Contracts with Framing

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Abstract

We study a model of contracts in which a profit-maximizing seller uses framing to influence buyers’ purchasing behavior. Framing temporarily affects how buyers evaluate different products, and buyers can renege on their purchases after the framing effect wears off. We characterize the optimal contracts with framing and their welfare properties in several settings. Framing that is not too strong reduces total welfare in regulated markets with homogenous buyers, but increases total welfare in markets with heterogenous buyers when the proportion of buyers with low willingness to pay is small. 

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1 Introduction

Sellers commonly use framing to influence buyers’ purchasing behavior. When presenting a product menu to buyers, for example, sellers often visually highlight a particular product by placing it in a prominent position, by coloring it differently from other products, or by other means.1 Sellers also tend to present information about products in a way that buyers find desirable, such as indicating the percentage of a dairy product that is “fat-free” rather than the actual fat content of the product.2 More subtle cues like the type of background music played in a store are also used to influence buyers’ behavior. For example, classical music has been shown to trigger buyers to purchase higher quality items.3 In all these cases, framing seems to influence buyers’ behavior by increasing the attractiveness of some product or product attribute.

This increased attractiveness is likely temporary. According to leading theories of cognition, the effect of various inputs on decision making depends on their relevance and frequency. Inputs that are less relevant or less frequent decay faster than more relevant or more frequent ones.4 Frames are payoff-irrelevant inputs that buyers encounter only at the point of sale, so their effect likely decays faster than that of more relevant inputs that enter decision making frequently. For example, the effect of music played in a store likely decays quickly after the buyer exits the store, because the buyer is no longer exposed to the music. The music is also less relevant than other inputs such as the purchased product’s characteristics.

After the framing effect wears off, the buyer may wish to, and often can, renege on his purchase. Many retailers in the US allow buyers to return products for a full refund within a certain time period, and require their franchisees to adopt the same policy. For example, Sam’s Club offers its members a “100% Membership Satisfaction Guarantee,” according to which the membership can be canceled at any time for a full refund. Incidentally, Sam’s Club membership brochure (see Appendix A) uses framing that highlights its premium “sam’s plus” membership.

There are also cases in which the law mandates return policies in order to protect buyers. In the European Union, for example, online and purchases other than in shops can be returned

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1 For an illustration, see the brochures in Appendix A.

2 This is similar to the positive versus negative framing of information in Tversky and Kahneman’s (1981) Asian disease problem.

3 The effect of background music on purchasing behavior has been studied extensively in the marketing literature. Some examples include Areni and Kim (1993), who showed that classical music led to more expensive wine purchases relative to top-40 music, North and Hargreaves (1998), who showed that classical music increased students’ purchase intentions in a cafeteria by approximately 20 percent, and North, Shilcock, and Hargreaves (2003), who showed that spending in a restaurant increased in the presence of classical music relative to pop music or no music.

4 One class of leading theories, developed by Anderson and colleagues (see e.g. Anderson (1993)), is the class of Adaptive Character of Thought (ACT) theories. A simplistic description of ACT is that the momentary activation level of a particular memory “chunk” is the sum of the base-level general usefulness of the “chunk” and the weighted average of inputs, which decay over time. Thus, the effect of a particular input declines if it is not repeated and as other inputs appear.
for any reason within a “cooling off” period of 14 days. Similarly, the US Department of Transportation requires airlines to allow buyers to renege on their flight ticket purchase within 24 hours.

In all these environments, the ability of buyers to return products makes the temporary nature of framing economically relevant because it naturally limits the ability of sellers to use framing to increase profit.

This paper studies the optimal design of product menus with frames in such environments. Our two main postulates are that (1) frames temporarily increase the attractiveness of some product or product attribute, and (2) after the framing effect wears off, buyers who overpaid for the product return it to the seller.

The seller in our model chooses a product menu and a frame in order to maximize profit. The different ways in which the seller can influence buyers’ behavior at the point of sale are captured by a collection of functions \( \{U_f\} \). Each function \( U_f \) describes how buyers evaluate products in the frame \( f \). Buyers’ preferences are captured by an additional function \( U \) that reflects how they evaluate products absent framing, that is, \( U \) reflects their true willingness to pay.

When making a choice among bundles in the frame \( f \) (a bundle is a product and its price), the buyer maximizes \( U_f \). That is, the frame influences the buyer at the point of sale. The buyer keeps his chosen product if doing so is \( U \)-superior to not buying anything, and otherwise returns it. That is, the framing effect is temporary, and after it wears off the buyer returns the purchased product if he overpaid for it. Thus, from a contracting perspective, frames relax buyers’ incentive compatibility constraint but not their participation constraint.

There is, of course, more than one way to model what the buyer does after returning a product. One possibility is that the buyer walks away without making another purchase. This may be the case when it is not clear to the buyer why he overpaid and he does not want to overpay again, or when the buyer believes the product he just returned is the best among the available ones and so there is no point in making another purchase. Another possibility, which reflects more sophistication on the part of the buyer, is that after returning the product the buyer makes another purchase according to his \( U \) preferences. This may be the case when the buyer internalizes ex-post how the frame affected his behavior, and is able to resist similar framing effects from that point on. There is also an intermediate possibility, in which the buyer makes another purchase according to the same choice procedure, ignoring the product he just returned.

The optimal contract in many of the settings we consider is robust to the exact post-return specification, because it does not involve returns. We therefore focus on the post-return specification in which the buyer walks away after returning the product. We also discuss settings in which the optimal contract involves returns, and demonstrate in Section 6 how increased sophistication of buyers in their post-return behavior may actually hurt them and increase the seller’s profit.

Welfare in the model is evaluated with respect to buyers’ preferences. This follows the view
that frames are details that are irrelevant to buyers’ intrinsic valuation of goods, and their effect is not persistent.\footnote{See Rubinstein and Salant (2008, 2012) for a detailed discussion of this approach to welfare analysis in the presence of framing, and Benkert and Netzer (2017) for an application of this approach to nudging.}

We analyze the profit and welfare implications of two types of framing. The first is framing that increases the attractiveness of a particular product attribute. Presenting information about products in a way that buyers find desirable and playing background classical music seem to have this effect. The second is framing that highlights a particular product. For example, Sam’s Club highlights its premium “sam’s plus” membership by putting it in a separate box and indicating that it is the “best value.”\footnote{See appendix A for visual illustrations of this and another example of highlighting.}

There are three main takeaways from our analysis. First, in regulated markets with homogeneous buyers, framing can undo the effect of regulations that aim to protect consumers, and can create efficiency distortions (Section 3). Second, in markets with heterogeneous buyers, framing can lead to a decrease in the efficiency distortions created by second-degree price discrimination (Section 4). Third, in markets in which the seller’s cost differs across buyers, e.g. insurance markets, framing can lead to advantageous - rather than adverse - selection (Section 5). We demonstrate the first two takeaways in the context of frames that increase the attractiveness of a particular product attribute and the third in the context of frames that highlight a product.

To demonstrate the first takeaway, we consider a regulated market with homogeneous buyers in which the seller is required to offer buyers a specific basic bundle, in addition to offering them other bundles of his choice. This is often the case in the cable-TV market, where regional cable providers have to offer customers a basic package of channels at a low rate, in addition to other packages of their choice.\footnote{See, for example, http://www.fcc.gov/guides/regulation-cable-tv-rates for cable-TV regulation in the US.} One rationale for this regulation in the absence of framing is that with homogeneous buyers it shifts surplus from the seller to buyers without creating efficiency distortions. This is because by offering an additional bundle, the seller can extract from buyers the entire social surplus from this bundle, up to an amount that makes them \(U\)-indifferent to the basic bundle. The seller therefore offers buyers the socially efficient product at a lower price than without the regulation.

The seller in such a market will choose to use framing that increases attractiveness. This is because such framing enables the seller to charge for the socially efficient product a higher price than without framing, and he can obtain an even higher profit in the optimal contract. More importantly, the optimal use of framing by the seller leads to non-trivial distortions: either the additional product offered by the seller and purchased by buyers is socially inefficient and buyers’ surplus is reduced, or the regulation fails to shift any surplus from the seller to buyers. That is, framing can create efficiency distortions and undo the effect of regulations that aim to increase consumer welfare in monopolistic settings.
To demonstrate the second takeaway, we consider a market with heterogeneous buyers. Framing that increases attractiveness is not necessarily profit-enhancing in this case, even though it relaxes buyers’ incentive compatibility constraint. This is because framing triggers buyers with low willingness to pay to perceive premium products, which are targeted at buyers with high willingness to pay, as more attractive than without framing. Since premium products may be priced above low-type buyers’ willingness to pay for them, framing may cause these buyers to forgo purchasing altogether, leading to an overall decrease in profit. Of course, the seller takes this effect into account when designing the optimal menu, but when framing is “sufficiently strong” his overall profit will nevertheless be lower than in the optimal contract without framing.\(^8\)

When framing is not too strong, it is profit-enhancing in this environment as well. The welfare implications of framing in this case are different for high- and low-type buyers. The product purchased by high-type buyers is always less efficient than in the optimal contract without framing, while the product purchased by low-type buyers is more efficient than in the optimal contract without framing when the proportion of these buyers is not large. Overall, framing increases total welfare when the proportion of low-type buyers is not large. That is, framing has the potential to mitigate the social inefficiencies created by second-degree price discrimination.

To demonstrate the third takeaway, we study an insurance market with heterogeneous buyers, in which the seller highlights a particular product. For example, the insurance provider “Insure My Rental Car” highlights its premium insurance policy by coloring it in a darker color and by adding a “check” mark to it. In the spirit of Tversky and Kahneman’s (1991) model of reference-dependent choice, we postulate that highlighting triggers buyers to anticipate regret if they are involved in an accident and have less coverage than in the highlighted insurance policy.

More specifically, we consider an insurance setting a-la Stiglitz (1977), in which a risk neutral insurance provider offers a menu of insurance bundles to a population of risk-averse buyers, and can choose to highlight one of them. We show that the highlighted bundle optimally coincides with the premium insurance policy targeted at high-risk buyers. This is in line with the real-world phenomenon, including the example mentioned above, that sellers tend to highlight premium bundles. We also show that in the optimal contract low-risk buyers are always partially insured, while high-risk buyers are either over-insured or do not purchase any insurance. Insuring low-risk buyers but not high-risk buyers is impossible in the optimal contract without framing, and is in line with the phenomenon of advantageous selection identified in the empirical literature (See Einav, Finkelstein, and Levin (2010) for a survey).

The paper proceeds as follows. Section 1.1 discusses the related literature. Section 2 introduces the framework. Section 3 analyzes regulated markets with homogeneous buyers. Section 4 studies markets with heterogeneous buyers. Section 5 studies the insurance setting. Section

\(^8\)This is one of the few results that depend on the specification of the buyer’s post-return behavior.
6 concludes. Appendix A contains visual illustrations of framing that highlights a product, and Appendix B contains proofs that do not appear in the main text.

