

Optimal Contracts for Experimentation^{*}

Marina Halac[†]

Navin Kartik[‡]

Qingmin Liu[§]

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Abstract

This paper studies long-term contracts for experimentation in a principal-agent setting with adverse selection (pre-contractual hidden information) about the agent's ability, dynamic moral hazard, and private learning about project quality. We show that profit maximization by the principal generally leads to under-experimentation by an agent of low ability, even though there would be no distortion in the absence of either adverse selection or moral hazard. The structure of optimal contracts is shaped by a variety of considerations including dynamic agency costs and the possibility of post-contractual hidden information about project quality. We derive two explicit menus of contracts that can be used to implement the second-best solution: “bonus contracts” and “clawback contracts”. Both feature history-contingent dynamic streams of transfers.

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[†]Graduate School of Business, Columbia University. Email: mhalac@columbia.edu.

[‡]Department of Economics, Columbia University. Email: nkartik@columbia.edu.

[§]Department of Economics, Columbia University. Email: qingmin.liu@columbia.edu.

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1 Introduction

This paper is concerned with the following contracting problem: a principal owns a project whose quality — viability, profitability, or difficulty — is unknown. There is an agent who has the expertise to work on or *experiment* with the project, but incurs a private cost in doing so. Both parties learn about the project’s quality over time as a function of output, the agent’s sequence of effort, and the agent’s (persistent) ability or skill. The expected benefit of effort at any time depends on these beliefs: if, at some point, beliefs about the project’s quality are sufficiently pessimistic relative to the agent’s ability, it would be optimal to abandon the project altogether. Both the agent’s ability and his effort choice in any period are unobservable to the principal, inducing both adverse selection and moral hazard.¹

These features are relevant in many contractual environments. Perhaps the most obvious application is the design of incentives within or across organizations for research and development (R&D). A related application is the testing of a breakthrough product, e.g. investigating potential side effects of a new drug. But there are other quite distinct applications: for example, a firm or other organization may hire a recruiting agency to search for an external candidate for its CEO or president position. The agency’s ability and effort combine with uncertain market conditions to determine when it is optimal to end the search and just promote the best internal candidate.

Although dynamic moral hazard, adverse selection, and learning are essential features of these agency relationships, there is virtually no existing theoretical work on contracting in such environments. It bears emphasis that “learning” here refers not to the principal updating about the agent’s ability over time, but rather both parties updating about a persistent state variable — the project’s quality — that affects the social value of effort given the agent’s ability. The socially optimal level of effort is history dependent, and private benefits are determined by a conjunction of hidden information, hidden action, and evolving beliefs about the state. Consequently, these are rich environments to contract in. How well can a principal incentivize the agent? What is the nature of distortions, if any, that arise? What are the qualitative properties of optimal incentive contracts?

Our main contribution is to provide answers to these questions in a simple yet canonical model of experimentation, the so-called “exponential bandit” model, which we now overview.

Modeling framework. The project at hand may either be good or bad (a persistent state). Time is discrete, and in each period, the agent can either exert effort (work) or not (shirk), a binary choice that is unobservable to the principal. The agent incurs a constant private cost in each period that he exerts effort. If the agent works in a period and the project is good, the project is successful in that period with some constant probability; if either the agent shirks or the project is bad, success cannot obtain in that period. Project success is publicly observable and obviates the need for any further effort.² The probability of success in

¹We use the term “adverse selection” synonymously with pre-contractual hidden information in a principal-agent relationship.

²Our analysis also applies if project success is privately observed by the agent but can be verifiably disclosed.

a period (conditional on the agent working and the project being good) depends on the agent's persistent ability, which, as usual, we refer to as his *type*. This is a binary variable — ability is either high or low — that is the agent's private information at the time of contracting. Project success yields a fixed social surplus that is directly accrued by the principal. Both parties are risk neutral and discount the future at a common rate.

Social optimum. Consider the first-best solution, i.e. when the agent's ability and effort are observable. The public belief that the project is good declines so long as effort has been exerted but success not obtained. Since effort is costly, the social optimum is characterized by a stopping time for each agent type: as a function of his ability, the agent keeps working (so long as he has failed to obtain a success in the past) up until some point at which the project is permanently abandoned. It turns out that the optimal stopping time is a non-monotonic function of the agent's ability. The intuition stems from two counter-vailing forces: on the one hand, for any given belief about the project's quality, a higher-ability agent has a greater marginal benefit of effort (since conditional on the project being good, he succeeds with a higher probability); but at any point in time, a higher-ability agent is also more pessimistic about the project's quality (conditional on having exerted effort and failed in all prior periods) because a failure is more informative about project quality when the agent's ability is higher. Thus, depending on parameter values, the optimal stopping time for a high-ability agent may be larger or smaller than that of a low-ability agent.

Agency issues. The contracting problem entails hidden information at the time of contracting (the agent's ability), dynamic hidden action (effort is costly for the agent and unobservable to the principal), and *private* learning (both parties update over time about project quality and hence the benefits of effort, as a function of their beliefs about ability and the history of effort). The principal's goal is to maximize profits, and to do so, she can commit ex-ante to a dynamic contract that specifies a sequence of transfers to the agent as a function of the publicly observable history, viz. project success/failure. More precisely, since there is hidden information at the time of contracting, the principal may offer the agent a *menu* of such dynamic contracts from which the agent can choose one.

Let us highlight some of the agency considerations that arise. First, since the agent chooses effort in a sequentially optimal fashion and there is moral hazard, a contract can induce the agent to choose different effort profiles depending on his ability; this is a key distinguishing feature from canonical static adverse selection problems. In particular, for an arbitrary contract, there is no analog of a “single-crossing property” to determine the relationship between the optimal effort profiles for the two types. Second, since the agent of either type is getting more pessimistic about the likelihood of project success as failures mount (so long as he keeps exerting effort), the static incentive constraint for effort becomes more demanding over time. While this suggests that it may be optimal to provide increasing rewards for success over time, there is also a dynamic incentive constraint that the agent should not prefer to postpone current effort to the future in order to benefit from a higher future reward for success. In other words, there can be a *dynamic agency cost*: the presence of a future reward for success makes it less costly for the agent to

forego a present reward for success and hence harder to prevent the agent from shirking in the present. Third, in addition to the pre-contractual hidden information about the agent's ability, private learning implies that there is also the possibility of post-contractual hidden information regarding beliefs about project quality.³ In particular, the principal's and the agent's beliefs about project quality would diverge whenever the agent deviates by either choosing a different contract from the menu than he is intended to and/or by shirking when the principal expects him to work.⁴ All these elements come into play when determining how to best incentivize the agent while minimizing his "information rent". Accordingly, part of this paper's contribution is methodological.

