

The Simple Economics of Conglomeration with Bankruptcy Costs: Separate or Joint Financing?*

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February 2009

Abstract

Which projects should be financed through separate non-recourse loans (or limited-liability companies) and which should be bundled into a single loan? In the presence of bankruptcy costs, this conglomeration decision trades off the benefit of *co-insurance* with the cost of *risk contamination*. This paper characterizes this tradeoff for projects with binary returns, depending on the mean, variability, and skewness of returns, the bankruptcy recovery rate, the correlation across projects, the number of projects, and their heterogeneous characteristics. In some cases, separate financing dominates joint financing, even though it increases the interest rate or the probability of bankruptcy.

Journal of Economic Literature Classification Codes: G32, G34.

Keywords: Bankruptcy, conglomeration, mergers, spin-offs, project finance.

*We thank Viral Acharya, Philip Bond, Patrick Bolton, Denis Gromb, Florian Heider, Roman Inderst, Fausto Panunzi, Sherrill Shaffer, Peter Norman Sørensen, Jean Tirole, Vish Viswanathan, and seminar participants at Arizona, Barcelona, Duke, Granada, London, Loughborough, Mannheim, Milan, and the European Winter Finance Conference 2008 for helpful feedback.

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Consider a firm with access to two risky projects with positive net present value. The firm has the choice of financing these projects jointly within a single company or financing them separately by setting up two independent companies. In either case, the firm can finance the projects only by borrowing from a competitive credit market through standard debt. If a company's returns fall below the debt repayment obligation, a fraction of the returns are lost because of bankruptcy costs. When should the firm finance the projects jointly and when separately? More generally, what is the optimal corporate structure in the presence of bankruptcy costs?

According to the conventional wisdom in corporate finance, conglomeration should bring about a reduction in the probability of bankruptcy by allowing the firm to use the proceeds of a successful project to save an unsuccessful one, which would have failed otherwise. By aggregating imperfectly correlated cash flows, the argument goes, the firm should be able to reduce expected bankruptcy costs and increase borrowing capacity—see Lewellen (1971). Conventional wisdom has largely settled on the view that the purely financial synergies from savings in bankruptcy costs achieved through conglomeration are always positive. As aptly summarized by Brealey, Myers, and Allen's (2006, page 880) textbook, “merging decreases the probability of financial distress, other things equal. If it allows increased borrowing, and increased value from the interest tax shields, there can be a net gain to the merger.”

This paper amends this conventional view by revisiting the *purely financial* effects of conglomeration in the presence of bankruptcy costs. We show that bankruptcy costs alone create a non trivial tradeoff for conglomeration, even abstracting from tax considerations. Our analysis clarifies conditions for when the logic of the conventional argument is reversed—sometimes when projects are financed jointly, failure of one project can drag down another profitable project that would have stayed afloat otherwise. We also show that this effect can be so strong to make it optimal in some cases for a firm to finance projects *separately*, even though joint financing would involve paying a *lower* repayment rate or would result in a *lower* probability of bankruptcy.

While the literature has focused mostly on the *co-insurance* effect of conglomeration, our analysis uncovers the *risk contamination* effect, whereby aggregating risky assets can generate incremental distress costs. Thus we formalize Esty's (2002) argument that “this phenomenon [risk contamination] must be balanced against the benefits of co-insurance

received from the project.”

Our results have implications for the conglomeration advantages and disadvantages of mergers and acquisitions, corporate spin-offs, and structured finance. Our model applies equally well to the choice of how a firm should bundle projects financed through non-recourse debt. For example, a firm with access to two projects could finance them either jointly or separately through two non-recourse loans. When a project is financed separately through non-recourse debt, if the firm does not meet the repayment obligation on one project, creditors do not have access to the returns of the other project. Thus, our results offer a simple explanation for the common use of project finance, which involves the transfer of a subset of a company’s assets into a special purpose vehicle financed with non-recourse debt.¹

To best uncover the tradeoff between co-insurance and risk contamination, we analyze the simplest model in which each project has binary returns, either low or high. We focus on the interesting case in which separate financing of a project results in default when the return is low. In that case, creditors are able to recover a fraction of the project’s return, while the remaining fraction is lost due to bankruptcy costs. Perfect competition among risk-neutral creditors drives down the required repayment obligation to a level at which creditors expect to exactly recoup the initial investment outlay (partly through the full repayment in case the project’s return is high and through the recovered fraction of the low return).

Next, suppose instead the two projects are financed jointly through a single loan. Clearly, default will result when both projects yield a low return. The interesting case to consider is when one project has a low return and the other has a high return. Suppose first that the sum of these two returns is sufficiently high to meet the required repayment obligation to creditors. If so, the high return project saves the low return project from bankruptcy. Given that the probability of bankruptcy is reduced by joint financing compared to separate financing, creditors are forced to further reduce the required repayment obligation. This is the logic of the “good” conglomeration stressed by Lewellen (1971).

If instead the sum of low and high returns is not sufficiently high to meet the required repayment obligation to creditors, conglomeration is “bad”. We derive conditions on the

¹See, for example, Gorton and Souleles (2006) for an introduction to project finance and a discussion of the importance of bankruptcy costs in that context.

exogenous parameters for good and bad diversification to arise, taking into account that the required repayment obligation is endogenously determined by the creditors' zero profit condition. We fully characterize when joint financing dominates separate financing, depending on the distribution of projects' returns, the recovery rate in case of bankruptcy, the correlation across projects, the number of projects, and the heterogeneous characteristics of projects.

An important practical implication of our result is that it is not always optimal for a firm to finance projects at the lowest repayment rate. We identify situations in which conglomeration is unprofitable, even though it entails a lower repayment rate. When the recovery rate in case of bankruptcy is sufficiently high, creditors are forced by competition to offer temptingly low repayment rates to firms that finance projects jointly. Given that creditors break even whether the projects are financed separately or jointly, the firm acts as residual claimant of the projects' expected returns net of the bankruptcy costs. Hence, the firm should optimally finance the projects separately at a higher repayment rate, thereby reducing expected bankruptcy costs. Even though the repayment rate is lower with joint financing, the firm bears the brunt for the inefficient increase in expected bankruptcy costs associated with joint financing.

In terms of the literature, we depart from Modigliani and Miller's (1958) world without financial synergies by introducing bankruptcy costs.² By clarifying the conditions for the value of conglomeration, this paper contributes to a voluminous literature on the purely financial motives for mergers. In his discussion to Lewellen (1971), Higgins (1971) notes that project bundling affects also the riskiness of the lender's returns—we instead abstract from risk by assuming risk neutrality. Scott (1977) and Sarig (1985) show that if cash flows can be negative, a firm can exploit the shelter of limited liability by financing projects in separate companies. In our analysis we explicitly abstract from this limited liability effect, so that the mode of financing does *not* affect the payoff of third parties, but only that of the firm and its creditor.

Our results are most closely related to three recent contributions to the corporate finance literature. First, Winton (1999) in the third case of his Proposition 3.1 discusses a situation in which a bank prefers to specialize even though the repayment rate for pooled

²In the absence of bankruptcy costs, diversification is not a valid argument for mergers. This is because investors can obtain the same diversification themselves by purchasing appropriate amounts of the unmerged firms. See, for example, Levy and Sarnat (1970).

projects is lower. Despite the differences in the model, our Proposition 7 on the occurrence of ugly conglomeration is similar to Winton’s earlier result. Second, Inderst and Müller (2003) analyze the pros and cons of project bundling in a two-project version of Bolton and Scharfstein’s (1990) dynamic model of debt. In their setting, financing two projects within the same company can reduce the firm’s ability to borrow when the firm is able to make follow-up investments without having to return to the capital market.³ This a very different channel through which bad conglomeration arises, as we explain in Appendix B. Third, Leland (2007) shows that financial separation can be beneficial when it allows a firm to fine tune the capital structure (mix of debt and equity) to the specific characteristics of projects with heterogeneous returns.⁴ In our paper, instead, we explicitly rule out the possibility of re-optimizing the capital structure by requiring projects to be financed with debt only. In addition, we provide simple analytical results which clarify what drives the sign of the profitability of conglomeration.⁵

The paper proceeds as follows. Section 1 formulates the model. Focusing on the baseline version of the model with two identically and independently distributed projects, Section 2 analyzes the conditions setting apart good from bad conglomeration and performs comparative statics with respect to the distribution (mean, variance, and skewness) of returns and the bankruptcy recovery rate. Section 3 analyzes the effect of correlation across projects. Section 4 turns to the economics of conglomeration when the number of projects is large. Section 5 extends the analysis to the case when projects have heterogeneous returns. Section 6 shows that separate financing might dominate joint financing even if it increases the repayment rate or the probability of bankruptcy. Section 7 concludes. Appendix A collects the proofs omitted from the text. Appendix B analyzes a dynamic version of the model with non-verifiable returns and optimal financial contracting.

³See also Faure-Grimaud and Inderst (2005).

⁴A number of papers (e.g., Higgins and Schall, 1975, and Kim and McConnell, 1977) have analyzed the effect of the current capital structure on merger incentives. These papers noted that, while mergers may increase total firm value, bondholders may gain at the expense of shareholders. We abstract from such a distributional conflict among (cashless) stakeholders, by considering the ex-ante choice of corporate structure by shareholders and forcing bondholders to compete and therefore obtain no surplus.

⁵Our results are very different from those of Shaffer (1994), who studies the effect of joint financing on the probability of *joint* failure. Instead, we compare the firm’s expected payoff when the interest rate is endogenously determined by competition among creditors.

1 Model

This section formulates our model of debt financing by a risk-neutral borrower endowed with n projects. This is the simplest possible model designed to analyze how a borrower should allocate projects to loans in a perfectly competitive credit market with bankruptcy costs. In the rest of the paper we derive results for special cases of the model.

Each project i requires at $t = 1$ an investment outlay normalized to $I = 1$ and yields at $t = 2$ a random return r^i with a binary distribution: the return is either low, $r^i = r_L^i > 0$, with probability $1 - p_i$, or high, $r^i = r_H^i > r_L^i$, with probability p_i . Each project has positive net present value, $(1 - p)r_L^i + pr_H^i - 1 > 0$. The low return is insufficient to cover the initial investment outlay, so that $r_L^i < 1$. Returns are possibly correlated across projects.⁶

The borrower needs to raise external finance from creditors at $t = 1$. The borrower chooses how to group projects into separate non-recourse loans. This means that creditors on each loan have access to the returns of all projects they finance through that loan, but they do not have access to the returns of other projects that are financed through other loans.