1.1 Related literature

The paper is related to several growing literatures. The first is the literature on individual choice with frames. Our specification of the buyer builds on Salant and Rubinstein (2008). The primitives that describe the buyer in our model correspond to their framework, but our specification of the buyer’s two-stage choice procedure is different. Other two-stage choice procedures were studied in the context of individual choice without framing. See, for example, Manzini and Mariotti (2007). Our approach to welfare analysis is based on Rubinstein and Salant (2008, 2012), who advocate evaluating welfare with respect to preferences rather than frame-dependent behavior. A related application of their approach is Benkert and Netzer (2017), who study the conditions under which a planner can identify from frame-dependent behavior an optimal nudge, i.e., a frame that triggers an individual to choose similarly to his preferences. In contrast to all these papers, we study the effect of frame-dependent behavior on the outcomes of strategic interactions.

In the context of strategic interactions with frame-dependent behavior, Piccione and Spiegler (2012) and Spiegler (2014) study competition between two firms in a complete-information setting in which frames influence consumers’ ability to compare the firms’ actions, such as prices. Firms choose “marketing messages,” in addition to actions, and these messages jointly determine the frame. The frame and the actions determine how the market is split between the firms. We study a different question, namely the optimal design of product menus with frames by a monopolistic seller in a regulated market or a market with incomplete information. Our model of consumer behavior is also different, since framing is not persistent and buyers can renge on their purchases.

Another related literature is the literature on behavioral contract theory (see K˝ oszegi (2014) for an excellent survey), and in particular the literature on screening agents with non-standard preferences. In this literature, the agent has at the outset some private information, either on his degree of inconsistency (see Eliaz and Spiegler (2006), Esteban and Miyagawa (2006), Esteban, Miyagawa, and Shum (2007), and Galperti (2015)), or on some payoff-relevant parameter, such as his willingness to pay (see Esteban and Miyagawa (2006) and Carbajal and Ely (2016)). The focus is on the design of an optimal product menu or menus from which the agent makes choices. In our framework the principal has an additional tool, frames, which he uses to temporarily influence how consumers evaluate different products. Our focus is on the optimal use of profit-enhancing frames, and product menus that complement them, to screen agents with payoff-relevant private information.

There are also papers that study implementation with boundedly-rational agents. De Clippel
(2014) studies implementation with general choice functions. Glazer and Rubinstein (2012) study a persuasion model in which agents are limited in their ability to find arguments that satisfy a set of rules specified by a principal in order to screen agents. We focus on framing as the cause for boundedly-rational behavior, and study the effect of frame-dependent behavior on the design of profit-maximizing contracts.

2 Framework

A profit-maximizing seller offers a contract \((M, f)\) to buyers. The menu \(M\) includes bundles \((x, t)\), where \(x \in [0, d] \subset \mathbb{R}\) is a product and \(t \in \mathbb{R}_+\) is a price. The frame \(f\) belongs to a set \(F\) of feasible frames.

Buyers. Frames affect how buyers evaluate bundles at the point of sale, and buyers can renege on their purchases after the framing effect wears off.

To capture this formally, let \(U(x, t, \theta) = u(x, \theta) − t\) describe the quasi-linear preferences over bundles of a type \(\theta\) buyer, where \(\theta \in \Theta\) is the buyer’s “taste” parameter, and let \(U^f(x, t, \theta) = u^f(x, \theta) − t\) describe how the buyer evaluates bundles in the frame \(f\). The functions \(u\) and \(u^f\) are differentiable and strictly increasing in \(x\). Let \(stayout = (0, 0)\) denote the buyer’s bundle if he does not purchase anything.

A buyer uses a two-stage choice procedure when making purchases. The first stage corresponds to the buyer’s behavior at the point of sale. In this stage, the buyer identifies a \(U^f\)-maximal bundle in the menu \(M\), and buys it if it is \(U^f\)-superior to \(stayout\). The second stage corresponds to the buyer’s behavior after the framing effect wears off. In this stage, the buyer reevaluates his purchase according to his \(U\)-preferences: He keeps the chosen bundle if it is \(U\)-superior to \(stayout\), and otherwise returns it.\(^9\)

This two-stage choice procedure is captured by the choice correspondence \(C^\theta\) that assigns to every contract \((M, f)\) the set \(C^\theta(M, f)\) of possible choices of a type \(\theta\) buyer, which consists of:

1. all the \(U^f\)-maximal bundles in \(M \cup \{stayout\}\) that are weakly \(U\)-superior to \(stayout\), and
2. \(stayout\) if it is weakly \(U\)-superior to some \(U^f\)-maximal bundle in \(M\).

An alternative interpretation of the choice correspondence is of ex-ante anticipation rather than ex-post return. In the ex-ante anticipation interpretation, which may fit situations in which a buyer is involved in similar interactions or communicates with other buyers, the buyer anticipates that he will be unable to resist the framing effect at the point of sale. That is, the buyer anticipates that he will maximize \(U^f\) at the point of sale. Understanding this, he chooses not to interact with the seller if the \(U^f\)-maximal bundle is \(U\)-inferior to not making a purchase.

\(^9\)The buyer does not incur a return cost. We discuss the effect of a return cost below.
Seller. The seller has a convex, differentiable, and strictly increasing cost \( c(x) \) of providing the product \( x \), with \( c(0) = 0 \). His full-information profit maximization problem subject to type \( \theta \) buyers obtaining a \( U \)-utility of \( U(0, 0, \theta) \) is strictly concave in \( x \) and has a unique “first-best” solution \((x_\theta^*, t_\theta^*)\). Note that the product \( x_\theta^* \) is socially efficient in the sense that it maximizes \( u(x, \theta) - c(x) \), the social surplus with respect to type \( \theta \) buyers.

The seller always has the option of offering a frameless contract to buyers by choosing the “null” frame \( \phi \in F \). In the null frame \( U^\phi = U \), so for any frameless contract \((M, \phi)\) the set \( C^\theta(M, \phi) \) is the set of \( U \)-maximal bundles in \( M \cup \{stayout\} \).

Implementation. An allocation rule \( g \) assigns to each \( \theta \in \Theta \) a bundle \( g(\theta) \). A contract \((M, f)\) (partially) implements \( g \) if \( g(\theta) \in C^\theta(M, f) \) for every \( \theta \in \Theta \). We then say that \( g \) is implementable with the frame \( f \). An allocation rule is implementable if it is implementable with some feasible frame. Finally, a contract is profit maximizing (or optimal) if it implements an allocation rule that maximizes the seller’s profit among all implementable allocation rules.

Frames increase attractiveness. Our main assumption on the seller’s framing technology is that frames increase the attractiveness of any increase in the product.\(^{11}\) For example, if products vary in quality, then the frame increases how much buyers are willing to pay for an increase in quality.

**Assumption A1.** Increased attractiveness: For every frame \( f \neq \phi \), every product \( x \), and every type \( \theta \), \( \partial u^f(x, \theta)/\partial x > \partial u(x, \theta)/\partial x \).

Assumption (A1) implies that if a bundle \((x, t)\) is \( U \)-superior to \( stayout \), then it is also \( U^f \)-superior to \( stayout \). This in turn implies that we can simplify the definition of the choice correspondence \( C^\theta \).

**Observation 1** Let \( \hat{C}^\theta \) be the choice correspondence that is obtained by replacing \( M \cup \{stayout\} \) with \( M \) in part (1) of the definition of \( C^\theta \) and leaving the rest of the definition unchanged. Then, \( \hat{C}^\theta = C^\theta \).

We will use the simplified definition of \( C^\theta \) (with \( M \) instead of \( M \cup \{stayout\} \)) in the reminder of the paper.

2.1 Benchmark analysis

We begin by examining a simple benchmark in which buyers are homogeneous, i.e., have the same type \( \theta \). The seller knows buyers’ type and has full discretion over the menu he offers.

\(^{10}\)Section 5 studies the implications of costs that depend on the buyer’s type.

\(^{11}\)We relax this assumption in Section 5.
Framing is not profit-enhancing in this case because it cannot trigger buyers to overpay. Indeed, if buyers choose the bundle \((x, t)\) with framing, then this bundle is weakly \(U\)-superior to not buying anything, so buyers will also choose it without framing if it is the only available bundle. Thus, whether or not the seller uses framing, he will offer buyers the first-best bundle \((x^*_\theta, t^*_\theta)\) and capture the entire surplus in excess of \(U(0, 0, \theta)\).

**Observation 2** Framing is not profit-enhancing in a complete information setting in which the seller has full discretion over the menu.

Observation 2 relies on returns being costless to buyers. When returns are costly, frames that increase attractiveness are profit enhancing. This is because with framing the seller can charge more than \(t^*_\theta\) for the first-best product \(x^*_\theta\), so his profit in the optimal contract is higher than without framing. Note, however, that the seller may not be able to extract the entire return cost from buyers when it is large relative to the increased attractiveness of the frames.\(^{12}\)

We proceed to discuss two other settings in which the conclusion of Observation 2 may fail. Section 3 studies a regulated market with homogeneous buyers in which the seller is required to offer buyers a specific bundle in the menu in addition to other bundles of his choice. Section 4 studies a market with heterogeneous buyers who have private information about their type. In both cases, the seller may optimally offer buyers a menu with more than one bundle, so framing has the potential to increase the seller’s profit.

### Section 3 Regulated market with homogeneous buyers

Monopolistic sellers are sometimes required by a regulator to offer consumers a specific bundle in the menu in addition to other bundles of their choice. This is the case, for example, in the cable-TV market, in which cable providers often have to offer customers a basic package of channels at a low rate in addition to other packages of their choice.\(^ {13}\) This section studies how framing changes the effectiveness of such regulation in markets with homogeneous buyers of the same known type \(\theta\).

Consider a regulation that requires the seller to include in the menu the bundle \((\bar{x}, \bar{t})\), which buyers strictly \(U\)-prefer to \((x^*_\theta, t^*_\theta)\). The product \(\bar{x}\) is basic in the sense that \(\bar{x} < x^*_\theta\). The seller can add other bundles to the menu.

In the absence of framing, this regulation is appealing because it changes the division of surplus between the seller and buyers without reducing social surplus. This is because the seller

\(^{12}\)This is because buyers cannot be convinced to pay \(t^*_\theta + c\) for the first-best product, where \(c\) is the return cost, when framing is not sufficiently strong, and producing above the first-best level to extract additional surplus from buyers may at some point be too costly for the seller relative to buyer’s increased willingness to pay in the frame.

\(^{13}\)Cable-TV providers offer a variety of contracts, some of which include no or small cancellation fees.
can charge for any product \( x \) a price \( t \) that is equal to the entire surplus \( u(x, \theta) - \bar{t} \). This price \( t \) makes buyers \( U \)-indifferent between \((x, t)\) and \((\bar{x}, \bar{t})\). The seller therefore chooses \( x \) to maximize \( u(x, \theta) - (u(\bar{x}, \theta) - \bar{t}) - c(x) \), and optimally offers the socially efficient product \( x_\theta^* \) at the price \( t = u(x_\theta^*, \theta) - (u(\bar{x}, \theta) - \bar{t}) \). Thus, the regulation increases consumer surplus at the expense of producer surplus without reducing social surplus.