Results. Consider first the case where the first-best stopping time for a high-ability agent is larger than that of a low-ability agent. In an optimal menu of contracts, the principal screens the agent types by offering two distinct contracts that satisfy the relevant self-selection or incentive compatibility constraints for contract choice. Each type's contract induces him to work for a sequence of consecutive periods (so long as success has not been obtained) at which point the project is abandoned. Compared to the social optimum, there is a distortion in the stopping time: while the high-ability type's stopping time is efficient, the low-ability type stops experimentation too early. This implies that it is not optimal to simply "sell the project" to the agent, because doing so would always induce the socially optimal stopping time. While this inefficiency result is reminiscent of the familiar "no distortion at the top but distortion below" in static models of adverse selection, the logic is rather different because of the varied considerations described above and elaborated on later. In particular, it is the *conjunction* of adverse selection and moral hazard that drives our results; without either one, the principal would maximize profits without causing any distortion from the first best.

We show how to implement the optimal solution in two different and economically interesting ways: a menu of *bonus contracts* and a menu of *clawback contracts*. In a bonus contract, the agent pays the principal an up-front fee and is then rewarded with a bonus that depends on when the project succeeds (if ever). For the low type, we characterize explicitly the unique time-dependent bonus that is optimal: it is increasing over time up until the second-best stopping time when it drops to zero, i.e. the contract terminates. For the high type, we show that the principal has latitude among a set of optimal bonus contracts.⁵ One way of interpreting bonus contracts is that the principal sells the project to the agent at the outset for a specified price, but commits to buy back the project's output (which obtains with a success) at time-dated future prices, so long as output is obtained by a certain date.

By contrast, in a clawback contract, the principal pays the agent an up-front amount, which can be

³Post-contractual hidden information is sometimes referred to as "hidden knowledge" or "moral hazard with hidden information" (e.g. [Hart and Holmström, 1987](#)).

⁴Of course, such deviations will not occur in an optimal menu of contracts; however, these off-path considerations matter when determining what the optimal menu is.

⁵The reason for the asymmetry is that the low type's bonus contract is uniquely pinned down by rent-minimization considerations, unlike the high type's contract. Of course, the high type's bonus contract cannot be arbitrary either.

viewed as a pre-payment for future success. Then, in each period in which a success does not obtain, the agent is required to pay the principal some time-dependent amount, up until either the project succeeds or the contract terminates. We characterize the unique sequence of payments from the agent to the principal that must be used in an optimal contract for the low-ability type: the payment sequence increases over time with a jump at the termination date. We call this a “clawback contract” based on the idea that clawbacks in practice involve recouping a payment already made to the agent (sometimes with added penalties) when there is some, perhaps inconclusive, evidence of the agent’s negligence. The evidence in our context is the lack of project success; it is important to note, however, that in equilibrium the agent does not shirk prior to the contract’s termination date. As with bonus contracts, there are multiple optimal clawback contracts that can be used for the high-ability type.

The case where the first-best stopping time for the high type is lower than that of the low type proves to be more challenging methodologically, for reasons best postponed until later. We fully characterize the solution here when there is no discounting. We find that the distortion (or lack thereof) in the duration of experimentation for each type is qualitatively the same as above. Nevertheless, the considerations driving the structure of optimal contracts have some important differences and implications.

Related literature. Broadly, this paper fits into the sizeable literature on dynamic moral hazard. Most contributions that study long-term contracting abstract from adverse selection, but recent exceptions include [Sannikov \(2007\)](#), [Fong \(2009\)](#), and [Gershkov and Perry \(2012\)](#).⁶ These papers are not concerned with experimentation and their settings differ from ours in multiple ways. Private learning from experimentation is a key feature of the economic environments we are interested in, and as we show, is qualitatively important to our results.⁷

A few authors have studied contracting for experimentation, but they address different issues than we do.⁸ In [Manso \(2011\)](#), a principal must not only incentivize an agent to work rather than shirk, but also to work on exploration of an uncertain technology rather than exploitation of a known technology. The latter concern, which we do not feature, is essential to his main insights; see also [Klein \(2012\)](#), and for a multiple-agent version, [Ederer \(2009\)](#). These papers do not have adverse selection and hence their focus

⁶Earlier papers with adverse selection and dynamic moral hazard, such as [Laffont and Tirole \(1988\)](#), often focus on the effects of short-term contracting.

⁷See [Demarzo and Sannikov \(2011\)](#) for a nice discussion of why private learning introduces analytical challenges in a model without adverse selection. [Chassang \(2011\)](#) considers a rather general environment and develops an approach to find detail-free contracts that are not optimal but instead guarantee some efficiency bounds so long as there is a long horizon and players are patient. Adverse selection and dynamic moral hazard also naturally arise in insurance problems, as surveyed recently by [Dionne et al. \(2012\)](#), but we are not aware of any related theoretical analysis on optimal contracts in this literature.

⁸Experimentation or “bandit” problems date back in economics to at least [Rothschild \(1974\)](#) and are surveyed by [Bergemann and Välimäki \(2008\)](#). Building on the Brownian model of [Bolton and Harris \(1999\)](#), games — as opposed to contracting problems — where players experiment with exponential bandits have been the focus of [Keller et al. \(2005\)](#), [Strulovici \(2010\)](#), [Bonatti and Hörner \(2011\)](#), and [Garfagnini \(2011\)](#), among others. [Bonatti and Hörner \(2012\)](#) study a career-concerns problem in the tradition of [Holmstrom \(1999\)](#), but model the agent’s effort as experimentation with an exponential bandit.

is quite different. Somewhat closer to our setting is [Hörner and Samuleson \(2012\)](#), who emphasize the dynamic agency cost mentioned earlier (see also [Bhaskar, 2009](#); [Mason and Välimäki, 2011](#)). Again, they do not have adverse selection. Motivated by venture capital financing (cf. [Bergemann and Hege, 1998, 2005](#)), Hörner and Samuelson examine a different aspect of agency by assuming short-term contracting with limited liability, specifically that the agent’s effort requires some fixed funding in each period and a spot contract specifies a profit-sharing arrangement. In this respect, we are closer to [Besanko et al. \(2012\)](#), but their framework does not have moral hazard and instead focuses on issues of ambiguity. [Gerardi and Maestri \(2012\)](#) analyze how an agent can be incentivized to acquire and truthfully report information over time using payments that compare the agent’s reports with the ex-post observed state of nature. By contrast, we assume that the state of nature is never observed when experimentation is terminated.

Our model can also be interpreted as a problem of delegated sequential search, as in one of the applications mentioned at the beginning of the introduction. [Lewis and Ottaviani \(2008\)](#) and [Lewis \(2011\)](#) are recent contributions on this topic. The main difference is that, in our context, these papers assume that the project’s quality is known and hence there is no learning about the likelihood of success (cf. [Subsection 6.2](#)); moreover, they do not have adverse selection.