Creditors lend money by way of standard debt contracts at $t = 1$. Creditors are risk neutral and operate in a perfectly competitive market. Without loss of generality we normalize the interest rate to $i = 0$.⁷ Therefore, creditors expect to make zero expected profits from lending. This is equivalent to assuming that the borrower makes a take-it-or-leave-it repayment offer to a single creditor for each loan j , promising to repay r_j^* at $t = 2$ for each unit borrowed at $t = 1$.⁸ For notational simplicity, we stipulate that the borrower accepts to be financed only when expecting to obtain *strictly* positive expected payoff. According to our accounting convention, the borrower's repayment obligation comprises principal as well as net interest—this repayment obligation can be interpreted as the gross interest rate, given our normalization of the investment outlay to 1.

⁶See Section 3.

⁷To see that $I = 1$ and $i = 0$ are innocuous normalizations, suppose that the investment outlay is equal to I and that the creditors' rate of time preference (or required interest rate) is $i > 0$. Denoting the random cash flow by R_i , the project's return in the model can be reinterpreted in terms of percentage gross return for each unit of period-2 equivalent outlay: $r_i = R_i/[I(1 + i)]$. Thus, it is without loss of generality to set $I = 1$ and $i = 0$.

⁸Thus, for the case in which each loan (or company) is financed by multiple creditors, we implicitly assume that there are no coordination failures across the creditors who syndicate the same loan.

On each loan, the borrower repays the creditor in full when the total realized return of the projects pledged is sufficient to cover the promised repayment, r_j^* . If instead the total realized return falls short of the repayment obligation, the borrower defaults and the ownership of the projects' realized returns is transferred to the creditor. Due to bankruptcy costs, following default the creditor is able to recover only a fraction β of the returns. The remaining fraction $1 - \beta$ of the returns is lost. The bankruptcy recovery rate $\beta \in [0, 1]$ measures the efficiency of bankruptcy and is industry specific.⁹

Note that the debt contract posited here is the optimal contractual arrangement if we assume that returns are privately observed by the borrower and can be verified only at a cost (equal to the bankruptcy cost), as in the costly state verification model of Townsend (1978) and Gale and Hellwig (1985). See Appendix B for an alternative derivation of the optimality of the debt contract in a dynamic version of our model developed along the lines of Bolton and Scharfstein (1990) and Hart and Moore (1998).

The borrower's problem analyzed here can be equivalently reinterpreted in terms of a company's decision to merge with (or spin off from) another company. Suppose that a borrower can assign its projects to different limited liability companies set up at no cost, and then seek independent financing for the projects assigned to each company. All the projects allocated to the same company are financed jointly, but independently from projects financed in the other companies set up by the same borrower. In this equivalent reformulation, the borrower's problem is to decide how to group projects into companies.

2 Two Identical and Independent Projects

This section analyzes the simplest possible specification of the model with $n = 2$ identically and independently distributed projects. Each project i yields a low return $r_L^i \equiv r_L$ with probability $1 - p_i \equiv 1 - p$ and a high return $r_H^i \equiv r_H > r_L$ with probability $p_i \equiv p$. We proceed by first examining the conditions for when the borrower is able to finance the two projects separately and jointly (Section 2.1). Second, we compare the profitability of separate and joint financing, when they are both feasible (Section 2.2). Third, we illustrate that separate financing can be easily optimal for empirically plausible parameter values (Section 2.3). Finally, we derive a set of comparative statics predictions for the occurrence

⁹For estimates of bankruptcy costs and other costs of financial distress across industries see, for example, Warner (1977), Weiss (1990), and Korteweg (2006).

of joint and separate financing (Section 2.4).

2.1 Financing Conditions

Consider first the possibility of financing the two projects through two separate non-recourse loans (or, equivalently, through two different limited liability companies). Given that the two projects are ex ante identical, when each of them is financed, financing will take place at the same equilibrium repayment rate (equal to the nominal repayment obligation). Such rate r_i^* must satisfy $r_L < 1 < r_i^* < r_H$, so that there is a positive probability that the loan is not repaid in full. Indeed, the firm would not accept to be financed at a rate above r_H , because this would result in zero payoff for the firm. Also, the rate must be above 1 because at rates at or below 1 the creditor would make negative expected profits (by obtaining a return never above the investment outlay of 1 and strictly below 1 with strictly positive probability) and therefore would not be willing to extend the loan.

In a competitive credit market, creditors make zero expected profits. Thus, the repayment requested by the creditor is r_i^* such that the gross profits, $pr_i^* + (1-p)\beta r_L$, are equal to the initial investment outlay 1. As a result, each project can be financed through a separate loan if and only if

$$r_i^* := \frac{1 - (1-p)\beta r_L}{p} < r_H. \quad (1)$$

Clearly, $r_i^* > 1$.

Next, consider joint financing of the two projects through a single loan (or, equivalently, within the same company). Denote by r_m^* the equilibrium repayment obligation *per unit of investment*, so that $2r_m^*$ is the total repayment promised to investors in return for the initial financing of the two projects, $2I = 2$. Two cases need to be distinguished, depending on whether or not the required repayment rate induces default when one project yields a high return while the other project yields a low return—this is the case with intermediate returns.

Suppose first that the equilibrium repayment rate r_m^* is such that $r_L < r_m^* < \frac{r_H+r_L}{2}$, so that there is no default with intermediate returns. As a result, the probability of default of the loan is reduced to $(1-p)^2$. Substituting again in the expected creditor profits, the

borrower would only be able to obtain this rate in a competitive market if and only if

$$r_m^* := \frac{1 - (1 - p)^2 \beta r_L}{1 - (1 - p)^2} < \frac{r_H + r_L}{2}. \quad (2)$$

Suppose now that the equilibrium repayment rate r_m^{**} is such that $\frac{r_H + r_L}{2} < r_m^{**} < r_H$ and therefore the borrower defaults in the event of a high and a low return. Hence, default occurs with probability $1 - p^2 = (1 - p)^2 + 2p(1 - p)$. In a competitive credit market, this case arises if and only if

$$r_m^{**} := \frac{1 - (1 - p) \beta (pr_H + r_L)}{p^2} < r_H. \quad (3)$$

Since the borrower's expected profits for a given distribution are decreasing in the equilibrium rate, if both conditions (2) and (3) are satisfied, the borrower prefers rate r_m^* to rate r_m^{**} .¹⁰ Summarizing the results so far, we have the following proposition.

Proposition 1 Projects can be financed separately if and only if $r_i^* < r_H$, in which case the equilibrium repayment rate is r_i^* . When the borrower seeks joint finance, if $r_m^* < (r_H + r_L)/2$, then the equilibrium rate is r_m^* ; if $r_m^* > (r_H + r_L)/2$ and $r_m^{**} < r_H$, then the equilibrium rate is r_m^{**} ; otherwise, the projects cannot be financed.

Figure 1 depicts how *per-project* expected returns are divided between borrower and creditor in the three scenarios described by Proposition 1. The area above the distribution function in all the panels is equal to the project's expected return. When the two projects are financed separately, the return of each project is a binary random variable with a cumulative distribution represented in Panel (a). When the two projects are bundled and financed jointly, there are three possible realized returns. Panels (b) and (c) display the cumulative distributions of the (per-project) returns resulting with joint financing for two different examples—in each graph the dashed distribution corresponds to the returns resulting with separate financing. Note that the distribution of (per-project) returns with separate financing is a mean-preserving spread of the distribution with joint financing. Intuitively, joint financing steepens the return distribution around the center by inducing an anti-clockwise rotation.

¹⁰It is straightforward to show that if $r_m^* > (r_H + r_L)/2$, then $r_m^{**} > (r_H + r_L)/2$. Therefore, if it is not possible to obtain r_m^* , then we can disregard the $r_m^{**} > (r_H + r_L)/2$ constraint.

For any given repayment rate r , the net expected return for the borrower corresponds to the area above the cumulative distribution of (per-project) returns at r , $F(r)$, and to the right of r (in blue). The gross expected return of the creditor is the sum of (i) the area above $F(r)$ and to the left of r (in yellow) and (ii) the fraction β of the area below $F(r)$ and above the distribution function (in pink). The first area is equal to pr , which is the full repayment of the outstanding obligation multiplied by the probability that the project stays afloat, p . The second area is equal to $(1-p)\beta r_L$, capturing the expected returns obtained in case of bankruptcy. The remaining fraction $1-\beta$ of the pink area is equal to the expected bankruptcy costs. This is also equal to the difference between the net present value of the company, the area above the distribution function and below 1, and the sum of creditor's and borrower's profits.

The equilibrium rate r^* in the three panels is such that the gross expected return of the creditor is equal to 1. Projects can be financed separately as long as the creditor's gross returns at a rate r_H are greater than 1, as in Panel (a). Projects can be financed jointly at a rate below the crossing point as long as the per-project creditor returns at $(r_H + r_L)/2$ are greater than 1, as in Panel (b). Projects can only be financed jointly at a rate above the crossing point if the per-project creditor returns at $(r_H + r_L)/2$ are lower than 1 and at r_H are greater than 1, as in Panel (c).

2.2 Good and Bad Conglomeration

When both separate and joint financing are feasible, which one should the borrower choose? Obviously, in the absence of bankruptcy costs (i.e., when $\beta = 1$) the borrower is indifferent between financing the projects separately or jointly. The next proposition states the gains and losses when $\beta < 1$.

Proposition 2 When the borrower can finance both projects separately and jointly:

(a) If the joint rate is r_m^* , then the borrower should finance the projects jointly because of the co-insurance effect. The per-project incremental surplus for the borrower of joint rather than separate financing is $p(1-p)(1-\beta)r_L$.

(b) If the joint rate is r_m^{**} , then the borrower should finance the projects separately because of the risk contamination effect. The per-project incremental surplus for the borrower of separate rather than joint financing is $p(1-p)(1-\beta)r_H$.

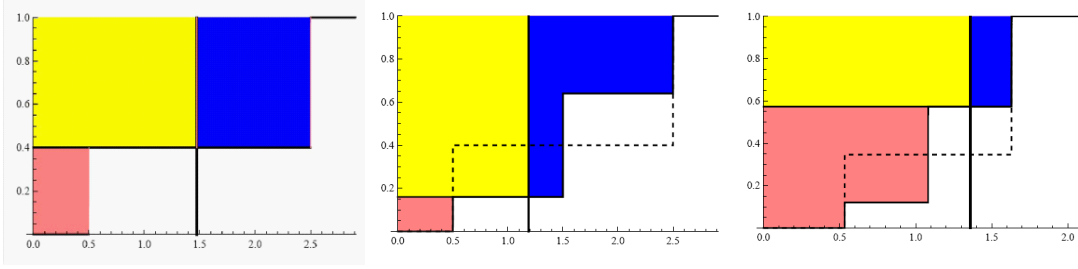


Figure 1: Distribution of Returns. The area above the distribution function represents the project’s expected return. For a given repayment rate r , the net expected return for the borrower corresponds to the area above the distribution function and to the right of r (in blue). The gross expected return for the creditor is the sum of (i) the area above $F(r)$ and to the left of r (in yellow) and (ii) the fraction β of the area below $F(r)$ and above the distribution function (in pink). The equilibrium rate r^* is such that the gross expected return for the creditor is equal to 1. Projects can be financed separately if the creditor’s gross expected return at the rate r_H are greater than 1, as in Panel (a). Projects can be financed jointly at a rate below the crossing point if the per-project creditor returns at $(r_H + r_L)/2$ are greater than 1, as in Panel (b). Projects can only be financed jointly at a rate above the crossing point if the creditor’s per-project returns at $(r_H + r_L)/2$ are smaller than 1 and at r_H are greater than 1, as in Panel (c).