The same regulation is less effective when the seller can use framing that increases attractiveness.

**Proposition 1** Suppose it is feasible to produce above the socially efficient level, i.e., \( x_\theta^* < d \). Then, in every optimal contract, either buyers have zero surplus or the product they purchase is strictly above the socially efficient level.

Thus, when the seller can use framing, the regulation either fails to redistribute surplus from the seller to buyers or creates efficiency distortions.

To see why this happens, we first observe that frames that increase attractiveness are profit-enhancing in this setting, because the seller can charge more for the product \( x_\theta^* \) than in the optimal frameless contract. He can obtain an even higher profit in the optimal contract.

The regulation fails to redistribute surplus from the seller to buyers when there is a frame \( f \) strong enough that buyers \( U_f \)-prefer the first-best bundle \((x_\theta^*, t_\theta^*)\) to \((\bar{x}, \bar{t})\). In this case the seller will offer buyers the first-best bundle with the frame \( f \), and buyers will choose it over \((\bar{x}, \bar{t})\).

If no frame is sufficiently strong, the seller will offer buyers a product above the socially efficient level. The intuition for this upward distortion is that at the socially efficient level \( x_\theta^* \), the seller’s marginal production cost is equal to buyers’ marginal \( U \)-willingness to pay, which in turn is strictly smaller than their marginal \( U_f \)-willingness to pay, by increased attractiveness. Thus, the seller can increase his profit by increasing \( x \) slightly above \( x_\theta^* \) and increasing the price by the marginal \( U_f \)-willingness to pay.

Figure 1 provides a graphical illustration for the case of linear utility. Figure 1.A corresponds to the case of a sufficiently strong frame. The solid black line through \((0, 0)\) describes buyers’ \( U \)-indifference curve through stayout. Buyers are willing to purchase bundles that are to the right and below this line. The parallel solid line describes buyers’ \( U \)-indifference curve through the regulator’s bundle \((\bar{x}, \bar{t})\). The dashed green line through \((\bar{x}, \bar{t})\) describes buyers’ \( U_f \)-indifference curve through the regulator’s bundle. This line is steeper than the solid lines, due to increased attractiveness. The convex solid curves through \( x_\theta^* \) describe the seller’s iso-profit curves. The seller’s profit increases above and to the left of the curves. Because the \( U_f \)-indifference curve through the regulator’s bundle is steep enough to cross the \( U \)-indifference curve through stayout at \( \hat{x} < x_\theta^* \), the seller can offer buyers the first-best bundle with the frame \( f \), and buyers will choose it over \((\bar{x}, \bar{t})\).
Figure 1.A Strong frame

Figure 1.B Weak frame

Figure 1.B corresponds to the case of a weak frame. The $U_f$-indifference curve through the regulator’s bundle is less steep, so it crosses the $U$-indifference curve through *stayout* at $\hat{x} > x^*_\theta$. This means that buyers will not purchase the first-best bundle if the seller offers it. The iso-profit curve is tangent to the $U_f$-indifference curve at $x^f_{\theta} > x^*_\theta$, implying that the seller can increase his profit by increasing $x$ above $x^*_\theta$ along the $U_f$-indifference curve. The profit maximizing bundle is $(x^f_{\theta}, t^f_{\theta})$, since this bundle is to the right and below buyers’ $U$-indifference curve through *stayout*. If $(x^f_{\theta}, t^f_{\theta})$ were above or to the left of buyers’ $U$-indifference curve through *stayout*, the profit maximizing bundle would be at the intersection of the $U_f$-indifference curve through $(\bar{x}, \bar{t})$ and the $U$-indifference curve through *stayout*.

Proposition 1 implies that when framing is not too strong it leads to a reduction in social surplus relative to the optimal frameless contract with a regulator’s bundle. Consumer surplus also goes down, because the seller’s profit goes up while the social surplus goes down, so redistribution of surplus is less effective than without framing.

In summary, the presence of framing in this setting either completely undoes the effect of regulation (when framing is sufficiently strong) or leads to inefficiencies (when framing is not too strong). In both cases, framing reduces the regulation’s effectiveness in shifting surplus to buyers.

4 Market with heterogeneous buyers

In the standard setting without framing, the seller often optimally offers heterogeneous buyers a menu with more than one product. Framing that increases attractiveness and is sufficiently strong may lead to profit reduction in this case. After explaining why this happens, this section characterizes the optimal contract and its welfare properties when framing is not too strong.

We consider a setting with two types of buyers, Low and High, where high-type buyers are
willing to pay more than low-type buyers for an increase in the product, both with and without framing. That is,

**Assumption A2.** Type ranking: For any frame $f$, including the null frame, and any product $x$, \( \partial u^f(x, H) / \partial x > \partial u^f(x, L) / \partial x \).

Assumption (A2) implies that when buyers of both types make a purchase, high-type buyers purchase a weakly larger product than low-type buyers.

A buyer’s type is his private information. The proportion of buyers of type $\theta \in \{L, H\}$ is $\pi_\theta > 0$, with $\pi_L + \pi_H = 1$.

### 4.1 Profit reduction

A frame that increases attractiveness (Assumption (A1)) has two effects on the seller’s profit. First, if a buyer chooses a product $x$ from a menu $M$ without the frame, then with the frame this product becomes more attractive relative to smaller products. Thus, with the frame the buyer will continue to choose $x$ over smaller products in the menu even if the price of $x$ increases slightly (and the prices of the smaller products do not decrease). This effect has favorable profit implications.

Second, the product $x$ becomes less attractive relative to larger products in the menu, whose prices may exceed the buyer’s willingness to pay. This implies that a buyer who made a purchase without the frame may not make a purchase with the frame, because the bundle he finds most attractive is too expensive. This effect did not have adverse profit implications in the regulated market setting, because the bundle intended for buyers was optimally larger than the regulator’s bundle. But with more than one type of buyer, adverse profit implications may arise, because the bundle intended for low-type buyers is often optimally smaller than the bundle intended for high-type buyers.

To analyze how the two effects interact, consider a contract $(M, f)$, where $M$ is part of a profit-maximizing frameless contract $(M, \phi)$ and $f$ increases attractiveness. If each buyer weakly $U^f$-prefers his chosen bundle in the frameless contract to larger bundles in $(M, f)$, then the second effect does not have adverse profit implications. In this case, every profit-maximizing contract involves framing. This is because the first effect implies that, similarly to the regulated market setting, the seller can increase the price of the largest chosen product in $M$ slightly so that every buyer will continue to purchase from the modified menu with the frame $f$ the same product he purchased in $(M, \phi)$. Intuitively, this scenario corresponds to a situation in which framing is not “too strong.”

On the other hand, when framing is strong enough that some buyers strictly $U^f$-prefer larger bundles in $(M, f)$ to their chosen bundle in $(M, \phi)$, an optimal contract with framing may generate strictly lower profit than the optimal frameless contract, despite the increased attractiveness.
Such profit reduction may arise when the seller’s ability to vary the products in the menu is limited, e.g., due to regulatory or technological constraints. In this case, the optimal frameless contract may involve selling a “basic” product to low-type buyers and a “premium” product to high-type buyers. With the frame, the seller may be forced to reduce the price of the basic product to make sure that low-type buyers do not find the premium product more attractive than the basic product.

Example 1 and Figure 2 illustrate that this price reduction may be so substantial when framing is strong that the seller may choose not to use such framing even though it increases attractiveness.\footnote{We thank Andrew Rhodes for developing this example.}

**Example 1 (Price discrimination with linear frames)** There are only two available products, a basic product $x_L$ and a premium product $x_H > x_L$, whose production is costless. Buyers’ utility $u(x, \theta)$ satisfies $u(0, \theta) = 0$. There is a single frame $f \in \mathbb{R}_+$ that increases attractiveness with $u^f(x, \theta) = u(x, \theta) + xf$, i.e., the frame interacts linearly with the product and does not interact with the type.

Suppose that $\pi_H \in \left\{ \frac{u(x_H, L) - u(x_L, L)}{u(x_H, H) - u(x_L, H)}, \frac{u(x_L, L)}{u(x_L, H)} \right\}$, so the optimal frameless contract offers both products.\footnote{The condition on $\pi_H$ is derived by comparing the profit in the optimal contract in which both products are bought to the optimal pooling contract and to the optimal contract in which low-type buyers do not purchase anything.} The basic product in this contract is bought by low-type buyers, and its price $u(x_L, L)$ is determined by their binding $U$-participation (or, individual rationality) constraint. The premium product is bought by high-type buyers and its price $u(x_H, H) - (u(x_L, H) - u(x_L, L))$ is determined by their binding $U$-incentive compatibility constraint. Figure 2.A depicts the optimal frameless contract for the case of linear utility.

![Figure 2.A Optimal frameless contract](image)

**Figure 2.B Optimal contract with weak frame**

**Figure 2.C Optimal contract with strong frame**

Notes: $IR^U_\theta$ denotes the $U$-participation constraint of type $\theta$; $IC^f_\theta$ denotes the incentive compatibility constraint of type $\theta$ in frame $f$; $\phi$ denotes the null frame.

With framing, the optimal contract changes as a function of the frame $f$. With a weak frame (Figure 2.B), the binding constraints are the $U$-participation constraint of low-type buyers and the...
Uⁿ-incentive compatibility constraint of high-type buyers. Profit increases relative to the optimal frameless contract. With a sufficiently strong frame (Figure 2.C), the binding constraints are the U-participation constraint of high-type buyers and the Uⁿ-incentive compatibility constraint of low-type buyers. Profit in the optimal contract decreases relative to the optimal frameless contract.

More formally, for \(0 \leq f \leq \frac{u(x_L,H) - u(x_L,L)}{x_H - x_L}\), \(t_L\) remains equal to \(u(x_L,L)\) but \(t_H\) increases by \(f(x_H - x_L)\), so the incentive compatibility constraint of high-type buyers with respect to \(f\) continues to bind. Relative to the optimal frameless contract, the profit increases by \(\pi_H f(x_H - x_L)\). See Figure 2.B.

At \(f = \frac{u(x_L,H) - u(x_L,L)}{x_H - x_L}\), we have that \(t_H = u(x_H,H)\) and the participation constraint of high-type buyers binds. Since the participation constraints of buyers of both types bind, this is the fully extractive separating contract. This remains the optimal contract for \(\frac{u(x_L,H) - u(x_L,L)}{x_H - x_L} < f \leq \frac{u(x_H,H) - u(x_H,L)}{x_H - x_L}\), since for these values of \(f\) the incentive compatibility constraints of buyers of both types are slack.