2 The Model

Environment. A principal needs to hire an agent to work on a project. The project’s quality — synonymous with the state of nature — may either be good or bad, a binary variable. Both parties are initially uncertain about the project’s quality; the common prior on the project being good is $\beta_0 \in (0, 1)$. The agent is privately informed about whether his ability is low or high, $\theta \in \{L, H\}$, where $\theta = H$ represents “high”. The principal’s prior on the agent’s ability being high is $\mu_0 \in (0, 1)$. In each period, $t \in \{1, 2, \dots\}$, the agent can either exert effort (work) or not (shirk); this choice is never observed by the principal. Exerting effort in any period costs the agent $c > 0$. If effort is exerted and the project is good, the project is successful in that period with probability λ^θ ; if either the agent shirks or the project is bad, success cannot obtain in that period. Success is observable and once a project is successful, no further effort is needed.⁹ We assume $1 > \lambda^H > \lambda^L > 0$. A success yields the principal a payoff normalized to 1; the agent does not intrinsically care about project success. Both parties are risk neutral, share a common discount factor $\delta \in (0, 1]$, and are expected-utility maximizers.

Contracts. We consider contracting at period zero with full commitment power from the principal. Since there is hidden information at the time of contracting, without loss of generality the principal’s problem is to offer the agent a *menu* of dynamic contracts from which the agent chooses one. A dynamic contract

⁹[Subsection 6.1](#) establishes that our results apply without change if success is privately observed by the agent but can be verifiably disclosed.

specifies a sequence of transfers as a function of the publicly observable history, which is simply whether or not the project has been successful to date. Throughout, we follow the convention that transfers are from the principal to the agent; negative values represent payments in the other direction. We assume that once the agent has accepted a contract, he is free to work or shirk in any period up until some termination date that is specified by the contract.¹⁰

Formally, a *contract* is given by $\mathbf{C} = (T, W_0, \mathbf{b}, \mathbf{l})$, where $T \in \mathbb{N}$ is the *termination date* of the contract, $W_0 \in \mathbb{R}$ is an up-front transfer (or wage) at period zero, $\mathbf{b} = (b_1, \dots, b_T)$ specifies a transfer $b_t \in \mathbb{R}$ conditional on the project being successful in period $t \leq T$, and analogously $\mathbf{l} = (l_1, \dots, l_T)$ specifies a transfer $l_t \in \mathbb{R}$ conditional on the project not being successful in period $t \leq T$ (nor in any prior period).¹¹ We refer to any b_t as a *bonus* and any l_t as a *penalty*; note, however, that b_t is not constrained to be positive nor must l_t be negative, although these cases will be focal and hence our choice of terminology. Without loss of generality, we assume that $T = \max\{t : \text{either } b_t \neq 0 \text{ or } l_t \neq 0\}$. The agent's actions are denoted by $\mathbf{a} = (a_1, \dots, a_T)$, where $a_t = 1$ if the agent works in period t and $a_t = 0$ if the agent shirks.

Payoffs. The principal's expected discounted payoff at time zero from a contract $\mathbf{C} = (T, W_0, \mathbf{b}, \mathbf{l})$, an agent of type θ , and a sequence of actions from the agent \mathbf{a} is denoted $\Pi_0^\theta(\mathbf{C}, \mathbf{a})$ and computed as follows:

$$\Pi_0^\theta(\mathbf{C}, \mathbf{a}) := -W_0 - (1 - \beta_0) \sum_{t=1}^T \delta^t l_t + \beta_0 \sum_{t=1}^T \delta^t \left[\prod_{s < t} (1 - a_s \lambda^\theta) \right] \left[a_t \lambda^\theta (1 - b_t) - (1 - a_t \lambda^\theta) l_t \right]. \quad (1)$$

To interpret the above formula, note first that W_0 is the up-front payment made from the principal to the agent. With probability $1 - \beta_0$ the state is bad, in which case the project never succeeds and the principal also pays the agent the entire sequence of penalties \mathbf{l} . Conditional on the state being good (which occurs with probability β_0), the probability of project success depends on both the agent's effort choices and his ability; $\prod_{s < t} (1 - a_s \lambda^\theta)$ is the probability that a success does not obtain between period 1 and $t - 1$ conditional on the good state. If the project were to succeed at time t , then the principal would earn a payoff of 1 in that period and would have paid the agent not only the up-front payment but also the sequence of penalties (l_1, \dots, l_{t-1}) followed by the bonus b_t .

Through analogous reasoning, bearing in mind that the agent does not directly value project success but incurs the cost of effort, the agent's expected discounted payoff at time zero given his type θ ,

¹⁰There is no loss of generality here. If the principal has the ability to block the agent from choosing whether to work in some period — “lock him out of the laboratory”, so to speak — this can just as well be achieved by instead stipulating that project success in that period would trigger a large payment to the principal.

¹¹Throughout, symbols in bold typeface denote vectors. W_0 and T are redundant because W_0 can be effectively induced by suitable modifications to b_1 and l_1 , while T can be effectively induced by setting $b_t = l_t = 0$ for all $t > T$. However, it is expositionally convenient to include these components explicitly in defining a contract. Furthermore, there is no loss in assuming that $T \in \mathbb{N}$; as shown below, it is always optimal for the principal to stop experimentation at a finite time, so she cannot benefit from setting $T = \infty$.

contract \mathbf{C} , and action profile \mathbf{a} is

$$U_0^\theta(\mathbf{C}, \mathbf{a}) := W_0 + (1 - \beta_0) \sum_{t=1}^T \delta^t (l_t - a_t c) + \beta_0 \sum_{t=1}^T \delta^t \left[\prod_{s < t} (1 - a_s \lambda^\theta) \right] \left[a_t (\lambda^\theta b_t - c) + (1 - a_t \lambda^\theta) l_t \right]. \quad (2)$$

If a contract is not accepted, both parties' payoffs are normalized to zero. Given any contract \mathbf{C} , it will be useful to denote $\alpha^\theta(\mathbf{C}) := \arg \max_{\mathbf{a}} U_0^\theta(\mathbf{C}, \mathbf{a})$ as the set of optimal action plans for the agent of type θ under this contract. With a slight abuse of notation, we will write $U_0^\theta(\mathbf{C}, \alpha^\theta(\mathbf{C}))$ for the type- θ agent's indirect utility at time zero from any contract \mathbf{C} .

3 Benchmarks

This section presents preliminaries concerning efficiency benchmarks and simple classes of contracts.

