed investors helps uninformed investors to be aware about ex-ante unknown firms, there is a reduction in the cost of equity.

EO show that the composition of information between public and private information affects positively the cost of capital, also that the dissemination of the
private information and the precision of the information negatively affect the cost of capital. Nevertheless, Hughes et al. (2005), using a noisy rational expectations model, find that information from private signals about idiosyncratic shocks has no effect on cost of capital, because it can be diversified.

Contradicting EO, Lambert et al. (2006) (henceforth, LLV) show that EO findings about the effect of the asymmetry in the cost of equity is an indirect effect, that is, the composition and the dissemination of the information indeed affects precision, which by its turn, affects the cost of equity. LLV’s idea is that when some investors acquire information, the overall degree of uncertainty decreases since all traders will have (partial) access to this information via price. LLV’s claim is that dissemination affects the precision of investors, and that is the reason for the reduction in the cost of equity.

2.2 Empirical research

Prior literature offers mixed conclusions on whether the information asymmetry is priced. BP find empirical evidence about the relation between information asymmetry and cost of equity, consistent with EO but inconsistent with other studies (e.g. LLV, and Leland, 1992.) Botosan (1997) examines the associating between disclosure level and the cost of equity, finding that for firms with low analyst following, greater disclosure is associated with lower cost of equity.

Verdi (2005) evaluates empirical proxies used to assess whether increased information risk increases the implied equity cost for capital. He documented that the relationship between asymmetry and cost of equity is sensitive to the use of alternative cost of equity measures, indicating a lack of consistency in the relation between proxies for information asymmetry and cost of equity. In conclusion, the results suggest that the choice of the proxies can affect the test.
3 The Model / Hypotheses development

In this section, I provide an extension on the rational expectations equilibrium proposed by EO. I relax the assumption that traders know their own types, but maintain the informed/uninformed type dichotomy.

As in EO, consider a two-period model where today investors choose portfolios and tomorrow the assets in these portfolios pay off. There is one risk-free asset, money, the \textit{numeraire}. There are $K$ risky stocks indexed by $k = 1, \ldots, K$. Future values, $v_k$, are independently, normally distributed with mean $\pi_k$ and precision $\rho_k$. The per capita supply of stock $k$, $x_k$, is also normally distributed with mean $\pi_k$ and precision $\eta_k$. Stock prices, $p_k$, are determined in the market. Traders trade today at prices $(1, p_1, \ldots, p_k)$ per share and receive tomorrow’s payoffs of $(1, v_1, \ldots, v_k)$ per share. Investors receive signals regarding future values of these stocks. There a total of $I_k$ signals for stock $k$. The signals are independently drawn from a normal distribution with mean $v_k$ and precision $\gamma_k$. The fraction of the signals that is private is $\alpha_k$ and the fraction of the signals that is public is $1 - \alpha_k$. All investors receive public signals before trade begins, but private signals are only observed by informed traders. Let $\mu_k$ be the fraction of traders who receives the private signals about stock $k$.

EO assumes that all investors know their own type. However, from the above setting, it seems reasonable to think that the ability to learn about one’s own type should be related to the knowledge about the total number of signals available at the beginning of the period, $I_k$. For instance, traders who observe a fraction $(1 - \alpha_k)I_k$ of the total number of signals and know that there are a total of $I_k$ signals automatically learn that they cannot be of a high type (informed). On the other hand, traders who observe all signals $I_k$ and know that this corresponds to the total number of
signals, immediately infer that they are of a high type.

In a setting where the total number of signals available about stock $k$ is unknown to some investors, we would expect then to behave differently from when there is certainty about types. This situation thus, gives rise to the existence of four possible existing types as illustrated in Figure 1: (A) being informed and knowing $I_k$, (B) being informed but not knowing $I_k$, (C) being uninformed but not knowing $I_k$ signals, and finally (D) being uninformed and knowing $I_k$.

Each investor solves for the optimal demand for assets $k = 1, \ldots, K$ maximizing his expected utility

$$\tilde{w}^j = \sum_k v_k z_k^j + m^j$$

subject to his budget constraint

$$m^j + \sum_k p_k z_k^j = \overline{m}^j$$

where $z_k^j$ is the number of shares of stock $k$ trader purchases, $m^j$ is the amount of money he holds and $\overline{m}^j$ is his initial wealth.

Substituting from $\overline{m}^j$ into the expected utility and supposing that conditional on all $j$ts information, he conjectures that payoffs on stocks are independent and that $v_k$ is distributed normal with mean $\nu_k^j$ and precision $\rho_k^j$. Assuming CARA utility, each investor solves

$$\begin{align*}
Max_{(z_k^j)_{k=1}^K} & \quad E \left[ \tilde{w}^j \right] - \frac{\delta}{2} Var^j \left[ \tilde{w}^j \right] \\
& \quad \text{subject to: } m^j + \sum_k p_k z_k^j = \overline{m}^j
\end{align*}$$

Thus, the investor’s demand for asset $k$ is

$$z_k^j = \frac{\nu_k^j - p_k}{\delta \left( \rho_k^j \right)^{-1}}$$
3.1 EO model - certainty about types

When certainty about types is assumed, as documented in EO, the scenario is the same as presented in Figure 1 with type B’s demand being equal to type A’s demand, and type C’s, equal type D’s\(^1\).

From Bayes’ Rule, if \( j \) is informed about asset \( k \) and about his type, he does not take into account the information content in price, and then his predicted distribution for \( v_k \) is normal with conditional mean and precision given by

\[
\bar{v}_k = \frac{\rho_k \bar{v}_k + \gamma_k \sum_{i=1}^{I_k} s_k}{\rho_k + \gamma_k I_k}, \quad \rho_j = \rho_k + \gamma_k I_k
\]  

(5)

The demand for the type (A) and (B) investors, \( z_k^A \) and \( z_k^B \), will be the same as in EO:

\[
z_k^B = z_k^A = \frac{\rho_k \bar{v}_k + \gamma_k \sum_{i=1}^{I_k} s_k - p_k (\rho_k + \gamma_k I_k)}{\delta}
\]  

(6)

On the other hand, traders who know that they are uninformed realize that there are some pieces of information that they do not observe. Also, they know that the demand of the informed traders affects the equilibrium price, and so they rationally make inferences about the underlying information through price. In a rational expectations equilibrium the conjecture from uniformed traders about the price function is self-fulfilling.

