Asset Revaluation Regulation

by

Joel S. Demski, Haijin Lin, and David E. M. Sappington[1]

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Abstract

GAAP mandates a variety of departures from historical cost asset valuation. Here we offer a simple model that leads to such variety, depending on regulatory objectives and the magnitude of various economic forces. The central feature of the model is entrepreneurial investment in an asset followed by private information that cannot be communicated. A lemons problem arises in the asset resale market, opening the door for mandated disclosure which takes the form of audited asset revaluation. We find that optimal revaluation policies can encompass the wide variety of seemingly disparate approaches to revaluation that is characteristic of GAAP.

1 Introduction

GAAP provides a bewildering and seemingly inconsistent array of asset revaluation requirements. These requirements include lower of cost or market for inventory, net realizable value for receivables, a less restrictive variant of lower of cost or market for long-lived assets, no revaluation for R&D, and fair value for a variety of financial instruments. We examine a model in which the scale of investment and asset prices are affected by the prevailing revaluation regulation. We find that all of the aforementioned policies can emerge as optimal policies, depending on regulatory objectives and model parameters. This finding suggests that standard economic considerations may instill a GAAP with a wide variety of asset revaluation requirements.

The central feature of the model is a simple lemons problem (Akerlof [1970]) where an investor or firm acquires an asset and then privately observes "value relevant" information

[1] University of Florida.

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before the asset resale market opens. Some potential sellers are forced by liquidity concerns to liquidate their holdings, while, thanks to their private information, others may opportunistically impersonate the liquidity constrained. And it is this impersonation that creates the lemons problem in the resale market. In turn, anticipation of this lemons problem may affect investment incentives, as entrepreneurs anticipate "mis-pricing" and recognize that if they ultimately become liquidity constrained, they may be unable to earn a reasonable return on their investment.

In turn, mandated revaluation of low-valued assets, in effect movement away from historical cost, can, in principle, lead to more accurate pricing, protect distressed investors, and enhance incentives for investment. However, the revaluation regulations also can impose costs on those required to undertake the mandated costly asset revaluation. These costs, in turn, have the potential to distort investment decisions and may influence incentives for voluntary asset revaluation. The optimal design of asset revaluation regulations requires a careful balancing of these benefits and costs.

We examine the optimal such balancing as follows. Section 2 provides an overview of our analysis and discusses related literature. Section 3 then lays out the key elements of the basic setting that we analyze. Section 4 presents key findings in this setting. Section 5 adds additional structure that facilitates a more complete characterization of equilibrium prices and optimal asset revaluation policies. Section 6 considers extensions of the basic setting. Section 7 offers concluding thoughts.

2 Overview and Literature Review

Departures from historical cost measurement are numerous, varied and seemingly ever increasing. Fair value reporting is applied to numerous assets (but has less force in liability measurement). Truncation approaches, such as lower of cost or market or asset impairment reporting, are also part of the scene. Similarly, R&D reporting is a de facto commitment to no revaluation whatsoever.

Numerous explanations for this heterogeneous mixture are possible, including irrationality, lack of political will, clever sequential response to ever more sophisticated transaction technologies, regulatory capture, and so on. Here we present a simple model in which rational regulation can lead to virtually any of these measurement approaches, depending on the relative importance of the economic forces at play and the regulatory objective.

The model itself relies on rational pricing, private information, costly verification, and an underlying and possibly distorted investment choice. Initially, and with no private information, entrepreneurs decide on the scale of a risky investment (e.g., Brown, Izan and Loh [1992] and Whittred and Chan [1992]). Subsequently they learn privately (1) whether they will be forced to liquidate their investment (due to unmodeled liquidity issues) and (2) an updated assessment of the true value of the project. At this point, a resale market opens, though with a classic "lemons" flavor (Akerlof [1970]), as those forced to liquidate cannot self-identify and are joined in the market by those who are not so forced but perceive their assets to be over over-valued. Rational expectations prevail in the resale market. Various departures from historical cost are then introduced by mandating, say, fair value or some type of truncation approach in reporting to the participants in the resale market.

As such, the model touches on a vast literature. One dimension concerns the nature of the friction that financial measurement is designed to alleviate. Managerial myopia (e.g., Bachar, Melumad and Weyns [1997] or Liang and Wen [2006]), informed and uninformed traders (e.g., Grossman and Stiglitz [1980]), capital structure tensions (e.g., Liang and Zhang [2006]), managerial opportunism (e.g., Dye [1988] or Newman, Patterson and Smith [2005]), leakage to product market players (e.g., Feltham and Xie [1992] or Verrecchia and Weber [2006]), market inefficiency (e.g., Aboody, Hughes and Liu) [2002], forced liquidation (e.g., Dye and Verrecchia [1995] or Kanodia, Sapra and Venugopalan [2004]) and risk sharing (e.g., Leland and Pyle [1977] or Datar, Feltham and Hughes [1991]) are familiar friction sources. Our model rests on risk neutrality and private information that cannot be communicated, coupled with probabilistic liquidation. As noted, one element of the private information is whether the entrepreneur is forced to liquidate. We assume his motivation for trading cannot be communicated, just as in, say, Dye's [1985] disclosure model where an agent cannot credibly communicate whether he has indeed received private information.[2] The other element of the private information is the updated assessment of the asset's value. Though we eventually analyze the case where this can be disclosed, for a price, we emphasize the alternative assumption that it cannot be disclosed through voluntary channels.

A second dimension of our model concerns the underlying accounting structure. By design, the model begins with an investment choice that leads to historical cost recognition thereof. Subsequently, when (and if) the asset is offered for sale, accounting regulations

[2] The resale market exhibits rational expectations. In particular, the equilibrium price incorporates the impact of opportunistic behavior by non-distressed entrepreneurs. This modelling approach allows opportunistic behavior to persist in equilibrium, performing much the same function as noise traders play in Grossman and Stiglitz's [1980] analysis.

may mandate some revision of this initial historical cost. This leads to fair value (e.g., Christensen and Frimor [2006] or Bachar, Melumad and Weyns [1997]) or, in a more limited sense, the truncation pattern of impairment type reporting, though asset write-ups (negative impairments so to speak) are not foreclosed.[3] Asset write-downs have an extensive history, especially in the long-standing use of lower of cost or market measurement and, more recently, the use of restructuring charges. FAS 121 and FAS 144 brought the latter (largely unregulated) activity under the regulatory umbrella.[4] Research on write-downs has been extensive, especially during the pre-FAS 121 period. This research has been largely empirical, with an emphasis on documenting valuation effects and possible opportunism. (See Alciatore *et al.* [1998] for an extensive review.) More recently, Riedl [2004] examines statistical properties of asset write-offs before and after the introduction of regulation (specifically, FAS 121). He finds macro and industry economic factors play relatively minor roles, while opportunistic considerations play more central roles, following the introduction of the regulation. The efficacy of such regulation and its effects on investment, however, remain open questions.

More broadly, the asymmetry of asset write-downs can be viewed as a form of conservatism, where bad news is recognized more aggressively than good news. (See, for example, Watts [2003a, 2003b], Basu [1997], Gigler and Hemmer [2001], Bachar, Melumad and Weyns [1997], Christensen and Demski [2004], and Lin [2006].) Likewise, asset write-downs can be viewed as a type of disclosure. (See Healy and Palepu [2001], Core [2001], Verrecchia [2001] and Dye [2001] for a recent review of this extensive literature.)[5] Our model is similar to Verrecchia's [1983] model of discretionary disclosure in that disclosure is costly but may nevertheless be pursued in both models. In contrast to Verrecchia, we emphasize investment effects, welfare-maximizing regulation, and the combined effects of mandated and discretionary disclosure.

Bachar, Melumad and Weyns' [1997] analysis is closest to ours, in terms of model setup and in terms of accounting structure. In their model, a firm's manager seeks to maximize the recorded value of the firm at the end of an initial time period. The manager employs

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[3]Walker [1992] provides an historical review of the SEC's various flutations with asset write-ups. To this must be added the current expansion into fair value reporting requirements.

[4]Loan impairments and impairment testing of goodwill provide additional illustrations.

[5]Bergemann and Valimaki [2006] provide a survey of the role of information and disclosure in mechanism design settings.

private information to determine when to either undertake costly selective auditing or sell assets in order to certify their value. The authors compare lower of cost or market, fair value, and historical cost methods, emphasizing the magnitude of the deadweight loss caused by the manager's myopic behavior. They also examine how certification costs affect policy performance, as do we.

Our analysis differs in three important respects, though. First, we consider the design of an optimal asset revaluation policy rather than comparing the performance of selected policies. Second, we focus on the lemons problem in the market for assets rather than on problems caused by managerial myopia. Third, we consider the impact of revaluation requirements on investment behavior, and allow for differential concern with the welfare of distressed and non-distressed investors.

Two main qualitative conclusions emerge in the basic setting where all risk-neutral investors are identical and where their investments are observable. First, the asset revaluation policy that maximizes aggregate expected surplus imposes no revaluation requirement whatsoever. This is the case because the losses the lemons problem imposes on distressed entrepreneurs are offset by the gains it affords to non-distressed entrepreneurs in the absence of any revaluation requirement. Consequently, the lemons problem does not affect the expected payoff from an investment and so the surplus-maximizing (first-best) level of investment arises when no revaluation occurs. Second, a non-trivial asset revaluation requirement is optimal when revaluation costs are sufficiently small and the welfare of distressed investors is valued more highly than the welfare of non-distressed investors. In this fashion, regulatory objectives come to the fore, as they address the lemons problem with an eye on a particular interest group. Moreover, when regulation is engaged, presumably based on these welfare concerns, either a "targeted" policy in which the auditor enforces an asset measurement policy that is ex ante welfare maximizing or a "proportional policy" in which the auditor enforces revaluation only when the asset is below a specified percentage of historical cost (e.g., LCM) can be optimal. In a stylistic sense, a policy based on principle may outperform one based on a rule, or vice versa.[6] And the variety we see in current day GAAP thus emerges, including so-called transaction redesign which takes the form of investment distortion in this setting.

Different findings can emerge in different settings. For example, non-trivial revaluation

[6] The reason here is not opportunism, but the differential feedback effect of each policy on the original investment choice.

requirements can increase total expected surplus when investors differ *ex ante* and when their capabilities or their investment levels are not observed publicly. Thus, although revaluation mandates do not increase surplus when market participants are only uncertain about investment outcomes, revaluation mandates can increase surplus when market participants also are imperfectly informed about key elements of the investment process. Similarly, if the original investment projects also vary in quality, surplus maximization may in and of itself lead to revaluation mandates.

In addition, the opportunity to voluntarily revalue high-value assets can reduce aggregate expected surplus. When the cost of voluntary revaluation is sufficiently low, a distressed investor with a high asset value will find it profitable to revalue his asset. The associated revaluation cost, coupled with the corresponding reduction in the equilibrium price of non-revalued assets, serves to reduce the aggregate expected surplus of entrepreneurs. Mandatory revaluation does not reduce the surplus reduction introduced by the possibility of voluntary revaluation. However, a prohibition on voluntary revaluation would increase aggregate expected surplus.[7]

Taken together, then, the simple investment choice, trading model leads to a setting where a revaluation mandate may (e.g. FAS 144) or may not (e.g., FAS 2) be optimal. When optimal it may be a highly specific rule (e.g., FAS 115 or 144) or may rely much more on judgment (e.g., FAS 5). And when it is optimal to mandate a highly specific rule, truncation occurs at virtually any point below or above historical cost. This reflects the underlying economic forces of an investment choice or project selection being affected by anticipated subsequent financial transactions and their attendant accounting-based costs and benefits.

3 The Basic Setting

We consider settings in which entrepreneurs decide initially how much to invest in a risky asset. In the basic setting on which we focus initially, the expected present value of the future cash flows associated with investment I by any risk-neutral entrepreneur is $\hat{x}(I)$, which is an increasing, concave function. The first best investment, denoted I^{FB}, is the solution to $\max_I \{\ \hat{x}(I) - I\ \}$. We assume $\hat{x}\left(I^{FB}\right) - I^{FB} > 0$.

[7] We also find that when a non-trivial asset revaluation regulation is imposed, the expected net payoff is always higher when the revaluation mandate is imposed only on asset sellers than when it is imposed on all asset owners.

The present value of the realized cash flows from investment I is modeled as $\hat{x}(I)+\widetilde{\mu}+\widetilde{\varepsilon}$, where $\widetilde{\mu}$ and $\widetilde{\varepsilon}$ are independent, mean-zero random variables with respective non-degenerate densities $h(\mu)$ and $g(\varepsilon)$. For simplicity, the scale of investment does not affect either $\widetilde{\mu}$ or $\widetilde{\varepsilon}$.

After spending I to purchase and develop an asset (which might be a firm, for example), each entrepreneur learns privately the realized value of $\widetilde{\mu}$ for his investment. Denote this realized value by μ. After observing μ, the expected value of the cash flows is revised to $E[x|I,\mu] = \hat{x}(I) + \mu$. At the same time, each entrepreneur discovers privately whether exogenous financial considerations compel him to sell his asset. Each entrepreneur becomes so distressed with probability $\pi \in (0,1)$. An entrepreneur is non-distressed (and so is not compelled to sell his asset) with probability $1-\pi$.[8]

A resale market for assets opens after each investor learns his $\widetilde{\mu}$ realization and whether he must sell his asset. Any asset offered for sale must be accompanied by an audited set of financial statements. The audited statements reveal the size of the investment (via the cash flow statement or, in our simple setting, the reported book value of the new asset). Consequently, all market participants know the initial investment associated with any asset that is offered for sale.[9] Absent revaluation, the accounting book value, v, of any asset offered for sale reflects its historical cost, i.e., $v = I$.

If an asset revaluation regulation is in effect, it is assumed to take the familiar form of fair value reporting or truncation (by requiring documentation of low asset values). Formally, the financial statements will report accounting book value

$$v = \begin{cases} \hat{x}(I) + \mu & \text{if } \hat{x}(I) + \mu \leq x_c(I) \\ I & \text{otherwise,} \end{cases}$$

where $x_c(I)$ is a bound on asset expected value below which any asset with book value I that is offered for sale must be revalued accurately for financial reporting purposes.[10] ($x_c(I)$

[8]The random distress in our model provides a simple means to introduce opportunism in the resale market while allowing a tractable analysis of the effects of revaluation regulation on investment behavior. Parallel modeling devices (e.g., noise traders or linear contracts) are employed in other studies.

[9]Because all entrepreneurs initially are identical in this basic setting, they all choose to undertake the same level of investment, I. Alternative settings are considered below.