3.1 The first best

Consider the first-best solution, i.e. when the agent's type θ is commonly known and his effort in each period is commonly observable. Since beliefs about the project quality (i.e. the state being good) decline so long as effort has been exerted but success not obtained, the first-best solution is characterized by a stopping rule such that an agent of ability θ keeps exerting effort so long as success has not obtained up until some period t^θ , whereafter effort is no longer exerted. Let β_t^θ be a generic belief on the state being good at the beginning of period t (which will depend on the history of effort), and $\bar{\beta}_t^\theta$ be this belief when the agent has exerted effort in all periods $1, \dots, t-1$. The optimal stopping time t^θ is given by

$$t^\theta = \max_{t \geq 0} \left\{ t : \bar{\beta}_t^\theta \lambda_\theta \geq c \right\}, \quad (3)$$

where, for each θ , $\bar{\beta}_0^\theta := \beta_0$, and for $t \geq 1$, Bayes' rule yields

$$\bar{\beta}_t^\theta = \frac{\beta_0 (1 - \lambda^\theta)^{t-1}}{\beta_0 (1 - \lambda^\theta)^{t-1} + (1 - \beta_0)}. \quad (4)$$

Note that (3) is only well-defined when $c \leq \beta_0 \lambda^\theta$; if $c > \beta_0 \lambda^\theta$, it would be optimal to never experiment, i.e. stop at $t^\theta = 0$. To focus on the most interesting cases, we assume the following:

Assumption 1. *Experimentation is efficient for both types: for $\theta \in \{L, H\}$, $\beta_0 \lambda^\theta > c$.*

In particular, the above assumption implies $c < 1$, where 1 is the social benefit from project suc-

cess. If parameter values are such that $\bar{\beta}_{t^\theta}^\theta \lambda^\theta = c$,¹² equations (3) and (4) can be combined to derive the following closed-form solution for the optimal stopping time for type θ :

$$t^\theta = 1 + \frac{\log\left(\frac{c}{\lambda^\theta - c} \frac{1 - \beta_0}{\beta_0}\right)}{\log(1 - \lambda^\theta)}. \quad (5)$$

Equation (5) yields intuitive monotonicity of the first-best stopping time as a function of the prior that the project is good, β_0 , and the cost of effort, c .¹³ But it also implies a fundamental non-monotonicity as a function of the agent's ability, λ^θ , as shown in Figure 1.¹⁴ This stems from the interaction of two countervailing forces. On the one hand, for any given belief about the state, the expected marginal benefit of effort is higher when the agent's ability is higher; on the other hand, the higher is the agent's ability, the more informative is a lack of success in a period in which he works. Hence, at any time $t > 1$, a higher-ability agent is more pessimistic about the state (given that effort has been exerted in all prior periods), which has the effect of decreasing the expected marginal benefit of effort. Altogether, this makes the first-best stopping time non-monotonic in ability; both $t^H > t^L$ and $t^H < t^L$ are robust possibilities that arise for different parameters. As we will see, this has substantial implications.

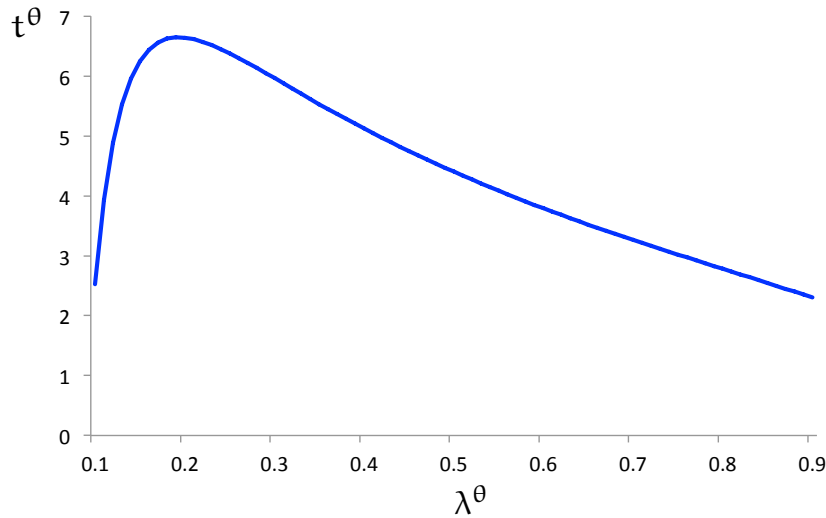


Figure 1 – The first-best stopping time.

¹²We do not assume this condition in the main analysis, but it is convenient to use for the discussion that follows in this subsection.

¹³One may also notice that the discount factor, δ , does not enter (5). In other words, unlike the traditional focus of bandit models, there is no tradeoff here between “exploration” and “exploitation”, as the first-best strategy is invariant to patience. Our model and subsequent analysis can be generalized to incorporate this tradeoff, but the additional burden does not yield commensurate insight.

¹⁴For simplicity, the figure ignores integer constraints on t^θ .

The first-best expected discounted surplus at time zero from type θ is

$$\sum_{t=1}^{t^\theta} \delta^t \left[\beta_0 (1 - \lambda^\theta)^{t-1} (\lambda^\theta - c) - (1 - \beta_0)c \right].$$

3.2 Simple contracts

One class of canonical contracts is where aside from the initial transfer, any other transfer occurs only when the agent obtains a success by the termination date:

Definition 1. A *bonus contract* is $\mathbf{C} = (T, W_0, \mathbf{b}, \mathbf{l})$ such that $l_t = 0$ for all $t \in \{1, \dots, T\}$. A bonus contract is a *constant-bonus contract* if, in addition, there is some constant b such that $b_t = b$ for all $t \in \{1, \dots, T\}$.

While a general bonus contract rewards the agent with a time-dependent bonus for success, a constant-bonus contract pays the same reward independent of when success is obtained up until the deadline. When the context is clear, we ease notation by denoting a bonus contract as just $\mathbf{C} = (T, W_0, \mathbf{b})$ and a constant-bonus contract as $\mathbf{C} = (T, W_0, b)$.

Another kind of contract that will prove useful is where the agent receives no payments for success, and instead is penalized for failure (i.e. lack of success). Such a contract will satisfy the agent's participation constraint only if he is paid some positive amount at time zero, which motivates the terminology of "clawbacks". Formally:

Definition 2. A *clawback contract* is $\mathbf{C} = (T, W_0, \mathbf{b}, \mathbf{l})$ such that $b_t = 0$ for all $t \in \{1, \dots, T\}$. A clawback contract is a *onetime-clawback contract* if, in addition, $l_t = 0$ for all $t \in \{1, \dots, T-1\}$.

Note that a clawback contract allows for $|l_t| > W_0$ in any t , i.e. clawbacks should be understood as allowing for penalties larger than what the agent initially received from the principal at time zero. Unlike bonus contracts, where at most one bonus is ever paid, a general clawback contract involves the possibility of transfers in multiple periods after experimentation has begun; in particular, the agent may be penalized for failure in every period from 1 to the termination date, T . However, in a onetime-clawback contract, this is not the case because the agent makes only one payment aside from any initial transfer, which is at time T (conditional on no success up to that point). When the context is clear, we ease notation by denoting a clawback contract as just $\mathbf{C} = (T, W_0, \mathbf{l})$ and a onetime-clawback contract as $\mathbf{C} = (T, W_0, l_T)$.