Following EO, suppose the price function has the following conjecture:

\[
p_k = a \bar{v}_k + b \sum_{i=1}^{\alpha_k I_k} s_{k_i} + c \sum_{i=\alpha_k I_k+1}^{I_k} s_{k_i} - d x_k + e \bar{x}_k
\]  

(7)

\(^1\)Comparing with EO, we would have \( \lambda_A + \lambda_B = \mu \) and \( \lambda_C + \lambda_D = 1 - \mu \).
Also, following the same methodology applied by Grossman and Stiglitz (1980), define the random variable $\theta_k$ as

$$
\theta_k = \frac{p_k - a\bar{v}_k - b \sum_{i=\alpha_k I_k+1}^{I_k} s_{k_i} + x_k(d - e)}{b\alpha_k I_k} = \frac{\alpha_k I_k}{\alpha_k I_k} = \left( \frac{d}{b\alpha_k I_k} \right) (x_k - \bar{x}_k) \quad (8)
$$

with mean $v_k$ and precision $\rho_{\theta_k}$ where

$$
\rho_{\theta_k} = \left[ \left( \frac{d}{b\alpha_k I_k} \right)^2 \eta_k^{-1} + \left( \frac{1}{\alpha_k I_k} \right) \gamma_k^{-1} \right]^{-1} \quad (9)
$$

Taking the information content in prices and its precision into account, it is possible to compute the posterior belief for uninformed traders. The conditional mean and variance from the perspective of the types $(C)$ and $(D)$ is given by

$$
\bar{v}_k^j = \frac{\rho_k v_k + \gamma_k \sum_{i=\alpha_k I_k+1}^{I_k} s_{k_i} + \rho_{\theta_k} \theta_k}{\rho_k + (1 - \alpha_k) \gamma_k I_k + \rho_{\theta_k}}, \quad \rho_k^j = \rho_k + (1 - \alpha_k) \gamma_k I_k + \rho_{\theta_k} \quad (10)
$$

$$
z_k^C = z_k^D = \frac{\rho_k v_k + \gamma_k \sum_{i=\alpha_k I_k+1}^{I_k} s_{k_i} + \rho_{\theta_k} \theta_k - p_k(\rho_k + (1 - \alpha_k) \gamma_k I_k + \rho_{\theta_k})}{\delta} \quad (11)
$$

Denoting $\lambda_j$ as the fraction of traders from type $(j)$, the equilibrium price $p_k$ for each asset $k$ is such that supply equals per-capita demand, then

$$
(\lambda^A_k + \lambda^B_k) z^A_k + (\lambda^C_k + \lambda^D_k) z^D_k = x_k \quad (12)
$$

From EO, we know that in such scenario, the equilibrium price and the required
return are as follows\(^2\)

\[
p_k = \frac{\rho_k \overline{v_k} + \gamma_k \sum_{i=\alpha_k I_k + 1}^{I_k} s_{k_i} + (\lambda_k^A + \lambda_k^B) \gamma_k \sum_{i=1}^{\alpha_k I_k} s_{k_i} + (\lambda_k^C + \lambda_k^D) (\rho_{\theta_k} \theta_k) - \delta \overline{x_k}}{C_k}, \tag{13}
\]

\[
E[v_k - p_k] = \frac{\delta \overline{x_k}}{\rho_k + (1 - \alpha_k) I_k \overline{\gamma_k} + (\lambda_k^A + \lambda_k^B) \alpha_k I_k \overline{\gamma_k} + (\lambda_k^C + \lambda_k^D) \rho_{\theta_k}} \tag{14}
\]

**EO Hypothesis - Certainty about Types**

In EO, all uninformed traders take prior, public, and price information into account when trading and all informed traders consider prior, public, and private information, therefore, the sum of the proportion of traders who take private information \((\lambda_k^A + \lambda_k^B)\); and price \((\lambda_k^C + \lambda_k^D)\) into account, must be equal to 1.

### 3.2 Modified EO model - uncertainty about types

As mentioned before, in the EO hypothesis, it is assumed that all traders know their own types and those types depend only on the number of signals that investors observe. However, we have argued that it is possible to allow for the situation where the total number of available signals about \(k\) (denoted in the model by \(I_k\)) may not be known. The knowledge of the total number available signals drives the certainty about types since, by definition, a trader is informed if he observes all available signals about asset \(k\), and so, if \(I_k\) is not known, traders never learn their real types. Therefore, in relaxing the assumption that all traders know \(I_k\) and arriving at four possible cases (Figure 1), I conjecture two possible scenarios that would alter some strategies derived in EO.

**Naive Hypothesis**

\(^2\)For further details, please refer to propositions 1 and 2 in EO.
The naive hypothesis presuppose that there might exist uninformed traders that may believe to be informed, investors who believe that price conveys no new information. This would happen due to the lack of knowledge about $I_k$, total number of signals available, and it may lead to strategies not considered by EO. Suppose that there are two uninformed traders with one of them believing that he is actually informed (type C). Holding this belief, the trader assumes that the signals he receives are indeed a sufficient statistic for future stock price, and so, by judging himself to have all the sufficient information to determine the stock price, he ends up not taking into account the information content in prices when trading. But, the other uninformed trader (type D), knowing his comparative disadvantage, will consider the information in price when trading. That is, $z^C_k \neq z^D_k$.

Type C traders are investors that are uninformed but believe to be informed, i.e. they do not know that the price conveys information that they do not observe. The demand for the type (C) investor will be based only on his observed signals:

\[
\begin{align*}
z^C_k &= \frac{\rho_k \bar{v}_k + \gamma_k \sum_{i=\alpha_k I_k+1}^{I_k} s_{k_i} - p_k (\rho_k + (1 - \alpha_k) \gamma_k I_k)}{\delta} \\
&= \frac{\rho_k \bar{v}_k + \gamma_k \sum_{i=\alpha_k I_k+1}^{I_k} s_{k_i} - p_k (\rho_k + (1 - \alpha_k) \gamma_k I_k)}{\delta} \quad(15)
\end{align*}
\]

For types (A) and (B), one should expect that since type (B) traders do not know their type, they would be willing to take price into account. However, since it is assumed that there is no extra information in price that is not known by informed traders, by considering or not the information in price, the demand for asset $k$ would be the same for informed traders. Hence, the relation between $z^A_k$, and $z^B_k$ would still hold, i.e. $z^A_k = z^B_k$. Proposition 1 details the equilibrium price and the required return in this scenario.

**Proposition 1** There exists a partially revealing rational expectations equilibrium
in which, for each asset \( k \), the equilibrium price and the required return are defined as follows

\[
p_k = \frac{\rho_k v_k + \gamma_k \sum_{i=0}^{N_k} s_{ki} + (\lambda^A_k + \lambda^B_k) \gamma_k \sum_{i=1}^{N_k} s_{ki} + (\lambda^B_k) (\rho_{\theta k} \theta_k) - \delta x_k}{C_k},
\]

\[
E[v_k - p_k] = \frac{\delta r_k}{\rho_k + (1 - \alpha_k) I_k \gamma_k + (\lambda^A_k + \lambda^B_k) \alpha_k I_k \gamma_k + (\lambda^B_k) \rho_{\theta k}}
\]

Therefore, in the naive hypothesis, only part of the uninformed traders take price information into account when trading and all informed traders consider private information. Therefore, the sum of the proportion of traders that takes private information, \((\lambda^A_k + \lambda^B_k)\), and the proportion that considers price, \((\lambda^C_k)\), is smaller than 1 \((\lambda^A_k + \lambda^B_k + \lambda^C_k = 1 - \lambda^D_k)\).