[10]It is indeed optimal in the present setting to impose asset revaluation requirements only on asset sellers, rather than on all asset owners. When revaluation requirements are imposed on all asset owners, non-distressed entrepreneurs are forced to incur revaluation costs. In our model, where the requirements

is specified before any investment is undertaken.) For example, an unbounded large $x_c(I)$ is a fair value requirement, while $x_c(I) = I$ is a lower of cost or market requirement and $x_c(I)$ slightly below I corresponds to the impairment test in FAS 144. Similarly, $x_c(I)$ unbounded low entails no revaluation whatsoever and is informationally equivalent to R&D reporting.

Thus, the required financial statements reveal the initial investment I for all assets. Further, if the asset's value is below $x_c(I)$, the financial statements reveal the expected present value of cash flows given μ. Market participants receive only the information in the financial statements, so accounting is the only source of information in the resale market in this basic model.[11]

The cost of auditing the financial statements is normalized to zero if the audit simply verifies the historical cost of the asset ($v = I$). For simplicity, we initially assume it is prohibitively costly to verify claims of asset values in excess of $x_c(\cdot)$. This simplifying assumption reflects the observation that, in practice, it is often more difficult to provide conclusive evidence that an asset has a particularly high value than it is to prove the asset has a particularly low value.[12] Audits can verify asset values below $x_c(\cdot)$, but such verification necessitates nontrivial incremental work with associated resource cost k. Compliance with the identified lower-tail revaluation policy is exogenously enforced (by SEC enforcement or

are imposed only on asset sellers, non-distressed entrepreneurs with particularly low asset values avoid revaluation costs without aggravating the lemons problem. The non-distressed entrepreneurs recognize they can only sell their low-value assets if they have their assets revalued, and so rationally choose to retain, rather than sell, their low-valued assets.

[11] Treating accounting as the sole information source for market participants is an often used assumption (e.g., Dye and Sridhar [2004], Liang and Zhang [2006] or, for that matter, earnings response studies). On the other hand, multiple sources of information are the central point in, say, Demski and Feltham [1994] and Christensen and Frimor [2006]. Our analysis in Section 6 allows investors to voluntarily certify (i.e., disclose) the value of the assets they offer for sale.

[12] High value often stems from such ethereal considerations as goodwill, while low value results from such readily observed, concrete considerations such as physical damage or product market collapse. FAS 144 and the predecessor FAS 121 reflect claims that the costs of certifying the fair value of a well-performing asset outweigh the corresponding benefits. For example, paragraph 141 of FAS 121 states "... Comment letters and ... testimony ... clearly indicated that a requirement to specifically test each asset or group of assets for impairment each period would not be cost-effective."

Of course, ascertaining realized losses from physical damage (e.g., from hurricanes, oil spills, or other environmental accidents) or other sources is far from trivial. However, such valuations are not uncommon in practice, and so are assumed to be feasible at cost k. Alternative audit technologies are considered below.

legal liability, for example).[13] For simplicity, we also assume that competition among audit firms eliminates rent for auditors.

If an asset is offered for sale in the resale market, then, the market participants know: (1) the entrepreneur chose to sell the asset; (2) the investment size is I; and (3) the expected value of the asset is $E[x|I,\mu] = \hat{x}(I) + \mu = v$ if $v < x_c(I)$ whereas the expected value is $E[x|I,\mu] \geq x_c(I)$ if $v = I$. Risk neutral pricing prevails. Let $P(v,I)$ denote the equilibrium price of an asset with underlying investment I and accounting book value v. An asset with accounting value $v < I$ that is offered for sale will sell at price $P(v,I) = v$ because its value is revealed accurately by the mandated audit. If an asset with reported book value $v = I$ is offered for sale, the lemons problem introduces uncertainty about the true prevailing asset value. All distressed entrepreneurs offer their assets for sale, as do non-distressed entrepreneurs with asset values that are intermediate between the entrepreneur's private assessment of the asset's value and the equilibrium price of assets with book value I ($P(I,I)$).

After observing $\widetilde{\mu} = \mu$, a non-distressed entrepreneur has the option of selling the asset. If $E[x|I,\mu] = \hat{x}(I) + \mu \leq x_c(I)$, selling the asset would secure a net return of $\hat{x}(I) + \mu - k = v - k$ for the entrepreneur while retaining the asset would net $\hat{x}(I) + \mu = v > v - k$. This is the case because a revalued low-value asset sells for its true value, and retaining the asset eliminates the audit cost k. If $\hat{x}(I) + \mu \in (x_c(I),\ P(I,I))$, the non-distressed entrepreneur will sell the asset at price $P(I,I)$. If $\hat{x}(I) + \mu \geq P(I,I)$, the non-distressed entrepreneur will retain his asset because its value exceeds the equilibrium price of assets with book value I.

The price of an asset with accounting book value $v = I$ that is offered for sale is simply the expected present value of the cash flows from the asset conditional on: (1) $\hat{x}(I) + \mu \geq x_c(I)$; (2) all distressed entrepreneurs with such assets have offered them for sale; and (3) all non-distressed entrepreneurs with such assets and private values of $\hat{x}(I) + \mu \in (x_c(I),\ P(I,I))$ also have offered their assets for sale. Let S denote the event of offering the asset for sale with a book value of $v = I$, which implies $\hat{x}(I) + \mu > x_c(I)$. Then the expected value of an asset with book value I offered for sale is:

$$E[x|S,I] \;=\; \hat{x}(I) + \int_{\varepsilon}\int_{\mu}[\mu + \varepsilon]h(\mu|S)g(\varepsilon)d\mu d\varepsilon \;=\; \hat{x}(I) + \int_{\mu}\mu h(\mu|S)d\mu\ ,$$

[13] This regulation requires the entrepreneur to reveal what he knows if his assessed value of the asset is below $x_c(I)$. This requirement appears to mirror institutional practice. If sufficient penalties were available and enforceable, if discovery were generally reliable, and if the legal infrastructure could commit itself to randomized discovery, full disclosure with random auditing would be an appealing policy. However, these assumptions, like the policy they could underlie, seem unrealistic.

where $h(\mu|S) = h(\mu, S)/\int_\mu h(\mu, S)d\mu$ is the probability that $\widetilde{\mu} = \mu$, given the asset is offered for sale. This probability is:

$$h(\mu|S) = \begin{cases} h(\mu) & \text{if } P(I,I) - \hat{x}(I) \geq \mu \geq x_c(I) - \hat{x}(I) \\ \pi h(\mu) & \text{if } P(I,I) - \hat{x}(I) \leq \mu \\ 0 & \text{otherwise.} \end{cases}$$

The equilibrium prices in the resale market will be:

$$P(v, I) = \begin{cases} v & \text{if } v \neq I \\ E[x|S, I] & \text{otherwise.} \end{cases}$$

An entrepreneur anticipates these equilibrium prices when choosing the investment level that maximizes his expected net return. Formally, after observing $x_c(\cdot)$, the entrepreneur chooses I to maximize:[14]

$$\begin{aligned} V(I) \;=\; & \pi\{ \int\limits_{\mu \leq x_c - \hat{x}(I)} [P(v, I) - k]h(\mu)d\mu + \int\limits_{\mu \geq x_c - \hat{x}(I)} P(I, I)h(\mu)d\mu\} + \\ & [1-\pi]\{\int\limits_{\varepsilon} \int\limits_{\mu \leq x_c - \hat{x}(I)} [\hat{x}(I) + \mu + \varepsilon]h(\mu)g(\varepsilon)d\mu d\varepsilon + \int\limits_{P(I,I) - \hat{x}(I) \geq \mu \geq x_c - \hat{x}(I)} P(I, I)h(\mu)d\mu + \\ & \int\limits_{\varepsilon} \int\limits_{\mu \geq P(I,I) - \hat{x}(I)} [\hat{x}(I) + \mu]h(\mu)g(\varepsilon)d\mu d\varepsilon\} - I\ . \end{aligned}$$

Before proceeding to characterize equilibrium outcomes in the basic setting, we review the timing of activities in this setting. The timing is as follows:

regulator sets revaluation policy, $x_c(\cdot)$	entrepreneur invests I	entrepreneur learns privately whether he is distressed	μ is realized and observed privately	distressed entrepreneur sells asset; non-distressed decides whether to sell

Notice the revaluation policy is designed to increase the information available to market

[14] For expositional simplicity, the dependence of $x_c(\cdot)$ on I is not stated explicitly in the following expression (and several subsequent expressions).

participants. Absent revaluation, the balance sheet may reflect an under- or an over-valued asset. In turn, increasing the information available reduces the amount of balance sheet error, and also has the potential to affect the initial investment choice because of the anticipated effect of the revaluation regulation.

4 Findings in the Basic Setting

We now describe some key equilibrium findings in the basic setting. Proposition 1 confirms the presence of a non-trivial lemons problem – assets that trade with historical cost balance sheets trade at a discount relative to *a priori* value – and thus a potential role for asset revaluation policy in this setting.

Proposition 1. *$P(I,I) < \hat{x}(I)$ in the basic setting for any investment I and revaluation policy $x_c(\cdot)$ such that revaluation is not certain (i.e., such that trade at price $P(I,I)$ occurs with positive probability).*

Proof. $P(I,I) \;=\; \hat{x}(I) + \int_\mu \mu\; h(\mu|S)d\mu \;<\; \hat{x}(I)$ because

$$\int_\mu \mu\; h(\mu|S)d\mu \quad = \int\limits_{x_c-\hat{x}(I)\le\mu\le P(I,I)-\hat{x}(I)} \mu\; h(\mu)d\mu \;+ \int\limits_{\mu\ge P(I,I)-\hat{x}(I)} \pi\;\mu\; h(\mu)d\mu$$

$$< \quad \int_\mu \mu\; h(\mu)d\mu \;=\; 0\;. \qquad \square$$

Central questions now are whether this lemons problem affects investment and welfare and, if so, whether asset revaluation policies can alleviate the problems that arise. To answer these questions, first suppose the relevant welfare measure is Marshallian welfare (total expected surplus). Because no expected rent accrues to any players other than the entrepreneurs, Marshallian welfare in the basic setting is simply the expected surplus captured by the entrepreneur.[15] As Proposition 1 implies, the distressed entrepreneur is ultimately forced to sell his asset for a price below its true value. However, the non-distressed entrepreneur may enjoy the advantage of selling his asset for a premium. Proposition 2 reports that rational pricing and risk neutrality ensure the expected losses for distressed entrepreneurs and the expected gains for non-distressed entrepreneurs are offsetting. Consequently, total expected

[15] Our market pricing assumption ensures the buyers in that market face a fair game, and thus in expectation acquire zero rent or surplus. Similarly, competition eliminates rent for auditors.

surplus is maximized and first-best investment is induced when no revaluation mandate is imposed in the basic setting.

Proposition 2. *Surplus-maximizing revaluation policy in the basic setting induces no revaluation by setting $x_c(I)$ arbitrarily low for all I. This policy induces all risk-neutral entrepreneurs to undertake the first-best investment level (I^{FB}) and secures the first-best level of expected surplus.*

Proof. With no revaluation requirement the entrepreneur's expected payoff reduces to:

$$V(I) = \pi P(I,I) + [1-\pi]\{ \int\limits_{\mu \leq P(I,I)-\hat{x}(I)} P(I,I)h(\mu)d\mu + \int\limits_{\mu \geq P(I,I)-\hat{x}(I)} [\hat{x}(I)+\mu]h(\mu)d\mu\} - I$$

$$= P(I,I)[\pi + (1-\pi) \int\limits_{\mu \leq P(I,I)-\hat{x}(I)} h(\mu)d\mu] + (1-\pi) \int\limits_{\mu \geq P(I,I)-\hat{x}(I)} [\hat{x}(I)+\mu]h(\mu)d\mu - I$$

$$= [\hat{x}(I) + \int_\mu \mu\ h(\mu|S)d\mu][\pi + \int\limits_{\mu \leq P(I,I)-\hat{x}(I)} (1-\pi)\ h(\mu)d\mu]$$
$$+ [1-\pi] \int\limits_{\mu \geq P(I,I)-\hat{x}(I)} [\hat{x}(I)+\mu]\ h(\mu)d\mu\} - I$$

$$= \hat{x}(I) + \int\limits_{\mu \leq P(I,I)-\hat{x}(I)} \mu\ h(\mu|S)d\mu + \pi \int\limits_{\mu \geq P(I,I)-\hat{x}(I)} \mu\ h(\mu|S)d\mu$$
$$+ [1-\pi] \int\limits_{\mu \geq P(I,I)-\hat{x}(I)} [\hat{x}(I)+\mu]\ h(\mu)d\mu - I = \hat{x}(I) - I\ ,$$

as $h(\mu|S) = h(\mu,S)/\int_\mu h(\mu,S)d\mu$ and $\int_\mu h(\mu,S)d\mu = \pi + [1-\pi] \int\limits_{\mu \leq P(I,I)-\hat{x}(I)} h(\mu)d\mu.$ □

Proposition 2 implies that no substantive reporting requirement should be imposed in the basic setting if the relevant social objective is to maximize total (unweighted) expected surplus. Reporting regulations, however, often have the flavor (if not the stated goal) of protecting the less fortunate, the less sophisticated, or some other target population. For example, the *Securities Act of 1933* states (section 7): "The Commission shall prescribe special rules ... as ... necessary or appropriate in the public interest or for the protection of investors." Presumably, investors are graded more important than, say, employees or customers. And in line with this orientation, the *Wall Street Journal* recently reported: "The nation's top securities regulator said he plans a "sustained and increasing focus" on protecting the assets of the aging baby-boomer population" (August 2, 2006). In a similar vein, the FASB emphasizes those with an information disadvantage: "The objectives in this Statement are those of general purpose external financial reporting by business enterprises. The objectives stem primarily from the informational needs of external users who lack the authority to prescribe the financial information they want from an enterprise and therefore must use the information that management communicates to them." (CON 1, paragraph 28). And the recently released FASB/IASB working draft of a unified conceptual framework reiterates this emphasis on "users who lack the ability to prescribe all the financial information they need."[16]

These observations suggest a regulatory goal in the financial reporting arena that is more focused than maximizing aggregate expected surplus. And this, in turn, raises the question of how regulatory mandates might be affected by the underlying regulatory goal of focusing on some specific, targeted interest group. Giving this standing in the model leads us to examine differential weighting of the plight of the distressed and non-distressed entrepreneurs. With this in mind, suppose social welfare increases dollar for dollar with the net payoff of distressed entrepreneurs but increases by only $w \in [0,1)$ dollars as the net payoff of non-distressed entrepreneurs increases by one dollar.[17] When the payoffs of non-distressed entrepreneurs are discounted in this manner, the gains they secure by selling their assets at a price above actual value confer a reduced social benefit. In this sense, the lemons problem imposes greater social losses. To limit these losses, the optimal asset revaluation regulation in this setting (which we call welfare-maximizing asset revaluation policy) imposes a non-trivial revaluation requirement on asset sellers when the incremental audit cost k is sufficiently small.