As one might expect, there is an isomorphism between clawback contracts and bonus contracts; furthermore, either class is "large enough" in a suitable sense. To make this notion precise, we introduce the following definition.

Definition 3. Two contracts, $\mathbf{C} = (T, W_0, \mathbf{b}, \mathbf{l})$ and $\widehat{\mathbf{C}} = (T, \widehat{W}_0, \widehat{\mathbf{b}}, \widehat{\mathbf{l}})$, *perform equivalently* or are simply *equivalent* if for all $\theta \in \{L, H\}$ and $\mathbf{a} = (a_1, \dots, a_T)$:

$$U_0^\theta(\mathbf{C}, \mathbf{a}) = U_0^\theta(\widehat{\mathbf{C}}, \mathbf{a}) \text{ and } \Pi_0^\theta(\mathbf{C}, \mathbf{a}) = \Pi_0^\theta(\widehat{\mathbf{C}}, \mathbf{a}).$$

In particular, for any given type of the agent, his set of optimal action plans is invariant across equivalent contracts.

Proposition 1. *For any contract, $\mathbf{C} = (T, W_0, \mathbf{b}, \mathbf{l})$, there exist both an equivalent clawback contract $\widehat{\mathbf{C}} = (T, \widehat{W}_0, \widehat{\mathbf{l}})$ and an equivalent bonus contract $\widetilde{\mathbf{C}} = (T, \widetilde{W}_0, \widetilde{\mathbf{b}})$.*

Proof. See [Appendix A](#).

Q.E.D.

[Proposition 1](#) implies that it is without loss to focus either on bonus contracts or on clawback contracts. The proof is constructive: given an arbitrary contract, it explicitly derives equivalent clawback and bonus contracts. The intuition is that all that matters in any contract is the induced vector of discounted transfers for success occurring in each possible period (and never). The proof also shows that when $\delta = 1$, onetime-clawback contracts are equivalent to constant-bonus contracts.

3.3 No adverse selection or no moral hazard

There are two sources of asymmetric information in our model: adverse selection and moral hazard. To see that their interaction is essential, it is useful to understand what would happen if only one of the two sources is present.¹⁵

Consider first the case without adverse selection, i.e. assume the agent's ability is observable but there is moral hazard. Constant-bonus contracts are then optimal for the principal. To see this, suppose the principal offers the agent of type θ a constant-bonus contract $\mathbf{C}^\theta = (t^\theta, W_0^\theta, \mathbf{1})$, where W_0^θ is chosen so that conditional on the agent exerting effort in each period up to the termination date (as long as success has not obtained), the agent's participation constraint at time 0 binds:

$$U_0^\theta(\mathbf{C}^\theta, \mathbf{1}) = \sum_{t=1}^{t^\theta} \delta^t \left[\beta_0 (1 - \lambda^\theta)^{t-1} (\lambda^\theta - c) - (1 - \beta_0)c \right] + W_0^\theta = 0,$$

where the notation $\mathbf{1}$ denotes the action profile of working in every period of the contract. This contract effectively sells the project to the agent at a price that extracts all the surplus. Plainly, this achieves the first-best level of experimentation and the principal cannot improve on this. [Proposition 1](#) implies that this optimum can also be implemented with clawback contracts; in fact, it can be shown that onetime-clawback contracts suffice.

Consider next the case with adverse selection but without moral hazard: the agent's effort in any period still costs him $c > 0$ but is observable and contractible. Formally this amounts to ignoring incentive compatibility constraints for effort. In this setting, the principal can use the two types' differing

¹⁵Learning about project quality is also important for our results. We discuss the case where both adverse selection and moral hazard are present but there is no learning about the project quality in [Subsection 6.2](#).

probabilities of success to screen the agent without creating any distortions, crucially exploiting the fact that under observable effort the agent can effectively be prohibited from adjusting his effort profile.¹⁶ To be more specific, suppose the principal offers the agent a choice between two constant-bonus contracts, both of which require the agent to work in every period until the termination date: $C^H = (t^H, W_0^H, b^H)$ and $C^L = (t^L, W_0^L, b^L)$, where $b^H > 0 > b^L$, and for each $\theta \in \{L, H\}$, W^θ is set so that $U_0^\theta(C^\theta, 1) = 0$, i.e. type θ 's participation constraint binds in the contract C^θ . Notice that contract C^H rewards the agent for success because $b^H > 0$ whereas C^L punishes the agent for success because $b^L < 0$. Under either contract, type H is more likely to incur these transfers than type L , given that the agent must work in each period. Consequently, by choosing the magnitudes of both b^H and b^L sufficiently large, one can ensure that type L finds C^H "too risky" to accept over C^L whereas type H finds C^L too risky to accept over C^H . We should emphasize that the bonuses in these contracts are not used to incentivize effort but are instead solely an instrument to screen the agent. The mechanism is reminiscent of [Cremer and McLean \(1985\)](#) because effectively the principal offers the agent a choice between two bets on whether success will be obtained when effort is exerted in every period, and exploits the two types' distinct success probabilities to extract all the surplus by pricing the bets differently.

An analogous argument can also be used to implement the optimum with clawback contracts rather than constant-bonus contracts, as predicted by [Proposition 1](#); indeed, onetime-clawback contracts again suffice. To summarize:

Theorem 1. *If there is either no moral hazard or no adverse selection, the principal optimally implements the first best and extracts all the surplus; either constant-bonus or onetime-clawback contracts can be used to achieve this.*

Proof. See [Appendix A](#).

Q.E.D.

4 Optimal Contracts when $t^H > t^L$

4.1 The solution

We develop our main results on optimal contracts for experimentation with both adverse selection and moral hazard by first studying the case where the first-best stopping times are ordered $t^H > t^L$, i.e. when the speed-of-learning effect that pushes the first-best stopping time down does not dominate the productivity effect that pushes in the other direction. We maintain this assumption implicitly throughout the discussion in this section.