**Career Concerns Hypothesis**

“Some operators have come to the conclusion that it is better to be wrong along with everybody else, rather than take the risk of being right, or wrong, alone... By its nature, trend following amplifies the imbalance that may at some point affect a market, potentially leading to vicious circles of price adjustments and liquidation of positions.”

The career concerns hypothesis suggests that there might be more information in price that can be observed in prior, public, and private information, such as short-term price trend. Hence, in this situation, not only uninformed but also some informed traders that are not absolutely certain about their type will consider the information content in price when trading. This hypothesis is based on documented

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findings that career concerns may induce some type of distortions in behavior, such as herd behavior (Scharfstein and Stein 1990; Zwiebel 1995; Prendergast and Stole 1996; Morris 1997; Avery and Chevalier 1999). In particular, Chevalier and Ellison (1999) suggest, based on empirical evidence, that because of career concerns, mutual fund managers might trade based on price even if it is conflicting with their private information. In the same direction, Dasgupta, et al. (2006) find a negative relationship between past net trade and future excess returns. They claim that a strategy consisting in selling stocks that have been bought by institutions in the last 5+ quarters and buying stocks that have been sold by institutions in the last 5+ quarters generates a cumulative abnormal return of 18% in the following two years, suggesting institutional investors herd.

In this scenario, one should expect that in the case of lack of knowledge about type, traders will in fact consider price content. Therefore, only type (A) traders will not consider information in price, but the others will trade based on the information they observe (type (B) -all signals available, and types (C) and (D)- only public signals) and on price. It is straightforward that the precision from private information together with price will be lower than the precision under only private information \( \alpha_k I_k \gamma_k + \rho_{\theta_k} \leq \alpha_k I_k \gamma_k \). The intuition is that when price is considered, there is noise increase in the information set of informed investors. On the other hand, for uninformed traders, there is a gain from taking price into account \( (1-\alpha_k) I_k \gamma_k + \rho_{\theta_k} \geq (1 - \alpha_k) I_k \gamma_k \).

Hence, the demand of each type, the equilibrium price, and the required return are presented in proposition 2.

**Proposition 2** There exists a partially revealing rational expectations equilibrium in which, for each asset \( k \), the demand of each type, the equilibrium price and the
required return are defined as follows

\[
\begin{align*}
\z_k^B &= \frac{\rho_k \bar{v}_k + \gamma_k \sum_{i=1}^{I_k} s_{ki} + \rho_{\theta_k} \theta_k - p_k (\rho_k + \gamma_k I_k + \rho_{\theta_k})}{\delta}, \\
\z_k^A &= \frac{\rho_k \bar{v}_k + \gamma_k \sum_{i=1}^{I_k} s_{ki} - p_k (\rho_k + \gamma_k I_k)}{\delta}, \\
\z_k^C &= \z_k^D = \frac{\rho_k \bar{v}_k + \gamma_k \sum_{i=\alpha_k I_k+1}^{I_k} s_{ki} + \rho_{\theta_k} \theta_k - p_k (\rho_k + (1 - \alpha_k) \gamma_k I_k + \rho_{\theta_k})}{\delta}.
\end{align*}
\]

\[p_k = \frac{\rho_k \bar{v}_k + \gamma_k \sum_{i=\alpha_k I_k+1}^{I_k} s_{ki} + (\lambda_k^A + \lambda_k^B) \sum_{i=1}^{\alpha_k I_k} s_{ki} + (\lambda_k^B + \lambda_k^C + \lambda_k^D) (\rho_{\theta_k} \theta_k) - \delta \bar{x}_k}{C_k},\]

\[E[v_k - p_k] = \frac{\delta \bar{x}_k}{\rho_k + (1 - \alpha_k) I_k \gamma_k + (\lambda_k^A + \lambda_k^B) \alpha_k I_k \gamma_k + (\lambda_k^B + \lambda_k^C + \lambda_k^D) \rho_{\theta_k}}.\]

Therefore, under the career concerns hypothesis, only type (A) do not take price information into account when trading, therefore, the sum of the proportion of traders that takes into account private information, \((\lambda_k^A + \lambda_k^B)\), and the proportion that considers price, \((\lambda_k^B + \lambda_k^C + \lambda_k^D)\), is greater than 1 \((\lambda_k^A + \lambda_k^B + \lambda_k^B + \lambda_k^C + \lambda_k^D = 1 + \lambda_k^B)\).

**Empirical Test**

Relaxed the assumption about certainty of types, I empirically compare EO model and Modified EO model. In order to do that, I estimate the following equation

\[r_{PEGPREM} = \frac{1}{b_0 + b_1 \cdot Public + b_2 \cdot Private + b_3 \cdot PIN}\]
Where
\[ \hat{b}_0 = \frac{\rho_k}{\sigma_k^2}; \quad \hat{b}_1 = \frac{\beta_1}{\sigma_k^2}; \quad \hat{b}_2 = \frac{\beta_2}{\sigma_k^2}; \quad \text{and} \quad \hat{b}_3 = \frac{\beta_3}{\sigma_k^2} \] and Public, Private and PIN follow BP and are described in section 4.

Then, I proceed the following joint hypothesis testing based on the discussion above

- **H$_{10}$**: EO model is valid: \( \beta_1 = 1, \beta_2 + \beta_3 = 1 \)
- **H$_{1a1}$**: Modified EO - Career C. - pertains: \( \beta_1 = 1, \beta_2 + \beta_3 > 1 \)
- **H$_{1a2}$**: Modified EO - Naive - pertains: \( \beta_1 = 1, \beta_2 + \beta_3 < 1 \)

### 3.3 Types disclosure

An interesting further analysis consists in examining firms’ optimal strategy considering their communication with traders. In particular, contexts in which firms reveal or do not reveal the traders’ type. Proposition 3 details firms’ optimal decision.

**Proposition 3** *In the absence of communication costs, in a rational expectations equilibrium, firms communicate traders’ types.*

**Proof.** The cost of capital equation defined under each hypothesis differs in their denominator, denoted average precision by LLV. A comparison among them, shows that uncertainty about types decreases the average precision and thus, increases the cost of capital.