[16] See paragraph OB11 of the FASB's July 6, 2006 Preliminary Views titled "Conceptual Framework for Financial Reporting: Objective of Financial Reporting and Qualitative Characteristics of Decision-Useful Financial Reporting Information." Relatedly, the Economist (December 20, 2003) reports private-equity funds are disuaded from opening their doors to retail investors because doing so would entail increased transparency and force them "to clean up their balance sheets."

[17] Recall that no other rents arise in the model, since auditors are assumed to receive no rent. Therefore, the reduced weighting of the surplus that accrues to non-distressed entrepreneurs enables identification of optimal policies that favor the distressed entrepreneur. More broadly, however, the important point is to document how the regulatory objectives that underlie GAAP affect the nature and consequences of GAAP. Cynically, for that matter, we might even entertain a form of regulatory capture.

Proposition 3. *When k is sufficiently small, the welfare-maximizing asset revaluation policy (for $0 \leq w < 1$) in the basic setting imposes non-trivial revaluation thresholds, $x_c(\cdot)$, that induce revaluation with strictly positive probability.*

Proof. From Proposition 1, the equilibrium price $(P(I, I))$ of an asset with book value I is less than its expected value $(\hat{x}(I))$ if no revaluation requirement is imposed. In contrast, if $x_c(I)$ is set at so high a level that all assets offered for sale are revalued, then the prices of these assets will be $P(v, I) = v = \hat{x}(I) + \mu$. The expected payoff from investment I for a distressed entrepreneur in this setting is $\hat{x}(I) - I - k$. A non-distressed entrepreneur will never sell his asset (so as to avoid the audit cost k), and so his expected return from investment I is $\hat{x}(I) - I$. Consequently, expected welfare under this revaluation policy is $\pi[\hat{x}(I) - k - I] + w[1 - \pi][\hat{x}(I) - I]$. When $k = 0$, this expected welfare strictly exceeds the corresponding measure under *laissez faire* $\pi[P(I, I) - I] + w[1 - \pi][\gamma - I]$, where, from Proposition 2, $\gamma = \frac{\hat{x}(I) - \pi P(I,I)}{1-\pi} > \hat{x}(I)$. □

Proposition 3 implies that non-trivial asset revaluation requirements can be optimal when the welfare of distressed entrepreneurs is valued more highly than the welfare of non-distressed entrepreneurs. Additional structure is required to identify the magnitude of audit costs that ensure non-trivial revaluation requirements and the specific investment distortions that arise from the requirements. The requisite additional structure is introduced in the next section.

5 Findings in the Structured Basic Setting

To obtain a more complete characterization of asset revaluation policies, we now introduce three forms of additional structure to the basic setting. First, as in Dye (2002), we assume investment I yields expected (gross) payoff $\hat{x}(I) = \frac{\beta}{\alpha} I^{\alpha}$, where α and β are strictly positive parameters. α is less than unity, reflecting diminishing expected payoffs to investment. Second, we assume the privately observed random variable, $\widetilde{\mu}$, is uniformly distributed on the interval $[-f,\ f]$, where f is a strictly positive constant. Thus, $h(\mu) = 1/[2f]$ for $\mu \in [-f, f]$. Consequently, the privately observed revision in value of the remaining cash flows, $v = E[x|I, \mu] = \hat{x}(I) + \mu$, follows a uniform distribution between $\hat{x}(I) - f$ and $\hat{x}(I) + f$. For later reference, denote the end points of this distribution by $\underline{x}(I) = \hat{x}(I) - f$ and $\bar{x}(I) = \hat{x}(I) + f$. Also let $\widehat{h}(v|I)$ denote the induced density on revised asset value $v : \widehat{h}(v|I) = 1/2f$, $v \in [\ \underline{x}(I), \bar{x}(I)\ ]$.

Third, we examine two competing revaluation policies. One, patterned after impairment regulations (e.g., LCM or FAS 144) sets the truncation threshold as a percentage of historical cost: $x_c(I) = zI$ for some to be determined percentage z. Institutionally, though, we recognize $z \geq 0$. We term this a *proportional policy*. It has the distinct flavor of an explicit rule. The second policy type is patterned after the seeming vagueness of FAS 5, where recognition is highly dependent on the nature of the transaction (a liability in this case) and requires considerable judgment on the part of the reporting firm and auditor, and simply sets $x_c(I) =$ "constant" *for this particular type of asset or transaction*. The idea is the auditor under this policy will set a truncation threshold that maximizes the regulatory objective for this specific type of investment and transaction. We term this the *targeted policy* because it allows GAAP to be fine tuned to the precise asset type and transaction. Relative to the flexible policy, this has more of the flavor of a principle than an explicit rule (which of course becomes an explicit mandate in any given setting).

Both policies potentially result in a truncation mandate. In the proportional regime, the auditor simply informs the entrepreneur that existing regulations require a truncation point of zI. Under the targeted regime, the auditor informs the entrepreneur that existing regulations call for him to exercise judgment for this type of investment, and that judgment leads him to the noted $x_c(I)$ (based on welfare maximization at the time the investment is contemplated).

Notice that either policy ends up prescribing a region in which revaluation takes place. As such they are similar to disclosure regions (e.g., Verrecchia 1983), but with the caveat they are driven by regulatory mandate. They also resemble common performance investigation policies (e.g., Townsend [1979]; Baiman and Demski [1980]; Lambert [1985]; and Dye [1986]), but with the caveat again of a regulatory mandate.

The uniform density for $\widetilde{\mu}$ In this "structured basic setting" admits a relatively simple expression for the equilibrium price in the resale market. Recall that the non-distressed entrepreneur offers his asset for sale only when its expected value $E[x|I, \mu] = \hat{x}(I) + \mu$ is intermediate between the critical cutoff value, x_c, and the equilibrium price, $P(I, I)$. Consequently, when $x_c \in [\underline{x}(I), \ \bar{x}(I)]$, the probability an asset with revised (private) value $v \geq x_c$ is offered for sale, given it is owned by a non-distressed entrepreneur, is:

$$\int_{x_c}^{P(I,I)} \widehat{h}(v|I) \, dv = \frac{P(I, I) - x_c}{2f} \, . \tag{1}$$

Because a distressed entrepreneur always sells his asset, the probability a distressed entrepreneur offers an asset with book value $v \geq x_c$ for sale at price $P(I, I)$ is:

$$\int_{x_c}^{\overline{x}(I)} \widehat{h}(v|I)\; dv \quad = \quad \frac{\overline{x}(I) - x_c}{2f} \; .$$

Therefore, the probability an asset with revised (private) value $v \geq x_c$ is traded at price $P(I, I)$ is:

$$q(I) \;=\; \pi \left[\frac{\overline{x}(I) - x_c}{2f} \right] + [1-\pi] \left[\frac{P(I) - x_c}{2f} \right] . \tag{2}$$

Thus, the expected value of a non-revalued I-asset traded at price $P(I, I)$ is

$$E[x|S, I] \;=\; \frac{\pi \left[\overline{x}(I) - x_c \right]}{2fq(I)} \left[\frac{x_c + \overline{x}(I)}{2} \right] + \frac{[1-\pi] \left[P(I, I) - x_c \right]}{2fq(I)} \left[\frac{x_c + P(I, I)}{2} \right] . \tag{3}$$

Equating $E[x|S, I]$ and $P(I, I)$ provides:

Lemma 1. *When $x_c \in [\underline{x}(I),\ \bar{x}(I)]$ in the structured basic setting, the equilibrium price of an asset with book value I is:*

$$P(I, I) = x_c + \frac{\sqrt{\pi}\,[\ \overline{x}(I) - x_c]}{1 + \sqrt{\pi}} \;=\; x_c \left[\frac{1}{1 + \sqrt{\pi}} \right] + \overline{x}(I) \left[\frac{\sqrt{\pi}}{1 + \sqrt{\pi}} \right] . \tag{4}$$

When $x_c \notin [\underline{x}(I),\ \bar{x}(I)]$, the corresponding equilibrium price is:

$$P(I, I) = \underline{x}(I) + \frac{\sqrt{\pi}\,[\ \overline{x}(I) - \underline{x}(I)\ ]}{1 + \sqrt{\pi}} \;=\; \widehat{x}(I) - f \;\left[\frac{1 - \sqrt{\pi}}{1 + \sqrt{\pi}} \right] . \tag{5}$$

Equation (5) in Lemma 1 reveals that if no asset revaluation is induced, the equilibrium price for assets with book value I will be less than the present value of future cash flows from the assets by the "lemons discount" of $f[1-\sqrt{\pi}]/[1+\sqrt{\pi}]$, as reported in Proposition 1. This discount is larger the more pronounced is the relevant information asymmetry about expected asset value (f) and the less likely is an entrepreneur to be distressed. When entrepreneurs are less likely to be distressed, there is an increased likelihood of opportunistic selling by non-distressed entrepreneurs which reduces the expected value, and thus the price, of assets with book value I.

Equation (4) in Lemma 1 provides corresponding insights for the case where non-trivial revaluation is induced in equilibrium. In this case, the equilibrium price of an asset with book value I is a weighted average of the specified revaluation threshold (x_c) and the largest possible value of the asset ($\overline{x}(I)$). x_c is weighted more heavily than $\overline{x}(\cdot)$, and the weight on x_c increases as entrepreneurs become less likely to be distressed (so π declines). Again, an increased presence of non-distressed entrepreneurs implies an increased likelihood of opportunistic selling of assets with book value I, and thus a lower equilibrium price for these assets.

The entrepreneur, of course, anticipates the forthcoming price behavior and the possibility of incurring revaluation consequences; and this, in turn, affects his investment choice. The effect, however, differs between the proportional and targeted policies. The reason is the truncation threshold is locally insensitive to investment scale in the investment choice subgame under the targeted policy (i.e., $\frac{dx_c(I)}{dI} = 0$) but moves proportionately with investment scale under the proportional policy (i.e., $\frac{dx_c(I)}{dI} = z$). This leads to the following pair of observations.

Proposition 4. *If revaluation occurs with strictly positive probability and trade at equilibrium price $P(I, I)$ occurs with strictly positive probability in the structured basic setting with a targeted truncation policy, investment (weakly) increases above the first-best level as x_c increases, attaining a maximum at $\left[\frac{\beta[2f+\pi k]}{2f}\right]^{\frac{1}{1-\alpha}} > I^{FB}$.*

Proposition 5. *If revaluation occurs with strictly positive probability and trade at equilibrium price $P(I, I)$ occurs with strictly positive probability in the structured basic setting with a proportional truncation policy, investment: (i) is greater than first-best investment (I^{FB}) if $z < 1$; (ii) is equal to I^{FB} if $z = 1$; and (iii) is less than I^{FB} if $z > 1$.*

Under the targeted policy, increasing investment scale lowers the odds of being forced to undergo costly revaluation. Near the first best investment scale, the effect of increasing investment has a second order effect on the expected present value of the remaining cash inflows, but a first order effect on reducing the expected outlay for revaluation. So investment increases. Eventually, however, this trade-off becomes uneconomic, and the maximum distortion takes over. Conversely, under the proportional policy, a small z renders the effect of increasing investment scale on the truncation point second order and we observe a pattern similar to that of the targeted policy. But a large z (i.e., $z > 1$) has the opposite effect, as here lowering investment scale at the margin saves more revaluation cost than it destroys present value of cash inflows. And the break point between the two effects is the LCM point of $z = 1$, where the truncation point moves one-for-one with the investment scale, and first best investment scale results.[18]

[18] The value of z in the proportional revaluation policy might be viewed as a tax on investment. The higher is z, the more pronounced is the increase in the upper bound of the region of mandated revaluation ($[\underline{x}(I), zI]$) as investment increases. Proposition 5 reveals that a high "investment tax" ($z > 1$) reduces investment below the first-best level. In contrast, a value of z below 1 spurs investment above the first-best level because the mandated revaluation region expands less rapidly than investment in this case.

From Proposition 3 we know that if k is sufficiently small, welfare maximization will lead to nontrivial revaluation prospects. In the structured basic setting coupled with the targeted policy, we are able to give precise meaning to "sufficiently small." Proposition 6 reports that when revaluation costs are below the identified bound, the welfare-maximizing policy displays nontrivial truncation and always induces the maximum investment level identified in Proposition 4.

Proposition 6. *When k is sufficiently small,*[19] *the welfare-maximizing revaluation policy $(0 \leq w < 1)$ in the structured basic setting with a targeted truncation policy specifies a threshold x_c that induces the maximum investment level $I = \left[\frac{\beta[2f+\pi k]}{2f}\right]^{\frac{1}{1-\alpha}}$, by setting $x_c(I) \neq I$) almost surely..*

Intuitively, when k is not too large, revaluation helps the distressed type because it eases the lemons problem in the resale market, even net of the related investment distortion and revaluation expenditure. And at the margin, it is welfare improving to increase x_c to the point in equilibrium we have revaluation with positive probability. Indeed, with "small" k it is apparent the (maximal) investment distortion is itself small. Moreover, this is accomplished by setting a truncation point that almost surely differs from the LCM point of $x_c(I) = I$, reflecting the balancing at the margin of certification cost and lessening of the lemon problem faced by the distressed entrepreneur.

The proportional policy is a different story. Here there is no tractable upper bound to what it means for k to be sufficiently small, though again from Proposition 3 we know welfare maximization will lead to nontrivial truncation provided k is "sufficiently small." Importantly, though, Proposition 7 reveals that it generally is not optimal to set $z = 1$. Therefore, the proportional revaluation policy, like the specific revaluation policy, generally will induce an investment distortion, and LCM is almost surely not the optimal proportional policy.

Proposition 7. *When k is sufficiently small, the welfare-maximizing revaluation policy $(0 \leq w < 1)$ in the structured basic setting with a proportional truncation policy induces an investment distortion (by setting $z \neq 1$) almost surely.*

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[19] The critical value of k is $\frac{2f\left[1-\sqrt{\pi}\right][1-w]\left[\pi-\sqrt{T}\right]}{\pi\left[1+\sqrt{\pi}\right]}$, where $T \equiv \frac{\pi\left[1+\sqrt{\pi}\right]C}{f[1-w]\left[1-\sqrt{\pi}\right]}$ and C is the product of $[\pi + w(1-\pi)]$ and $\left\{\frac{1-\alpha}{\alpha} - \left[\frac{2f-\alpha[2f+\pi k]}{\alpha[2f+\pi k]}\right]\left[\frac{\pi k+2f}{2f}\right]^{\frac{1}{1-\alpha}}\right\}\beta^{\frac{1}{1-\alpha}}$.