Intuitively, when the borrower obtains a rate that avoids intermediate bankruptcy, the probability of default under joint financing is lower than under separate financing. The low-return project is saved from default when the other project yields a high return, thereby reducing the inefficiency associated with bankruptcy. Per-project expected savings when the projects are financed jointly rather than separately—the “co-insurance effect”—are equal to the probability that the first project yields a low return while the second project yields a high return, $p(1 - p)$, multiplied by the avoided losses due to bankruptcy costs, $(1 - \beta)r_L$. Graphically, per-project savings due to the co-insurance effect associated with joint financing are represented by a fraction $(1 - \beta)$ of the red area in Panel (a) of Figure 2.

If, instead, the borrower obtains a joint rate that does not avoid intermediate bankruptcy, a low-performing project drags down the other, increasing the probability of default. Per-project expected losses when projects are financed jointly rather than separately—the “risk contamination effect”—are equal to the probability that the first project yields a high return while the second project yields a low return, $p(1 - p)$, multiplied by the additional losses in bankruptcy costs incurred, $(1 - \beta)r_H$. Graphically, the per-project costs due to the risk contamination effect associated with joint financing are represented

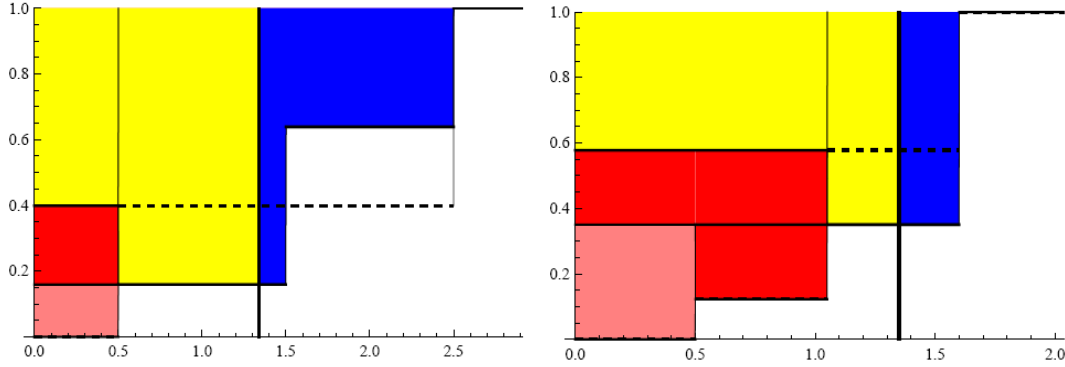


Figure 2: **Good and Bad Conglomeration.** Panel (a): Projects can be financed jointly at a rate below the crossing point. The reduction in expected bankruptcy costs obtained with joint rather than separate financing (co-insurance effect) is equal to the red area. Panel (b): Projects cannot be financed jointly at a rate below the crossing point. The increase in expected bankruptcy costs obtained with joint rather than separate financing (risk contamination effect) is equal to the red area.

by a fraction $(1 - \beta)$ of the red area in Panel (b) of Figure 2.

The key question is whether the equilibrium repayment rate for joint financing is below or above the crossing point, $(r_H + r_L)/2$. In conclusion, joint financing is optimal in case (a) when condition (2) is satisfied—otherwise separate financing is optimal.

Notice that the crossing point is not necessarily at the mean. In particular, if $p > 1/2$, so that the distribution is skewed to the left (negatively skewed), the crossing point is below the mean. As a result, equilibrium rates above the crossing point are consistent with a probability of default below 50%. The resulting default probabilities would be $1 - p$ for separate financing and $1 - p^2$ for joint financing, which for p high enough may be very low, as illustrated in the following numerical example.

2.3 Illustration

We now present an illustration of how conglomeration can result in an increase in expected bankruptcy costs for empirically plausible parameter values. To this end, Figure 3 calibrates the four parameters of our baseline model (r_H , r_L , p , and β) using representative values obtained from the empirical literature. To identify our four parameters, we use the probability of bankruptcy, the internal rate of return, the loss given default, and the bankruptcy recovery rate. The key assumption for this calibration exercise is

CALIBRATED VARIABLE	PARAMETRIZATION
1. Probability of bankruptcy	$1 - p$
2. Internal rate of return (IRR)	$\frac{pr_H + (1-p)r_L - 1}{1}$
3. Bankruptcy recovery rate	β
4. Bankruptcy costs as a fraction of firms' value prior to default	$\frac{(1-\beta)r_L}{pr_H + (1-p)r_L}$
SOURCE	VALUE
1. Longstaff et al. (2005): BBB-rated firms, 5 year horizon (10%)	0.10
2. IRR rules: go ahead if IRR >10-15% (depending on risk)	0.175
3. Alderson and Betker (1995) (65%)	0.65
4. Altman (1984): 11-17% of firms' value up to 3 years before default	0.10

Figure 3: **Parameter Calibration.**

that returns are binary. The calibrated values are $r_H = 1.22$; $r_L = 0.49$; $p = 0.9$; $\beta = 0.65$, for which projects could be financed separately since $r_i^* = 1.076 < 1.22 = r_H$ and jointly $r_m^{**} = 1.107 < 1.22 = r_H$ but not at the rate below the crossing point $r_m^* = 1.006 > 0.855 = (r_H + r_L)/2$. In this illustration, the risk contamination effect identified in Proposition 2 is

$$p(1-p)(1-\beta)r_H = (.9)(1-.9)(1-0.65)1.22 \approx 3.8\%$$

of the investment outlay $I = 1$, corresponding to $.038/0.175 \approx 22\%$ of the project's net present value.

2.4 Comparative Statics Predictions

We now derive comparative statics predictions with respect to changes in the characteristics of the projects: the recovery rates and the distribution of returns (mean, variability, and skewness). For each attribute, we study whether separate or joint financing is optimal for a larger range of the remaining parameters. Again, the key aspect is whether it becomes easier or more difficult to obtain a repayment rate for joint financing below the crossing point. In turn, this depends on how parameter changes affect the crossing point as well as the amount the firm can pledge to the creditors at that point.

Prediction 1 *For lower bankruptcy costs (higher β) then (a) financing, both jointly and separately, can be obtained for a larger region of parameters and (b) joint financing is preferred for a larger region of the remaining parameters.*

A lower bankruptcy cost increases the maximum pledgeable income both jointly and separately since the recovered returns in case of default are higher. With joint financing, this is represented in Figure 4(a) as a lower discount in the pink area.

Prediction 2 *For higher probability of high return (higher p) then (a) financing can be obtained for a larger region of parameters, both jointly and separately and (b) joint financing is optimal for a larger region of the remaining parameters.*

If the probability of a high return increases it becomes easier to finance the project as well as to finance at a repayment rate that avoids intermediate bankruptcy. Graphically, this lowers all the horizontal lines in the graph increasing the expected value, the area above the distribution. Financing is eased and, in particular, financing at a rate that avoids intermediate bankruptcy is eased because the maximum expected return pledgeable to creditors (the sum of the yellow area, the red area, and a fraction $1 - \beta$ of the pink area) also increases.

Neither the bankruptcy recovery rate nor the probability of success affect the crossing point, $(r_H + r_L)/2$. Changes in the variability of the project's return instead also affect the crossing point when the distribution of returns is asymmetric, $p \neq 1/2$.

Prediction 3 *Consider the effect of a mean-preserving spread in the project's return consisting in an increase in the high return r_H and a reduction in the low return r_L so as to maintain the mean return constant. Then, there exists $\bar{p} < 1/2$ such that the region of parameters for which separate financing is optimal increases if and only if $p > \bar{p}$.*

To derive this result, first consider the effect of a mean preserving spread for the special case with a symmetric distribution ($p = 1/2$). In this case, the mean preserving spread consists in an increase in r_H exactly equal to the reduction in r_L . While the crossing point is clearly unaffected, according to equation (2), the joint financing rate that avoids intermediate bankruptcy becomes more difficult to obtain. Thus, a mean preserving spread in the distribution of returns tends to favor separate financing. Indeed, low returns are even lower and therefore the pledgeable returns before the crossing point are lower. In the graph, the pink area, and therefore the area to the right of the crossing point, shrinks.

Turning to the case of asymmetric distributions, $p \neq 1/2$, a mean-preserving spread also affects the crossing point, $(r_H + r_L)/2$. If the distribution of returns is negatively skewed and therefore the mean is higher than the crossing point ($p > 1/2$), the crossing point is decreased. Indeed, to maintain the mean constant, a given increase in r_H must be combined with a larger decrease in r_L , resulting in a reduction in the crossing point.¹¹ Thus, it becomes even more difficult to obtain joint financing below the crossing point. Thus, this second effect also favors separate financing.

The sign of the second effect is reversed if the distribution of returns is instead positively skewed ($p < 1/2$) and therefore the mean return lies below the crossing point. In this case, the two effects go in opposite directions. If the distribution is sufficiently skewed ($p < \bar{p} < 1/2$), the second effect is stronger than the first. In the graph, the increase in the yellow area more than compensates the decrease in the pink area.

Prediction 4 *Consider the effect of a mean-preserving increase in negative skewness in the project's return consisting in a reduction in the low return level r_L and an increase in the probability of high return p so as to maintain the mean return constant. Then, it becomes optimal to finance the projects separately for a larger region of parameters if and only if the high return level r_H is sufficiently large.*

Consider first the case in which bankruptcy is extremely costly and the recovery rate is zero ($\beta = 0$). In this case, an increase in the negative skewness has two conflicting effects. On the one hand, as r_L decreases, the crossing point is reduced, so that joint financing at the rate avoiding intermediate bankruptcy becomes more difficult. On the other hand, as p increases so as to keep the mean constant, the probability that both projects' returns are low is reduced, so that it becomes easier to finance the projects at the rate avoiding intermediate bankruptcy. Graphically, the yellow area representing the creditor's expected returns at the crossing point is less wide (lower crossing point) but higher (higher probability of staying afloat).

Which of these two effects dominates depends on the level of the high return r_H . For a larger r_H , the same reduction in the probability of high return p needs a higher reduction in

¹¹Formally, from $r'_H = r_H + \varepsilon$ and $r'_L = r_L - \varepsilon p / (1 - p)$, we have $(r'_H + r'_L) / 2 = (r_H + r_L) / 2 - \varepsilon (2p - 1) / 2 (1 - p)$.

the low return realization to ensure a constant mean. As a result, for the same increase in the probability of staying afloat (height of the rectangle), we have a higher reduction in the crossing point (width of the rectangle). Hence, the increase in negative skewness makes it more difficult to finance the projects jointly at a repayment rate below the crossing point.