At \(f = \frac{u(x_H,H) - u(x_H,L)}{x_H - x_L}\) the incentive compatibility constraint of low-type buyers binds. To maintain the incentive compatibility constraint of low-type buyers for \(f > \frac{u(x_H,H) - u(x_H,L)}{x_H - x_L}\) in a separating contract, \(t_L\) must be decreased. The optimal separating contract has \(t_L = u(x_H,H) - (u(x_H,L) - u(x_L,L)) - f(x_H - x_L)\), so the incentive compatibility constraint of low-type buyers binds. See Figure 2.C. For large enough \(f\), the profit in this contract is smaller than in the optimal frameless contract described above. And since in order to generate more profit than the optimal frameless contract the optimal contract with the frame \(f\) must be a separating contract in which low-type buyers buy the basic product and high-type buyers buy the premium product, we conclude that for large enough \(f\) the optimal frameless contract generates a higher profit than any contract with the frame \(f\).

Profit reduction may also arise when the seller is able to change the products he offers. In this case, with strong framing he will offer low-type buyers a better product than in the optimal frameless contract, rather than reducing the price of the basic product as in Figure 2.C. The prices of these better products will be relatively low, because the \(U\)-willingness to pay of low-type buyers is low. In a setting similar to that of Example 1, this will imply that framing increases the seller’s profit, because production is costless. But when producing better products is costly, offering them at relatively low prices may decrease the seller’s profit more than the gain due to increased attractiveness. Example 3 in the Appendix illustrates this channel for profit reduction.

To summarize,

Observation 3 If there exists an optimal frameless contract \((M, \phi)\) and a frame \(f\) that increases attractiveness such that every type weakly \(Uⁿ\)-prefers in \((M,f)\) the product he chooses in \((M,\phi)\) to larger products, then every optimal contract involves framing. If this is not the case, then it may be that every optimal contract is frameless.
To determine whether framing is used in the optimal contract, Observation 3 requires solving for the optimal frameless contract. Our third assumption avoids this difficulty by identifying a condition that relies only on the primitives of the model and guarantees that framing does not decrease profit.

**Assumption A3.** Limited distortion: For any frame $f$, $U^f(x_L^*, t_L^*, H) > U^f(x_H^*, t_H^*, H)$.

Assumption (A3) formalizes the notion that framing is not too strong. Graphically, the assumption means that for any frame $f$ the $U^f$-indifference curve of high-type buyers through $(x_L^*, t_L^*)$ crosses their $U$-indifference curve through *stayout* at a point $\tilde{x} > x_H^*$, as depicted in Figure 3. While stated with respect to high-type buyers, this assumption guarantees that the adverse profit implications of framing due to the behavior of low-type buyers can be mitigated in the optimal contract, as we will demonstrate after stating Proposition 2.

![Figure 3. Graphical illustration of (A3)](image)

### 4.2 Characterization of the optimal contract

We now characterize the set of optimal contracts and their welfare properties under Assumptions (A1)-(A3).

The set of optimal contracts may include pooling and separating contracts. If some optimal contract is pooling, then it implements the allocation rule $g(\theta) = (x_L^*, t_L^*)$, because the bundle $(x_L^*, t_L^*)$ is the profit-maximizing bundle subject to low-type buyers being $U$-indifferent between making and not making a purchase. In particular, framing does not influence the seller’s profit in this case. On the other hand,

**Proposition 2** Any optimal contract that is separating involves framing.

Figure 4 provides a graphical illustration for the case of linear utility. In an optimal frameless contract, the binding constraints are the $U$-participation constraint of low-type buyers ($IR_L^U$) and
the $U$-incentive compatibility constraint of high-type buyers ($IC^\phi_H$). To demonstrate that the profit is higher in an optimal contract with a frame $f$, we consider two cases. In the first case (Figure 4.A), which corresponds to a weak frame, in order to arrive at a higher profit it suffices to increase $t_H$ until the $U^f$-incentive compatibility constraint of high-type buyers ($IC^f_H$) through $(x_L, t_L)$ binds. In the second case (Figure 4.B), which corresponds to a stronger frame, even when $t_H$ is increased until the $U$-participation constraint of high-type buyers binds, high-type buyers still $U^f$-prefer $(x^*_H, t^*_H)$ to $(x_L, t_L)$. We thus increase the bundle $(x_L, t_L)$ along the low-type buyers’ $U$-indifference curve through stayout until high-type buyers are $U^f$-indifferent between the two bundles. Assumption (A3) guarantees that this happens before we arrive at $(x^*_L, t^*_L)$, so this process increases profit.

An immediate implication of Proposition 2 is that framing is profit-enhancing whenever pooling contracts are not optimal. Corollary 1 identifies sufficient conditions for this to happen.

**Corollary 1** Framing is profit-enhancing when $x^*_L < d$ or when there exists a frame $f$ such that $\partial u(d, L)/\partial x < \pi_L(\partial c(d)/\partial x) + \pi_H(\partial u^f(d, H)/\partial x)$.

In both cases identified in the corollary, the optimal pooling contract is dominated by a separating contract in which low-type buyers are offered a product that is slightly lower than $x^*_L$ at a price that equals their $U$-willingness to pay, and high-type buyers are offered the product $x^*_L$ at a price that makes them $U^f$-indifferent to the low-type buyers’ bundle. Every optimal contract is therefore separating, so by Proposition 2 framing is profit-enhancing.

Another implication of Proposition 2 relates to participation.

**Corollary 2** Both types of buyers purchase positive products in any optimal contract.
This result contrasts with the model without framing, in which the optimal frameless contract excludes low-type buyers when their proportion in the population is small in order to eliminate the information rents of high-type buyers.

To see why such exclusion is never optimal with framing, consider a frameless contract that excludes low-type buyers and extracts the maximum surplus from high-type buyers by offering them the first-best bundle \((x^*_H, t^*_H)\). By Proposition 2, this contract generates strictly less profit than any optimal contract with framing. Because the maximum surplus is already extracted from high-type buyers in the frameless contract, the only way to increase profit in an optimal contract with framing is to offer low-type buyers a positive product. This can be done due to increased attractiveness.\(^{16}\)

Because buyers of both types purchase positive products in an optimal contract with framing, it suffices to focus on contracts with menus \(\{(x_L, t_L), (x_H, t_H)\}\), where \((x_\theta, t_\theta)\) is the bundle purchased by type \(\theta\). For a frame \(f\), the seller’s profit maximization problem can then be written as follows:

Choose \(\{(x_L, t_L), (x_H, t_H)\}\) to maximize \(\pi_L(t_L - c(x_L)) + \pi_H(t_H - c(x_H))\) subject to:

\[\text{IR}_L: U(x_\theta, t_\theta, \theta) \geq U(0, 0, \theta) \text{ for } \theta \in \{L, H\},\]

\[\text{IC}_H: U^f(x_\theta, t_\theta, \theta) \geq U^f(x_{\theta'}, t_{\theta'}, \theta') \text{ for } \theta, \theta' \in \{L, H\} \text{ and } \theta' \neq \theta.\]

Note that the \(U^f\)-participation constraint of type \(\theta\) buyers does not appear in the seller’s problem, because by Assumption (A1) and Observation 1 this constraint is implied by the buyers’ \(U\)-participation constraint.

By Assumption (A2), we have that \((x_H, t_H) \geq (x_L, t_L)\), so we refer to \((x_L, t_L)\) as the basic bundle and to \((x_H, t_H)\) as the premium bundle. Our next proposition identifies the binding constraints in the seller’s profit maximization problem.

**Proposition 3** In an optimal contract with a frame \(f\), the binding constraints are \(\text{IR}_L^U\) and \(\text{IC}_H^f\), i.e., low-type buyers are \(U\)-indifferent between buying the basic bundle and not buying anything, and high-type buyers are \(U^f\)-indifferent between buying the premium bundle and the basic bundle.

The binding constraints in Proposition 3 are similar to the binding constraints in the optimal frameless contract. But in contrast to the optimal frameless contract, the constraints do not imply that whenever the basic product is positive high-type buyers strictly \(U\)-prefer the premium bundle to not making a purchase, so they do not necessarily obtain information rents in the form of positive surplus. That is, high-type buyers may be \(U\)-indifferent between purchasing the premium bundle and not making a purchase, even when low-type buyers purchase a positive product.

\(^{16}\)The seller can also exclude high-type buyers in the model with framing by offering them a bundle that is \(U^f\)-superior to the other bundles in the menu but is \(U\)-inferior to stayout, but this is dominated by the optimal pooling contract, because the production cost is type-independent. Section 5 considers a setting with type-dependent cost, in which excluding high-type buyers may be optimal.
4.3 Welfare properties

Framing that is profit-enhancing has several welfare implications. The first relates to the efficiency of the basic product. When the proportion of low-type buyers \( \pi_L \) is small, the basic product with framing is more efficient than without framing, in the sense that it generates a larger social surplus with respect to low-type buyers, \( \pi_L(u(x, L) - c(x)) \). This is because the optimal frameless contract excludes low-type buyers when \( \pi_L \) is small in order to eliminate the information rents of high-type buyers, while the optimal contract with framing always offers low-type buyers a positive product. This positive product is smaller than \( x^*_L \), and thus generates positive social surplus. By Proposition 3, the entire surplus gain goes to the seller.

A second welfare implication relates to the efficiency of the premium product. Without framing, this product is efficient in the sense that it maximizes the social surplus with respect to high-type buyers, \( \pi_H(u(x, H) - c(x)) \). This is because for any basic bundle, the seller can extract from high-type buyers the entire social surplus generated from the premium bundle, up to an amount that makes high-type buyers \( U \)-indifferent between the premium bundle and the basic bundle. In contrast,

**Proposition 4** The premium product in an optimal separating contract is strictly above the efficient level \( x^*_H \) when \( x^*_H < d \), and is efficient when \( x^*_H = d \).

The reason for this efficiency distortion is that for any basic bundle \((0, 0) < (x_L, t_L) < (x^*_L, t^*_L)\) such that low-type buyers are \( U \)-indifferent between this bundle and not purchasing anything, increasing the premium product slightly above the efficient level along the high-type’s \( U \)-indifference curve does not decrease the seller’s profit to a first-order. But such an increase makes the premium product strictly more \( U^f \)-attractive to high-type buyers relative to the basic bundle, so the basic bundle can be increased along the low-type’s \( U \)-indifference curve through it without reducing the price of the premium product, which results in a first-order gain to the seller.\(^{18}\)

A third welfare implication relates to the information rents of high-type buyers. Without framing, high-type buyers always obtain a strictly positive surplus whenever the basic product is positive. This is because they can mimic low-type buyers, so by choosing the premium bundle they must obtain the surplus they would obtain from choosing the basic bundle. The most the seller can therefore charge for the premium product is the high-type buyer’s \( U \)-willingness to pay for it minus the difference between the high type’s \( U \)-willingness to pay for the basic product and the basic product’s price. But with framing, the seller can charge for the premium product the high-type buyer’s \( U^f \)-willingness to pay for it minus the difference between his \( U^f \)-willingness to pay for the basic product and the high-type buyer’s \( U^f \)-willingness to pay for it minus the difference between his \( U^f \)-willingness to pay for the basic product and the

\(^{17}\)Otherwise, decreasing the basic bundle along the \( U \)-indifference curve of low-type buyers through *stayout* would increase profit without violating any constraints.