Again, the welfare improvement flows from revaluation easing the lemons problem that taxes the distressed type, but must be balanced against investment distortion. Here, using Proposition 5, we know investment distortion occurs unless $z = 1$; but $z = 1$ imposes potentially a small or a large relative truncation point, depending on the investment rent (α and β) and the project's risk (f). This effectively removes anything but a measure zero connection to LCM.

But the question remains, which of the two policies is preferred? The answer is moot if the regulatory objective is Marshallian expected surplus (Proposition 2), but hardly so if the regulatory objective differentially weights the plight of the distressed type (Proposition 3). In this case, two factors come into play: the investment distortions differ between the two policies and the proportional policy has a lower bound of $x_c(I) = zI = 0$, as $z \geq 0$.

The lower bound of $x_c(I) = 0$ most readily explains why it is possible to exhibit settings where the targeted policy is preferred to the proportional policy, though preference for the targeted policy is not confined to this particular region in parameter space. The reverse, however, is also possible, because the targeted policy, with its lack of direct feedback from investment choice to x_c can invite excessive investment distortion, relative to the proportional case, for a given unit of reduction in the lemons problem in the resale market.

To illustrate, let $\alpha = .5$, $\beta = 10$, $\pi = .7$, $k = 20$ and $f = 150$. First best investment is $I^{FB} = 100$. Under the targeted policy, $x_c(I) = 134.4453$, $I = 109.5511 > 100$ and the expected payoff to the distressed type is 87.2785. But under the proportional policy, $z = 1.2257$, $I = 98.0177 < 100$, $x_c(98.0177) = 120.1365$ and the expected payoff to the distressed type is $87.4850 > 87.2785$.[20] Thus, if we heavily weight the distressed type (discount the non-distressed type), the proportional policy outperforms the targeted policy. Conversely, if we set $k = 150$ and $f = 1,000$, we find just the opposite conclusion, because the $z \geq 0$ constraint in the proportional policy is binding.[21] For that matter, $k = 5$ and $f = 150$ also leads to preference for the targeted policy, and the $z \geq 0$ constraint is not there binding. This provides the following conclusion.

Proposition 8. *When k is sufficiently small that both the targeted and proportional policies*

[20] It is also readily verified the expected payoff for the distressed type is even lower if no revaluation whatever occurs, or if revaluation occurs with probability 1.

[21] In this case, the targeted policy uses $x_c(I) = -476$ while the proportional policy uses $x_c(I) = 0$ based on $z = 0$.

are welfare improving, neither one dominates the other.

Finding elegant, intuitive sufficient conditions to ensure one dominates the other (and vice versa) remains an open question. However, more mechanical answers are available. For this purpose, and to reduce notational clutter,we assume $\alpha = .5$ and the welfare weight on the non-distressed type is $w = 0$. This considerably reduces the notation, but does not affect the general tenor of follows. Denote the following:

$$I^{SC} \equiv I^{FB}(1 + \pi k/2f)^2$$

$$A \equiv \overline{x}(I^{SC}) - \frac{(1+\sqrt{\pi})k}{1-\sqrt{\pi}}$$

$$\widehat{P} \equiv \frac{A + \overline{x}(I^{SC})\sqrt{\pi}}{1+\sqrt{\pi}}$$

$$B \equiv \frac{-\pi k}{2f}\widehat{x}(I^{SC})$$

$$G \equiv -B\underline{x}(I^{SC}) + (B - I^{SC})\widehat{P} + \overline{x}(I^{SC})\frac{I^{SC} + \sqrt{\pi}B}{1+\sqrt{\pi}} - \frac{k}{2f}\left(I^{SC} - B\right) + 2\pi k I^{SC}$$

Proposition 9. *When $\alpha = .5$, $w = 0$, and k is sufficiently small that both the targeted and proportional policies are welfare improving, $A = 0$ and $G > 0$ implies the proportional policy is locally preferable to the targeted policy for welfare maximization purposes, while $A < 0$ and $G < 0$ implies the opposite.*

Proof. Given $w = 0$ and $= .5$, I^{SC} is the induced investment choice under the targeted regime, as noted in Proposition 6. Similarly, from the proof of Proposition 6, $A = x_c(I^{SC})$. Now, with $w = 0$, and again from the proof of Proposition 6, the welfare measures reduces to

$$W^{SC}(I, x_c) = \frac{\pi}{2f}\left[\frac{1}{2}(x_c^2 - \underline{x}^2) - k(x_c - \underline{x}) + P(\overline{x} - x_c)\right] \text{-}\pi I$$

and under the proportional regime we have $x_c = zI$, for $z \geq 0$. From here, and letting I be a function of z, some algebraic manipulation reveals

$$\frac{dW^{SC}(I(z), zI(z))}{dz}|_{z=0} = (2f/\pi)G$$

It is also routine to verify that at $z = 0$, $I(z) = I^{SC}$, which implies the two regimes are welfare equivalent if $x_c(I) = 0 = z$. Now, $A = 0$ and $G > 0$ implies the optimal targeted solution can be improved upon by $z > 0$ in the proportional regime. Likewise, $A < 0$ but

$G < 0$ implies the targeted regime at some negative $x_c(I) < 0$ is superior to $x_c(I) = 0$, and any feasible proportional solution at best locally matches the $x_c(I) = 0$ solution in the targeted case.

Finally, the noted inequalities are not empty sets. It is readily verified that $\beta = 10$, $f = 521.7889$ and $k = 57.5822$ satisfies the first set of conditions and $f = 451.5070$ and $k = 59.9337$ satisfies the second, that in both cases the claimed $x_c(I)$ is optimal in the targeted regime, and neither no revaluation nor complete revaluation are superior welfare maximizing policies. □

Continuity, of course, implies the conditions in Proposition 9 are far from vacuous.

Stepping back, we see that the model supports a wide variety of potential regulatory solutions. Under the proportional policy, the LCM setting of $z = 1$ is a measure zero phenomenon, and both $z < 1$ and $z > 1$ are possible. In addition, preference between a targeted and a proportional policy is not definitive. A wide array of prescriptions is present, much as we see in present day GAAP.

6 Extensions of the Model

Various assumptions are used to paint this picture. We now turn to a sensitivity exploration, designed to document the role played by various assumptions: no voluntary disclosure, a single type of entrepreneur, observable investment, and risk neutrality. In each instance we concentrate on a regulatory objective of expected surplus maximization. We also, where appropriate, emphasize the targeted revaluation policy.

6.1 Voluntary Disclosure of (Audited) Asset Value

We have assumed voluntary disclosure is impossible, and this is a potential driving force in the conclusion (Proposition 2) that surplus maximization entails no regulation whatever. We now examine the implications of voluntary audited disclosure. To do so most simply, we continue with all of the features of the structured basic setting except that auditors can always verify accurately the value of any asset at cost k, regardless of its privately observed value. This, of course, takes us into the voluntary disclosure realm.

In such a setting, an entrepreneur can voluntarily disclose via audited revaluation of his asset ($x > x_c$) at personal cost k if he chooses to do so. Consequently, a distressed entrepreneur will undertake voluntary revaluation if the revised estimated value of his asset exceeds the equilibrium price for non-certified assets by more than the cost of revaluation. Such voluntary revaluation is profitable because it allows the entrepreneur to increase the revenue from the sale of his relatively valuable asset by more than the cost of revaluation.

In contrast, a non-distressed entrepreneur will never undertake (disclosure via) revaluation in this setting, as revaluation generates a net expected payoff that is k below its counterpart from retaining the asset. The non-distressed entrepreneur will continue to opportunistically sell his asset with book value I whenever its privately observed value is between x_c and the prevailing price for assets with book value I.[22]

Proposition 10 considers the impact of voluntary revaluation disclosure when no mandatory revaluation is imposed. The proposition reveals that the opportunity to voluntarily revalue assets can be detrimental for investors. When the cost of voluntary revaluation is sufficiently low, investors will find it profitable to revalue assets with sufficiently high value. By removing the high-value assets from the pool of assets with book value I, such voluntary certification reduces the expected value, and thus the equilibrium price, of an asset with book value I. The combination of revaluation costs for high-value assets and a lower price for assets with book value I causes aggregate expected surplus to decline. Thus, investors would be better off *ex ante* if they could credibly promise to never avail themselves of the opportunity to voluntarily revalue assets.

Proposition 10. *Suppose in the structured basic setting entrepreneurs can voluntarily revalue their assets at cost* $k < \frac{2f}{1+\sqrt{\pi}}$*, but no mandatory certification is imposed. Then entrepreneurs will voluntarily revalue assets with expected values above* $\underline{x}(I^{FB}) + k\,[1+\sqrt{\pi}]$*, undertake the first-best level of investment* (I^{FB})*, and achieve expected surplus* $\hat{x}(I^{FB}) - I^{FB} - \frac{\pi k}{2f}\,[2f - k\,(1+\sqrt{\pi})] < V^{FB}$.

Conceivably, mandatory revaluation of low-value assets could increase expected surplus in this setting with voluntary revaluation. Mandatory revaluation would both raise the equilibrium price of assets with book value I and reduce the incidence of (costly) voluntarily revaluation. However, mandatory revaluation only reduces voluntary revaluation costs by introducing corresponding mandatory revaluation costs. Furthermore, although any diminution of the lemons problem can benefit distressed entrepreneurs, it can harm non-distressed

[22] We continue to assume an entrepreneur that chooses to sell an asset with actual value below x_c must certify the value of the asset (at personal cost k).

entrepreneurs. On balance, as Proposition 11 reports, mandatory revaluation affects neither the entrepreneur's investment level nor his expected net payoff in this setting.

Proposition 11. *Suppose in the structured basic setting entrepreneurs can voluntarily revalue their assets at cost* $k < \frac{2f}{1+\sqrt{\pi}}$*. Then non-trivial mandatory revaluation via the targeted policy affects neither the entrepreneur's investment level* (I^{FB}) *nor his expected surplus.*

The "irrelevance" of mandatory revaluation suggested by Proposition 11 should be interpreted with caution because the conclusion reflects all of the maintained simplifying assumptions, including constant revaluation costs. If high asset values are significantly more costly to audit than low asset values, voluntary revaluation of high-value assets generally will not exactly offset mandated revaluation of low-value assets. Consequently, a meaningful role for non-trivial revaluation mandates can emerge even when voluntary certification is feasible in this setting. The more robust conclusion is that the design of asset revaluation regulation can become more complex and more subtle when voluntary revaluation is a viable option for investors, and must deal with subgame temptations to engage in ex ante inefficient voluntary disclosure via the revaluation option.

6.2 Unobserved Investor Type

We have also assumed all entrepreneurs are identical and the productivity of each entrepreneur's investment is common knowledge. This, too, is a potential driving force in the conclusion (Proposition 2) that surplus maximization entails no regulation whatever. Consider, then, the possibility that entrepreneurs differ and each entrepreneur is privately informed about the expected payoff from his investment.

For simplicity, suppose each entrepreneur is either a high (H) or a low productivity (L) type. The present value of the future cash flows from investment I by a type $i \in \{\text{L, H}\}$ investor is $x_i(I) = \frac{\beta_i}{\alpha} I^{\alpha} + \mu + \varepsilon$, where $\beta_H > \beta_L$ and where μ and ε are the realizations of zero-mean independent random variables $\widetilde{\mu}$ and $\widetilde{\varepsilon}$. The random variable $\widetilde{\mu}$ is again uniformly distributed on $[-f,\ f]$ and $\widetilde{\varepsilon}$ has unbounded support (to preclude perfect inference of the entrepreneur's type from the realized expected payoff from investment).

Recall that the surplus-maximizing revaluation policy in the basic setting effectively imposes no revaluation requirement. Absent any such requirement, all (identical) entrepreneurs undertake the first-best investment level (I^{FB}) and the equilibrium price for assets with book value I^{FB} is $\widehat{x}(I^{FB}) - f[1 - \sqrt{\pi}]/[1 + \sqrt{\pi}]$ (from Lemma 1). The parallel outcome in the present setting would entail first-best investment by both investor types ($I_i^{FB} \equiv \arg\max_I \{\frac{\beta_i}{\alpha} I^{\alpha} - I\}$ for $i = \text{L, H}$) and equilibrium prices for assets with book value

I_i^{FB} equal to $\widehat{x}_i(I_i^{FB}) - f[1-\sqrt{\pi}]/[1+\sqrt{\pi}]$, where $\widehat{x}_i(I) \equiv \frac{\beta_i}{\alpha} I^\alpha$ for $i =$ L, H. This outcome may not arise in the absence of revaluation mandates in the present setting, though. L types may undertake investment I_H^{FB} in order to masquerade as H types and thereby effectively exaggerate the expected value of their assets.

The equilibrium ability and incentive of L types to masquerade as H types depends in part on the inferences drawn from off-equilibrium behavior. Suppose that absent any asset revaluation, asset buyers in the resale market believe that any investment at or above I_H^{FB} was undertaken by H types, while any investment below I_H^{FB} was undertaken by L types. Further suppose $\widehat{x}_H(I) - \widehat{x}_L(I) < 2f/[1+\sqrt{\pi}]$ for all $I \geq I_H^{FB}$, so that the difference in expected expected payoff from investment according to the investor's type is not too pronounced. In addition, for convenience, suppose $\alpha = 0.5$, $f = 2\beta_H^2$, and β_L is sufficiently close to zero, so the L type is productive but at a considerably lesser magnitude than the H type. These conditions ensure that L types will either invest I_L^{FB} or I_H^{FB} in this setting with unknown investor type.

Lemma 2. *If no revaluation mandate is imposed in the setting with unknown investor types: (i) a separating equilibrium (with $I_L = I_L^{FB} < I_H^{FB} = I_H$) arises when π is sufficiently close to zero; whereas (ii) a pooling equilibrium (with $I_L = I_H = I_H^{FB}$) arises when π is sufficiently close to unity.*

When π is close to zero, investors are unlikely to become distressed. The prevalence of non-distressed investors in this case creates a severe lemons problem that leads to a low price for assets with book value I_H^{FB}. In light of the meager financial expected payoff L types anticipate from undertaking investment I_H^{FB}, they prefer to reduce their investment cost by undertaking the lower investment, I_L^{FB}. In contrast, when π is close to unity, few opportunistic non-distressed entrepreneurs are present, and so the market price for assets with book value I_H^{FB} will be relatively high. This relatively high price induces L types to invest I_H^{FB}, leading to the pooling equilibrium identified in Lemma 2.