For the general case with a positive recovery rate $\beta > 0$, there is a third effect that makes it even more difficult to finance the projects at the rate avoiding intermediate bankruptcy because the increase in negative skewness reduces the recovered returns. Indeed, the pink area (the expected returns conditional on default) shrinks by becoming less wide (lower r_L) and less high (lower $1 - p$). Therefore, the threshold level of r_H above which an increase in negative skewness favors separate financing decreases in β .

3 Correlated Returns

We now consider the effect of correlation in the joint distribution of returns. Suppose that the probability of having two high returns is equal to $p[1 - (1 - p)(1 - \rho)]$, the probability of two low returns is equal to $(1 - p)[1 - p(1 - \rho)]$, whereas the probability that one of the projects yields a high return whereas the other yields a low one is equal to $p(1 - p)(1 - \rho)$. In that case, ρ would be the correlation coefficient between the two projects. In order to be well-defined, it is necessary to assume that $\rho \geq \max\langle -(1 - p)/p, -p/(1 - p) \rangle$. Clearly, if $\rho = 0$ we are back to the case with independent returns.

Proposition 3 *If the correlation between the projects increases (ρ is larger), then separate financing is preferred for a larger set of parameters.*

The probability of having two high returns and the probability of having two low returns increase simultaneously with ρ . As a result, the repayment rate when intermediate bankruptcy is avoided is higher because the probability of two low returns is higher. When intermediate bankruptcy cannot be avoided, instead, the repayment rate is lower because the probability of two high returns also increases. As a consequence, the financing conditions avoiding intermediate bankruptcy are tighter and those not avoiding it looser.

The effects of correlation on the optimality conditions are also intuitive. In the extreme case in which one project has a high return the other necessarily has a low one (i.e., if

$\rho = -1$ and $p = 1/2$), projects can always be jointly financed at a rate that avoids intermediate bankruptcy.¹² Projects, therefore should always be financed jointly. As correlation increases above $\rho = -1$, conglomeration is less likely to be optimal. If both projects are perfectly correlated ($\rho = 1$), the conditions for joint and separate financing are identical and the firm is clearly indifferent between them.

4 Large Number of Projects

Consider a borrower with access to a large number of projects with independent returns. We show that if the number of projects is sufficiently large, it always becomes possible for the borrower to finance all the projects with a single loan. This result exploits the law of large numbers. Namely, as the number of projects n increases, the probability that the average number of projects with high returns differs from p , the probability of a high return, by more than a small amount ε tends to zero. We can then construct a rate offer to finance all projects jointly that is acceptable for the creditors. The borrower's returns when financing all projects jointly is then arbitrarily close to the first best as the number of projects grows large. Therefore, for a large number of projects financing all the projects jointly is approximately optimal for the borrower, because it yields a payoff that is close to the highest possible level.

Proposition 4 *There exists n' and $q \in (0, p)$ such that for $n > n'$ a joint loan comprising all projects can be financed at a repayment rate that avoids bankruptcy when nq projects have high returns. The per-project return achieved in this way approaches the net present expected value of each project as n grows.*

5 Heterogeneous Projects

In a recent paper, Leland (2007) stresses a different benefit of financial separation from ours. Financial separation allows firms with different return profiles to choose different capital structures. We have abstracted so far from this effect by assuming that projects are ex-ante symmetric. In this section, we extend the model to allow for heterogeneity across projects.

¹²This is not true for $p \neq 1/2$ because either the probability of two high realizations or the probability of two low realizations is greater than 0, even when the correlation is at the lowest possible level.

5.1 Financing Conditions

Focus on the case with $n = 2$ heterogeneous (and independently distributed) projects $i = 1, 2$. Project i yields returns r_H^i with probability p_i and r_L^i with probability $1 - p_i$. Without loss of generality, assume that $r_H^1 + r_L^2 > r_L^1 + r_H^2$, interchanging the indices if necessary. With asymmetric projects, four (rather than three) levels of combined returns are possible, adding an extra case to the conditions for joint financing. Now, the possibility arises that default is avoided if project 1 yields a high return and project 2 a low return, whereas default is not avoided if the reverse occurs (case (b) in the following proposition).

Proposition 5 *There exists r'_i such that project i can be financed separately if and only if $r'_i < r_H^i$, in which case, the equilibrium repayment rate is r'_i . If the firm seeks joint finance, there exist r'_m, r''_m and r'''_m such that*

- (a) *if $r_L^1 + r_H^2 > 2r'_m$, then the equilibrium rate is r'_m ;*
- (b) *if $r_H^1 + r_L^2 > 2r''_m$ and $r_L^1 + r_H^2 < 2r'_m$, then the equilibrium rate is r''_m ;*
- (c) *if $r_H^1 + r_H^2 > 2r'''_m$, $r_L^1 + r_H^2 < 2r'_m$, and $r_H^1 + r_L^2 < 2r''_m$, then the equilibrium rate is r'''_m ;*
- (d) *if $r_H^1 + r_H^2 < 2r'''_m$, $r_L^1 + r_H^2 < 2r'_m$, and $r_H^1 + r_L^2 < 2r''_m$, then the projects cannot be financed.*

5.2 Good and Bad Conglomeration

We now turn to the question of whether the borrower should finance the projects jointly or separately when both financing modes are feasible. As in the symmetric case, if a rate that avoids bankruptcy in both intermediate situations can be obtained (case (a) in Proposition 5), then projects co-insure each other and therefore should be financed jointly. If, instead, the firm can only obtain a rate that does not avoid bankruptcy in any of the intermediate situations (case (c)), then the projects should be financed separately because they drag down each other. If bankruptcy can only be avoided for the more favorable intermediate situation, then both co-insurance and contamination effects are present at the same time. On the one hand, project 1, when it yields a high return, saves project 2 when project 2 yields a low return; on the other hand, project 1, when it yields a low return, contaminates project 2 when project 2 yields a high return. The optimality of separate or joint financing depends on whether the gains from co-insurance dominate the losses from risk contamination.

Proposition 6 *If the borrower can finance both projects separately and jointly, then*

(a) *If the joint rate is r'_m , then the borrower should finance the projects jointly because of the co-insurance effect. The gain in expected payoff from joint rather than separate financing is $(1 - p_1)p_2(1 - \beta)r_L^1 + p_1(1 - p_2)(1 - \beta)r_L^2$.*

(b) *If the joint rate is r''_m , then the borrower should finance the projects separately if and only if the risk contamination effect dominates the co-insurance effect: $(1 - p_1)p_2(1 - \beta)r_H^2 > p_1(1 - p_2)(1 - \beta)r_L^2$.*

(c) *If the joint rate is r'''_m , then the borrower should finance the projects separately because of the risk contamination effect. The gain in expected payoff from separate rather than joint financing is $p_1(1 - p_2)(1 - \beta)r_H^1 + (1 - p_1)p_2(1 - \beta)r_H^2$.*

Note that if the two projects have the same probability of success, then the risk contamination effect always dominates the co-insurance effect in case (b). With joint financing, the probabilities of saving and dragging down project 2 are the same but the co-insurance gains are outweighed by the contamination losses, because the project is saved when it has a low return but it is dragged down following a high return. Hence, separation is optimal unless a joint-financing rate that avoids bankruptcy with both intermediate returns can be obtained.

5.3 Comparative Statics Predictions

For the case in which one project is a mean preserving spread of the other, the next result establishes that more risk typically induces even more separation.

Prediction 5 *If project 1 second-order stochastically dominates project 2, and therefore $p_1 = p_2$ and $r_H^1 = r_H^2 + \varepsilon$ and $r_L^1 = r_L^2 - \frac{p_1}{1-p_1}\varepsilon$ for $\varepsilon > 0$, then projects should be financed separately unless the rate r'_m can be obtained with joint financing. The region of parameters for which separation is optimal increases with the spread of the risky project.*

The area in which joint financing is optimal shrinks as the spread of the risky project increases, as the condition for obtaining the rate r'_m (case (a) in Proposition 5) becomes more stringent. Indeed, the less favorable intermediate returns $(r_L^1 + r_H^2)$ decrease in the spread of project 1 and the repayment rate (r'_m) increases, as the creditor recovers less in

the event of bankruptcy (when both projects yield low returns). In addition, it becomes easier to finance the projects separately as the increase in the high realization of the return is not compensated by the increase in the repayment rate (r'_i), making condition (a) in Proposition 5 easier to satisfy.

When projects are heterogeneous, separation has the additional advantage of allowing for project-specific loans. If one of the projects has a low expected return, it might be better to finance only the other project rather than financing both of them with the same loan, even if this is possible. This reduces the attractiveness of joint financing, even when a rate that avoids bankruptcy for all intermediate returns can be obtained, as illustrated by the following result.

Prediction 6 *If project 1 first-order stochastically dominates project 2, and in particular, $r_H^1 = r_H^2$ and $r_L^1 = r_L^2 = r_L$ and $p_1 > p_2$, then if both projects can be financed separately they should be financed separately unless the rate r'_m can be obtained. If only the high-mean project can be financed separately, then the borrower should only finance this project unless (i) the rate r'_m can be obtained and (ii) the ex-post net present value of financing the low-mean project separately is compensated by the co-insurance effects, i.e. if and only if $[(1 - p_2)p_1 + (1 - p_1)p_2](1 - \beta)r_L > 1 - p_2r_H - (1 - p_2)\beta r_L$.*

If both projects can be financed separately ($p_2r_H + (1 - p_2)\beta r_L - 1 > 0$) then they should be financed separately unless the rate joint rate r'_m can be obtained (here r''_m is never obtained, as there is only one level of intermediate returns). If, instead, the low-mean project has negative ex-post returns ($p_2r_H + (1 - p_2)\beta r_L - 1 < 0$), then this project cannot be financed separately. However, it might still be possible to finance this project jointly with the high-mean project if a joint rate r'_m can be obtained, as projects might save each other when they generate a low return. The borrower should indeed opt for joint financing rather than financing only the high-mean project if the co-insurance benefits more than compensate the ex-post negative returns of the low-mean project.