Without loss, we assume that the principal specifies a desired effort profile along with a contract. An optimal menu of contracts maximizes the principal's ex-ante expected profit subject to incentive

¹⁶For example, the principal can stipulate severe penalties if the agent does not use the desired effort profile.

compatibility constraints for effort (IC_a^θ below), participation constraints (IR^θ below), and self-selection constraints for the agent's choice of contract ($\text{IC}^{\theta,\theta'}$ below). Formally, the principal's program is:

$$\max_{(\mathbf{C}^H, \mathbf{C}^L, \mathbf{a}^H, \mathbf{a}^L)} \mu_0 \Pi_0^H(\mathbf{C}^H, \mathbf{a}^H) + (1 - \mu_0) \Pi_0^L(\mathbf{C}^L, \mathbf{a}^L)$$

subject to, for all $\theta, \theta' \in \{L, H\}$,

$$\mathbf{a}^\theta \in \alpha^\theta(\mathbf{C}^\theta), \quad (\text{IC}_a^\theta)$$

$$U_0^\theta(\mathbf{C}^\theta, \mathbf{a}^\theta) \geq 0, \quad (\text{IR}^\theta)$$

$$U_0^\theta(\mathbf{C}^\theta, \mathbf{a}^\theta) \geq U_0^{\theta'}(\mathbf{C}^{\theta'}, \alpha^{\theta'}(\mathbf{C}^{\theta'})). \quad (\text{IC}^{\theta,\theta'})$$

Adverse selection is reflected in the self-selection constraints ($\text{IC}^{\theta,\theta'}$), as is familiar. Dynamic moral hazard is reflected directly in the constraints (IC_a^θ) and, importantly, also indirectly in the constraints ($\text{IC}^{\theta,\theta'}$) via the term $\alpha^\theta(\mathbf{C}^{\theta'})$. To get a sense of how these matter, consider the problem of incentivizing the agent to work in some period t . The agent's incentive to work at t is shaped not only by period t transfers (b_t and l_t) but also by the subsequent sequences of transfers through their effect on continuation values. In particular, *ceteris paribus*, raising the continuation value (say, by increasing either b_{t+1} or l_{t+1}) makes reaching period $t + 1$ more attractive and hence reduces the incentive to work in period t : this is a dynamic agency effect. Note moreover that the continuation value at any point in a contract depends on the agent's type and his effort profile; hence it is not sufficient to simply work with a single continuation value at each period. Furthermore, besides having an effect on continuation values, the agent's type also affects current incentives for effort because the expected marginal benefit of effort in any period differs for the two types. Altogether, the optimal plan of action will generally be different for the two types of the agent, i.e. for an arbitrary contract \mathbf{C} , we may have $\alpha^H(\mathbf{C}) \cap \alpha^L(\mathbf{C}) = \emptyset$.

Our result on second-best efficiency is as follows:

Theorem 2. Assume $t^H > t^L$. In any optimal menu of contracts, each type $\theta \in \{L, H\}$ is induced to work for some number of periods, \bar{t}^θ ; if $\delta < 1$, the periods of work are $1, \dots, \bar{t}^\theta$.¹⁷ Relative to the first-best stopping times, t^H and t^L , the second-best has $\bar{t}^H = t^H$ and $\bar{t}^L \leq t^L$.

Theorem 2 says that relative to the first best, there is no distortion in the stopping time of the high-ability agent whereas the low-ability agent stops experimenting too early. It is interesting that this is a familiar “no distortion (only) at the top” result from static models of adverse selection, even though the inefficiency arises here from the *conjunction* of adverse selection and dynamic moral hazard (cf. **Theorem 1**). The reason is that only in the presence of moral hazard does the high type extract an “information rent” here. As we will see subsequently, distorting the duration of experimentation from the low type allows the

¹⁷Generically, \bar{t}^θ is unique for each $\theta \in \{L, H\}$. In the non-generic cases where multiple optima exist, we focus on the highest optimum for each type.

principal to reduce this information rent. The optimal \bar{t}^L trades off this information rent with efficiency or social surplus from the low type. For typical parameters, it will be the case that $\bar{t}^L \in \{1, \dots, t^L - 1\}$; however, it is possible that the low type will be induced to not experiment at all ($\bar{t}^L = 0$) and it is also possible to have no distortion in the low type's stopping time ($\bar{t}^L = t^L$). The former possibility arises for reasons akin to exclusion in the standard model (e.g. the prior, μ_0 , on the high type is sufficiently high); the latter possibility is because time is discrete. Indeed, if the length of each time interval shrinks and one takes a suitable continuous-time limit, then there will be some distortion, i.e. $\bar{t}^L < t^L$; a formal proof is available from the authors on request.

[Theorem 2](#) is established through a characterization of a class of optimal menus:

Theorem 3. Assume $t^H > t^L$. There is an optimal menu in which the principal separates the two types using clawback contracts. In particular, the optimum can be implemented using a onetime-clawback contract for type H , $\mathbf{C}^H = (t^H, W_0^H, l_{t^H}^H)$ with $l_{t^H}^H < 0 < W_0^H$, and a clawback contract for type L , $\mathbf{C}^L = (\bar{t}^L, W_0^L, \mathbf{l}^L)$, such that:

1. For all $t \in \{1, \dots, \bar{t}^L\}$,

$$l_t^L = \begin{cases} -(1 - \delta) \frac{c}{\bar{\beta}_t^L \lambda^L} & \text{if } t < \bar{t}^L, \\ -\frac{c}{\bar{\beta}_{\bar{t}^L}^L \lambda^L} & \text{if } t = \bar{t}^L; \end{cases} \quad (6)$$

2. $W_0^L > 0$ is such that the participation constraint, [\(IR^L\)](#), binds;

3. Type H gets an information rent: $U_0^H(\mathbf{C}^H, \alpha^H(\mathbf{C}^H)) > 0$;

4. $\mathbf{1} \in \alpha^H(\mathbf{C}^H)$; $\mathbf{1} \in \alpha^L(\mathbf{C}^L)$; and $\mathbf{1} = \alpha^H(\mathbf{C}^L)$.

Generically, the above contract is the unique optimal contract for type L within the class of clawback contracts.

Proof. See [Appendix B](#).

Q.E.D.

According to [Theorem 3](#), the high type gets an information rent in any optimal menu of contracts. The contract for the low type characterized by [\(6\)](#) is a clawback contract in which the penalty is increasing in magnitude in each period $t < \bar{t}^L$ at which the project does not succeed (since $\bar{\beta}_t^L > \bar{\beta}_{t+1}^L$), followed by a larger penalty that “jumps” in the final period \bar{t}^L conditional on no success then. [Figure 2](#) below depicts this contract graphically; the comparative statics seen in the figure will be discussed subsequently. Only when there is no discounting does the low type's contract reduce to a onetime-clawback contract where a penalty is paid only if the project has not succeeded by \bar{t}^L . For any discount factor, the high type's contract characterized in [Theorem 3](#) is a onetime-clawback contract in which he only pays a penalty to the principal if there is no success by the first-best stopping time t^H . On the equilibrium path, both types exert effort in every period until their respective stopping times; moreover, were type H to take type L 's contract (off the equilibrium path), he would also exert effort in every period of the contract. This

implies that type H gets an information rent because he would be less likely than type L to incur any of the penalties in C^L .