\(^{18}\)Over-consumption arises for other reasons in Carbajal and Ely (2016) and Galperti (2015).
pay for the basic product and its price, subject to not exceeding high-type buyers’ $U$-willingness to pay for the premium product. When this last constraint binds, high-type buyers do not obtain any surplus. In fact, even a frame that creates a small distortion can eliminate the entire surplus of high-type buyers, as the following example illustrates.

**Example 2 (Price discrimination with linear utility)** Suppose that production is costless, that $L < H \in \mathbb{R}_+$, that $w^f(x, \theta) = u(x, \theta) + xf$ as in the previous example, and that $u(x, \theta) = x\theta$. The first-best product is then $x^* = d$ independently of buyers’ types. Fix some frame $f > 0$ and assume that $\pi_H \in \left(\frac{L}{H+f}, \frac{H}{H+f}\right)$.

Using well-known properties of the optimal contract without framing,\(^{(19)}\) one can show that because $\pi_H < \frac{L}{H}$, the optimal frameless contract is a pooling contract with the bundle $(d, dL)$. The surplus of high-type buyers in this contract is $d(H - L) > 0$.

In the optimal contract with framing, we have that $x_H = d$, because the optimal pooling contract includes the bundle $(d, dL)$ and Proposition 4 implies that $x_H = x_H^* = d$ in an optimal separating contract. By Proposition 3, the price of the basic product is $x_L L$, and the price of the premium product satisfies $d(H + f) - t_H = x_L (H + f) - t_L$, so $t_H = d(H + f) - x_L (H + f - L)$. In addition, the price of the premium bundle cannot exceed the $U$-willingness to pay of high-type buyers, i.e., $t_H \leq dH$. The minimal $x_L$ that satisfies these conditions is $\frac{df}{H + f - L}$, and it is straightforward to verify that because $\pi_H > \frac{L}{H + f - L}$, this $x_L$ is profit-maximizing.

We thus obtain that the uniquely optimal contract is $\{(\frac{df}{H + f - L}, \frac{dL}{H + f - L}), (d, dH)\}, f$). In contrast to the optimal frameless contract, the surplus of high type buyers in this contract is $0$.

A fourth welfare implication relates to the effect of framing on total welfare. As indicated above, framing increases efficiency with respect to the basic bundle when the proportion of low-type buyers is small, but decreases efficiency with respect to the premium bundle. The overall effect on welfare $\pi_L(u(x_L, L) - c(x_L)) + \pi_H(u(x_H, H) - c(x_H))$ is positive when the proportion of low-type buyers is small. To see why, recall that when the proportion of low-type buyers is small, the optimal frameless contract excludes them and offers the first-best bundle $(x_H^*, t_H^*)$ to high-type buyers. Now fix a frame $f$, and let $(x_L, t_L)$ denote a basic bundle such that high-type buyers are $U^f$-indifferent between this bundle and $(x_H^*, t_H^*)$ and low-type buyers are $U$-indifferent between this basic bundle and not making a purchase. By increased attractiveness for high-type buyers, $(x_L, t_L) \succ (0, 0)$, and by assumptions (A2) and (A3), $(x_L, t_L) \prec (x_H^*, t_H^*)$.\(^{(20)}\) The total welfare in the contract $\{(x_L, t_L), (x_H^*, t_H^*)\}, f)$ is higher than in the optimal frameless contract because the premium product is unchanged and the basic product is more efficient. The profit-maximizing contract with framing further increases welfare: it weakly increases the seller’s profit.

\(^{(19)}\)See, for example, Fudenberg and Tirole (1991, Chapter 7.1.1).

\(^{(20)}\)If $(x_L, t_L) \succ (x_H^*, t_H^*)$, then low-type buyers $U^f$-prefer $(x_L, t_L)$ to $(x_H^*, t_H^*)$ because they are $U$-indifferent between these two bundles. Because high-type buyers have the same $U^f$-ranking of these two bundles, and because they are $U^f$-indifferent between $(x_H^*, t_H^*)$ and $(x_L, t_L)$, we obtain that they $U^f$-prefer $(x_H^*, t_H^*)$ to $(x_L, t_L)$, contradicting Assumption (A3).
by definition, and it gives buyers of both types a weakly higher $U$-utility than what they get in the above contract, in which they are $U$-indifferent to not buying anything.

## 5 Monopolistic insurance with a highlighted bundle

Sellers often highlight a premium product in the menu. For example, Sam’s Club highlights its premium “sam’s plus” membership by putting it in a separate box and by indicating that it is the “best value” (Figure A.1 in Appendix A). The premium membership includes a 2 percent annual rebate and early shopping hours, which are not included in the basic membership. Similarly, the online insurance provider “Insure My Rental Car” highlights its premium policy over the basic one by coloring it in a darker color and adding a “check” mark to it (Figure A.2). The premium policy covers personal property and hotel burglary, which are not covered by the basic policy. In both examples, product returns are straightforward. Sam’s Club offers a “100% Membership Satisfaction Guarantee,” according to which you can cancel your membership at any time for a full refund, and Insure My Rental Car offers a ten day window for a full refund of the insurance premium.

In the spirit of Tversky and Kahneman’s (1991) model of reference-dependent choice, buyers may treat the highlighted product as a reference point. They may thus anticipate a mental loss or regret in case they need a feature that is included in the highlighted product but not in the product they purchase. For example, buyers may anticipate a mental loss if they purchase a basic insurance policy but need coverage for an event that is only included in the highlighted premium policy. In contrast, as indicated by Kahneman, Knetsch, and Thaler (1990), buyers often do not view the money they spend in the course of a transaction as a loss.

This section studies framing that highlights a particular product in the context of Stiglitz’s (1977) monopolistic insurance setting.\(^\text{21}\) Buyers treat the highlighted product (but not its price) as a reference point, to which they compare other products.\(^\text{22}\) This mechanism for forming a reference point departs from the literature on expectation-based loss-aversion (Kőszegi and Rabin (2006)) and pricing with expectation-based loss-averse consumers (Heidhus and Kőszegi (2008)). We postulate, following Kahneman and Tversky’s (1984) discussion of the Asian disease example, that the framing of the decision problem influences how consumers form their reference point when making choices.

**Stiglitz’s monopolistic insurance model.** A risk-neutral profit-maximizing insurance provider offers a menu of insurance bundles to a population of risk-averse individuals. Each individual

\(^{21}\) For related work on competitive insurance markets with over-confident consumers, see Sandroni and Squintani (2007).

\(^{22}\) Our results extend to cases in which highlighting also operates on the money dimension, but the analysis is more complicated.
has initial wealth \(w\), and may suffer an accident of size \(A > 0\). An individual’s privately-known probability of an accident is \(\theta \in \{L, H\}\), with \(0 < L < H < 1\). The proportion of Low-risk individuals in the population is \(\pi_L > 0\) and of High-risk individuals is \(\pi_H = 1 - \pi_L > 0\). Each individual’s preferences over wealth are summarized by a strictly increasing, strictly concave, and continuously differentiable function \(u\).

An insurance bundle is a pair \((x, t)\), where \(t\) is the premium paid by the individual to the insurance provider upfront and \(x \geq 0\) is the amount paid by the provider to the individual if the accident occurs. The expected utility of an individual with risk level \(\theta\) from the bundle \((x, t)\) is

\[
U(x, t, \theta) = \theta u(w - t - A + x) + (1 - \theta) u(w - t).
\]

Because buyers are risk averse and the insurance provider is risk neutral, the first-best bundles include full coverage, i.e., \(x_H^* = x_L^* = A\).

**Frames that highlight a bundle.** We enrich Stiglitz’s model by assuming that in addition to offering a menu of insurance bundles the provider can also highlight one bundle in the menu. The highlighting changes the “reference point” to which buyers compare insurance bundles, so that instead of evaluating bundles relative to the outside option, buyers evaluate them relative to the highlighted bundle. This leads buyers to anticipate a mental loss in case of an accident if they buy less coverage than in the highlighted bundle. After the highlighting effect wears off, buyers evaluate bundles relative to the outside option, i.e., according to \(U\).

Formally, let \(f = (x_f, t_f)\) denote the highlighted bundle. A contract is a pair \((M, f)\) where \(f \in M\). That is, the highlighted bundle has to be offered in the menu. The set \(F\) of frames is the set of all possible bundles, with \(f = (0, 0)\) being the null frame.

A buyer anticipates that if he purchases an insurance bundle \((x, t)\) with coverage \(x \leq x_f\), he will experience a mental loss or regret of \(r(x_f - x)\) in case an accident occurs, in addition to the material effect of the accident on his wealth. That is, in the frame \(f\) an individual chooses from the menu a bundle \((x, t)\) that maximizes

\[
U^f(x, t, \theta) = \theta (u(w - t - A + x) - 1_{x \leq x_f} r(x_f - x)) + (1 - \theta) u(w - t).
\]

In the spirit of Tversky and Kahneman (1991), the regret function \(r\) satisfies the following properties:

- \(r'(\Delta) > 0\) for \(\Delta \geq 0\): Regret is increasing in the difference in coverage \(\Delta = x_f - x\) between the highlighted coverage and the chosen coverage,
- \(r''(\Delta) < 0\) for \(\Delta > 0\): Marginal regret is decreasing, and
- \(r(0) = 0\): There is no regret if the chosen coverage is equal to the highlighted coverage.
As indicated above, the loss function \( r \) operates on the product dimension \( x \) and not on the money dimension \( t \).

**Departures from the environment of Section 4.** The Stiglitz setting with highlighting departs from the environment of Section 4 in several ways.

First, the seller’s cost \( c(x, \theta) = x\theta \) is type-dependent, because high-risk individuals are more costly to serve than low-risk individuals. This changes the analysis of the optimal contract because the seller may wish to exclude the costly high-risk individuals. This is impossible in the standard model, but can be done (and is sometime optimal) with framing.

Second, frames do not increase the attractiveness of every marginal increase in coverage as Assumption (A1) postulates. Rather, they increase the attractiveness of additional coverage as long as the total coverage is less than the reference coverage. That is,

**Assumption A1’. Increased attractiveness:** For every frame \( f = (x_f, t_f) \neq \phi \), every bundle \((x, t)\) such that \( x < x_f \), and every type \( \theta \),

\[
-\frac{\partial U_f(x, t, \theta)/\partial x}{\partial U_f(x, t, \theta)/\partial t} > -\frac{\partial U(x, t, \theta)/\partial x}{\partial U(x, t, \theta)/\partial t}.
\]

Third, the functions \( U \) and \( \{U_f\}_{f \in F} \) are not quasi-linear, and do not satisfy Assumption (A2). They nonetheless satisfy the natural extension of this assumption to non quasi-linear environments, in that high-risk individuals value an additional unit of coverage more than low-risk individuals.