- **EO Hypothesis:**
  \[
  \text{Average Precision}^{\text{EO}} = (\lambda^A_k + \lambda^B_k)(\rho_k + I_k \gamma_k) + (\lambda^C_k + \lambda^D_k)(\rho_k + (1 - \alpha_k) I_k \gamma_k + \rho \theta_k)
  \]

- **Naive Hypothesis:**
  \[
  \text{Average Precision}^{\text{Naive}} = (\lambda^A_k + \lambda^B_k)(\rho_k + I_k \gamma_k) + \lambda^C_k (\rho_k + (1 - \alpha_k) I_k \gamma_k) + \lambda^D_k (\rho_k + (1 - \alpha_k) I_k \gamma_k + \rho \theta_k)
  \]

- **Career Concerns:**
  \[
  \text{Average Precision}^{\text{CC}} = (\lambda^A_k)(\rho_k + I_k \gamma_k) + \lambda^B_k (\rho_k + I_k \gamma_k + \rho \theta_k) + (\lambda^C_k + \lambda^D_k) (\rho_k + (1 - \alpha_k) I_k \gamma_k + \rho \theta_k)
  \]
Neglecting costs, firms will always choose to inform traders about their types if and only if

\[ \text{Average Precision}^{EO} \geq \text{Average Precision}^{Naive} \quad \text{and} \]

\[ \text{Average Precision}^{EO} \geq \text{Average Precision}^{CC} \]

Indeed, simple manipulations show that

\[ \text{Average Precision}^{EO} - \text{Average Precision}^{Naive} = \lambda_k^C \rho_{\theta_k} \geq 0 \]

\[ \text{Average Precision}^{EO} - \text{Average Precision}^{CC} = \lambda_k^B I_k \gamma_k - \lambda_k^B (I_k \gamma_k + \rho_{\theta_k}) \geq 0 \quad \blacksquare \]

Proposition 3 shows that, in the absence of communication cost, firms minimizing the cost of equity would choose, in this rational-expectation equilibrium, to inform traders about their types.

In the Modified EO, revealing types consists in announcing the total number of available signals. In practice, we observe that firms tend to disclosure this information through special reports, holding conference calls, attracting analysts and adopting good corporate governance practices, among others.

### 3.4 Effect of each information attribute on cost of equity

BP’s empirical estimation of EO analyzes the effect of three information attributes: composition, dissemination and precision on the cost of equity, assuming a linear relationship between them and the cost of equity. However, the relationship derived in EO is non-linear (Eq.14). Assuming linearity, might in fact lead to inconsistent conclusions. As an example of possible misleading inferences due to misspecification choice, although in another context, is given by Hanlon et al. (2003), where they point out:

"...assuming a linear relation between ESO (employee stock option) grant values and future earnings (as done in prior studies) might result..."
in misleading inferences about the nature of the payoff to option grants. Because the conclusions are somewhat sensitive to whether the model assumes linear or non-linear relation between future earnings and ESO grant values, researchers might want to consider this specification choice carefully in their work."

Hanlon et al. (2003) argue that if linearity is assumed, the relation between ESO grant values and future earnings is negative but when non-linear relation is considered, the relationship becomes positive, which ultimately modifies the whole interpretation of the results.

I reexamine BP estimation by considering non-linearity in the relationship between information attributes and cost of capital. One way to see the importance of doing this is by analyzing an extreme case where dissemination is very high, i.e., almost all traders are informed and so, private information is almost totally revealed by prices. In this case, an increase in private information’s precision would lead to an increase in the overall precision and even uninformed traders would be better off from this increase.

On the other hand, in the case where the fraction of informed traders is small, i.e., dissemination is small, an increase in private information precision benefits basically investors who observe it and, given the small number of traders, their trade may not be significant enough to make price reflect their private information. Thus, given their relative disadvantage, uninformed traders would require a higher return to trade, increasing the cost of equity. The difference in the magnitude of the impact of change in composition on the cost of capital described above can be seen by the following partial derivative
\[
\frac{\partial E[v_k - p_k]}{\partial \alpha_k} = \frac{\delta \pi_k (1 - \mu_k) I_k \gamma_k}{(C_k)^2 (1 + \alpha_k I_k \eta_k \mu_k^2 \gamma_k \delta^2)^2} > 0 \tag{21}
\]

As \(\mu_k\) approaches zero, ceteris paribus, cost of equity is always higher.

In this sense, it appears that the analysis and estimations of the model should not be conducted by assuming that the effect of each attribute on the cost of capital is constant, as in BP. Thus, in the remaining, I relax linearity and test the direction and significance of composition, dissemination, and precision on cost of equity using a non-parametric sign test. As pointed out by MacKinlay (1997), and Dixon and Mood (1946) a sign test is commonly used to specify statistical significance independently of an assumption concerning the distribution. I apply sign test to computed year-over-year changes in the characteristics of information environment and in cost of capital. I focus on the direction and significance of the effect of each variable, given the magnitude is not constant. For each observation, it is obtained the change in each variable considering the current year and the previous year for each firm (e.g. \(\Delta \text{Variable} = \text{Variable}_t - \text{Variable}_{t-1}\)). Setting \(K = 0.5\) if \(\text{Variable}_t > \text{Variable}_{t-1}\) \(-0.5\) otherwise.

I then, compute the number of times that the sign of the changes in each variable coincides with the sign of the change in the cost of equity. By knowing that, I assess the probability that each characteristic changes to the same direction as the cost of equity. To exemplify, let \(P_{\text{compos}}^{\text{pegprem}}\) be 1 if composition changed from year \(t - 1\) to year \(t\) in the same direction as cost of capital, estimated by \(r_{\text{PEGPREM}}\). Figure 2 presents the intermediate steps for the case of composition.

Finally, after computing the \(Ks\), we can calculate

\[
P_{\text{compos}}^{\text{pegprem}} = \frac{\sum_{n=1}^{N} K_{\text{compos}}^{n} + K_{\text{peg}}^{n}}{N},
\]

18
where $Prob_{\text{compos}}$ is the probability that composition changes in the same direction of $r_{\text{PEGPREM}}$.

In order to examine the direction and significance of how changes in each attribute is related to changes in cost of capital, I test the following hypothesis

- $H_0$ - no correlation: $Prob_{\text{attribute}}^{\text{cost of cap.}} = 0.5$
- $H_{a1}$ - positive correlation: $Prob_{\text{attribute}}^{\text{cost of cap.}} > 0.5$
- $H_{a2}$ - negative correlation: $Prob_{\text{attribute}}^{\text{cost of cap.}} < 0.5$

For this test, the predictions are that composition (positive), dissemination (negative), and precision (negative) will follow the same relation described in EO and Modified EO.

The conclusions drawn from this test are subject to failure in considering multivariate setting. However, since each effect is already dictated by analytical model, this shortcoming might be overcome.

**Lambert, Leuz, and Verrecchia Interpretation about EO model**

Recently, EO model have received some critics regarding whether composition and dissemination affect cost of capital directly or via average precision. In particular, LLV (2006) point out

“A shift in information asymmetry can appear to affect the cost of capital, but only because the average precision of information is also changing simultaneously. In other words, the dissemination of more information to more investors reduces the cost of capital, but only because it increases the average precision of investors, not because it reduces information asymmetry. For example, it is possible to change the information structure such that information asymmetry increases despite
the fact that average precision also increases. In these settings, we show the cost of capital decreases; this is inconsistent with the information asymmetry hypothesis.\textsuperscript{4}

LLV’s main point is the analysis of no informed traders versus some informed traders. Their intention is to suggest that the existence of informed traders lowers cost of capital, and not in spite of it. In my interpretation, EO is clear that, \textit{ceteris paribus}, an increase in dissemination (in this case from $\mu = 0$ to $\mu > 0$) results in an decrease in cost of capital\textsuperscript{5}. That is, in this particular case, the inclusion of informed traders results in an increase in the overall precision (negative effect on cost of equity), and an increase in dissemination (negative effect), but no change in composition. So, according to EO, the result effect is negative.