When pooling would otherwise arise, revaluation requirements can increase expected surplus, as Proposition 12 reports. The proposition considers a revaluation policy that links the critical revaluation threshold ($x_c(\cdot)$) to the observed investment level, in effect employing the targeted policy, with two distinct asset scale categories. The additional flexibility in the revaluation policy is valuable in the presence of asymmetric knowledge of both the *ex ante* and the *ex post* expected payoff on investment.

Proposition 12. *Suppose pooling would arise in the setting with unknown investor types in* *the absence of any revaluation requirement. Then when audit costs k are sufficiently large, a surplus-maximizing revaluation policy sets $x_c(I_L^{FB}) << 0$, $x_c(I_L^{FB}) = \widehat{x}_H(I_H^{FB}) - f$, and $x_c(I)$ arbitrarily high for $I \notin \{I_L^{FB}, I_H^{FB}\}$. In equilibrium, separation occurs, first best investment and surplus obtain, and no revaluation is ever observed.*

The revaluation policy identified in Proposition 12 relieves an investor of all revaluation obligations if he invests I_L^{FB}. In contrast, entrepreneurs that invest I_H^{FB} are required to revalue assets with expected value below $\widehat{x}_H(I_H^{FB}) - f$. This requirement imposes revaluation costs on L types who invest I_H^{FB} (when $E[x|I_H^{FB}, \mu] \in [\widehat{x}_L(I_H^{FB}) - f, \widehat{x}_H(I_H^{FB}) - f))$ without imposing any corresponding revaluation costs on H types who invest I_H^{FB}. H types are certain to avoid revaluation costs in this setting because the (interim) expected value of an asset derived from investment I_H^{FB} by an H type is never below $\widehat{x}_H(I_H^{FB}) - f$ (since $\mu \geq -f$). When revaluation costs k are sufficiently large, the policy described in Proposition 12 reduces the expected payoff the L type anticipates from undertaking investment I_H^{FB} below the corresponding expected payoff from investing I_L^{FB}. Consequently, the policy will induce a separating equilibrium in which first-best investments are undertaken, first-best level of expected surplus is achieved, and, in equilibrium, revaluation will never occur.

The uniform distribution for $\widetilde{\mu}$ and the corresponding moving support for $E[x|I, \mu]$ in this setting with unknown investor type thus admit a revaluation policy that secures the first-best level of expected surplus. More generally, revaluation policies like the one described in Proposition 12 may impose revaluation costs on H types in equilibrium and thereby fail to secure the first-best level of surplus. However, the key features of the revaluation in Proposition 12 will persist more generally. By imposing more stringent revaluation requirements on assets with large underlying investment than on assets with small underlying investment, low-productivity entrepreneurs can be deterred from over-investing in an attempt to exaggerate the value of their assets. Notice that the more stringent revaluation policy imposed on large investments does not reflect any special concern with the welfare of "small" investors here. Instead, the policy reflects the gain in total surplus that arises when low-productivity types are deterred from over-investing in an attempt to exaggerate the value of their assets.

6.3 Unobserved Investment

We have also assumed investment is observable (as in, say, Newman, Patterson and Smith [2005] but not in, say, Dye and Sridhar [2004] or Liang and Wen [2006]). In principle, the critical investment in question could entail effort that is inherently difficult to measure. To briefly consider this possibility formally, return to the basic setting but now suppose the scale of investment is privately known by the entrepreneur and cannot be communicated. Proposition 13 reports that entrepreneurs will under-invest when the scale of their investment is unobservable.

Proposition 13. *If investment is not observed and no revaluation is possible in the basic setting, equilibrium investment will be below first-best investment.*

Proof. Let I^e denote the market's conjecture about the level of investment undertaken by the (identical) entrepreneurs. Also let $P(I^e)$ denote the equilibrium price of an asset offered for sale. The expected expected payoff to investment in this setting with no revaluation and unobserved investment is:

$$V(I) = \pi P(I^e) + [1-\pi]\{ \int\limits_{\mu \leq P(I^e)-\hat{x}(I)} P(I^e)h(\mu)d\mu + \int\limits_{\mu \geq P(I^e)-\hat{x}(I)} [\hat{x}(I) + \mu]h(\mu)d\mu\} - I. \quad (6)$$

Differentiating (6) provides:

$$\frac{dV(I)}{dI} = [1-\pi]\{-\hat{x}'(I)\,P(I^e)h(P(I^e) - \hat{x}(I)) + \hat{x}'(I)\,P(I^e)h(P(I^e) - \hat{x}(I))$$
$$+[1 - H(P(I^e) - \hat{x}(I))]\hat{x}'(I)\} - 1 = [1-\pi][1 - H(P(I^e) - \hat{x}(I)]\hat{x}'(I) - 1.$$

Hence, $\hat{x}'(I) > 1$ at $I = I^e$, since the conjectured and the actual investment coincide in equilibrium. □

Proposition 13 implies that for k sufficiently small, surplus maximizing regulation will induce non-trivial revaluation in order to increase equilibrium investment and the resulting expected surplus. Therefore, revaluation policies can enhance total surplus when investment is unobservable even though they do not do so in the basic setting where investment is observable.

6.4 Risk Aversion

We have also assumed the entrepreneurs and asset resale pricing reflect risk neutrality. Now return, again, to the basic setting (where investment is observed publicly) but suppose entrepreneurs are strictly risk averse, while pricing continues to reflect risk neutrality. An entrepreneur's expected cash flow from investment I in this setting is $\hat{x}(I) - I$ in the absence of any revaluation requirements. If $k = 0$ and revaluation were always mandated in this setting, an entrepreneur would receive the same expected cash flow as a function of I.

However, the cash flow stream under revaluation would be more risky because it would be a mean preserving spread of the corresponding stream under no revaluation (Rothschild and Stiglitz [1970]). The same holds for any truncated revaluation requirement. Therefore, no revaluation would be imposed if the objective were to maximize the expected surplus of strictly risk averse entrepreneurs.

The difficulty here is mandated revaluation exposes the seller to revaluation risk, a risk he does not face in the absence of mandated revaluation. More deeply, though, we would now be interested in portfolios, options and other risk mitigating devices before getting too excited about our pithy observation.

7 Conclusions

GAAP contains a bewildering array of prescriptions, ranging from seemingly principle-based to explicit rule-based mandates coupled with a seemingly inconsistent approach to asset valuation. While the explanations for this consistently inconsistent set of prescriptions are many and varied, we focus on asset revaluation and offer a simple economic trading model in which virtually any element in the entire array of prescriptions can surface as optimal regulatory prescription. Thus, what we termed a targeted (or principle based) policy might or might not be preferred to a proportional (rule-based) policy, and within a given policy vehicle, the truncation pattern of extant revaluation regulations surfaces, but with heavy dependence on the local economic forces *and* the regulatory objective.

The model itself is simple, by design. It relies on liquidity constrained (or distressed) and non-distressed players, coupled with an inability to communicate whether in fact you are so distressed. This coupled with private valuation information results in an investment followed by resale model that hosts our exploration of various facets of GAAP.

Of course, the down side to a simple model is the temptation to append additional forces. Here we have documented the effect of appending voluntary disclosure, productivity types, unobservable investment scale and risk aversion. All quantitatively alter the analysis, but qualitatively lead to the same point: the apparent inconsistencies in GAAP reflect economic forces in the model.

Questions to be explored also remain, including deepening the various aspects of our analysis. We also suspect analysis of alternative audit cost structures[23], the possibility of

[23]Notice, for example, that optimal revaluation policies may no longer mandate revaluation of only the least

evading revaluation regulations, and capital market tensions (e.g., Whittred and Chan [1992]) are important. Combinations of regulatory issues, such as impairment and revenue recognition, are also of considerable interest. Future research also might extend the basic models analyzed here by incorporating managerial incentives (along the lines of Kirschenheiter and Melumad (2002) and Arya and Glover (2003), for example).

valuable assets if revaluation costs are lower for highly valuable assets than for less valuable assets.

8 References

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Appendix

Proof of Lemma 1.

$P(I,I) = P$ is determined by:

$$P = \frac{\pi\,[\,\overline{x}(I) - x_c\,]}{2fq}\left[\frac{x_c + \overline{x}(I)}{2}\right] + \frac{[1-\pi]\,[P - x_c]}{2fq}\left[\frac{x_c + P}{2}\right]. \tag{A1}$$

Rearranging (A1) and employing $\overline{x}$ to denote $\overline{x}(I)$ provides:

$$4fqP = \pi[\overline{x} - x_c][x_c + \overline{x}] + [1-\pi][P^2 - x_c^2]\ . \tag{A2}$$

Since $4fq = 2\pi[\overline{x} - x_c] + 2[1-\pi][P - x_c]$, (A2) can be written as:

$$2\pi[\overline{x} - x_c]P + [1-\pi][2P^2 - 2Px_c] = \pi[\overline{x}^2 - x_c^2] + [1-\pi][P^2 - x_c^2]. \tag{A3}$$

Rearranging terms provides:

$$2\pi[\overline{x} - x_c]P + [1-\pi][P^2 - 2Px_c + x_c^2] - \pi[\overline{x}^2 - x_c^2] = 0. \tag{A4}$$

(A4) can be rewritten as:

$$[1-\pi][P - x_c]^2 + 2\pi[\overline{x} - x_c][P - x_c] + 2\pi[\overline{x} - x_c]x_c - \pi[\overline{x}^2 - x_c^2] = 0. \tag{A5}$$

Since $2\pi[\overline{x} - x_c]x_c - \pi[\overline{x}^2 - x_c^2] = -\pi[\overline{x}^2 - 2\overline{x}x_c + x_c^2]$, (A5) can be rewritten as:

$$[1-\pi][P - x_c]^2 + 2\pi[\overline{x} - x_c][P - x_c] - \pi[\overline{x} - x_c]^2 = 0. \tag{A6}$$

Solving (A6) provides:

$$P - x_c = \frac{-2\pi[\overline{x} - x_c] \pm 2\sqrt{\pi}[\overline{x} - x_c]}{2[1-\pi]} = \frac{[-\pi \pm \sqrt{\pi}][\overline{x} - x_c]}{1-\pi}, \text{ or} \tag{A7}$$

$$P = x_c + \frac{[-\pi \pm \sqrt{\pi}][\overline{x} - x_c]}{1-\pi}. \tag{A8}$$

Therefore:

$$P = x_c + \frac{\sqrt{\pi}[\overline{x} - x_c]}{1+\sqrt{\pi}}, \text{ or } P = x_c - \frac{\sqrt{\pi}[\overline{x} - x_c]}{1-\sqrt{\pi}}. \tag{A9}$$

$P = x_c - \frac{\sqrt{\pi}[\overline{x}-x_c]}{1-\sqrt{\pi}}$ is not possible because the expected value of the non-certified asset cannot be below x_c. Therefore,

$$P = x_c + \frac{\sqrt{\pi}[\overline{x} - x_c]}{1+\sqrt{\pi}} = \frac{x_c + \sqrt{\pi}\overline{x}}{1+\sqrt{\pi}}. \quad \blacksquare \tag{A10}$$

Proof of Proposition 4.

Recall $\widehat{h}(x) = \frac{1}{2f}$ is the density function for x. Given x_c (and integrating out the $\widetilde{\varepsilon}$ disturbance) the entrepreneur's expected profit given investment I is:

$$\begin{aligned} V^{NC}(I;x_c) &\equiv \pi P + [1-\pi]\left[\int_{\underline{x}}^{P} P\widehat{h}(x)dx + \int_{P}^{\overline{x}} x\widehat{h}(x)dx\right] - I \\ &= \widehat{x}(I) - I \qquad if \quad x_c \leq \underline{x} = \widehat{x}(I) - f \ ; \end{aligned} \tag{A11}$$

$$\begin{aligned} V^{FC}(I;x_c) &\equiv \pi \int_{\underline{x}}^{\overline{x}} [x-k]\widehat{h}(x)dx + [1-\pi]\int_{\underline{x}}^{\overline{x}} x\widehat{h}(x)dx - I \\ &= \widehat{x}(I) - I - \pi k \qquad if \quad x_c \geq \overline{x} = \widehat{x}(I) + f \ ; \quad \text{and} \end{aligned} \tag{A12}$$

$$\begin{aligned} V^{SC}(I;x_c) \equiv \pi &\left[\int_{\underline{x}}^{x_c} [x-k]\widehat{h}(x)dx + \int_{x_c}^{\overline{x}} P\widehat{h}(x)dx - I\right] \\ &+ [1-\pi]\left[\int_{\underline{x}}^{x_c} x\widehat{h}(x)dx + \int_{x_c}^{P} P\widehat{h}(x)dx + \int_{P}^{\overline{x}} x\widehat{h}(x)dx - I\right] \end{aligned} \tag{A13}$$

$$= \ \widehat{x} - I - \frac{\pi k[x_c - \underline{x}]}{2f} \qquad if \quad \underline{x} = \widehat{x}(I) - f \leq x_c \leq \widehat{x}(I) + f = \overline{x}. \tag{A14}$$

(A14) follows from (A13) because (A13) implies,

$$\begin{aligned} V^{SC}(I;x_c) = &\frac{\pi}{2f}\left[\frac{1}{2}[x_c^2 - \underline{x}^2] - k[x_c - \underline{x}] + P[\overline{x} - x_c]\right] \\ &+ \frac{[1-\pi]}{2f}\left[\frac{1}{2}[x_c^2 - \underline{x}^2] + P[P - x_c] + \frac{1}{2}[\overline{x}^2 - P^2]\right] - I. \end{aligned} \tag{A15}$$

Substituting from (A10) into (A15) provides:

$$
\begin{aligned}
V^{SC}(I,x_c) &= \frac{1}{2f}\left\{\frac{1}{2}[x_c^2-\underline{x}^2]-\pi k[x_c-\underline{x}]+\pi\overline{x}\left[\frac{x_c+\sqrt{\pi}\overline{x}}{1+\sqrt{\pi}}\right]-x_c\left[\frac{x_c+\sqrt{\pi}\overline{x}}{1+\sqrt{\pi}}\right]\right.\\
&\quad\left.+\left[\frac{1-\pi}{2}\right]\left[\overline{x}^2+(\frac{x_c+\sqrt{\pi}\overline{x}}{1+\sqrt{\pi}})^2\right]\right\}-I\\
&= \frac{1}{2f}\left\{\frac{1}{2}x_c^2-\frac{1}{2}\underline{x}^2-\pi k[x_c-\underline{x}]+\frac{\pi\overline{x}x_c}{1+\sqrt{\pi}}+\frac{\pi\sqrt{\pi}\overline{x}^2}{1+\sqrt{\pi}}-\frac{x_c^2}{1+\sqrt{\pi}}\right.\\
&\quad\left.-\frac{\sqrt{\pi}\overline{x}x_c}{1+\sqrt{\pi}}+\left[\frac{1-\pi}{2}\right]\overline{x}^2+\frac{[1-\sqrt{\pi}]}{2[1+\sqrt{\pi}]}[x_c^2+\pi\overline{x}^2+2\sqrt{\pi}\overline{x}x_c]\right\}-I\\
&= \frac{1}{2f}\left[\frac{1}{2}\overline{x}^2-\frac{1}{2}\underline{x}^2-\pi k[x_c-\underline{x}]\right]-I=\hat{x}-I-\frac{\pi k[x_c-\underline{x}]}{2f}. \qquad \text{(A16)}
\end{aligned}
$$

For a given x_c, the entrepreneur will select I to maximize $\max\{V^{NC}(x_c),V^{FC}(x_c),V^{SC}(x_c)\}$, where $V^{NC}(x_c)$ denotes $\max_I V^{NC}(I;x_c)$ subject to $\widehat{x}(I)-f\geq x_c$; $V^{FC}(x_c)$ denotes $\max_I V^{FC}(I;x_c)$ subject to $x_c\geq\widehat{x}(I)+f$; and $V^{SC}(x_c)$ denotes $\max_I V^{SC}(I;x_c)$ subject to $\widehat{x}(I)-f\leq x_c\leq\widehat{x}(I)+f$.