Although the optimal contract for type L is generically unique among clawback contracts, there are a variety of optimal contracts for type H (even generically). The reason is that the low type's optimal contract is pinned down by the need to simultaneously incentivize the low type's effort and yet minimize the information rent obtained by the high type. This leads to a sequence of penalties for the low type, given by (6), that make the low type indifferent between working and shirking in each period of the contract, as we explain further in Subsection 4.2. On the other hand, the high type's contract only needs to be suitably designed to make it unattractive for the low type, subject to incentivizing effort from the high type and delivering to him a utility level given by his information rent. There is latitude in how this can be done: the high type's onetime-clawback contract of Theorem 3 is chosen to make it "too risky" for the low type, analogous to the logic used in Theorem 1.

Remark 1. Our proof of Theorem 3 provides a simple algorithm to solve for an optimal menu of contracts. For any $\hat{t} \in \{0, \dots, t^L\}$, we characterize an optimal menu that solves the principal's program subject to an additional constraint that the low type must experiment until period \hat{t} . The low type's contract in this menu is given by (6) with the termination date \hat{t} rather than \bar{t}^L . An optimal menu is then obtained by maximizing the principal's objective function over $\hat{t} \in \{0, \dots, t^L\}$.

The characterization in Theorem 3 yields the following comparative statics:

Proposition 2. *Assume $t^H > t^L$. The second-best stopping time for the low type, \bar{t}^L , is weakly increasing in β_0 and weakly decreasing in both c and μ_0 .*

Proof. See Appendix C.

Q.E.D.

Figure 2 illustrates these comparative statics. The comparative static in μ_0 is intuitive: the higher the ex-ante probability of the high type, the more the principal benefits from reducing the high type's information rent and hence the more she distorts the low type's length of experimentation. Matters are more subtle for the other parameters. Consider, for example, an increase in β_0 . On the one hand, this increases the social surplus from experimentation, which suggests that \bar{t}^L should increase. But there are two other effects: holding fixed \bar{t}^L , penalties of lower magnitude can be used to incentivize effort from the low type because the project is more likely to succeed (cf. Equation (6)), which has an effect of decreasing the information rent for the high type; yet, a higher β_0 also has a direct effect of increasing the information rent because the differing probability of success for the two types is only relevant when the project is good. Despite these multiple effects, the proof of Proposition 2 shows that in net, it is always optimal to (weakly) increase \bar{t}^L when β_0 increases. Related points apply to changes in the cost of effort, c .

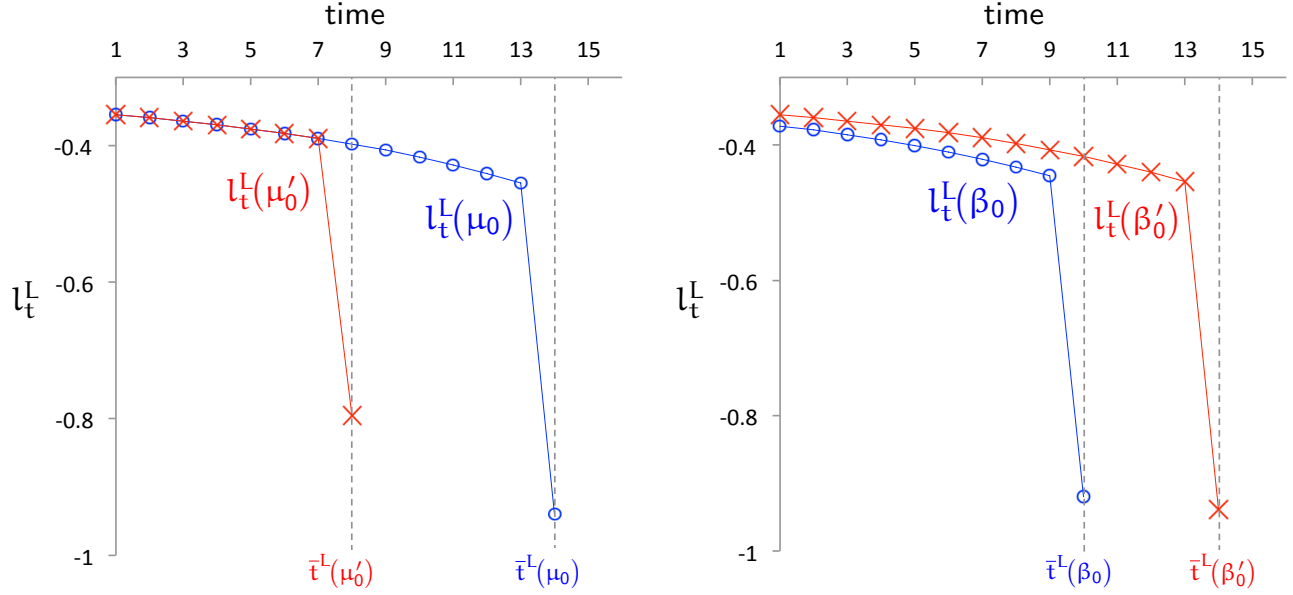


Figure 2 – The low type’s optimal clawback contract under different values of μ_0 and β_0 . Both graphs have $\delta = 0.5$, $\lambda^L = 0.1$, $\lambda^H = 0.12$, and $c = 0.06$. The left graph has $\beta_0 = 0.89$, $\mu_0 = 0.3$, and $\mu'_0 = 0.6$; the right graph has $\beta_0 = 0.85$, $\beta'_0 = 0.89$, and $\mu_0 = 0.3$. The first-best entails $t^L = 15$ on the left graph, and $t^L = 12$ (for β_0) and $t^L = 15$ (for β'_0) on the right graph.

4.2 Sketch of the proof

As the proof of [Theorem 3](#) (and hence [Theorem 2](#)) is a central contribution of this paper, we sketch in some detail the methodology used. This subsection also serves as a guide to follow the formal proof in [Appendix B](#).

While we have defined a contract as $\mathbf{C} = (T, W_0, \mathbf{b}, \mathbf{l})$, it will be useful in this subsection alone (so as to parallel the formal proof) to consider a larger space of contracts, where a contract is given by $\mathbf{C} = (\Gamma, W_0, \mathbf{b}, \mathbf{l})$. The first element here is a set of periods, $\Gamma \subseteq \mathbb{N}$, at which the agent is not “locked out,” i.e. at which he is allowed to choose whether to work or shirk. As discussed in [fn. 10](#), endowing the principal with this additional instrument does not yield her any benefit, but it will be notationally convenient in the proof. The termination date of the contract is now $\max\{t : t \in \Gamma\}$. We say that a contract is *connected* if $\Gamma = \{1, \dots, T\}$ for some T ; in this case we refer to T as the *length* of the contract, and T is also the termination date. The agent’s actions are denoted by $\mathbf{a} = (a_t)_{t \in \Gamma}$.