In my interpretation, EO model does not presuppose that the increase in the level of information asymmetry necessarily results in an increase in cost of capital, but it does suggest that it will be generated possibly three effects (from composition, dissemination, and precision) and the final result, whether increase or decrease cost of equity, depends on their strength. Therefore, the test of the effect of each attribute on cost of capital remains pertinent.

\section{Sample selection and variable construction}

I collect data from six databases. Firms’ stock price and return information are obtained from \textit{CRSP}, financial data from \textit{Compustat}, intra-day transaction data from \textit{Trade And Quote (TAQ)}, analysts’ coverage data from \textit{First Call} database, price forecast and firms’ characteristics from \textit{Value Line}, and risk-free interest rate from

\textsuperscript{4}LLV, pages 3-4.

\textsuperscript{5}EO, page 1573
US Federal Reserve. The sample consists of 4,709 firm-year observations covering the period from 1993 to 2003.

Following Botosan and Plumlee (2006), the estimate for cost of equity capital, $r_{PEGPREM}$, is obtained by subtracting the risk free rate of interest from $r_{PEG}$ (Easton, 2004), where $r_{PEG}$ is defined as

$$r_{PEG} = \sqrt{\frac{E_0 (\text{eps}_5) - E_0 (\text{eps}_4)}{P_0}}$$

where $\text{eps}_t$ is earnings per share at time $t$ (Value Line).

The market beta ($BETA$) is calculated using the market model with a minimum of 30 out of 60 monthly returns and market index return equal to the value weighted NYSE/AMEX return (CRSP). The estimation period for $BETA$ ends on June 30th of the year cost of equity capital is estimated. The long-term growth in earnings ($LGROW$) is the natural log of the 3-5 year annual rate of change in expected earnings included in Value Line. The market value of equity ($LMKVL$) computed by taking the log of the variable obtained by multiplying the number of common shares outstanding by stock price at the quarter-end immediately prior to June 30th of the year cost of equity capital is estimated. The quarterly data is drawn from Compustat, but we use CRSP if Compustat data is not available. Book-to-price ($BP$) is calculated by scaling the book value of the firm’s common equity (Compustat) at quarter-end immediately prior to June 30th of the year cost of equity capital is estimated.

Regarding the attributes of information, composition ($COMPOS$), which is the fraction of the information set that is private, is derived from Barron et al. (1998) model, and is given by
\[
\frac{\alpha_k I_k \gamma_k}{\alpha_k I_k \gamma_k + (1 - \alpha_k) I_k \gamma_k} = \frac{PRIV ATE_{it}}{PUBLIC_{it} + PRIV ATE_{it}}
\]

where

\[
PUBLIC = \frac{(SE - \frac{D}{N})}{\left[(SE - \frac{D}{N} + D)^2 \right]}, PRIV ATE = \frac{D}{\left[(SE - \frac{D}{N} + D)^2 \right]}
\]

\(SE\): squared error in the mean forecast. \(SE = \left(\overline{F_{it}} - A_{it}\right)^2\)

\(D\): forecast dispersion. \(D = \frac{1}{N-1} \sum_{i=1}^{N} (\overline{F_{it}} - F_{ijt})^2\)

\(N\): number of forecasts

\(\overline{F_{it}}\): mean forecast for firm \(i\), quarter \(t\)

\(A_{it}\): actual earnings for firm \(i\), quarter \(t\)

\(F_{ijt}\): analyst \(j\)'s forecast of earnings for firm \(i\), quarter \(t\).

\(SE\) and \(D\) were obtained from most recent one-quarter ahead forecasts of quarterly earnings (First Call) and a minimum of three forecasts is required. We used time series average of four successive quarterly values of \(PUBLIC\) and \(PRIV ATE\) that precede the 3rd quarter of the calendar year in which cost of equity capital is estimated.

Dissemination of private information, which is the fraction of informed traders (DISSEM), is derived according to methodology proposed by Easley et al. (1997) (EKO) which assumes informed and uninformed traders and an equally uninformed market maker. Information events occur daily with probability \(\alpha\) and are independent. The news about a stock is classified as either good, with probability \(1 - \delta\) or bad, with probability \(\delta\). The market maker set prices and execute orders as they arrive. On the same day the information event occurs, the informed traders will buy a stock for which the news is good and will sell it otherwise. The rate of informed trading is \(\mu\) and the rates of uninformed buy and sell orders are \(\varepsilon_b\) and \(\varepsilon_s\), respectively. EKO assume the number of buy and sell trades to be independent
of one another, to follow a Poisson processes for a particular trading day and to be independent across trading days. EKO assume the number of buy and sell trades to be independent of one another, to follow a Poisson processes for a particular trading day and to be independent across trading days. Hence, the unconditional likelihood function for a single trading day is a mixed process can be written as:

$$L(\theta|B, S) = (1 - \alpha) e^{-\varepsilon_b} \frac{\varepsilon_b^B}{B!} e^{-\varepsilon_s} \frac{\varepsilon_s^S}{S!} + \alpha \delta e^{-\varepsilon_b} \frac{\varepsilon_b^B}{B!} e^{-(\mu + \varepsilon_s)} \frac{(\mu + \varepsilon_s)^s}{S!} + a (1 - \delta) e^{-(\mu + \varepsilon_b)} \frac{(\mu + \varepsilon_b)^B}{B!} \frac{\varepsilon_s^S}{S!}$$

where B and S are the total number of buy and sell trades and $\theta = (\alpha, \mu, \varepsilon_b, \varepsilon_s, \delta)$ is the parameter vector. Assuming independence across days, the likelihood function over I days is:

$$V = L(\theta|M) = \prod_{i=1}^{I} L(\theta|B_i, S_i)$$

where $(B_i, S_i)$ are day $i$ trades $M = ((B_1, S_1), \ldots, (B_I, S_I))$ is the data set. The parameter set is estimated by maximizing the likelihood over $\theta$.

There is a truncation problem when ones attempts to estimate the parameters of EKO with a large number of daily buys and sells. To deal with this, Easley et al. (2001) assumes the arrival rate of uninformed buyers and sellers equal to each other, such that, $\varepsilon = \varepsilon_b = \varepsilon_s$. Thus, consistent with BP, my proxy for DISSEM is

$$DISSEM = \frac{\mu}{\mu + 2\varepsilon}.$$ 

As suggested by previous studies (Chen et. al 2005), the proxy for the precision of private information reflected in stock price ($\rho_{bk}$) is given by the probability of informed trading ($PIN$),
Finally, the overall precision of the information set \((\text{PRECIS}_t)\) is given by \(\text{PUBLIC}_t + \text{PRIVATE}_t\).

Descriptive Statistics

Table 1 provides descriptive statistics of the cost of capital estimate and independent variables. The mean of \(r_{\text{PEGPREM}}\) is 0.063, consistent with previous research (BP, Francis et al. 2004). Mean Beta for my sample is 1.048, suggesting no bias regarding firm level of risk. The mean of Composition, Dissemination and Precision are also consistent with previous findings.