Because $\hat{x}\left(I\right)=\frac{\beta}{\alpha}I^{\alpha}$, we have,

$$I^{FB}=\beta^{\frac{1}{1-\alpha}}\ \text{ and }\ \widehat{x}^{FB}\equiv\widehat{x}(I^{FB})=\frac{\beta}{\alpha}\beta^{\frac{\alpha}{1-\alpha}}=\frac{1}{\alpha}\beta^{\frac{1}{1-\alpha}}. \qquad \text{(A17)}$$

and

$$V^{NC}(x_c)=\left\{\begin{array}{cc}\widehat{x}^{FB}-I^{FB}, & if\ x_c\leq\widehat{x}^{FB}-f\\ x_c+f-I,\ where\ I=[\frac{\alpha}{\beta}(x_c+f)]^{\frac{1}{\alpha}}, & otherwise\end{array}\right\}. \qquad \text{(A18)}$$

The first line of (A18) follows from solving $\max_I\ V^{NC}(I;x_c)$ with the constraint $x_c\leq\widehat{x}(I)-f$ omitted. The second line of (A18) holds because when the constraint $x_c\leq\widehat{x}(I)-f$ binds, $x_c=\widehat{x}(I)-f$, $I=[(\alpha/\beta)(x_c+f)]^{1/\alpha}$. Analogous logic reveals,

$$V^{FC}(x_c)=\left\{\begin{array}{cc}\widehat{x}^{FB}-I^{FB}-\pi k, & if\ x_c\geq\widehat{x}^{FB}+f\\ x_c+f-I-\pi k,\ where\ I=[\frac{\alpha}{\beta}(x_c-f)]^{1/\alpha}, & otherwise\end{array}\right\}. \qquad \text{(A19)}$$

Similarly, differentiating (A14) gives,

$$\frac{\partial V^{SC}(I,x_c)}{\partial I}=\widehat{x}'\left[1+\frac{\pi k}{2f}\right]-1=0, \qquad \text{(A20)}$$

$$\widehat{x}'(I)=\beta I^{\alpha-1}=\frac{1}{1+\frac{\pi k}{2f}}, \qquad \text{(A21)}$$

$$I^{SC} = \left[\frac{\beta[2f + \pi k]}{2f}\right]^{\frac{1}{1-\alpha}} > I^{FB}. \tag{A22}$$

Thus,

$$V^{SC}(x_c) = \left\{ \begin{array}{cc} \widehat{x}(I^{SC}) - I^{SC} - \frac{\pi k}{2f}[x_c - \underline{x}(I^{SC})], & if\ \widehat{x}(I^{SC}) - f \leq x_c \leq \widehat{x}(I^{SC}) + f \\ \widehat{x}(I) - I,\ where\ I = [\frac{\alpha}{\beta}(x_c + f)]^{\frac{1}{\alpha}}, & if\ x_c \leq \widehat{x} - f \\ \widehat{x}(I) - I - \pi k,\ where\ I = [\frac{\alpha}{\beta}(x_c - f)]^{1/\alpha}, & if\ x_c \geq \widehat{x} + f \end{array} \right\}. \tag{A23}$$

■

Proof of Proposition 5.

An entrepreneur's expected net payoff under truncation, recall, is:

$$\begin{aligned} V^{SC}(I; x_c(I)) \equiv\ & \pi\left[\int_{\underline{x}(I)}^{x_c(I)} [x - k]\widehat{h}(x)dx + \int_{x_c(I)}^{\overline{x}(I)} P(I)\widehat{h}(x)dx - I\right] \\ & +[1-\pi]\left[\int_{\underline{x}(I)}^{x_c(I)} x\widehat{h}(x)dx + \int_{x_c(I)}^{P(I)} P(I)\widehat{h}(x)dx + \int_{P(I)}^{\overline{x}(I)} x\widehat{h}(x)dx - I\right]. \end{aligned}$$

This expression can be simplified as:

$$V^{SC}(I; x_c(I)) = \widehat{x}(I) - I - \frac{\pi k[x_c(I) - \underline{x}(I)]}{2f}.$$

Differentiating provides:

$$\frac{\partial V^{SC}(I; x_c(I))}{\partial I} = \widehat{x}'(I) - 1 - \frac{\pi k[z - \widehat{x}'(I)]}{2f} = \widehat{x}'\left[1 + \frac{\pi k}{2f}\right] - 1 - \frac{\pi k z}{2f} = 0,$$

which implies $I^{SC} = [\frac{2f+\pi kz}{\beta(2f+\pi k)}]^{\frac{1}{\alpha-1}} = I^{FB}[\frac{2f+\pi k}{2f+\pi kz}]^{\frac{1}{1-\alpha}}$. Hence, $I^{SC} > I^{FB}$, if $z < 1$; $I^{SC} = I^{FB}$, if $z = 1$; and $I^{SC} < I^{FB}$, if $z > 1$. As a side issue, for truncation to obtain in equilibrium we require $\frac{\beta}{\alpha}[I^{SC}]^{\alpha-1} - \frac{f}{I^{SC}} < z < \frac{\beta}{\alpha}[I^{SC}]^{\alpha-1} + \frac{f}{I^{SC}}$. ■

Proof of Proposition 6.

If the regulator sets x_c sufficiently low, no revaluation will be undertaken, and the price of a non-certified asset will be $P^{NC} = \widehat{x}(I^{FB}) - [\frac{1-\sqrt{\pi}}{1+\sqrt{\pi}}]f$. Furthermore, expected welfare will be:

$$W^{NC} = \pi[P^{NC} - I] + w[1-\pi]\left[\int_{\underline{x}}^{P^{NC}} P^{NC}\widehat{h}(x)dx + \int_{P^{NC}}^{\overline{x}} x\widehat{h}(x)dx - I\right]. \tag{A24}$$

Performing the integration in (A24) and substituting for P^{NC} provides:

$$\begin{aligned}
W^{NC} &= \pi[P - I] + \frac{w[1-\pi]}{2f}\left[P[P - \underline{x}] + \frac{1}{2}[\overline{x}^2 - P^2] - I\right] \\
&= \pi\left[\widehat{x} - \left(\frac{1-\sqrt{\pi}}{1+\sqrt{\pi}}\right)f\right] + \frac{w[1-\pi]}{2f}\left\{\frac{1}{2}\left[\widehat{x} - \left(\frac{1-\sqrt{\pi}}{1+\sqrt{\pi}}\right)f\right]^2\right. \\
&\qquad \left. - \left[\widehat{x} - \left(\frac{1-\sqrt{\pi}}{1+\sqrt{\pi}}\right)f\right]\underline{x} + \frac{1}{2}\overline{x}^2\right\} - [\pi + w(1-\pi)]I \\
&= [\pi + w(1-\pi)][\widehat{x} - I] - \pi f\left[\frac{1-\sqrt{\pi}}{1+\sqrt{\pi}}\right] + \frac{w[1-\pi]\pi f}{[1+\sqrt{\pi}]^2}. \qquad (A25)
\end{aligned}$$

If the regulator sets x_c sufficiently high, all distressed entrepreneurs will certify their asset values in equilibrium, and expected surplus will be:

$$W^{FC} = [\hat{x}(I^{FB}) - I^{FB}][\pi + w(1-\pi)] - \pi k. \tag{A26}$$

If the regulator sets $x_c > \underline{x}(I^{FB})$ and thereby induces selective revaluation and/or investment in excess of I^{FB} to avoid revaluation costs, expected welfare given investment I and critical asset value x_c will be:

$$\begin{aligned}
W^{SC}(I, x_c) &= \pi\left[\int_{\underline{x}}^{x_c}[x - k]\widehat{h}(x)dx + \int_{x_c}^{\overline{x}} P\widehat{h}(x)dx - I\right] \\
&\quad + w[1-\pi]\left[\int_{\underline{x}}^{x_c} x\widehat{h}(x)dx + \int_{x_c}^{P} P\widehat{h}(x)dx + \int_{P}^{\overline{x}} x\widehat{h}(x)dx - I\right]. \qquad (A27)
\end{aligned}$$

Performing the integration in (A27) and rearranging provides:

$$\begin{aligned}
W^{SC}(I, x_c) = {} & \frac{\pi}{2f}\left[\frac{1}{2}(x_c^2 - \underline{x}^2) - k(x_c - \underline{x}) + P(\overline{x} - x_c)\right] \\
& + \frac{w(1-\pi)}{2f}\left[\frac{1}{2}(x_c^2 - \underline{x}^2) + P(P - x_c) + \frac{1}{2}(\overline{x}^2 - P^2)\right] - [\pi + w(1-\pi)]I \\
= {} & \frac{\pi}{2f}\left[\frac{1}{2}(x_c^2 - \underline{x}^2) - k(x_c - \underline{x}) + P(\overline{x} - x_c)\right] \\
+ {} & \frac{w(1-\pi)}{2f}\left[\frac{1}{2}(x_c^2 - \underline{x}^2) - Px_c + \frac{1}{2}(\overline{x}^2 + P^2)\right] - [\pi + w(1-\pi)]I.
\end{aligned} \tag{A28}$$

From the second line of (A23), when the critical asset value $x_c > \underline{x}(I^{FB})$ induces precisely the minimum investment level ($I^B > I^{FB}$) required to avoid revaluation, the entrepreneur's expected profit is:

$$V^{SC}(x_c) = x_c + f - I^B, \; where \; I^B = [(\alpha/\beta)(x_c + f)]^{1/\alpha}. \tag{A29}$$

From the first line of (A23), when the critical asset value x_c induces selective revaluation, the entrepreneur's expected profit is:

$$\begin{aligned}
V^{SC}(x_c) &= \widehat{x}(I^{SC}) - I^{SC} - \frac{\pi k[x_c - \underline{x}(I^{SC})]}{2f}, \; where \\
I^{SC} &= \left[\frac{\beta[2f + \pi k]}{2f}\right]^{\frac{1}{1-\alpha}} \;\; > \; I^{FB}.
\end{aligned} \tag{A30}$$

The entrepreneur will prefer investment I^B to investment I^{SC} if and only if:

$$\begin{aligned}
x_c + f - I^B \geq \widehat{x}(I^{SC}) - I^{SC} - \left[\frac{\pi k[x_c - \underline{x}(I^{SC})]}{2f}\right] & \\
\Leftrightarrow \;\; f - \widehat{x}(I^{SC}) + I^{SC} - \frac{\pi k \underline{x}(I^{SC})}{2f} \geq I^B - x_c - \frac{\pi k x_c}{2f} & \\
\Leftrightarrow \;\; I^{SC} - \left[1 + \frac{\pi k}{2f}\right]\underline{x}(I^{SC}) \geq I^B - \left[1 + \frac{\pi k}{2f}\right]x_c & \\
\Leftrightarrow \; I^{SC} - I^B \geq \left[1 + \frac{\pi k}{2f}\right][\underline{x}(I^{SC}) - x_c]. &
\end{aligned} \tag{A31}$$

(A31) holds as an equality if and only if $x_c = \frac{\beta}{\alpha}\left[\frac{\beta(2f+\pi k)}{2f}\right]^{\frac{\alpha}{1-\alpha}} - f$. Rearranging the terms in (A31) gives:

$$I^{SC} - \left[1 + \frac{\pi k}{2f}\right]\underline{x}(I^{SC}) \geq I^B - \left[1 + \frac{\pi k}{2f}\right]x_c. \tag{A32}$$

The left hand side of (A32) is independent of x_c and the right hand side of (A32) is increasing in

x_c. Therefore, the inequality in (A31) holds if and only if:

$$x_c \leq \frac{\beta}{\alpha}\left[\frac{\beta[2f+\pi k]}{2f}\right]^{\frac{\alpha}{1-\alpha}} - f. \tag{A33}$$

It follows that when x_c induces either selective revaluation or precisely the minimum investment level (I^B) that avoids revaluation, the entrepreneur's investment increases with x_c for $x_c \leq \frac{\beta}{\alpha}[\frac{\beta(2f+\pi k)}{2f}]^{\frac{\alpha}{1-\alpha}} - f$ and otherwise does not vary with x_c. These two regions are now considered in turn in demonstrating that when $k \in (0, \frac{2f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}]$ and either selective revaluation or investment I^B is induced, the welfare-maximizing regulator will set revaluation level $x_c = \overline{x} - \frac{[1+\sqrt{\pi}]k}{[1-\sqrt{\pi}][1-w]}$.

In the region where investment increases with x_c, substituting $\underline{x} = x_c$, $\overline{x} = x_c + 2f$ and $P = x_c + \frac{\sqrt{\pi}[\overline{x}-x_c]}{1+\sqrt{\pi}}$ into (A28) provides:

$$\begin{aligned} W^{SC}(I, x_c) = {} & \pi\left[x_c + \frac{2\sqrt{\pi}f}{1+\sqrt{\pi}}\right] + \frac{w[1-\pi]}{2f}\left\{-x_c\left[x_c + \frac{2\sqrt{\pi}f}{1+\sqrt{\pi}}\right]\right. \\ & \left. + \frac{1}{2}[x_c+2f]^2 + \frac{1}{2}\left[x_c + \frac{2\sqrt{\pi}f}{1+\sqrt{\pi}}\right]^2\right\} - [\pi + w(1-\pi)]I. \end{aligned} \tag{A34}$$

Differentiating (A34) with respect to x_c provides:

$$\begin{aligned} \frac{\partial W^{SC}(I, x_c)}{\partial x_c} & = \pi + \frac{w\,[1-\pi]}{2f}\left[-(x_c + \frac{2\sqrt{\pi}f}{1+\sqrt{\pi}}) - x_c + x_c + 2f + x_c + \frac{2\sqrt{\pi}f}{1+\sqrt{\pi}}\right] \\ & = \pi + w[1-\pi] > 0. \end{aligned} \tag{A35}$$

(A35) implies for any x_c that induces I^B, expected welfare would increase if x_c were raised. Therefore, $x_c > \underline{x}(I^B)$.