6 Ugly Conglomeration

We now turn to the managerial implications of our analysis. At first it seems plausible that (i) the financing option with the lowest repayment rate has the lowest likelihood

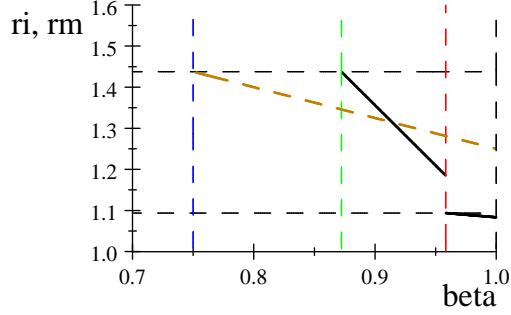


Figure 4: **Rates for Ugly Conglomeration.** This figure plots the rates for separate and joint financing depending on the bankruptcy recovery rate. In this illustration, $r_L = 3/4$, $r_H = 23/16$, and $p = 1/2$. Separate financing corresponds to the brown line and joint financing to the black lines. Separate financing is obtained for $\beta > \beta_i^*$, represented by the blue line. Joint financing at a rate that does and does not avoid intermediate bankruptcy is obtained for $\beta > \beta_m^*$ (red line) and for $\beta > \beta_m^{**}$ (green line), respectively.

of bankruptcy and (ii) the financing option with the lowest probability of bankruptcy is optimal. In this section, we show that these two rules of thumb are false in general.

6.1 Repayment Rates and Bankruptcy Probability

Figure 4 depicts the financing conditions and the repayment rates charged as a function of the recovery rate β , for a given homogeneous combination of returns, r_H and r_L , and probability of high return, p . Separately, financing for each project can be obtained after the blue line threshold at the rate depicted by the brown line. Jointly, financing at a rate that does and does not avoid intermediate bankruptcy can be obtained after the red and green thresholds, respectively, at a repayment rate depicted by the black line.

After the red threshold, the borrower obtains lower repayment rates with joint financing and, as we have seen before, joint financing is optimal. Nevertheless, below this threshold the loan rates are not necessarily lower with separate financing, although this is the optimal financing choice. The following proposition formalizes this result; borrowers should not always accept the loan with the lowest repayment rate.

Proposition 7 (Homogeneous) projects should be financed separately despite having higher repayment rates if and only if (i) the joint rate is r_m^{**} and (ii) the separate rate is such that $r_i^* < \beta r_H$.

Suppose that the borrower has the choice of financing the projects independently and jointly, although only at a rate with intermediate bankruptcy. In this region with bad conglomeration, the low return project drags down the high return one. The borrower should finance the projects separately because the losses from bankruptcy are lower. However, if, at the same time, the returns recovered from a bankrupt high value project are higher than what the creditor can charge for separate loans ($\beta r_H > r_i^*$), the creditor has higher returns if the projects are financed jointly, even though bankruptcy is more likely. As a result, the repayment rates are lower with joint than with separate financing. The borrower might feel tempted to finance the projects jointly, but this is suboptimal. Low interest rate associated with joint financing here are deceptively attractive—while it might look good, conglomeration is bad. In this case, one could say that conglomeration is “ugly”.

The logic of ugly conglomeration can be further illustrated by Panel (b) of Figure 2. For an (exogenous) repayment rate above the crossing point, $r > (r_H + r_L)/2$, as the one depicted, the creditor’s expected returns might be higher if projects are financed jointly in spite of the increased occurrence of bankruptcy. Indeed, with joint financing, the creditor obtains the part of the yellow area above the dashed line as well as a fraction β of the red and pink areas. With separate financing, the creditor obtains the yellow area and the upper part of the red area fully and a fraction a fraction β of the pink area. The creditor’s returns at this interest rate are higher if proceeds from the fraction β of the red area, $p(1-p)\beta r_H$, are greater than the sum of the upper part of the red area and the part of the yellow area below the dashed line, $p(1-p)r$. This is the case if and only if $\beta r_H > r$, which holds if bankruptcy costs are sufficiently low. If the creditor breaks even at rate $r_i^* = r$ in the equilibrium with separate financing, the equilibrium rate with joint financing must be lower, so that $r_m^{**} < r_i^*$. In this case, equilibrium interest rate for joint financing is lower despite higher probability of bankruptcy. Intuitively, creditors can obtain higher expected proceeds from bankruptcy with joint financing, and so are forced by competition to offer lower interest rate—however, the borrower obtains higher expected payoff with separate financing at a higher interest rate.¹³

¹³Note if the distribution of returns was continuous (rather than discrete, as in our model with binary returns), the extra losses from higher probability of bankruptcy if the equilibrium rate with joint financing was marginally above the crossing point will always be compensated by the increased proceeds from bankruptcy. Therefore, ugly conglomeration always appears when the project’s returns are continuously distributed, because then there would be no discrete jump in the probability of bankruptcy at the crossing point (as there is with binary returns, for which ugly conglomeration therefore does not always arise).

6.2 Bankruptcy Probability and Optimal Conglomeration

Secondly, the option with the lowest probability of bankruptcy might not be optimal. This is due to the fact that, despite having a higher probability, the benefits of co-insurance might be outweighed by the costs of risk contamination.

Proposition 8 Separate financing is optimal even though it results in higher probability of bankruptcy if and only if (i) the joint rate is r_m'' and the contamination losses dominates co-insurance gains: $(1 - p_1)p_2(1 - \beta)r_H^2 > p_1(1 - p_2)(1 - \beta)r_L^2$; and (ii) the probability of the former is lower than that of the latter ($p_1 > p_2$).

When the joint rate is r_m'' , we have that (i) if project 1 yields a low return, it drags down project 2's high return (that would have stayed afloat with separate financing) and (ii) if project 1 yields a high return, it saves project 2's low return (that would have defaulted with separate financing).

On the one hand, with separate financing, the probability of default is reduced when project 1 fails and project 2 succeeds—as the now separate project 2 is not dragged down by the failing project 1, as instead would not have happened with joint financing. According to this first effect, the probability of default with separate financing is reduced by $(1 - p_1)p_2$ compared to joint financing. On the other hand, with separate financing, the probability of default is increased when project 2 fails and project 1 succeeds—as the failing project 2 is not saved by the successful project 1, as instead would have happened with joint financing. According to this second effect, the probability of default with separate financing is increased by $p_1(1 - p_2)$ compared to joint financing. Overall, the probability of default with separate financing is higher than with joint financing if $(1 - p_1)p_2 < p_1(1 - p_2)$, i.e., if $p_2 < p_1$. Indeed, with joint financing project 2's probability of staying afloat goes up from p_2 to p_1 , whereas project 1's probability is the same.

Despite this, it might still be that the risk contamination effect dominates the co-insurance effect, and therefore the projects should be financed separately. We have that $p_1 > p_2$ and therefore $p_1(1 - p_2) > p_2(1 - p_1)$ but $p_1(1 - p_2)(1 - \beta)r_L^2 < (1 - p_1)p_2(1 - \beta)r_H^2$ provided that r_H^2 is sufficiently high compared to r_L^2 . Even though the probability of the co-insurance outcome is higher than that of risk contamination, if the level of bankruptcy

costs conditional on default are sufficiently greater when project 2's return is high, the risk contamination losses outweigh the co-insurance gains.

7 Conclusion

This paper addresses the classic question of the value of conglomeration with bankruptcy costs. By focusing on the simplest setting with binary returns, qualify the long-standing claim that joint financing generates financial benefits by economizing on bankruptcy costs. The same logic that allows conglomeration to create co-insurance savings in expected bankruptcy costs also results in additional risk contamination losses. We provide a full characterization of the conditions for which combining two (high-risk low-return) projects results in an increase in expected bankruptcy costs. We derive the following predictions:

- An increase in the bankruptcy recover rate favors joint financing.
- An increase in the probability of a high return favors joint financing.
- An increase in the riskiness of (sufficiently negatively skewed) projects favors separate financing.
- An increase in the negative skewness of projects (with sufficiently high return) favors separate financing.
- An increase in the correlation of projects favors separate financing.
- Joint financing of a sufficiently large number of independent projects is preferred.

In addition, our analysis uncovers additional advantages of separate financing when projects are heterogeneous. We also characterize situations in which projects should be financed in separate companies, even though this involves paying a higher interest rate than under joint financing. This is the case when the recovery rate in case of bankruptcy is sufficiently high, creditors are forced by competition to offer a more favorable interest rate for joint than separate financing. These results have clear implications for project finance and securitization.

Appendix A: Proofs

Proof of Proposition 3: Clearly, separate financing is not affected by correlation. The joint financing repayment rates, r_m^* and r_m^{**} in Proposition 1, and the corresponding financing conditions, are now replaced by $r_{m,\rho}^*$ and $r_{m,\rho}^{**}$, respectively, where

$$r_{m,\rho}^* := \frac{1 - (1-p)[1-p(1-\rho)]\beta r_L}{1 - (1-p)[1-p(1-\rho)]} < \frac{r_H + r_L}{2},$$

and

$$r_{m,\rho}^{**} := \frac{1 - (1-p)\beta r_L}{p[1 - (1-p)(1-\rho)(1-\beta)]} < r_H.$$

Note that $r_{m,\rho}^*$ and $r_{m,\rho}^{**}$ are respectively increasing and decreasing in ρ . Q.E.D.

Proof of Proposition 4: First statement. Define $g(\gamma) := \gamma r_H + (1-\gamma)r_L$. We have that $g(p) > 1$ because of the positive net present value condition, and trivially $g(0) = r_L < 1$ and $g'(\gamma) > 0$. Then there exists a unique $\gamma^* \in (0, p)$ such that $g(\gamma^*) = 1$. For a fixed rational number ε (small) define $q := \gamma^* + \varepsilon$. Clearly, $qr_H + (1-q)r_L > 1$

Take any number of projects n such that nq is an integer number. Suppose that we were to finance all these n projects jointly at an interest rate that avoids bankruptcy when at least nq of them have high returns. This is possible if and only if the per-project repayment satisfies

$$r_n^* \leq qr_H + (1-q)r_L.$$

But, the creditor's zero profit condition implies that

$$r_n^* := \frac{1 - \beta \left(\sum_{k=0}^{nq-1} f(k) \frac{kr_H + (n-k)r_L}{n} \right)}{1 - F(nq-1)},$$

where $f(m)$ and $F(m)$ are the probability density and distribution that m out of the n projects have high returns, i.e.

$$f(m) := \binom{n}{m} p^m (1-p)^{n-m} \quad \text{and} \quad F(m) := \sum_{k=0}^m f(k).$$

Given that the returns recovered in the event of bankruptcy are positive we have that

$$r_n^* \leq \frac{1}{1 - F(nq-1)} < \frac{1}{1 - F(nq)}.$$

From the law of large numbers we have that $F(nq)$ tends to 0 as n grows large (remembering that $q < p$). Therefore r_n^* is bounded above by a number that is arbitrarily close to 1.

Given that $qr_H + (1 - q)r_L > 1$, there exists n' such that for all $n > n'$ then r_n^* is such that

$$r_n^* \leq qr_H + (1 - q)r_L,$$

as was to be shown.