5 Empirical results

Modified EO

In order to test whether the Modified EO can be supported by the data, I estimate the following equation

\[
r_{\text{PEGPREM}} = \frac{\delta \tau_k}{\beta_0 + \beta_1 \text{Public} + \beta_2 \text{Private} + \beta_3 \rho_{\theta_k}} \tag{22}
\]

And test whether \(\beta_2 + \beta_3 = 1\). However, given that there is no proxy for \(\delta \tau_k\), I estimate the following alternative equation

\[
r_{\text{PEGPREM}} = \frac{1}{b_0 + b_1 \text{Public} + b_2 \text{Private} + b_3 \text{PIN}} + \xi \tag{23}
\]
consistent with EO specification where \( \xi \) is the error term, assumed to be normally distributed with mean zero and variance \( \sigma^2 \), by Maximum Likelihood (MLE). To do that, I rewrite EQ (23) as

\[
\xi = r_{PEGREM} - \frac{1}{b_0 + b_1 Public + b_2 Private + b_3 PIN} \sim N(0, \sigma^2) \quad (24)
\]

I construct the log-likelihood function as follows

\[
\ln L(r_{PEGREM}, Public, Private, PIN, \sigma) = -0.5*\ln(2\pi) - \ln(\sigma) - \frac{1}{2\sigma^2} \left( r_{PEGREM} - \frac{1}{b_0 + b_1 Public + b_2 Private + b_3 PIN} \right)
\]

Interpretation of the results from this estimation depends on \( \delta \overline{x}_k \), as shown in EQ (22). To solve for this unknown, I assume that the model is well-specified, that the measurement errors have the same effect on the coefficients and \( \delta \overline{x}_k \) is constant along firms.

Table 2 (Panel A) presents the results from MLE. Under both EO and Modified EO, the coefficient \( \beta_1 \), which is the proportion of traders that take into account public information, must be, by construction, equal to 1. In order to test the null joint hypothesis,

\[
\beta_1 = 1, \quad \beta_2 + \beta_3 = 1
\]

I do the following transformation.

\[
\beta_1 = 1, \quad \beta_2 + \beta_3 = 1 \iff \beta_2 + \beta_3 = \beta_1
\]

Dividing all terms by \( \delta \overline{x}_k \), we end up with

\[
\frac{\beta_2}{\delta \overline{x}_k} + \frac{\beta_3}{\delta \overline{x}_k} = \frac{\beta_1}{\delta \overline{x}_k}
\]
Then, the null hypothesis becomes

\[ \iff \hat{b}_2 + \hat{b}_3 = \hat{b}_1 \]

Table 2 (Panel B) presents the statistical test of the transformed null hypothesis. For a test of a size 0.99, the null hypothesis is rejected, and the \( H1_{a1} \) (Modified EO - Career Concerns) pertains.

Due to the non restriction concerning \( \delta x_k \) and the presence of measurement errors, one could claim against the relevance of the test. However, recall that EO model is a special case of Modified EO. Therefore, the above test does not intend to imply that EO is not correct, but rather to document the possibility of the existence of a set of traders behaving differently from another that observes the same set of signals, because of differences in beliefs about the total number of signals, in a rational expectation equilibrium.

**Information attributes and cost of capital**

To reexamine the direction and the significance on the relationship between each information attribute and cost of capital considering non-linearity, I applied a sign test, as described in previous section. The basis of the test is that, under the null hypothesis, it is equally probable that changes in attributes are in the same or opposite direction that changes in cost of capital. Failure in rejecting the null hypothesis leads to the conclusion that the attribute has no effect on cost of capital. On the other hand, under \( Ha_1 \), the effect of the attribute on cost of capital is negative, and under \( Ha_2, positive \).

The test statistic, denote by \( \theta_1 \), to test the null hypothesis is defined as follows
\[ \theta_1 = \frac{\text{Pro}^\text{attribute}_{\text{cost of cap.}} - 0.5}{\frac{0.5}{\sqrt{N}}} - N(0, 1). \]

This distributional result is asymptotic. For a test of size 0.95, \( \alpha = 0.05 \), \( H_0 \) is rejected if \( \theta_1 > \Phi^{-1}(\alpha) = 1.96. \)

As presented in Table 3, the effect of composition on cost of capital measured by \( r_{\text{PEGPREM}} \) is positive and significant, and the effect of dissemination and precision are negative and significant, all consistent with EO and BP.

### 6 Conclusions

A better understanding of the relationship between information and cost of capital, the incentives involved and consequences of asymmetry seems important from the point of view of both the growing body of accounting theory and a more effective design of regulation policies from agencies.

This paper theoretically analyzed the consequences of uncertainty about types in the relation between information and cost of capital. Expanding from EO model, we have showed that uncertainty about types results in a higher cost of capital, that is, investors on average require higher return to hold position if some investors have uncertain about their type.

Under the two conjectured hypotheses that address the potential effect given uncertainty about types: the naive hypothesis that considers uninformed traders behaving as informed and career concerns hypothesis where informed traders, because of career concerns, consider price when trading, the paper provide evidence suggesting optimal behaviors not documented in the previous literature.

As a final contribution, the estimation of the relation between cost of equity and information attributes using a non-parametric test, sign test, shows results
consistent with EO model, namely, composition affects positively cost of capital, and dissemination and precision, negatively.

A further analysis in detailing how the effect of composition, dissemination, and precision are affected by the uncertainty about types, and how firms assess the optimal level of this uncertainty in its market and how firms’ characteristics change the cost of credible disclosure is an interest continuation of the topic.

7 Appendix

Proof of Proposition 1: Under the naive hypothesis, we need to show that there is a solution to the market clearing equation. Substituting for the demands of the four categories of types, we have

\[
\begin{align*}
\lambda_k^A \left( \rho_k \overline{v}_k + \sum_{i=1}^{I_k} \alpha_k \bar{I}_k s_i + \gamma_k \sum_{i=\alpha_k I_k}^{I_k} s_i - p_k (\rho_k + \gamma_k \bar{I}_k) \right) + \\
\lambda_k^B \left( \rho_k \overline{v}_k + \sum_{i=1}^{\alpha_k I_k} s_i + \gamma_k \sum_{i=\alpha_k I_k}^{I_k} s_i + \rho_k \theta_k - p_k (\rho_k + \alpha_k \bar{I}_k - (1 - \alpha_k) \bar{I}_k) \rho \theta_k \right) + \\
\lambda_k^C \left( \rho_k \overline{v}_k + \gamma_k \sum_{i=\alpha_k I_k+1}^{I_k} s_i - p_k (\rho_k + (1 - \alpha_k) \gamma_k \bar{I}_k) \right) + \\
(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C) \left( \rho_k \overline{v}_k + \gamma_k \sum_{i=\alpha_k I_k+1}^{I_k} s_i + \rho_k \theta_k - p_k (\rho_k + (1 - \alpha_k) \gamma_k \bar{I}_k + \rho \theta_k) \right) \\
= \delta x_k
\end{align*}
\]
Thus,
\[
\rho_k \overline{v}_k + \gamma_k \sum_{i=\alpha_k I_k}^{I_k} s_{ki} + \left(\lambda_k^A + \lambda_k^B\right) \gamma_k \sum_{i=1}^{\alpha_k I_k} s_{ki} + \left(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C\right) (\rho_{\theta k} \theta_k)
\]
\[
= p_k \left( \rho_k + I_k(1 - \alpha_k) \gamma_k + \left(\lambda_k^A + \lambda_k^B\right) \alpha_k I_k \gamma_k + \rho_{\theta k} \left(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C\right) \right)
\]