**Assumption A2’. Type ranking:** For any frame \( f \) (including the null frame) and any bundle \((x, t)\),

\[
-\frac{\partial U_f(x, t, H)/\partial x}{\partial U_f(x, t, H)/\partial t} > -\frac{\partial U_f(x, t, L)/\partial x}{\partial U_f(x, t, L)/\partial t}.
\]

Note that Assumption (A3) continues to hold in the Stiglitz setting with highlighting, because \( x^*_H = x^*_L = A \) and \( t^*_H > t^*_L \).

Fourth, there is a dependency between the menu and the frame, because the reference bundle has to be offered in the menu. Thus, an optimal contract may in principal require three bundles: a basic one targeted at low-risk individuals, a premium one targeted at high-risk individuals, and a highlighted bundle. The seller’s profit-maximization problem then has the additional incentive compatibility constraints that type-\( \theta \) buyers \( U_f \)-prefer the bundle \((x_\theta, t_\theta)\) to the highlighted bundle.

**Optimal insurance contract.** In solving for the optimal contract, we first observe that Proposition 2 extends to the Stiglitz setting with highlighting.

**Proposition 2’** Any optimal insurance contract that is separating involves highlighting.

Because the optimal frameless contract in Stiglitz’s original setting is separating, we obtain that

**Property 1** Every optimal insurance contract involves highlighting.
Because it is always possible to exclude low-risk individuals and extract the maximum surplus from high-risk buyers in a frameless contract, Property 1 implies that

**Property 2** In an optimal insurance contract, low-risk individuals purchase insurance regardless of the distribution of types.

Property 2 implies that highlighting increases the social surplus with respect to low-risk individuals when their proportion in the population is small. This is because in this case the optimal frameless contract excludes them. However, the entire surplus gain goes to the seller because, as we will show below, low-risk individuals obtain no surplus in an optimal contract.

High-risk individuals may or may not buy insurance in an optimal contract. For the case in which they do, we identify the binding constraints in the seller’s profit-maximization problem in two steps. First, we consider the profit maximization problem of Section 4, i.e., we ignore the additional incentive compatibility constraints that type-θ buyers $U^f$-prefer the bundle $(x_θ, t_θ)$ to the highlighted bundle. This enables us to restate Proposition 3 using Assumptions (A1)’ and (A2)’ instead of Assumptions (A1) and (A2). Then, we solve for the optimal highlighted bundle, and show that the constraints we ignored are satisfied with respect to this bundle.

**Proposition 3’** For a given highlighted bundle $f$, the binding constraints in the profit maximization problem of Section 4 are $IR_L^U$ and $IC_H^f$.

The following property characterizes the coverage in an optimal highlighted bundle.

**Property 3** The highlighted coverage is identical to the premium coverage in any optimal contract in which high-risk individuals purchase insurance.

Property 3 is in line with the real-world phenomenon that the highlighted product often coincides with the premium product. This property arises because the marginal sensitivity to losses is decreasing, so by setting the highlighted coverage to be equal to the premium coverage the seller minimizes the attractiveness to high-risk individuals of the basic insurance bundle relative to the premium bundle.

Property 3 implies that in an optimal contract in which high-risk individuals purchase insurance, the constraints that type-θ buyers $U^f$-prefer the bundle $(x_θ, t_θ)$ to the highlighted bundle are satisfied, because the highlighted bundle can be chosen to be identical to the bundle of high-risk individuals. We thus obtain, similarly to Section 4, that in an optimal insurance contract with highlighting in which high-risk individuals buy insurance, low-risk individuals are $U$-indifferent between buying basic insurance and not buying insurance, and high-type buyers are $U^f$-indifferent between buying premium insurance and basic insurance.

But high-risk individuals may not buy insurance in an optimal contract.
Property 4 *In an optimal contract, high-risk individuals are either strictly over-insured, or do not purchase insurance.*

Property 4 implies that framing reduces the social surplus with respect to high-risk individuals. One channel for inefficiency is overinsurance of high-risk individuals, which enables the seller to increase the coverage of low-risk individuals, similarly to Proposition 4. A new channel for inefficiency is exclusion. Because high-risk individuals are more costly to serve than low-risk individuals, the insurance provider may want to exclude them and only serve low-risk individuals. This is impossible without framing, but can be done with framing by offering high-risk individuals a premium insurance bundle that they *Uf*-prefer to the basic bundle but that is *U*-inferior to not purchasing insurance.

Taken together, Properties 2 and 4 imply that with framing the seller may optimally choose to have advantageous selection, i.e., serve only low-risk individuals.\(^{23}\) This happens, for example, when the proportion of high- and low-risk individuals in the population is similar, high-risk individuals are almost certain to have an accident, so *H* is close to 1, and low-risk individuals are at an intermediate risk of having an accident, so *L* is close to 1/2. In this case, the profit *P(L)* from providing the first-best insurance to low-risk individuals is significantly higher than the profit *P(H)* from providing the first-best insurance to high risk individuals. The insurance provider can achieve *P(L)* by (1) offering the first best insurance to low-risk individuals and (2) excluding high-risk individuals by offering them a highlighted bundle (*xf, tf*) that they weakly *Uf*-prefer to the low-risk individuals’ insurance bundle and is *U*-inferior to not buying insurance. In order to achieve a higher profit in a contract without exclusion, low-risk individuals must be almost fully insured, and high-risk individuals must be even more insured. But if regret is moderate, selling a large amount of insurance to high-risk individuals at a price that makes them *Uf*-indifferent to the bundle of low-risk individuals leads to a loss.

6 Conclusion

This paper introduced framing into contract theory. We postulated an environment in which a profit-maximizing seller uses framing to influence buyers’ behavior, and studied the optimal design of contracts in this environment.

The effect of framing on buyers’ behavior is captured through the buyer’s increased willingness to pay at the point of sale, which relaxes buyers’ incentive compatibility constraint. However, the increased willingness to pay is temporary, and buyers can renege on their purchase

\(^{23}\)Sandroni and Squintani (2013) show that advantageous selection may arise in competitive, but not monopolistic, insurance markets with over-confident consumers. Aperjis and Balestrieri (2014) show that advantageous selection may also arise when consumers are expectation-based loss averse as in Köszegi and Rabin (2006) and face modest scale risk.
after the effect wears off. This is captured by setting buyers’ participation constraint according to their true willingness to pay. The temporary change in willingness to pay complicates the seller’s design of the optimal product menu, because it implies that low-type buyers, who may wish to purchase a premium product at the point of sale, may renege on their purchase after the framing effect wears off. This force nullifies the positive implications of the increased willingness to pay when framing is sufficiently strong.

When framing is not too strong, it leads to efficiency distortions at the top in the sense that high-type buyers over-consume relative to the setting without framing, and to potential efficiency improvement at the bottom, in the sense that low-type buyers consume more efficiently when their proportion in the population is sufficiently small.

There are other ways in which sellers can influence buyers’ purchase behavior, and in particular their incentive compatibility constraint. For example, sellers may offer menus with a large number of bundles, thus triggering buyers to construct a consideration set, which includes a subset of the menu’s bundles, and choose from the consideration set. Alternatively, sellers may use different measurement units for different products in the menu in order to make it harder for buyers to compare bundles. In both cases, buyers may not compare all the bundles in the menu according to their underlying preferences, so their incentive compatibility constraint will change.

There are also alternative ways to model the temporary effect of the frame. One possibility that reflects greater sophistication on the part of the buyers is that instead of walking away after returning a product that they overpaid for, buyers internalize how framing influenced them at the point of sale, and become immune to the framing effect from that point on. This can be modeled by the choice correspondence that assigns to every contract \((M, f)\) the set of:

1. all the \(U_f\)-maximal bundles in \(M \cup \{\text{stayout}\}\) that are weakly \(U\)-superior to \(\text{stayout}\), and
2. all the \(U\)-maximal bundles in \(M \cup \{\text{stayout}\}\) if \(\text{stayout}\) is weakly \(U\)-superior to some \(U_f\)-maximal bundle in \(M\).

We conclude with a discussion of how such increased sophistication affects the predictions of our model.

Our first observation is that the increased sophistication of buyers does not change the characterization of the optimal contract in Section 3, in Section 4 when framing is not too strong (as captured by Assumption (A3)), and in Section 5 when high-risk individuals purchase insurance. Consider, for example, the optimal allocation rule in Section 4. Any contract that implements this allocation rule in our model also implements it in the model with increased sophistication. And to verify that this allocation rule is profit-maximizing with increased sophistication, note that in the model with increased sophistication, we have that (i) a type-\(\theta\) buyer purchases \((x_\theta, t_\theta)\) only if it is \(U\)-superior to not buying anything, and (ii) if low-type buyers make a purchase, then high-type buyers purchase the premium bundle only if it is \(U_f\)-superior to the basic bundle. The constraints (i) and (ii) are the only relevant constraints in the original model.
Our second observation is that the increased sophistication may actually increase the seller’s profit in other settings. This can happen when framing is strong enough that Assumption (A3) is violated. In this case, the seller in the original model is worried about low-type buyers not making a purchase because they are attracted to the premium bundle, which is too expensive for them. He therefore has to offer them a basic bundle of higher quality at a relatively low price to make sure they make a purchase. But in the alternative model, the seller does not have to worry about this. Buyers who overpay will internalize the framing effect, return the premium product and then buy another product according to their $U$-preferences. Thus, increased sophistication may actually hurt buyers because it enables the seller to screen them better, and have high-type buyers make purchases according to $U^f$ and low-type buyers according to $U$. 
A  Examples of highlighting

![Sam's Club membership brochure]

**Figure A.1** Sam’s Club membership brochure

![Insurance policies at InsureMyRentalCar.com]

**Figure A.2** Insurance policies at InsureMyRentalCar.com

B  Proofs

**Proof of Observation 1.** Fix a contract \((M, f)\). By Assumption (A1), if a bundle \((x, t)\) is \(U\)-superior to \(stayout\), then it is also \(U^f\)-superior to \(stayout\). Thus, \((x, t)\) is \(U^f\)-maximal in \(M\)
and $U$-superior to $stayout$ if and only if $(x, t) \in U^f$-maximal in $M \cup \{stayout\}$ and $U$-superior to $stayout$. Therefore, for any $(x, t) \neq stayout$, $(x, t) \in C^\theta(M, f)$ if and only if $(x, t) \in \hat{C}^\theta(M, f)$.