Now consider the region in which the investment does not vary with x_c, where $I^{SC} = \left[\frac{\beta[2f+\pi k]}{2f}\right]^{\frac{1}{1-\alpha}} > I^{FB}$ from (A22). Since $P = \frac{x_c+\sqrt{\pi}\overline{x}}{1+\sqrt{\pi}}$,

$$\frac{dP}{dx_c} = \frac{1}{1+\sqrt{\pi}}. \tag{A36}$$

Differentiating (A28), using (A36), provides:

$$\begin{aligned}\frac{\partial W^{SC}(I,x_c)}{\partial x_c} &= \frac{\pi}{2f}\left[x_c - k + \left[\frac{\overline{x}-x_c}{1+\sqrt{\pi}}\right] - \frac{x_c+\sqrt{\pi}\overline{x}}{1+\sqrt{\pi}}\right] \\ &+ \frac{w(1-\pi)}{2f}\left[x_c - \left[\frac{x_c+\sqrt{\pi}\overline{x}}{1+\sqrt{\pi}}\right] - \frac{x_c}{1+\sqrt{\pi}} + \frac{x_c+\sqrt{\pi}\overline{x}}{(1+\sqrt{\pi})^2}\right] \\ &= \frac{\pi}{2f}\left[-k + \frac{[1-w]\,[1-\sqrt{\pi}]\,[\overline{x}-x_c]}{1+\sqrt{\pi}}\right]. \end{aligned} \tag{A37}$$

The welfare-maximizing revaluation requirement under selective revaluation is identified by $\frac{\partial W^{SC}(I,x_c)}{\partial x_c} = 0$. Therefore, from (A37), the welfare-maximizing revaluation requirement that induces selective revaluation is:

$$x_c = \overline{x}(I^{SC}) - \frac{[1+\sqrt{\pi}]k}{[1-\sqrt{\pi}][1-w]}\ . \tag{A38}$$

From (A38), $x_c \geq \underline{x}(I^{SC})$ if and only if $k \leq \frac{2f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}$, since

$$x_c \geq \underline{x}(I^{SC}) \ \Leftrightarrow\ \frac{[1+\sqrt{\pi}]k}{[1-\sqrt{\pi}][1-w]} \leq 2f \ \Leftrightarrow\ k \leq \frac{2f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}. \tag{A39}$$

We now derive the maximum level of expected welfare that can be achieved by inducing selective revaluation and investment I^{SC} when $k \in (0, \frac{2f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}]$. To do so, first substitute (A38) into $P = \frac{x_c+\sqrt{\pi}\overline{x}}{1+\sqrt{\pi}}$ to obtain:

$$P = \frac{\overline{x} - \frac{[1+\sqrt{\pi}]k}{[1-\sqrt{\pi}][1-w]} + \sqrt{\pi}\overline{x}}{1+\sqrt{\pi}} \ = \ \overline{x} - \frac{k}{[1-\sqrt{\pi}][1-w]}. \tag{A40}$$

Define $S \equiv \frac{k}{[1-\sqrt{\pi}][1-w]}$. Substituting $x_c = \overline{x} - [1+\sqrt{\pi}]S$ and $P = \overline{x} - S$ into (A28) provides:

$$\begin{aligned} W^{SC}(I^{SC},x_c) &= \frac{\pi}{2f}\left\{\frac{1}{2}[\overline{x}-(1+\sqrt{\pi})S]^2 - \frac{1}{2}\underline{x}^2 - k[\overline{x}-(1+\sqrt{\pi})S-\underline{x}]\right. \\ &\left. +[\overline{x}-S][1+\sqrt{\pi}]S\right\} + \frac{w[1-\pi]}{2f}\left\{\frac{1}{2}[\overline{x}-(1+\sqrt{\pi})S]^2 - \frac{1}{2}\underline{x}^2\right. \\ &\left. -[\overline{x}-S][\overline{x}-(1+\sqrt{\pi})S] + \frac{1}{2}\overline{x}^2 + \frac{1}{2}[\overline{x}-S]^2\right\} - [\pi + w(1-\pi)]I \\ &= \frac{\pi}{2f}\left[2f\widehat{x} - 2fk + \frac{1}{2}[\pi-1]S^2 + [1+\sqrt{\pi}]kS\right] \\ &+ \frac{w[1-\pi]}{2f}\left[2f\widehat{x} + \frac{1}{2}\pi S^2\right] - [\pi + w(1-\pi)]I. \end{aligned} \tag{A41}$$

Substituting $S \equiv \frac{k}{[1-\sqrt{\pi}][1-w]}$ into (A41) provides,

$$
\begin{aligned}
W^{SC}(I^{SC}, x_c) &= \frac{\pi}{2f}\left[2f\widehat{x} - 2fk + \frac{1}{2}[\pi-1]\frac{k^2}{[1-\sqrt{\pi}]^2[1-w]^2} + [1+\sqrt{\pi}]k\frac{k}{[1-\sqrt{\pi}][1-w]}\right] \\
&\quad + \frac{w[1-\pi]}{2f}\left[2f\widehat{x} + \frac{1}{2}\pi\frac{k^2}{[1-\sqrt{\pi}]^2[1-w]^2}\right] - [\pi + w(1-\pi)]I
\end{aligned}
$$

$$
= [\pi + w(1-\pi)][\widehat{x} - I^{SC}] + \frac{\pi}{2f}\left[-2fk + \frac{[1+\sqrt{\pi}]k^2}{2[1-\sqrt{\pi}][1-w]}\right]. \tag{A42}
$$

(A42) is the maximum level of expected welfare that can be achieved by inducing selective revaluation and investment I^{SC} when $k \in (0, \frac{2f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}]$.

(A25) and (A26) imply no revaluation ensures a higher level of expected welfare than full revaluation (i.e., $W^{NC} > W^{FC}$) if and only if $k > \frac{f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}$. Notice $\frac{f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}} < \frac{2f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}$. Consequently, when $k > \frac{2f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}$, expected welfare is maximized when no revaluation requirement is imposed.

It remains to verify that expected welfare is higher when selective revaluation is induced than when no revaluation is induced. From (A25) and (A41), this will be the case if,

$$
\begin{aligned}
& [\pi + w(1-\pi)][\widehat{x}(I^{SC}) - I^{SC}] + \frac{\pi}{2f}\left[-2fk + \frac{[1+\sqrt{\pi}]k^2}{2[1-\sqrt{\pi}][1-w]}\right] \\
> \;\; & [\pi + w(1-\pi)][\widehat{x}(I^{FB}) - I^{FB}] - \frac{[1-\sqrt{\pi}]\pi f[1-w]}{1+\sqrt{\pi}}.
\end{aligned} \tag{A43}
$$

$$
\begin{aligned}
&\Leftrightarrow \frac{\pi}{2f}\left[-2fk + \frac{[1+\sqrt{\pi}]k^2}{2[1-\sqrt{\pi}][1-w]}\right] + \frac{[1-\sqrt{\pi}]\pi f[1-w]}{1+\sqrt{\pi}} \\
&> [\pi + w(1-\pi)]\{[\widehat{x}(I^{FB}) - I^{FB}] - [\widehat{x}(I^{SC}) - I^{SC}]\}.
\end{aligned} \tag{A44}
$$

Recall from (A17) and (A22),

$$
\widehat{x}(I^{FB}) - I^{FB} = \left[\frac{1}{\alpha} - 1\right] I^{FB}, \text{ and} \tag{A45}
$$

$$
\widehat{x}(I^{SC}) - I^{SC} = \left[\frac{2f}{\alpha\,[2f+\pi k]} - 1\right] I^{SC} = \left[\frac{2f}{\alpha[2f+\pi k]} - 1\right]\left[1 + \frac{\pi k}{2f}\right]^{\frac{1}{1-\alpha}} I^{FB}. \tag{A46}
$$

Using (A45) and (A46), the right hand side of (A44) can be written as:

$$C = [\pi + w(1-\pi)]\{\frac{1}{\alpha} - 1 - \left[\frac{2f}{\alpha[2f+\pi k]} - 1\right]\left[1 + \frac{\pi k}{2f}\right]^{\frac{1}{1-\alpha}}\}\beta^{\frac{1}{1-\alpha}} > 0. \tag{A47}$$

Thus, (A44) can be written as:

$$-\pi k + \frac{[1+\sqrt{\pi}]\pi k^2}{4f[1-\sqrt{\pi}][1-w]} + \frac{[1-\sqrt{\pi}]\pi f[1-w]}{1+\sqrt{\pi}} - C > 0. \tag{A48}$$

(A48) holds if:

$$k < \frac{2f[1-\sqrt{\pi}][1-w]}{[1+\sqrt{\pi}]\pi}\left[\pi - \sqrt{\frac{\pi[1+\sqrt{\pi}]C}{f[1-\sqrt{\pi}][1-w]}}\right] \equiv Z(w)\ . \tag{A49}$$

It is readily verified that $Z(w)$ is 0 at $w = 1$, and is decreasing in w. Therefore, there always exists a critical level of k $(= Z(w))$ such that the regulator will prefer selective revaluation to no revaluation if k is less than this critical value. ■

<u>Proof of Proposition 7</u>.

Paralleling the proof of Proposition 6, but employing $x_c(I) = zI$, tedious algebra reveals the following first order condition for the regulator's choice of z, presuming truncation takes place:

$$\begin{aligned}\frac{dW^{SC}(z)}{dz} \quad = \quad & \frac{[1-\pi][1-w]\pi^2 k^2 I[1-z]}{[2f+\pi k][2f+\pi kz][\alpha-1]} - \frac{\pi k I}{2f} \\ & -\frac{\pi[1-\sqrt{\pi}][1-w][\widehat{x} - zI + f]}{2f(1+\sqrt{\pi})}\{\frac{2f\pi k I[1-z]}{[2f+\pi k][2f+\pi kz][\alpha-1]} - I\} = 0.\end{aligned}$$

From here, the analysis follows in straightforward fashion, with the exception that any k that leads to $\frac{dW^{SC}(z)}{dz}|_{z=1} = 0$ is in fact inconsistent with truncation. ■

<u>Proof of Proposition 10</u>.

If some non-certified assets are sold in equilibrium when no mandatory revaluation requirement is imposed, the probability a non-certified asset is traded at price $P = P(I, I)$ in equilibrium is:

$$q^{vn} \equiv \frac{P - \underline{x}(I^{vn}) + \pi k}{2f}\ , \tag{A50}$$

where I^{vn} is the level of investment induced in the setting with voluntary revaluation when no mandatory revaluation is imposed, i.e.,

$$\pi \int_{\underline{x}}^{P+k} \frac{dx}{2f} + [1-\pi] \int_{\underline{x}}^{P} \frac{dx}{2f} \;=\; \pi \left[\frac{P+k-\underline{x}}{2f} \right] + [1-\pi] \left[\frac{P-\underline{x}}{2f} \right] \;=\; \frac{P-\underline{x}+\pi k}{2f}. \tag{A51}$$

Furthermore, when $k < \frac{2f}{1+\sqrt{\pi}}$, voluntary revaluation will be undertaken with strictly positive probability, and the equilibrium price of non-certified assets will be:

$$P = \underline{x}(I^{vn}) + k\sqrt{\pi}. \tag{A52}$$

This is the case because the equilibrium price of a non-certified asset will be its expected value:

$$EV^N \equiv \frac{1}{q^{vn}} \left[\pi \left[\frac{P+k-\underline{x}}{2f} \right] \left[\frac{P+k+\underline{x}}{2} \right] + [1-\pi] \left[\frac{P-\underline{x}}{2f} \right] \left[\frac{\underline{x}+P}{2} \right] \right]. \tag{A53}$$

Rearranging provides:

$$P = \frac{1}{4fq^{vn}} \left[\pi[P-\underline{x}+k][P+\underline{x}+k] + [1-\pi]\,[P-\underline{x}]\,[P+\underline{x}] \right]. \tag{A54}$$

Solving for P provides:

$$4fq^{vn}P = \pi\,[P-\underline{x}]\,[P+\underline{x}] + \pi k[P-\underline{x}] + \pi k[P+\underline{x}+k] + [1-\pi]\,[P-\underline{x}]\,[P+\underline{x}]\;. \tag{A55}$$

Substituting from (A52) and simplifying (A55) provides:

$$\begin{aligned} 2P[P-\underline{x}+\pi k] &= [P-\underline{x}]\,[P+\underline{x}] + \pi k[2P+k] \\ &\Leftrightarrow 2P^2 - 2P\underline{x} + 2P\pi k = P^2 - \underline{x}^2 + 2P\pi k + \pi k^2 \\ &\Leftrightarrow\; P^2 - 2P\underline{x} + \underline{x}^2 - \pi k^2 = 0. \end{aligned} \tag{A56}$$

Solving (A56) for P provides:

$$P = \frac{1}{2}\left[2\underline{x} \pm \sqrt{4\underline{x}^2 - 4[\underline{x}^2 - \pi k^2]} \right] = \underline{x} \pm k\sqrt{\pi}\;. \tag{A57}$$

Since P must exceed $\underline{x}$, (A57) implies $P = \underline{x} + k\sqrt{\pi}$. Moreover, voluntary revaluation occurs in equilibrium when:

$$x_u \equiv P + k \;=\; \underline{x} + k[1+\sqrt{\pi}] < \overline{x} \quad \Leftrightarrow \; k < \frac{2f}{1+\sqrt{\pi}} \;. \tag{A58}$$

When $k \in (0, \frac{2f}{1+\sqrt{\pi}})$ in this setting, the expected net return from investment I is:

$$V^{vn}(I, x_c) = \pi E[v^{Dn}] + (1-\pi)E[v^{Nn}], \tag{A59}$$

where:

$$E[v^{Dn}] = \frac{1}{2f}\left[\int_{\underline{x}}^{x_u} P dx + \int_{x_u}^{\overline{x}} (x-k)dx\right] - I \;; \tag{A60}$$

$$E[v^{Nn}] = \frac{1}{2f}\left[\int_{\underline{x}}^{P} P dx + \int_{P}^{\overline{x}} x dx\right] - I \;; \text{ and} \tag{A61}$$

$$P = \underline{x} + k\sqrt{\pi}, \text{ and } x_u = P + k = \underline{x} + k[1+\sqrt{\pi}]. \tag{A62}$$