Second statement: From the loan described above, the borrower obtains a per-project gross profit

$$\pi_n = \beta \sum_{k=0}^{nq-1} f(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n}\right) r_L \right] + \sum_{k=nq}^n f(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n}\right) r_L \right].$$

Fix a small rational number ε and an integer n such that $n(p - \varepsilon)$ and $n(p + \varepsilon)$ are integer numbers. Then, given that $q < p - \varepsilon$, and that all terms in the first and in the second sum are positive, we have that

$$\pi_n \geq \sum_{k=n(p-\varepsilon)}^{n(p+\varepsilon)} f(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n}\right) r_L \right].$$

Given that the terms in the second factor in the sum are larger for larger k , the sum is reduced by replacing the summand of a given k by that of $n(p - \varepsilon)$, the smallest term. Then, rearranging,

$$\pi_n \geq [(p - \varepsilon)r_H + [1 - (p - \varepsilon)]r_L] [F[n(p + \varepsilon)] - F[n(p - \varepsilon)]].$$

From the law of large numbers, $F[n(p + \varepsilon)] - F[n(p - \varepsilon)]$ tends to 1 as n grows. Indeed, from elementary statistics we know that

$$F[n(p + \varepsilon)] - F[n(p - \varepsilon)] \geq 1 - \frac{(p + \varepsilon)(1 - p)}{n\varepsilon^2} - \frac{(1 - p + \varepsilon)p}{n\varepsilon^2} = 1 - \frac{2p(1 - p) + \varepsilon}{n\varepsilon^2}$$

and therefore

$$\pi_n \geq [pr_H + (1 - p)r_L - \varepsilon(r_H - r_L)] \left(1 - \frac{2p(1 - p) + \varepsilon}{n\varepsilon^2}\right).$$

That is for n large, the gross per-project profit differs from the (gross) present value of each project by an amount that is arbitrarily small, $\varepsilon(r_H - r_L)$. Similarly,

$$\frac{\pi_n}{\pi^*} \geq \left(1 - \frac{\varepsilon(r_H - r_L)}{pr_H + (1 - p)r_L}\right) \left(1 - \frac{2p(1 - p) + \varepsilon}{n\varepsilon^2}\right)$$

where π^* is equal to first-best gross profits, $\pi^* = pr_H + (1 - p)r_L$. Q.E.D.

Proof of Proposition 5: Following the same procedure as in the symmetric case, the repayment rate should satisfy $1 < r'_i < r^i_H$. The creditor's zero profit condition is now

$$pr'_i + (1 - p_i)\beta r^i_L - 1 = 0, \quad (\text{A1})$$

and project i can be financed (at r'_i) if and only if

$$r'_i := \frac{1 - (1 - p_i)\beta r^i_L}{p_i} < r^i_H. \quad (\text{A2})$$

There are three cases in which joint financing is feasible depending on whether bankruptcy can be avoided in both cases with intermediate returns, or only when project 1 yields a high return and 2 a low return, or in neither case. In the former case, competitive credit markets imply that

$$[1 - (1 - p_1)(1 - p_2)]2r'_m + (1 - p_1)(1 - p_2)\beta(r^1_L + r^2_L) - 2 = 0, \quad (\text{A3})$$

and therefore this is possible if and only if

$$r'_m := \frac{1 - (1 - p_1)(1 - p_2)\beta\frac{r^1_L + r^2_L}{2}}{1 - (1 - p_1)(1 - p_2)} < \frac{r^1_L + r^2_H}{2}. \quad (\text{A4})$$

If default can be avoided with high intermediate returns but not with low intermediate returns, then

$$p_1 p_2 2r''_m + p_1(1 - p_2)2r''_m + (1 - p_1)p_2\beta(r^1_L + r^2_H) + (1 - p_1)(1 - p_2)\beta(r^1_L + r^2_L) - 2 = 0, \quad (\text{A5})$$

and therefore this is possible if and only if

$$\frac{r^1_L + r^2_H}{2} < r''_m := \frac{1 - (1 - p_1)p_2\beta\frac{r^1_L + r^2_H}{2} - (1 - p_1)(1 - p_2)\beta\frac{r^1_L + r^2_L}{2}}{p_1} < \frac{r^1_H + r^2_L}{2}.$$

If default cannot be avoided with either intermediate returns, then

$$p_1 p_2 2r'''_m + p_1(1 - p_2)\beta(r^1_H + r^2_L) + (1 - p_1)p_2\beta(r^1_L + r^2_H) + (1 - p_1)(1 - p_2)\beta(r^1_L + r^2_L) - 2 = 0, \quad (\text{A6})$$

and therefore this is possible if and only if

$$\frac{r^1_H + r^2_L}{2} < r'''_m < \frac{r^1_H + r^2_H}{2}, \quad (\text{A7})$$

where

$$r'''_m := \frac{1 - p_1(1 - p_2)\beta\frac{r^1_H + r^2_L}{2} - p_2(1 - p_1)\beta\frac{r^1_L + r^2_H}{2} - (1 - p_1)(1 - p_2)\beta\frac{r^1_L + r^2_L}{2}}{p_1 p_2}.$$

Again, since the borrower obtains all the ex-post net present value, rate r'_m is preferred to r''_m and r''_m is preferred to r'''_m . To complete the proof we only need to show that the lower bound conditions for r''_m and r'''_m are irrelevant. From (A3) and (A5), and rearranging, we have

$$p_1(r'_m - r''_m) = p_2(1 - p_1) \left[\beta \left(\frac{r_L^1 + r_H^2}{2} \right) - r'_m \right],$$

and therefore if $r'_m > \frac{r_L^1 + r_H^2}{2}$ then the right hand side is negative. As a consequence, we have $r''_m > r'_m > \frac{r_L^1 + r_H^2}{2}$. Similarly, from (A5) and (A6) and rearranging, we have

$$p_2(r''_m - r'''_m) = (1 - p_2) \left[\beta \left(\frac{r_H^1 + r_L^2}{2} \right) - r''_m \right]$$

and therefore if $r''_m > \frac{r_H^1 + r_L^2}{2}$ then the right hand side is negative. As a consequence, we have $r'''_m > r''_m > \frac{r_H^1 + r_L^2}{2}$. Q.E.D.

Proof of Proposition 6: Substituting r'_m in the right hand side of (A3) and r'_i in the right hand side of (A1) and subtracting the latter from the former, we have

$$p_2(1 - p_1)(1 - \beta)r_L^1 + p_1(1 - p_2)(1 - \beta)r_L^2 (> 0).$$

Similarly, substituting r''_m in the right hand side of (A5) and subtracting again the ex-post net present value of financing the two projects separately from this, we obtain

$$-(1 - p_1)p_2(1 - \beta)r_H^2 + p_1(1 - p_2)(1 - \beta)r_L^2,$$

which can be positive or negative. Lastly, substituting r'''_m in the right-hand side of (A6) and subtracting the ex-post net present value of financing the two projects separately from this, we have

$$-p_1(1 - p_2)(1 - \beta)r_H^1 - p_2(1 - p_1)(1 - \beta)r_H^2 (< 0),$$

as desired. Q.E.D.

Proof of Prediction 5: Given that one project is obtained from an elementary increase in risk from the other and returns should still be binary, we must have that $p_1 = p_2$. Letting ε be such that $r_H^1 = r_H^2 + \varepsilon$, we have $r_L^1 = r_L^2 - \frac{p}{1-p}\varepsilon$. Indeed, $p(r_H^2 + \varepsilon) + (1 - p)r_L^1 = pr_H^2 + (1 - p)r_L^2$. We can also check that $r_L^1 + r_H^2 = r_L^2 - \frac{p}{1-p}\varepsilon + r_H^2 < r_L^2 + \varepsilon + r_H^2 = r_H^1 + r_L^2$.

As shown in the previous proposition, given that the probabilities of success are equal, we have that, when both projects can be financed separately as well as jointly, joint

financing is only optimal if a rate r'_m can be obtained. Moreover, the region for which joint financing is optimal shrinks as the repayment rate r'_m is more difficult to obtain if ε increases. Indeed, the left-hand side of condition (A4) decreases in ε and the repayment rate (the right-hand side) increases in ε .

On the other hand, the region for which separate financing is possible expands if ε increases. Indeed, the derivative of the left-hand side of condition (A2) is equal to β whereas the right hand-side is equal to 1. Hence, this condition is more easily satisfied as ε increases. Q.E.D.

Proof of Prediction 6: If both projects can be financed separately then, Proposition 5 implies that the borrower should finance them jointly if and only if a repayment rate r'_m can be obtained. Indeed, case (b) never occurs if the ex-post returns are the same as there is only one level of intermediate returns.

Suppose now that only one project can be financed separately, i.e. $p_i r_H + (1-p_i)\beta r_L > 1$ and $p_j r_H + (1-p_j)\beta r_L < 1$ for $i \neq j$, where we denote again $r_H := r_H^i$ and $r_L := r_L^i$ for $i = 1, 2$. Then the expected surplus from funding the project separately is $p_i r_H^i + (1-p_i)\beta r_L^i - 1$. Subtracting this from the expected surplus from joint financing in the case in which the repayment rate r'_m can be obtained and simplifying, we have

$$p_j r_H + (1-p_j)\beta r_L - 1 + [(1-p_j)p_i + (1-p_i)p_j](1-\beta)r_L.$$

On the other hand, if we subtract this from the expected surplus from joint financing in the case in which the repayment rate r'_m can be obtained and simplifying, we obtain

$$p_j r_H + (1-p_j)\beta r_L - 1 - [(1-p_j)p_i + (1-p_i)p_j](1-\beta)r_H,$$

so that separate financing is optimal because both terms are negative.

Suppose that none of the two projects can be financed separately, if a rate r'_m can be obtained, i.e. if condition (A4) is satisfied, then from (A3), we have that the ex-post net present value of the joint combination is positive. On the other hand, it cannot be that a rate that does not avoid intermediate bankruptcy is obtained since (the second) inequality in (A4) implies that

$$p_1 p_2 2r_H + [p_1(1-p_2) + p_2(1-p_1)]\beta(r_L + r_H) + (1-p_1)(1-p_2)\beta 2r_L - 2 > 0,$$

which implies that the ex-post net present value is positive. This condition is equivalent to

$$p_1 r_H + (1 - p_1) \beta r_L - 1 + p_2 r_H + (1 - p_2) \beta r_L - 1 - [p_1 (1 - p_2) + (1 - p_1) p_2] (1 - \beta) r_H > 0,$$

which contradicts the fact that the two projects cannot be financed independently. Q.E.D.