So,
\[
\rho_k \overline{v}_k + \gamma_k \sum_{i=\alpha_k I_k+1}^{I_k} s_{ki} + \left(\lambda_k^A + \lambda_k^B\right) \gamma_k \sum_{i=1}^{\alpha_k I_k+1} s_{ki} + \left(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C\right) (\rho_{\theta k} \theta_k) - \delta x_k
\]
\[
p_k = \frac{C_k}{\sum_{i=1}^{\alpha_k I_k}}
\]

since
\[
\theta_k = \frac{\sum_{i=1}^{\alpha_k I_k} s_{ki}}{\alpha_k I_k} - \left(\frac{d}{b \alpha_k I_k}\right)(x_k - \overline{x}_k)
\]

we have
\[
\rho_k \overline{v}_k + \gamma_k \sum_{i=\alpha_k I_k+1}^{I_k} s_{ki} + \left(\lambda_k^A + \lambda_k^B\right) \gamma_k \sum_{i=1}^{\alpha_k I_k+1} s_{ki} + \left(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C\right) \rho_{\theta k} \left(\sum_{i=1}^{\alpha_k I_k} s_{ki} - \left(\frac{d}{b \alpha_k I_k}\right)(x_k - \overline{x}_k)\right) - \delta x_k
\]
\[
p_k = \frac{C_k}{\sum_{i=1}^{\alpha_k I_k}}
\]

rearranging the terms
\[
p_k C_k = \rho_k \overline{v}_k + \gamma_k \sum_{i=\alpha_k I_k+1}^{I_k} s_{ki} + \left[\left(\lambda_k^A + \lambda_k^B\right) \gamma_k + \left(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C\right) (\rho_{\theta k} \theta_k) \right] \sum_{i=1}^{\alpha_k I_k} s_{ki} +
\]
\[
- \left[\left(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C\right) \rho_{\theta k} d \frac{d}{b \alpha_k I_k} - \delta\right] x_k + \left[\left(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C\right) \rho_{\theta k} d \frac{d}{b \alpha_k I_k} - \delta\right]
\]

Comparing the coefficients
\[ d = \frac{(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C)(\rho_{\theta k})}{\alpha_k I_k} - \delta \]

\[ b = \frac{(\lambda_k^A + \lambda_k^B) \gamma_k + (1 - \lambda_k^A - \lambda_k^B - \lambda_k^C)(\rho_{\theta k})}{\alpha_k I_k} \]

\[ \iff C_k b - (\lambda_k^A + \lambda_k^B) \gamma_k = \frac{(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C)(\rho_{\theta k})}{\alpha_k I_k} \]

Thus, substituting into \( d \), we get

\[ C_k d = \frac{(C_k b - (\lambda_k^A + \lambda_k^B) \gamma_k) b}{\delta C_k} + \delta \iff C_k db = C_k db - (\lambda_k^A + \lambda_k^B) \gamma_k d + \delta b \]

\[ \Rightarrow \frac{d}{b} = \frac{\delta}{(\lambda_k^A + \lambda_k^B) \gamma_k} \]

So,

\[ C_k d = \frac{(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C)(\rho_{\theta k})}{\alpha_k I_k \gamma_k (\lambda_k^A + \lambda_k^B)} - \delta \]

\[ C_k e = \frac{(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C)(\rho_{\theta k})}{\alpha_k I_k \gamma_k (\lambda_k^A + \lambda_k^B)} \]

Having established the equilibrium, let us show how the return depends on the information structure, with the level of public and private information influencing the cross-sectional equilibrium return demanded by investors. The random return per share to holding asset \( k \) is \( v_k - p_k \). The expected return per share to holding asset \( k \) for an investor with information set \( I \), price and public information for an uninformed investor or all information for an informed investor is \( E[v_k|I] - p_k \). The average return per share for this investor is thus, \( E[E[v_k|I] - p_k] = E[v_k - p_k] \), where the expectation is computed with respect to prior information. This, leads
us to the second proposition

\[ E[v_k - p_k] = E\left[ v_k - a\bar{v}_k - b\sum_{i=1}^{\alpha_k I_k} s_{ki} - c\sum_{i=\alpha_k I_k + 1}^{I_k} s_{ki} + d.x_k - e.x_k \right] \]

since \( E[s_{ki}] = \bar{v}_k \) and \( E[x_k] = \bar{x}_k \), we have

\[ E[v_k - p_k] = \bar{v}_k [1 - a - b\alpha_k I_k - c (1 - \alpha_k) I_k \gamma_k] + \bar{x}_k (d - e) \]

substituting for the coefficients from Proposition 1

\[ E[v_k - p_k] = \bar{v}_k \left[ 1 - \frac{\rho_k}{C_k} - \frac{\lambda_k^A + \lambda_k^B}{C_k} \alpha_k I_k \gamma_k + \frac{(1 - \lambda_k^A - \lambda_k^B - \lambda_k^C)}{C_k} \rho_{th} - \frac{(1 - \alpha_k) I_k \gamma_k}{C_k} \right] + \bar{x}_k (d - e) \]

the term in the brackets is equal to zero and \( d - e = \frac{\delta}{C_k} \).

Therefore,

\[ E[v_k - p_k] = \frac{\delta \bar{x}_k}{\rho_k + (1 - \alpha_k) I_k \gamma_k + (\lambda_k^A + \lambda_k^B) \alpha_k I_k \gamma_k + (1 - \lambda_k^A - \lambda_k^B - \lambda_k^C) \rho_{th}} \]

\textit{Proof of Proposition 2:} Considering the career concerns hypothesis we need to show that there is a solution to the market clearing equation. Substituting for the demands of the four categories of types, we have
From the equilibrium price that clears the market, the steps are the same presented in the proof of proposition 1.