Simplifying (A60), using (A62), provides:

$$\begin{aligned} E[v^{Dn}] =& \frac{1}{2f}\left[P[x_u - \underline{x}] + \frac{1}{2}[\overline{x}^2 - x_u^2] - k[\overline{x} - x_u]\right] - I \\ =& \frac{1}{2f}\left[2f\widehat{x} - 2fk + \frac{1}{2}[1+\sqrt{\pi}]^2 k^2\right] - I = \widehat{x} - I + \frac{1}{4f}[1+\sqrt{\pi}]^2 k^2 - k. \end{aligned} \tag{A63}$$

Similarly, simplifying (A61), using (A62), provides:

$$\begin{aligned} E[v^{Nn}] =& \frac{1}{2f}\left[P[P - \underline{x}] + \frac{1}{2}[\overline{x}^2 - P^2]\right] - I \\ =& \frac{1}{2f}\left[\frac{1}{2}[\overline{x}^2 - \underline{x}^2] + \frac{1}{2}\pi k^2\right] - I = \frac{1}{2f}\left[2f\widehat{x} + \frac{1}{2}\pi k^2\right] - I = \widehat{x} - I + \frac{\pi k^2}{4f}. \end{aligned} \tag{A64}$$

Substituting (A63) and (A64) into (A59) provides:

$$\begin{aligned} V^{vn}(I, x_c) =& \pi\left[\widehat{x} - I + \frac{1}{4f}(1+\sqrt{\pi})^2 k^2 - k\right] + [1-\pi]\left[\widehat{x} - I + \frac{\pi k^2}{4f}\right] \\ =& \widehat{x} - I - \frac{\pi k}{2f}\left[2f - k[1+\sqrt{\pi}]\;\right]. \end{aligned} \tag{A65}$$

Differentiating (A65) with respect to I provides $\frac{\partial V^v(I,x_c)}{\partial I} = \hat{x}'(I) - 1$, which implies the induced investment level is first best. Furthermore, $2f - k\,[1+\sqrt{\pi}] > 0$ since $k < \frac{2f}{1+\sqrt{\pi}}$. Therefore, (A65) implies $V^{vn}(I^{FB}, x_c) < \hat{x}(I^{FB}) - I^{FB}$, and so the entrepreneur's expected profit is reduced by the possibility of voluntary revaluation. ■

Proof of Proposition 11.

When selective revaluation is induced in the setting with voluntary revaluation, the probability a non-certified asset is traded at price $P = P(I, I)$ is:

$$\pi\left[\frac{P+k-x_c}{2f}\right] + [1-\pi]\left[\frac{P-x_c}{2f}\right] \;=\frac{P-x_c+\pi k}{2f} \equiv q^v. \tag{A66}$$

Furthermore, the expected value of a non-certified asset will be:

$$EV \equiv \frac{1}{q^v}\left[\pi[\frac{P+k-x_c}{2f}][\frac{P+k+x_c}{2}] + [1-\pi][\frac{P-x_c}{2f}][\frac{x_c+P}{2}]\right]\;. \tag{A67}$$

Equating EV and P provides:

$$P = \frac{1}{4fq^v}\left[\pi[P-x_c+k][P+x_c+k] + [1-\pi]\,[P-x_c]\,[P+x_c]\right], \tag{A68}$$

or

$$4fq^vP = \pi\,[P-x_c]\,[P+x_c] + \pi k[P-x_c] + \pi k[P+x_c+k] + [1-\pi]\,[P-x_c]\,[P+x_c]\;. \tag{A69}$$

Substituting from (A66) and simplifying (A69) provides:

$$\begin{aligned} 2P[P-x_c+\pi k] &= [P-x_c]\,[P+x_c] + \pi k[2P+k] \\ &\Leftrightarrow\; 2P^2 - 2Px_c + 2P\pi k = P^2 - x_c^2 + 2P\pi k + \pi k^2 \\ &\Leftrightarrow\; P^2 - 2Px_c + x_c^2 - \pi k^2 = 0. \end{aligned} \tag{A70}$$

Solving (A70) for P provides:

$$P = \frac{1}{2}\left[2x_c \pm \sqrt{4x_c^2 - 4[x_c^2 - \pi k^2]}\right] = x_c \pm k\sqrt{\pi}. \tag{A71}$$

Since P must exceed x_c, (A71) implies $P = x_c + k\sqrt{\pi}$.

The expected net return from investment I in this setting is:

$$V^v(I, x_c) = \pi E[v^d] + [1-\pi]E[v^n]. \tag{A72}$$

where:

$$E[v^d] = \frac{1}{2f}\left[\int_{\underline{x}}^{x_c}[x-k]dx + \int_{x_c}^{x_u} P dx + \int_{x_u}^{\overline{x}}(x-k)dx\right] - I \ ; \tag{A73}$$

$$E[v^n] = \frac{1}{2f}\left[\int_{\underline{x}}^{x_c} x dx + \int_{x_c}^{P} P dx + \int_{P}^{\overline{x}} x dx\right] - I \ ; \tag{A74}$$

$$P = x_c + k\sqrt{\pi}, \text{ and } x_u = P + k = x_c + k[1+\sqrt{\pi}]. \tag{A75}$$

Simplifying (A73), using (A75), provides:

$$\begin{aligned} E[v^d] &= \frac{1}{2f}\left[\frac{1}{2}[x_c^2 - \underline{x}^2] - k[x_c - \underline{x}] + P[x_u - x_c] + \frac{1}{2}[\overline{x}^2 - x_u^2] - k[\overline{x} - x_u]\right] - I \\ &= \frac{1}{2f}\left[2f\widehat{x} - 2fk + \frac{1}{2}[1+\sqrt{\pi}]^2 k^2\right] - I = \widehat{x} - I + \frac{1}{4f}\left[1+\sqrt{\pi}\right]^2 k^2 - k. \end{aligned} \tag{A76}$$

Similarly, simplifying (A74), using (A75), provides:

$$\begin{aligned} E[v^n] &= \frac{1}{2f}\left[\frac{1}{2}[x_c^2 - \underline{x}^2] + P[P - x_c] + \frac{1}{2}[\overline{x}^2 - P^2]\right] - I \\ &= \frac{1}{2f}\left[\frac{1}{2}[\overline{x}^2 - \underline{x}^2] + \frac{1}{2}\pi k^2\right] - I = \frac{1}{2f}\left[2f\widehat{x} + \frac{1}{2}\pi k^2\right] - I = \widehat{x} - I + \frac{\pi k^2}{4f} \ . \end{aligned} \tag{A77}$$

For a given level of I, neither (A76) nor (A77) is the function of x_c. Substituting (A76) and (A77) into (A72) provides:

$$\begin{aligned} V^v(I, x_c) &= \pi\left[\widehat{x} - I + \frac{1}{4f}[1+\sqrt{\pi}]^2 k^2 - k\right] + [1-\pi][\widehat{x} - I + \frac{\pi k^2}{4f}] \\ &= \widehat{x} - I - \frac{\pi k}{2f}\left[2f - k(1+\sqrt{\pi})\right] \ . \end{aligned} \tag{A78}$$

Differentiating (A78) with respect to I provides $\frac{\partial V^v(I,x_c)}{\partial I} = \widehat{x}'(I) - 1$, which implies the induced investment level is first best. ■

<u>Proof of Lemma 2</u>.

Given the off equilibrium beliefs, the price anticipated by the entrepreneur at the time of investment is

$$P(v,I)=\begin{cases} v \text{ if } v\neq I \\ \hat{x}_L\left(I\right)-\frac{f(1-\sqrt{\pi})}{1+\sqrt{\pi}} \text{ if } v=I \text{ and } I<I_H^{FB} \\ \hat{x}_H\left(I\right)-\frac{f(1-\sqrt{\pi})}{1+\sqrt{\pi}}\text{if } v=I \text{ and } I\geq I_H^{FB} \end{cases}$$

First consider the high type. Absent any revaluation, and facing the noted price structure, his expected net payoff from investing I reduces to

$$V^H(I)) \;\equiv\; \pi P(I,I)+[1-\pi]\{\int\limits_{P(I,I)-\hat{x}_H(I)\geq\mu} P(I,I)h(\mu)d\mu+\int\limits_{\mu\geq P(I,I)-\hat{x}_H(I)}[\hat{x}_H\left(I\right)+\mu]h(\mu)d\mu\}-I$$

Using the uniform density, this simplifies to

$$V^H(I)) \;\equiv\; \pi P(I,I)+\frac{1-\pi}{2f}[P(I,I)(P(I,I)-\hat{x}_H\left(I\right)+f)]+\frac{1-\pi}{4f}[(\hat{x}_H\left(I\right)+f)^2-P(I,I)^2]-I$$

For $I\geq I_H^{FB}$, this further reduces to $\hat{x}_H\left(I\right)-I$, which implies choice of I_H^{FB} given$I\geq I_H^{FB}$. Conversely, for $I<I_H^{FB}$ we have the fact (verified by differentiation) that $V^H(I))$ is increasing in $P(I,I)$, but in this region the price is lower ($\hat{x}_L\left(I\right)-\frac{f(1-\sqrt{\pi})}{1+\sqrt{\pi}}<\hat{x}_H\left(I\right)-\frac{f(1-\sqrt{\pi})}{1+\sqrt{\pi}}$). Thus, any choice of I in this region is inferior to what it would be with a price of $\hat{x}_H\left(I\right)-\frac{f(1-\sqrt{\pi})}{1+\sqrt{\pi}}$ and with this higher price the entrepreneur would still choose I_H^{FB}.

Next consider the low type. Paralleling the above, his expected net payoff from investing I reduces to

$$V^L(I)) \;\equiv\; \pi P(I,I)+\frac{1-\pi}{2f}[P(I,I)(P(I,I)-\hat{x}_L\left(I\right)+f)]+\frac{1-\pi}{4f}[(\hat{x}_L\left(I\right)+f)^2-P(I,I)^2]-I$$

For $I<I_H^{FB}$ this reduces to $V^L(I)) =\hat{x}_L\left(I\right)-I$, and thus results in choice of $I=I_L^{FB}<I_H^{FB}$. Conversely, in the region where $I\geq I_H^{FB}$ we have (with $P+P(I,I)$)

$$\begin{aligned}\frac{dV^L(I))}{dI} \;&=\; \frac{1-\pi}{2f}[\hat{x}_L\left(I\right)+f-P]\frac{d\hat{x}_L\left(I\right)}{dI}+[\pi+\frac{1-\pi}{2f}(P-\hat{x}_L\left(I\right)+f)]\frac{dP}{dI}-1= \\ &\frac{d\hat{x}_H\left(I\right)}{dI}-1+\frac{1-\pi}{2f}[\hat{x}_L\left(I\right)+f-P][\frac{d\hat{x}_L\left(I\right)}{dI}-\frac{d\hat{x}_H\left(I\right)}{dI}]\end{aligned}$$

and this is negative as long as $\hat{x}_L\left(I\right)+f>P(I,I)=\hat{x}_H\left(I\right)-\frac{f(1-\sqrt{\pi})}{1+\sqrt{\pi}}$, or $\hat{x}_H\left(I\right)-\hat{x}_L\left(I\right)<$

$\frac{2f}{1+\sqrt{\pi}}$. Thus, with this latter condition, the low type will select between separating, via $I = I_L^{FB} < I_H^{FB}$ and mimicing, via $I = I_H^{FB}$. Let $P^H = \hat{x}_H\left(I_H^{FB}\right) - \frac{f(1-\sqrt{\pi})}{1+\sqrt{\pi}}$ denote the equilibrium price $P(I_H^{FB}, I_H^{FB})$. The low type's expected payoff from mimicing the high type is

$$V^m = \pi P^H + \frac{1-\pi}{2f} P^H[P^H - \hat{x}_L\left(I_H^{FB}\right) + f)] + \frac{1-\pi}{4f}[(\hat{x}_L\left(I_H^{FB}\right) + f)^2 - P^{H2}] - I_H^{FB}$$

If $\hat{x}_L\left(I_L^{FB}\right) - I_L^{FB} \geq V^m$, we have a separating equilibrium. Otherwise, mimic dominates separation.

Mimic dominates separation if, say, π is arbitrarily close to unity, implying $V^m \approx P^H - I_H^{FB} \approx \hat{x}_H\left(I_H^{FB}\right) - I_H^{FB} > \hat{x}_L\left(I_L^{FB}\right) - I_L^{FB}$. Similarly, separation dominates mimic if, say, π is arbitrarily close to zero, $\alpha = .5$, $f \approx 2\beta_H^2$, and $\beta_L = \varepsilon$. This implies $P^H \approx (\hat{x}_H\left(I_H^{FB}\right) - f$ and $V^m \approx \frac{1}{2f}P^H[P^H - f)] + \frac{1}{4f}[f^2 - P^{H2}] - I_H^{FB} = -\beta_H^2/2 < \hat{x}_L\left(I_L^{FB}\right) - I_L^{FB} \approx 0$.

Finally, suppose $V^M > \hat{x}_L\left(I_L^{FB}\right) - I_L^{FB}$. Further suppose the revaluation point is set at $x_c(I_L^{FB}) << 0$, $x_c(I_H^{FB}) = \hat{x}_H\left(I_H^{FB}\right) - f$, and arbitrarily high otherwise (to avoid off equilibrium issues). (Notice this requires use of the targeted policy.) If the low type does not mimic, revaluation never occurs. But if he elects to mimic, he incurs revaluation with probability $[\hat{x}_H\left(I_H^{FB}\right) - \hat{x}_L\left(I_H^{FB}\right)]/2f$, and if k is sufficiently large the expected revaluation cost will ensure $V^m < \hat{x}_L\left(I_L^{FB}\right) - I_L^{FB}$. ■

Proof of Proposition 12.

Continuing with the notation in the proof of Proposition 11, now suppose $V^M > \hat{x}_L\left(I_L^{FB}\right) - I_L^{FB}$. Further suppose the revaluation point is set at $x_c(I_L^{FB}) << 0$, $x_c(I_H^{FB}) = \hat{x}_H\left(I_H^{FB}\right) - f$, and arbitrarily high otherwise (to avoid off equilibrium issues). (Notice this requires use of the targeted policy.) If the low type does not mimic, revaluation never occurs. But if he elects to mimic, he incurs revaluation with probability $[\hat{x}_H\left(I_H^{FB}\right) - \hat{x}_L\left(I_H^{FB}\right)]/2f$, and if k is sufficiently large the expected revaluation cost will ensure $V^m < \hat{x}_L\left(I_L^{FB}\right) - I_L^{FB}$. ■