If $stayout \in \hat{C}^\theta(M, f)$ then, by definition of $\hat{C}^\theta$, it is $U$-superior to some $U^f$-maximal bundle $(x, t)$ in $M$. If $(x, t)$ is also $U^f$-maximal in $M \cup \{stayout\}$ then $stayout$ is $U$-superior to some $U^f$-maximal bundle in $M \cup \{stayout\}$, and hence $stayout \in C^\theta(M, f)$ by part (2) of the definition of $C^\theta$. And if $(x, t)$ is not $U^f$-maximal in $M \cup \{stayout\}$ then $stayout$ is $U^f$-superior to $(x, t)$, and hence $U^f$-maximal in $M \cup \{stayout\}$, and is in $C^\theta(M, f)$ by part (1) of the definition.

If $stayout \in C^\theta(M, f)$ and is not $U^f$-maximal in $M \cup \{stayout\}$ then it is $U$-superior to some $U^f$-maximal bundle in $M$. In this case, $stayout \in \hat{C}^\theta(M, f)$. And if $stayout$ is $U^f$-maximal in $M \cup \{stayout\}$ then it is also $U$-maximal $M \cup \{stayout\}$. (Otherwise, there is a bundle $(x, t) \in M$ that is $U$-superior to $stayout$ but $U^f$-inferior to $stayout$ in contradiction to Assumption (A1).)

We thus obtain that $stayout \in \hat{C}^\theta(M, f)$ in this case as well.

**Proof of Proposition 1.** We proved in the main text that the optimal contract involves framing, and that if there exists a frame $f$ such that buyers $U^f$-prefer the first-best bundle $(x_0^*, t_0^*)$ to $(\bar{x}, \bar{t})$, then buyers purchase the first-best bundle in the optimal contract. We now proceed to examine the case in which there is no such frame.

Denote by $(x, t)$ a bundle chosen by a buyer in an optimal contract. It cannot be that $x < x_0^*$, because then replacing $(x, t)$ with the bundle $(x_0^*, t + \Delta)$, where $\Delta = u(x_0^*, \theta) - u(x, \theta)$, would increase the seller’s profit (by increased attractiveness and the concavity of the seller’s profit-maximization problem). If $x = x_0^* < d$, then $t < t_0^*$ (otherwise $(x_0^*, t_0^*)$ is implementable), so $U(x_0^*, t, \theta) > U(x_0^*, t_0^*, \theta) = U(0, 0, \theta)$, and by optimality of the contract $U^f(x_0^*, t, \theta) = U^f(\bar{x}, \bar{t}, \theta)$. For small $\varepsilon > 0$, let $\Delta = u^f(x_0^* + \varepsilon, \theta) - u^f(x_0^*, \theta)$. Thus, $U^f(x_0^*, t, \theta)$ = $U^f(x_0^* + \varepsilon, t + \Delta, \theta)$. In addition, $\partial u^f(x_0^*, \theta)/\partial x$ = $\partial u^f(x_0^*, \theta)/\partial x$ = $c'(x_0^*)$ (the equality follows from the definition of $x_0^* < d$), so for sufficiently small $\varepsilon$ we have that $U^f(x_0^* + \varepsilon, t + \Delta, \theta) = U^f(\bar{x}, \bar{t}, \theta)$, $U(x_0^* + \varepsilon, t + \Delta, \theta) > U(0, 0, \theta)$, and $c(x_0^* + \varepsilon) - c(x_0^*) < \Delta$. Thus, replacing the bundle $(x, t)$ with $(x + \varepsilon, t + \Delta)$ increases the seller’s profit.

**Example 3.** Consider the price discrimination setting of Example 1, with $u(x, \theta) = x\theta$, $c'(x) = x$ for $x \leq 1$, and $c'(x) = 1 + (x - 1)/B$ for $x > 1$, where $B$ is large.$^{24}$ Suppose that the seller can only increase attractiveness substantially. Specifically, suppose that $F = \{\phi, f\}$, where $f = 9$. Suppose also that high type buyers’ $U$-willingness to pay for quality is much higher than that of low type buyers. Specifically, $L = 1$ and $H = 2$. Finally, suppose that $\pi_L > 1/2$.

We now specify two frameless contracts $D$ and $E$ by describing their menus $D$ and $E$, and show that the profit that any contract with the frame $f$ generates is strictly lower than the maximum of the profits that these two contracts generate. Let $D = \{(x_L^*, t_L^*), (x_H^*, t_H^*)\}$, where $x_L^* = 1$, $x_H^* = 1 + B$, $t_L^* = 1$, and $t_H = 2B + 1$. Then, $(x_L^*, t_L^*) \in C^\theta(D, \phi)$ and

$^{24}$The cost of producing $x$ units is therefore $c(x) = x^2/2$ for $x \leq 1$ and $c(x) = (1 - B + 2(B - 1)x + x^2)/2B$ for $x > 1$. 28
(x_H^*, t_H^*) \in C^H(D, \phi). When buyers choose these bundles, D generates profit \pi_H (B + 1) / 2 from high type buyers, which is only \pi_H less than the first-best profit from selling to high type buyers, and generates the first-best profit from low type buyers. Let E = \{(\varepsilon, \varepsilon), (x_H^*, t_H^* - \varepsilon)\} for some small \varepsilon > 0. Then, (\varepsilon, \varepsilon) \in C^L(E, \phi) and (x_H^*, t_H^* - \varepsilon) \in C^H(E, \phi). When buyers choose these bundles and \varepsilon is sufficiently small, the profit that E generates is strictly higher than the first-best profit from selling to high type buyers, because \pi_L > \pi_H.

Consider a contract with the frame f that excludes buyers of some type, i.e., these buyers choose stayout. If the contract excludes high type buyers, then the profit it generates is bounded above by the first-best profit from selling to low type buyers, which is strictly lower than the profit generated by D. If it excludes low type buyers, then the profit it generates is bounded above by the first-best profit from selling to high type buyers, which is strictly lower than the profit generated by E.

Now consider a non-excluding contract G with the frame f, denote by (x_\theta^*, t_\theta^*) \neq stayout the bundle that buyers of type \theta choose, and suppose that G generates more profit than any excluding contract. To generate more profit than D, the contract G must generate a profit of at least \pi_H (B + 1) / 2 from high type buyers, because D already generates the first-best profit from low type buyers. This implies that x_H' > B/4, because H = 2. Because low type buyers weakly U^f-prefer (x_L', t_L') to (x_H', t_H'), we must also have that x_L' (1 + f) - t_L' \geq x_H' (1 + f) - t_H'. Because t_L' \geq 0 (otherwise, excluding low type buyers and selling the first-best to high type buyers is profit enhancing), t_H' \leq 2x_H' (otherwise, high type buyers would strictly U-prefer stayout to (x_H', t_H')), and f = 9, we obtain that x_L' \geq 4x_H'/5 > B/5. But for B large enough, every unit above B/8 sold to low type buyers leads to a loss of at least 1/16, even if low type buyers are charged L = 1 per unit. This implies that for a large enough B the loss in G on low type buyers is larger than the possible gain on high type buyers.

**Proof of Proposition 2.** The proof of this result is identical to the proof of Proposition 2' below with the only changes being that (1) any frame f \neq \phi can be used in the modified contract and it should not be adjusted throughout the proof, and (2) Assumptions (A1) and (A2) replace Assumptions (A1)' and (A2)'.

**Proof of Proposition 2'.** Suppose to the contrary that there is an optimal separating contract that is frameless, and denote by g(\theta) = (x_\theta, t_\theta) the optimal allocation it implements. The standard theory tells us that x_L \leq x_L^*, x_H = x_H^*, low-type buyers are U-indifferent between (0, 0) and (x_L, t_L), and high-type buyers are U-indifferent between (x_L, t_L) and (x_H, t_H).

Consider a modified contract with the same menu \{(x_L, t_L), (x_H, t_H)\} and the frame f = (x_H, t_H). By Assumption (A1)', high-type buyers strictly U^f-prefer (x_H, t_H) to (x_L, t_L), but low-type buyers may also U^f-prefer (x_H, t_H) to (x_L, t_L). We now modify the bundles and the frame in a way that increases profit. First, increase t_H (in the bundle (x_H, t_H) and in f) until high-type buyers are either U^f-indifferent between (x_H, t_H) and (x_L, t_L) or are U-indifferent
between \((x_H, t_H)\) and \((0,0)\). If the latter occurs before the former, increase \(x_L\) and \(t_L\) along the 
U-indifference curve of low type buyers through \((0,0)\). By Assumption (A3), high-type buyers 
will be \(U^f\)-indifferent between \((x_H, t_H) = (x^*_H, t^*_H)\) and \((x_L, t_L)\) before \((x_L, t_L)\) reaches \((x^*_L, t^*_L)\).

By Assumption (A2)', in the modified contract a type \(\theta\) buyer chooses \((x_\theta, t_\theta)\). Moreover, 
because \(t_H\) is higher and \((x_L, t_L)\) generates more profit in the modified contract, the modified 
contract generates a strictly higher profit than the original contract, a contradiction.

**Proof of Proposition 3.** If \(f = \phi\), then we are in the standard setting, in which these properties 
are well known. Suppose that \(f \neq \phi\). If the contract is a pooling one, then the properties follow 
immediately. It thus remains to consider a separating contract in which all buyers choose a 
positive product and \(f \neq \phi\). The proof of this part is identical to the proof of Proposition 3' 
below with Assumptions (A1) and (A2) replacing Assumptions (A1)' and (A2)'.

**Proof of Proposition 3'.** Fix a frame \(f \neq \phi\). The seller's profit-maximization problem 
(conditional on \(f\)) is:

Choose \(((x_L, t_L), (x_H, t_H))\) to maximize \(\pi_L(t_L - c(x_L)) + \pi_H(t_H - c(x_H))\) subject to:

\[
\begin{align*}
\text{IR}_{\theta}^U & : U(x_\theta, t_\theta, \theta) \geq U(0,0, \theta) \quad \text{for} \quad \theta \in \{L, H\}, \\
\text{IC}_{\theta}^f & : U^f(x_\theta, t_\theta, \theta) \geq U^f(x_{\theta'}, t_{\theta'}, \theta) \quad \text{for} \quad \theta, \theta' \in \{L, H\} \quad \text{and} \quad \theta' \neq \theta.
\end{align*}
\]

Considering an optimal contract, we first note that if \(\text{IC}_{\theta}^f\) holds strictly, then \(\text{IR}_{\theta}^U\) binds, 
otherwise \(t_\theta\) can be increased slightly without violating any of the constraints. This implies 
that either \(\text{IC}_{\theta}^f\) or \(\text{IC}_{L}^f\) bind. Otherwise, because by Assumption (A3) \(((x_L, t_L), (x_H, t_H)) \neq 
((x^*_L, t^*_L), (x^*_H, t^*_H))\), some \(x_\theta\) can be increased or decreased slightly along the \(U\)-indifference 
curve of agent \(\theta\) to decrease \(|x_\theta - x^*_\theta|\), which increases the principal's profit, without violating 
any of the constraints.