Proof of Proposition 7: To prove this, suppose first that a rate below the crossing point can be obtained. We have that

$$r_m^* = \frac{1 - (1 - p)^2 \beta r_L}{1 - (1 - p)^2} < \frac{1 - (1 - p) \beta r_L}{p} = r_i^*,$$

because $1 > \beta r_L$. Suppose now that only a rate r_m^{**} can be obtained and therefore the probability of bankruptcy is higher with joint financing. r_m^* associated with joint financing is, nevertheless, lower than r_i^* associated with separate financing whenever

$$r_m^{**} = \frac{1 - (1 - p) \beta (p r_H + r_L)}{p^2} < \frac{1 - (1 - p) \beta r_L}{p} = r_i^*,$$

or equivalently

$$\beta r_H > \frac{1 - (1 - p) \beta r_L}{p} = r_i^*.$$

Appendix B

Optimal Contracting with Non-Verifiable Returns

The debt contract we have adopted in this paper is the optimal financial arrangement in the “costly state verification” model (see Townsend, 1978, and Gale and Hellwig, 1985), when creditors can verify company returns at a cost equal to the bankruptcy cost. As it is well known, however, debt is no longer optimal when the possibility of renegotiation is introduced, because verification is ex-post suboptimal in equilibrium.

This appendix revisits our analysis of conglomeration in an alternative model in which debt is not only the optimal financial arrangement, but is also robust to the introduction of renegotiation. This model is a two-project extension of Bolton and Scharfstein’s (1990) dynamic model of debt with non-verifiable returns.

Suppose that (1) projects generate unverifiable returns not for one but potentially for two periods and (2) creditors can threaten continuation from the first to the second period by withholding required intermediate funding. Formally, assume that each of the $n = 2$ projects available requires an up-front investment I at time 0 and generates income R_H with probability p and R_L with probability $1 - p$, with $R_H > R_L$, at time 1. Conditional on an additional investment of L at time 1, each project generates, at time 2, an expected income R_H^2 if the first-period income was R_H and expected income R_L^2 if it was R_L , where $R_H^2 > R_L^2$. Although projects’ returns are correlated across periods, the additional investment required is independent from the first-period return. Termination is inefficient in any case, as $R_H^2 > R_L^2 > L$. Therefore, in this setting bankruptcy costs are defined as the loss in expected net present value from early termination of the project.¹⁴

Returns are not verifiable and therefore the borrower will repay nothing in the second (and last) period. In other words, R_j^2 is a private benefit for the borrower. Creditors, however, can induce a truthful report of the first period by committing (ex-ante) to provide additional funds at time 1. However, it is not possible to always guarantee extra funding, as $R_L < I + L =: I^n$. As in the baseline model, we assume away discounting and assume that the borrower has all the bargaining power.

When deciding whether to finance the two projects separately or jointly, the borrower faces the following trade off. On the one hand, by financing the two projects jointly,

¹⁴As in the baseline model, more resources are lost when a good project is terminated. However, the creditor obtains the same amount whether the project is good or bad.

the borrower can achieve the benefits of coinsurance. On the other hand, by financing each project separately in a stand-alone company, the borrower reduces the possibility of risk contamination, the phenomenon whereby a failing asset drags an otherwise healthy sponsoring firm into distress. We assume that, when the projects are financed jointly, it is impossible to terminate one project without terminating the other or, equivalently, that a common refinancing probability must be used. If this was not the case, then it would always be optimal to finance the projects jointly as, at the very least, one could replicate the separate contracts. Indeed, if the projects are financed separately, one of them can be terminated without terminating the other and therefore separate refinancing probabilities can be used. Finally, to abstract from the problems of internal financing, studied by Inderst and Müller (2003), we assume that self-refinancing is not possible.

We proceed by first deriving the optimal contracts for separate and joint financing, and then comparing the expected benefits of these two options. The proofs are collected at the end of the section.

Optimal Contracts

For each separate project, the borrower maximizes the expected surplus subject to the individual rationality and the incentive compatibility constraints. Denote as D_1 the payment at time 1 if a high return is announced and D_0 if a low is announced. Let the continuation probabilities be y_1 in case of a high announcement and y_0 in case of a low. The borrower's problem is given by

$$\begin{aligned} \underset{y_0, y_1, D_1, D_0}{Max} \quad & p(R_H - D_1 + y_1 R_H^2) + (1-p)(R_L - D_0 + y_0 R_L^2) \\ \text{s.t.} \quad & (IC_h) \quad (y_1 - y_0) R_H^2 \geq D_1 - D_0 \\ & (IR) \quad p(D_1 + (1 - y_1)L) + (1-p)(D_0 + (1 - y_0)L) \geq I^n, \end{aligned}$$

where we are implicitly assuming that $D_1 \leq R_H$, $D_0 \leq R_L$ (and $D_1 > D_0$) and that IC_l , $(y_0 - y_1)R_L^2 \geq D_0 - D_1$, is satisfied.

Lemma 1 *IC_h and IR constraints are binding and $y_1 = 1$.*

In the high state, it is better to have a higher continuation probability than a lower payment (better for efficiency and for incentives). In contrast, in the low state, it might

be better to have a low probability and a high payment to improve incentives. As stated in the IC_h constraint, the difference in probability should be high enough relative to the difference in payments to induce truth-telling.¹⁵

Proposition 9 *Assume that*

$$\frac{I^n - R_L}{pR_H^2 + (1-p)L}R_H^2 < R_H - R_L \quad (\text{B8})$$

then the optimal contract satisfies

$$\begin{aligned} D_1 &= \frac{I^n - R_L}{pR_H^2 + (1-p)L}R_H^2 + R_L; \quad D_0 = R_L; \\ 1 - y_1 &= 0; \quad 1 - y_0 = \frac{I^n - R_L}{pR_H^2 + (1-p)L}. \end{aligned}$$

To minimize inefficiencies, the borrower tries to set the continuation probability in the low state as high as possible. With respect to the payments, it is optimal to have a high payment in the low state and a not so high payment in the high state. If the difference is small, then the continuation probability difference can also be small and therefore the probability of continuation in the low state can be high.

Similarly, the optimal contract for joint financing should maximize the borrower's expected surplus subject to individual rationality and the incentive compatibility constraints. Denote $D_{1,1}$ the payment in period 1 if two high returns are announced, $D_{1,0}$ if a high and a low are announced and $D_{0,0}$ if two lows are announced and the respective continuation probabilities as $y_{1,1}$, $y_{1,0}$ and $y_{0,0}$. The problem becomes

$$\begin{aligned} & \max_{y_{0,0}, y_{1,0}, y_{1,1}, D_{1,1}, D_{0,0}, D_{1,0}} p^2(2R_H - D_{1,1} + y_{1,1}2R_H^2) + \\ & 2(1-p)p(R_L + R_H - D_{1,0} + y_{1,0}[R_H^2 + R_L^2]) + (1-p)^2(2R_L - D_{0,0} + y_{0,0}2R_L^2) \\ \text{s.t. } & (IC_{h,h}) \quad (y_{1,1} - y_{1,0})2R_H^2 \geq D_{1,1} - D_{1,0} \\ & (IC_{h,l}) \quad (y_{1,0} - y_{0,0})(R_H^2 + R_L^2) \geq D_{1,0} - D_{0,0} \\ & (IR) \quad p^2(D_{1,1} + (1 - y_{1,1})2L) + 2p(1-p)(D_{1,0} + (1 - y_{1,0})2L) + \\ & \quad + (1-p)^2(D_{0,0} + (1 - y_{0,0})2L) \geq 2I^n, \end{aligned}$$

¹⁵We show that we can still view the optimal contract should as a debt contract since, in the sense that the probability of termination is 0 in case of a repayment.

where, we are again implicitly assuming that the limited liability ($D_{1,1} \leq 2R_H, D_{0,0} \leq 2R_L, D_{1,0} \leq R_L + R_H$) and the other IC constraints are satisfied.¹⁶

The following proposition outlines the optimal contract.

Proposition 10 *If $2p(1-p)(R_H^2 + R_L^2 - 2L) - p^2(R_H^2 - R_L^2) > 0$ then the optimal contract satisfies*

$$(1 - y_{1,1}) = 0;$$

$$(1 - y_{1,0}) \text{ as low as possible subject to } D_{1,0} < R_H + R_L \text{ and } D_{1,1} < 2R_H$$

$$(1 - y_{0,0}) = \frac{2I^n - 2R_L + (1 - y_{1,0}) [2p(1-p)(R_H^2 + R_L^2 - 2L) - p^2(R_H^2 - R_L^2)]}{(R_H^2 + R_L^2)(1 - (1-p)^2) + (1-p)^2 2L}$$

$$D_{1,1} = (1 - y_{0,0})(R_H^2 + R_L^2) + (1 - y_{1,0})(R_H^2 - R_L^2) + 2R_L$$

$$D_{1,0} = (1 - y_{0,0})(R_H^2 + R_L^2) - (1 - y_{1,0})(R_H^2 + R_L^2) + 2R_L; \quad D_{0,0} = 2R_L.$$

Note that in the case of independent projects we assumed that the limited liability constraint is satisfied for the high repayment. Here we do not make that assumption a priori, but it must be checked that this condition holds.

Separate or Joint Finance?

Again, the expected inefficiency might be greater by pooling the projects and therefore separation might be optimal. It is possible that a good and a bad realization ends up with a joint failure, whereas if the projects had been independent, the good one would have been saved. As opposed to the baseline model, a high and a low realization is not a sure failure but a strictly positive probability of failure. Still, for other parameter values, a good and a bad never go bankrupt, the bad one is saved by the high one with probability one. As a result, the expected inefficiency is lower with joint projects and therefore they should be financed jointly.

For the comparison we will compare the inefficiency arising from joint projects, denoted as A , and that from separation, denoted by B , where

$$A := (1-p)^2(1-y_{0,0})2(R_L^2 - L) + 2p(1-p)(1-y_{1,0})(R_H^2 + R_L^2 - 2L),$$

$$B := 2(1-p)(1-y_0)(R_L^2 - L).$$

¹⁶We can easily check that $2[I + p^2L + 2p(1-p)L + (1-p)^2L] = 2I^n$.

The comparison is driven by the previous proposition. If bankruptcy does not result following a high and a low realization, joint financing is optimal. If instead bankruptcy does result in that instance, separate financing might dominate.

Lemma 2 $y_{1,0}$ can be equal to 1 only if

$$\frac{2I^n - 2R_L}{(R_H^2 + R_L^2)(1 - (1 - p)^2) + (1 - p)^2 2L} (R_H^2 + R_L^2) < R_H - R_L. \quad (\text{B9})$$

This condition is more stringent (in terms of $R_H - R_L$) than condition (B8).

We analyze two cases, depending on whether this condition is satisfied, always assuming that condition (B8) in Proposition 9 for financing independent projects holds. Note that in the baseline case the key to decide whether to bundle the projects was whether bankruptcy results when one project yields a high return and the other low return. Here, if the condition is satisfied, the two projects are again saved when one of them yields a low return so that joint financing is optimal:

Proposition 11 *If condition (B9) is satisfied, then financing the projects jointly is optimal.*

If, on the other hand, the condition is not satisfied, then it might be that separation is optimal. To show this, we focus on the special case in which L is close to R_L^2 . In this case, the condition in Proposition is equivalent to $p < 2/3$.