**Proposition 4** For any stock $k$, and provided that $(\lambda^A_k + \lambda^B_k) < 1$, under the naive hypothesis, the difference in the effect of composition on cost of equity compared to the effect in EO model, is increasing in the proportion of traders that deviate from the strategy defined in the certainty-about-type scenario. That is,

$$\frac{\partial E [v_k - p_k]}{\partial \alpha}^{Naive} - \frac{\partial E [v_k - p_k]}{\partial \alpha}^{EO} = \varepsilon + \lambda^C_k \left( \frac{\delta x_k}{C_k^2} \frac{\partial \rho_{\theta_k}}{\partial \alpha_k} \right)$$
Proof.

\[
\frac{\partial E [v_k - p_k]}{\partial \alpha_k} = \frac{-\delta T_k \left[ -I_k \gamma_k + \left( \lambda_k^A + \lambda_k^B \right) I_k \gamma_k + (1 - \lambda_k^A - \lambda_k^B - \lambda_k^C) \frac{\partial \rho_{\theta_k}}{\partial \alpha_k} \right]}{C_k^2} \]

\[
= \frac{-\delta T_k \left[ I_k \gamma_k \left( -1 + \lambda_k^A + \lambda_k^B \right) I_k \gamma_k + (1 - \lambda_k^A - \lambda_k^B - \lambda_k^C) \frac{\partial \rho_{\theta_k}}{\partial \alpha_k} \right]}{C_k^2} \]

\[
= \frac{-\delta T_k \left[ I_k \gamma_k \left( -1 + \lambda_k^A + \lambda_k^B \right) I_k \gamma_k + (1 - \lambda_k^A - \lambda_k^B) \frac{\partial \rho_{\theta_k}}{\partial \alpha_k} \right]}{C_k^2} + \frac{\lambda_k^C \delta T_k \partial \rho_{\theta_k}}{C_k^2 \partial \alpha_k} \]

\[
= \frac{\partial E [CE]^{EO}}{\partial \alpha} + \varepsilon + \lambda_k^C \left( \frac{\delta T_k \partial \rho_{\theta_k}}{C_k^2 \partial \alpha_k} \right) \]

Thus,\[
\frac{\partial E [v_k - p_k]^{Naive}}{\partial \alpha} - \frac{\partial E [v_k - p_k]^{EO}}{\partial \alpha} = \varepsilon + \lambda_k^C \left( \frac{\delta T_k \partial \rho_{\theta_k}}{C_k^2 \partial \alpha_k} \right) \]

\[
\]

8 References


access to disclosure” Journal of Accounting and Economics 34, 181-187.


Figure 1
Description of four possible existing types given the uncertainty about types: (A) being informed and knowing $\lambda$, (B) being informed but not knowing $\lambda$, (C) being uninformed but not knowing $\lambda$, signals, and finally (D) being uninformed and knowing $\lambda$. 
Figure 2
Detailing the intermediate steps for assessing the probability that the information attribute changes in the same direction of the cost of capital.

$P_{COMPOS}^{P2G} = |k_{COMPOS}^t + k_{PBG}^t| = 1$

$P_{COMPOS}^{PBG} = |k_{COMPOS}^t + k_{PBG}^t| = 0$

$P_{COMPOS}^{PBG} = |k_{COMPOS}^t + k_{PBG}^t| = 0$

$P_{COMPOS}^{PBG} = |k_{COMPOS}^t + k_{PBG}^t| = 1$
### Table 1

Panel A: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>rpegprem</td>
<td>4662</td>
<td>.063</td>
<td>.0433</td>
</tr>
<tr>
<td>beta</td>
<td>4571</td>
<td>1.048</td>
<td>.2923</td>
</tr>
<tr>
<td>lgrow</td>
<td>4672</td>
<td>2.522</td>
<td>.5659</td>
</tr>
<tr>
<td>lmkvl</td>
<td>4686</td>
<td>7.826</td>
<td>1.3456</td>
</tr>
<tr>
<td>BP</td>
<td>4695</td>
<td>.499</td>
<td>.3402</td>
</tr>
<tr>
<td>compos</td>
<td>4695</td>
<td>.105</td>
<td>.1420</td>
</tr>
<tr>
<td>dissem</td>
<td>4695</td>
<td>.272</td>
<td>.1365</td>
</tr>
<tr>
<td>rprecis</td>
<td>4695</td>
<td>.500</td>
<td>.2829</td>
</tr>
<tr>
<td>pin</td>
<td>4695</td>
<td>.149</td>
<td>.0618</td>
</tr>
</tbody>
</table>

*PEGPREM* is the estimated risk premium based on the PEG method (Easton 2004). BETA is capital market beta estimated via the market model. LGROW is the log of the *Value Line* long-range earnings growth forecasts. LMKVL is the market value of equity. BP is the book value of common equity scaled by the market value of common equity. ALPHA is the probability of a private information event. DISSEM fraction of investors who are informed. PRECIS is total information precision, and PIN is the probability of informed trade.
Table 2

Panel A: Maximum Likelihood Estimation of cost of capital on information attributes*

\[
\hat{r} = \frac{1}{b_0 + b_1 \cdot Public + b_2 \cdot Private + b_3 \cdot PIN}
\]

| VAR  | Estimate | STD  | z     | p>|Z| |
|------|----------|------|-------|-----|
| \(b_1\) | 5.298    | 0.550| 9.640 | 0.000 |
| \(b_2\) | 7.158    | 0.550| 13.010| 0.000 |
| \(b_3\) | 5.499    | 2.398| 2.290 | 0.022 |
| \(b_0\) | 9.425    | 0.426| 22.130| 0.000 |
| **sigma** | **0.042** | **0.000** | **96.560** | **0.000** |

Panel B: Testing the null hypothesis \(H_{10}\).

\[
H_{10} : \beta_1 = 1, \ \hat{b}_2 + \hat{b}_3 = \hat{b}_1
\]

\[
\text{chi2}(1) = 8.03 \\
\text{Prob} > \text{chi2} = 0.0046
\]

*The sample includes 4709 firm-year observations from 1993-2003. Panel A is estimated using Maximum Likelihood. Public (Private) is the precision of public (private) information available for firm \(k\) (Earron et al., 1998). PIN is the probability of informed trade (Easloy et al. 1997), used as estimate for information content in price, and \(r\) is the estimated risk premium (Easton, 2004). In Panel B s tested the null hypothesis \(H_{10}\).
Table 3

Reexamintion of the direction and the significance of the relationship between each information attribute and cost of capital considering non-linearity, using Sign Test. The test statistic, denote by $\theta_1$, to test the null hypothesis is defined as follows

$$\theta_1 = \frac{\text{Prob}_{\text{attribute}} - 0.5}{\frac{0.5}{\sqrt{N}}} \sim N(0, 1)$$

<table>
<thead>
<tr>
<th>Variable$^1$</th>
<th>N</th>
<th>Mean</th>
<th>$\theta_1$</th>
<th>Hyp.</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob compost</td>
<td>3461</td>
<td>0.5233</td>
<td>2.737</td>
<td>$H_{a_2}$</td>
<td>(+)</td>
</tr>
<tr>
<td>Prob dissem</td>
<td>3461</td>
<td>0.4805</td>
<td>-2.295</td>
<td>$H_{a_1}$</td>
<td>(−)</td>
</tr>
<tr>
<td>Prob precis</td>
<td>3461</td>
<td>0.4643</td>
<td>-4.199</td>
<td>$H_{a_1}$</td>
<td>(−)</td>
</tr>
</tbody>
</table>

$^1$ Where each variable represents the probability that the information attribute changes in the same direction of the cost of capital.