In fact, \(\text{IC}_{L}^f\) must bind. Indeed, suppose that \(\text{IC}_{L}^f\) binds. By Assumption (A2)', because 
\(x_L < x_H\), \(\text{IC}_{L}^f\) holds strictly, so \(\text{IR}_{\theta}^U\) binds. We now modify the bundles in a series of steps in a 
way that increases profit, such that either at some point along the sequence all the constraints 
are satisfied, so the modified bundles generate more profit than the optimum, a contradiction, 
or the modified bundles are \((x^*_\theta, t^*_\theta)\) and \(\text{IC}_{L}^f\) holds, which contradicts Assumption (A3). The 
first step applies if \(x_H > x^*_H\). In this case, decrease \((x_H, t_H)\) continuously along the high type's 
\(U\)-indifference curve until either \(\text{IC}_{L}^f\) binds or \(x_H = x^*_H\). In the former case, Assumption (A2)' 
implies that \(\text{IC}_{L}^f\) holds, so all the constraints are satisfied and the principal's profit increases, 
a contradiction. We therefore have that \(x_H \leq x^*_H\) and \(\text{IC}_{L}^f\) holds strictly. Now increase \(t_L\) until 
\(\text{IR}_{\theta}^U\) binds. This further relaxes \(\text{IC}_{H}^f\). Finally, if \(x_L < x^*_L\), increase \((x_L, t_L)\) continuously along 
the low type's \(U\)-indifference curve until either \(\text{IC}_{H}^f\) binds or \(x_L = x^*_L\). In the former case, we

\[25\] It must be that \(x_H \geq x_L\), because by \(\text{IR}_{L}^f\) and (A2)' \((x_L, t_L)\) lies below the high type's \(U\)-indifference curve 
through \((0,0)\), so \(\text{IC}_{H}^f\) binds before \(x_H\) reaches \(x_L\).
obtain a contradiction as in the first step. We have therefore reached a situation in which (i) \( x_L \geq x_L^* \) and \( \text{IR}_L^U \) binds, (ii) \( x_H \leq x_H^* \) and \( \text{IR}_H^U \) binds, and (iii) \( \text{IC}_L^H \) holds strictly. Now, (i), Assumption (A1)', and Assumption (A2)' imply that

\[
U(x_L, t_L, L) = U(x_L^*, t_L^*, L) \Rightarrow U(x_L, t_L, H) \geq U(x_L^*, t_L^*, H) \Rightarrow U^f(x_L, t_L, H) \geq U^f(x_L^*, t_L^*, H),
\]

and (ii) and Assumption (A1)' imply that

\[
U(x_H^*, t_H^*, H) = U(x_H, t_H, H) \Rightarrow U^f(x_H^*, t_H^*, H) \geq U^f(x_H, t_H, H),
\]

so by (iii) we have \( U^f(x_H^*, t_H^*, H) > U^f(x_H^*, t_h^*, H) \), which contradicts Assumption (A3).

Because \( \text{IC}_L^f \) binds, by (A2)' we have that \( \text{IC}_L^f \) holds strictly, so \( \text{IR}_L^U \) binds.

**Proof of Proposition 4.** First observe that \( x_L \leq x_L^* \) and \( x_H \geq x_H^* \). Indeed, if \( x_L > x_L^* \), then decrease \( (x_L, t_L) \) slightly along low-type buyers’ \( U \)-indifference curve so that \( \text{IC}_L^f \) continues to hold. By Assumption (A1) and (A2) this relaxes \( \text{IC}_H^f \), so all constraints hold and the profit increases, a contradiction. If \( x_H < x_H^* \), then increase \( (x_H, t_H) \) slightly along the high type’s \( U \)-indifference curve so that \( \text{IC}_L^f \) continues to hold. By Assumption (A1) this relaxes \( \text{IC}_H^f \), so all constraints hold and the profit increases, a contradiction.

Finally, suppose that \( x_H^* < d \) and \( x_H = x_H^* \). If \( \text{IR}_H^U \) holds strictly, then \( x_L < x_L^* \), similarly to the standard setting.\(^{26} \) And if \( \text{IR}_H^U \) binds, then \( x_L < x_L^* \), because Assumption (A3) implies that \( \{(x_L, t_L), (x_H, t_H)\} \neq \{(x_L^*, t_L^*), (x_H^*, t_H^*)\} \). But \( x_L < x_L^* \) implies that the principal’s marginal profit at \( x_L \) along the low type’s \( U \)-indifference curve is positive, while \( x_H = x_H^* \) implies that the principal’s marginal profit at \( x_H \) along the high type’s \( U \)-indifference curve is 0. Therefore, the profit can be increased by increasing \( x_H \) slightly along the high type’s \( U \)-indifference curve, which relaxes \( \text{IC}_H^f \) and makes it possible to increase \( x_L \) along the low type’s \( U \)-indifference curve.\(^{27} \)

**Proof of Property 3.** Consider the profit-maximization problem:

\({}^{26}\)If \( x_L^* = x_L \), then \( x_L^* = x_L < x_H = x_H^* < d \), because the contract is separating. That \( x_L^* < d \) implies that the principal’s marginal cost at \( x_L^* \) is equal to low type buyers’ marginal \( U \)-utility, whereas \( x_L^* < x_H^* \) implies that high type buyers’ marginal \( U \)-utility at \( x_L^* \) is strictly higher (because the profit function is concave along each type’s \( U \)-indifference curve). Therefore, by Assumption (A2), decreasing \( x_L \) by some small \( \varepsilon \) along the low type’s \( U \)-indifference curve decreases high type buyers’ \( U \)-utility from the bundle \( (x_L, t_L) \) by at least \( \delta \varepsilon \) for some \( \delta > 0 \) that is independent of \( \varepsilon \). This decreases means that \( t_L \) can be increased by \( \delta \varepsilon \) without violating \( \text{IC}_H^f \). Thus, for sufficiently small \( \varepsilon \) this leads to an increase in the principal’s profit, because to a first order the change in profit from changing the the low type’s bundle is 0, and this change allows an increase in profit from the high type that is positive to a first order.

\({}^{27}\)More precisely, increasing \( x_H \) by some small \( \varepsilon \) along the high type buyers’ \( U \)-indifference curve increases their \( U \)-utility from the bundle \( (x_H, t_H) \) by at least \( \delta \varepsilon \) for some \( \delta > 0 \) that is independent of \( \varepsilon \). And increasing \( x_L \) by some small \( \gamma \) along the low type buyers’ \( U \)-indifference curve increases the high type buyers’ \( U \)-utility from the bundle \( (x_L, t_L) \) by no more than \( \alpha \gamma \) for some \( \alpha > 0 \). Thus, the increase of \( x_H \) by \( \varepsilon \) allows to increase \( x_L \) by at least \( \delta \varepsilon / \alpha \). And because the marginal effect on the profit of such an increase in \( x_H \) is 0, whereas the marginal effect on the profit of the increase in \( x_L \) is positive, for small \( \varepsilon \) the profit increases.
Choose \(((x_L, t_L), (x_H, t_H), f = (x_f, t_f))\) to maximize \(\pi_L(t_L - c(x_L)) + \pi_H(t_H - c(x_H))\) subject to:

\[
\text{IR}_U^\theta : U(x_\theta, t_\theta, \theta) \geq U(0, 0, \theta) \quad \text{for} \quad \theta \in \{L, H\},
\]

\[
\text{IC}_f^\theta : U_f(x_\theta, t_\theta, \theta) \geq U_f(x_{\theta'}, t_{\theta'}, \theta) \quad \text{for} \quad \theta, \theta' \in \{L, H\} \quad \text{and} \quad \theta' \neq \theta.
\]

Assume to the contrary that there exists a contract that solves this problem in which the reference coverage \(x_f\) differs from the high type’s coverage \(x_H\). (Strictly speaking, this is not a contract yet because it ignores the constraints that individuals should \(U^f\)-prefer their bundle to the reference bundle.)

By Proposition 3’, \(\text{IC}_H^f\) holds with equality in this contract, and because \(x_H > x_L\), by Assumption (A2)’ \(\text{IC}_L^f\) holds strictly. We now modify this contract by modifying the reference coverage to derive a contradiction to Proposition 3’. If \(x_f < x_H\), then increase \(x_f\) slightly to \(x_\tilde{f}\) (or slightly above the low-risk individual’s coverage \(x_L\) if \(x_f < x_L\)) so \(\text{IC}_L^f\) still holds. This increases the regret associated with purchasing the low-risk individuals’ bundle (but not with purchasing the high-risk individuals’ bundle), so \(\text{IC}_H^f\) holds strictly, and the constraints \(\text{IR}_U^\theta\) continue to hold because they are with respect to \(U\). If \(x_f > x_H\), then decrease \(x_f\) slightly to \(x_\tilde{f}\) so \(\text{IC}_L^f\) still holds. This makes low-risk individuals’ bundle less attractive relative to that of high-risk individuals, because \(r\) is concave and \(x_H > x_L\). Again, this implies that \(\text{IC}_H^f\) holds strictly and all other constraints hold. In both cases, the new separating contract generates the same profit as the original one, and is therefore optimal, but in contradiction to Proposition 3’, the constraint \(\text{IC}_H^f\) holds strictly.

We thus obtain that in any contract that solves the above problem, \(x_f = x_H\), and \(t_f\) can be chosen to be equal to \(t_H\). Consequently, any pair of bundles that solves the above problem together with the frame \(f = (x_H, t_H)\) also solves the same problem with the added constraint that individuals \(U^f\)-prefer their bundle to the reference bundle. Therefore, \(x_f = x_H\) in any contract that solves the same problem with the added constraint.\(^{28}\)

**Proof of Property 4.** Consider an optimal contract in which high-risk individuals buy insurance. As in the proof of Proposition 4, we have that \(x_L \leq A = x_L^*\) and \(x_H \geq A = x_H^*\). Thus, to complete the proof it suffices to verify that any contract in which low-risk individuals are partially insured, high-risk individuals are fully insured, and full coverage is highlighted is not optimal.

Consider such a contract, and increase the high-risk individuals’ coverage and premium slightly along their \(U\)-indifference curve. This does not change the provider’s profit to a first order, because when high-risk individuals are fully insured their willingness to pay for an additional unit of insurance is equal to the provider’s cost of providing this unit. Because the new

\(^{28}\)This is because both problems maximize the same function but the domain of the former is a superset of the latter. Thus, the fact that a maximizer of the former lies in the domain of the latter implies that all maximizers of the latter are maximizers of the former.
coverage is larger than the reference coverage, $U$-indifference implies that a high-risk individual is also $U^f$-indifferent between his original bundle and the new bundle, so $IC^f_H$ continues to hold; and $IC^f_L$ continues to hold because it held strictly before the change. Now increase $x_f$ to equal the new coverage $x_f^\sim$ for the high-risk individual. Then $IC^f_H$ holds strictly, and $IC^f_L$ continues to hold if the change in coverage is small enough. Finally, increase the low-risk individuals’ coverage and premium slightly along their $U$-indifference curve, which strictly increases profit to a first order and does not violate any of the constraints.

References