Proposition 12 *If condition (B9) is not satisfied, there are cases in which it is optimal to finance the two projects separately. For example, if L is close to R_L^2 and if the following two conditions are satisfied*

$$p < 2/3 \text{ and } \frac{I^n - R_L}{p^2 R_H^2 + (1 - p^2) R_L^2} < \frac{R_H^2 + R_L^2 - (1 - p)^2 (R_H^2 - R_L^2)}{R_H^2 + R_L^2 - (1 - 2p^2) (R_H^2 - R_L^2)},$$

separate financing is optimal if and only if

$$\frac{I^n - R_L}{p + (1 - p) \frac{R_L^2}{R_H^2}} < R_H - R_L < \frac{2I^n - 2R_L}{1 - (1 - p)^2 \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}}.$$

Proofs for Appendix B

Proof of Lemma 1: The IR constraint is binding because if not, we could lower D_1 and still IC_h would be satisfied while the optimum would be higher. $y_1 = 1$ because, if not ($1 > y_1 > y_0$) an increase in y_1 by a small amount ε and an increase in D_1 by εL would keep IR and IC_h satisfied (the latter since $R_H^2 > L$) and the borrower's utility would increase by $p(R_H^2 - L)\varepsilon > 0$. IC_h is binding because, if not, we could raise y_0 by ε and increase D_1 by $\varepsilon L(1-p)/p$ so that to IR is still satisfied. Borrower's utility would increase by $-p\varepsilon L(1-p)/p + (1-p)\varepsilon R_L^2 = (1-p)\varepsilon(R_L^2 - L) > 0$. Q.E.D.

Proof of Proposition 9: From the previous lemma, we have from the IR and the IC constraints

$$\begin{aligned}(1 - y_0) R_H^2 + D_0 &= D_1, \\ pD_1 + (1 - p)(D_0 + (1 - y_0)L) &= I^n.\end{aligned}$$

Substituting the first into the second and rewriting, we have that

$$\begin{aligned}D_1 &= (1 - y_0)R_H^2 + D_0 \\ 1 - y_0 &= \frac{I^n - D_0}{pR_H^2 + (1 - p)L}.\end{aligned}$$

(Notice that the constraint $(y_0 - y_1)R_L^2 \geq D_0 - D_1$ is satisfied. Indeed $D_1 = (1 - y_0)R_H^2 + D_0 \geq (1 - y_0)R_L^2 + D_0$.) Substituting the D_1 in the objective function

$$pR_H + (1 - p)R_L - D_0 + y_0 [pR_H^2 + (1 - p)R_L^2]$$

and substituting y_0 we have

$$pR_H + (1 - p)R_L - D_0 + \left[1 - \frac{I^n - D_0}{pR_H^2 + (1 - p)L}\right] [pR_H^2 + (1 - p)R_L^2].$$

Deriving with respect to D_0 we have

$$-1 + \frac{pR_H^2 + (1 - p)R_L^2}{pR_H^2 + (1 - p)L}$$

which is positive because L is lower than R_L^2 and therefore the denominator is lower than the numerator. That should be as high as possible, $D_0 = R_L$. This is possible because the assumption ensures that $D_1 < R_H$. Q.E.D.

Proof of Proposition 10: First, following the same reasoning as before, we can show that the constraints are binding and $y_{1,1} = 1$. Substituting them into the the IC constraints and rewriting

$$D_{1,1} = (1 - y_{0,0})(R_H^2 + R_L^2) + (1 - y_{1,0})(R_H^2 - R_L^2) + D_{0,0}, \quad (\text{B10})$$

$$D_{1,0} = (1 - y_{0,0})(R_H^2 + R_L^2) - (1 - y_{1,0})(R_H^2 + R_L^2) + D_{0,0}. \quad (\text{B11})$$

Substituting these and $y_{1,1} = 1$ into the IR constraint

$$\begin{aligned} (IR) \quad & D_{0,0} + (1 - y_{0,0}) [(R_H^2 + R_L^2)(1 - (1 - p)^2) + (1 - p)^2 2L] \\ & - (1 - y_{1,0}) [2p(1 - p)(R_H^2 + R_L^2 - 2L) - p^2(R_H^2 - R_L^2)] = 2I^n \end{aligned}$$

and then in the objective function, we have

$$\begin{aligned} & p^2(2R_H) + 2(1 - p)p(R_L + R_H) + (1 - p)^2(2R_L) + p2R_H^2 + 2R_L^2(1 - p) - D_{0,0} \\ & - (1 - y_{1,0})p^2(R_H^2 - R_L^2) - (1 - y_{0,0}) [(R_H^2 + R_L^2) - (1 - p)^2(R_H^2 - R_L^2)]. \end{aligned}$$

Now, by increasing $D_{0,0}$ by εs (where $s = (R_H^2 + R_L^2)(1 - (1 - p)^2) + (1 - p)^2 2L$) and $y_{0,0}$ by ε we have the same IR and the objective function increases by $\varepsilon [(R_H^2 + R_L^2) - (1 - p)^2(R_H^2 - R_L^2) - (R_H^2 + R_L^2)(1 - (1 - p)^2) - (1 - p)^2 2L] = (1 - p)^2 [2R_L^2 - 2L] > 0$.

The IR can be written as

$$(1 - y_{0,0}) = \frac{2I^n - 2R_L + (1 - y_{1,0}) [2p(1 - p)(R_H^2 + R_L^2 - 2L) - p^2(R_H^2 - R_L^2)]}{(R_H^2 + R_L^2)(1 - (1 - p)^2) + (1 - p)^2 2L}$$

and therefore if $y_{1,0}$ is higher $y_{0,0}$ is higher and both the $y_{1,0}$ and $y_{0,0}$ terms increase the objective function. Provided that the condition on the statement of the proposition is true, we have that the numerator is positive. Q.E.D.

Proof of Lemma 2: We have that $y_{1,0}$ can be equal to 1 only if $D_{1,0} < R_H + R_L$, which simplifying is equivalent to

$$\frac{2I^n - 2R_L - (1 - y_{1,0}) [p^2 2R_H^2 + (1 - p^2) 2L]}{(R_H^2 + R_L^2)(1 - (1 - p)^2) + (1 - p)^2 2L} R_H^2 + R_L^2 < R_H - R_L.$$

Substituting $y_{1,0} = 1$, this condition is exactly as in the statement in the text.

The second part of the Lemma follows since the left hand side of the statement in the Lemma is higher than the left hand side of the statement in Proposition 2 as long as $(1 - p)L [R_L^2 + pR_H^2] + p^2 \frac{1}{2} (R_H^2 + R_L^2) R_H^2 > 0$, which is clearly true. Q.E.D.

Proof of Proposition 11: Substituting $y_{1,0} = 1$ into $(1 - y_{0,0})(R_H^2 + R_L^2) + (1 - y_{1,0})(R_H^2 - R_L^2) + 2R_L < 2R_H$ and simplifying

$$\frac{2I^n - 2R_L}{(R_H^2 + R_L^2)(1 - (1 - p)^2) + (1 - p)^2 2L} R_H^2 + R_L^2 < 2R_H - 2R_L,$$

and clearly if (B9) is satisfied this is also satisfied because this is less stringent.

Ex-ante inefficiencies from joint financing are

$$A = \frac{I^n - R_L}{\left(\frac{R_H^2 + R_L^2}{2}\right) \frac{(1 - (1 - p)^2)}{(1 - p)^2} + L} 2(R_L^2 - L),$$

Ex-ante inefficiencies from separate financing are

$$B = \frac{I^n - R_L}{\frac{p}{(1 - p)} R_H^2 + L} 2(R_L^2 - L).$$

Clearly $B > A$ because B has higher numerator and a lower denominator than A . Q.E.D.

Proof of Proposition 12: Substituting $D_{1,0} = R_L + R_H$ into the equation of the proposition, we have that

$$(1 - y_{1,0}) = (1 - y_{0,0}) - \frac{(R_H - R_L)}{(R_H^2 + R_L^2)}.$$

If we have that $0 < (1 - y_{1,0}) < (1 - y_{0,0}) < 1$ and that $D_{1,1} < R_H$ we have that the ex-ante inefficiencies are

$$A = (1 - p)^2 (1 - y_{0,0}) 2(R_L^2 - L) + 2p(1 - p) \left[(1 - y_{0,0}) - \frac{(R_H - R_L)}{(R_H^2 + R_L^2)} \right] (R_H^2 + R_L^2 - 2L)$$

and, for the case of separate projects,

$$B = \frac{I^n - R_L}{\frac{p}{(1 - p)} R_H^2 + L} 2(R_L^2 - L).$$

Clearly if L is close to R_L^2 and $R_H^2 > R_L^2 = L$, we would have that

$$B = 0 \text{ and } A = 2p(1 - p)(R_H^2 - R_L^2)(1 - y_{1,0}) > 0$$

and therefore separation is optimal.

We now need to check that $(1 - y_{1,0}) > 0$ (clearly, then $(1 - y_{0,0}) > 0$). Substituting $R_L^2 = L$ in the expression above, we have that $(1 - y_{1,0}) > 0$ as long as

$$\frac{2I^n - 2R_L}{1 - (1 - p)^2 \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}} > R_H - R_L. \quad (\text{B12})$$

Similarly, we have that $(1 - y_{0,0}) < 1$ (and therefore $(1 - y_{1,0}) < 1$) if and only if (remember that we have $(2 - 3p) > 0$),

$$\frac{2I^n - 2R_L - p^2 2R_H^2 - (1 - p^2) 2R_L^2}{p(2 - 3p) \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}} < R_H - R_L. \quad (\text{B13})$$

We also need that $D_{1,1} = (1 - y_{0,0})(R_H^2 + R_L^2) + (1 - y_{1,0})(R_H^2 - R_L^2) + 2R_L < 2R_H$, which substituting is equal to

$$\frac{2I^n - 2R_L}{p^2 + (1 - p^2) \frac{R_L^2}{R_H^2} + \left[1 - (1 - p)^2 \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}\right]} < R_H - R_L. \quad (\text{B14})$$

Notice that if we have that the left-hand side of (B13) is lower than the one of independent projects then, by assuming the latter the former becomes irrelevant. This is true if the second condition of the statement of the proposition is satisfied, i.e. the second condition is equivalent to

$$\frac{2I^n - 2R_L - p^2 2R_H^2 - (1 - p^2) 2R_L^2}{p(2 - 3p) \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}} < \frac{2I^n - 2R_L}{1 - (1 - p)^2 \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}}.$$

It is easy to check that if the condition for the statement for separate projects is satisfied then the (B14) also becomes irrelevant. Finally, it is easy to check that the condition on separation can be satisfied simultaneously with (B14). Summarizing, it can indeed be that $R_H - R_L$ satisfy

$$\frac{I^n - R_L}{p + (1 - p) \frac{R_L^2}{R_H^2}} < R_H - R_L < \frac{2I^n - 2R_L}{1 - (1 - p)^2 \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}},$$

and therefore all the conditions are satisfied. Q.E.D.

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