The principal’s problem is to find an optimal menu of contracts, $\mathbf{C}^\theta = (\Gamma^\theta, W_0^\theta, \mathbf{b}^\theta, \mathbf{l}^\theta)$ for each $\theta \in \{L, H\}$. Without loss of generality by [Proposition 1](#), we focus on menus of clawback contracts: for each $\theta \in \{L, H\}$, $\mathbf{C}^\theta = (\Gamma^\theta, W_0^\theta, \mathbf{l}^\theta)$. Clawback contracts are analytically convenient to deal with the combination of adverse selection and dynamic moral hazard for reasons explained in Step 5 below.

Step 1: It is without loss to focus on contracts for type L that induce him to work in every non-

lockout period, i.e. on contracts in the set $\{C^L : \mathbf{1} \in \alpha^L(C^L)\}$. The idea is as follows: fix any contract, C^L , in which there is some period, $t \in \Gamma^L$, such that it would be suboptimal for type L to work in period t . Since type L will not succeed in period t , one can modify C^L to create a new contract, \hat{C}^L , in which $t \notin \hat{\Gamma}^L$, and l_t^L is “shifted up” by one period with an adjustment for discounting. This ensures that the incentives for type L in all other periods remain unchanged, and critically, that no matter what behavior would have been optimal for type H under contract C^L , the new contract is less attractive to type H , i.e. $U_0^H(C^L, \alpha^H(C^L)) \geq U_0^H(\hat{C}^L, \alpha^H(\hat{C}^L))$.

Step 2: Given Step 1, the principal can optimize over menus of clawback contracts in which the low type’s contract induces him to work in every (non-lockout) period, subject to, for each $\theta \in \{L, H\}$, (IC_a^θ) , (IR^θ) , and $(IC^{\theta, \theta'})$. Call this program [P]. Since it is not obvious *a priori* which constraints in this program bind, and in particular what action plan the high type may use when taking the low type’s contract, we focus instead on a relaxed program, [RP1], that (i) ignores (IR^H) and (IC^{LH}) , and (ii) replaces (IC^{HL}) by a relaxed version, called $(Weak-IC^{HL})$, that only requires type H to prefer taking his contract and following an optimal action plan over taking type L ’s contract and working in every period. Formally, (IC^{HL}) requires $U_0^H(C^H, \alpha^H(C^H)) \geq U_0^H(C^L, \alpha^H(C^L))$ whereas $(Weak-IC^{HL})$ requires only $U_0^H(C^H, \alpha^H(C^H)) \geq U_0^H(C^L, \mathbf{1})$. We emphasize that this restriction on type H ’s action plan under type L ’s contract is not without loss for an arbitrary contract C^L ; i.e., given an arbitrary C^L with $\mathbf{1} \in \alpha^L(C^L)$, it need not be the case that $\mathbf{1} \in \alpha^H(C^L)$. The reason is that because of their differing probabilities of success from working in future periods (conditional on the good state), the two types trade off current and future penalties differently when considering exerting effort in the current period. In particular, the desire to avoid future penalties provides more of an incentive for the low type to work in the current period than the high type.¹⁸

The relaxation of (IC^{HL}) to $(Weak-IC^{HL})$ is a critical step in making the program tractable, and we will see that this turns out to work because $t^H > t^L$. In the relaxed program [RP1], it is straightforward to show that $(Weak-IC^{HL})$ and (IR^L) must bind at an optimum: otherwise, time-zero transfers in one of the two contracts can be profitably lowered without violating any of the constraints. Consequently, one can substitute from the binding version of these constraints to rewrite the objective function as the sum of total surplus less an information rent for the high type, as in the standard approach.¹⁹ We are left with a relaxed program, [RP2], whose objective is to maximize social surplus less the high type’s information

¹⁸To substantiate this point, consider any two-period clawback contract under which it is optimal for both types to work in each period. It can be verified that changing the first-period penalty by $+\varepsilon_1 > 0$ while simultaneously changing the second period penalty by $-\varepsilon_2 < 0$ would preserve type θ ’s incentive to work in period one if and only if $\varepsilon_1 \leq (1 - \lambda^\theta)\delta\varepsilon_2$. Note that because $-\varepsilon_2 < 0$, both types will continue to work in period two independent of their action in period one. Consequently, the initial contract can always be modified in a way that preserves optimality of working in both periods for the low type, but makes it optimal for the high type to shirk in period one and work in period two.

¹⁹It is worth emphasizing, however, that this approach only works in the relaxed program, [RP1]. In the full program [P], one cannot directly establish that either (IR^L) or (IC^{HL}) must bind. This contrast with the standard approach is because of dynamic moral hazard.

rent, and whose only constraints are the direct moral hazard constraints (IC_a^H) and (IC_a^L) , where type L must work in all periods. This program is tractable because it can be solved by separately optimizing over each type's clawback contract. The following steps, 3–6, derive an optimal contract for type L in program [RP2] that has useful properties.

Step 3: We show that there is an optimal clawback contract for type L that is connected. A rough intuition is as follows.²⁰ Because type L is required to work in all non-lockout periods, the value of the objective function in program [RP2] can be improved by removing any lockout periods in one of two ways: either by “shifting up” the sequence of effort and penalties or by terminating the contract early (suitably adjusting for discounting in either case). Shifting up the sequence of effort and penalties eliminates inefficient delays in type L 's experimentation, but it also increases the rent given to type H , because the penalties — which are more likely to be borne by type L than type H — are now paid earlier. Conversely, terminating the contract early reduces the rent given to type H by lowering the total penalties in the contract, but it also shortens experimentation by type L . It turns out that either of these modifications may be beneficial to the principal, but at least one of them will be if the initial contract is not connected.

Step 4: Given any termination date T^L , there are many penalty sequences that can be used by a connected clawback contract of length T^L to induce the low-ability agent to work in each period $1, \dots, T^L$. We construct the unique sequence, call it $\bar{l}(T^L)$, that ensures that the low type's incentive constraint for effort binds in each period of the contract, i.e. in any period $t \in \{1, \dots, T^L\}$, the low type is indifferent between working (and then choosing any optimal effort profile in subsequent periods) and shirking (and then choosing any optimal effort profile in subsequent periods), given the past history of effort. The intuition is straightforward: in the final period, T^L , there is obviously a unique such penalty as it must solve $\bar{l}_{T^L}^L(T^L) = -c + (1 - \bar{\beta}_{T^L}^L \lambda^L) \bar{l}_{T^L}^L(T^L)$. Iteratively working backward using a one-step deviation principle, this pins down penalties in each earlier period through the (forward-looking) incentive constraint for effort in each period. Naturally, for any T^L and $t \in \{1, \dots, T^L\}$, $\bar{l}_t^L(T^L) < 0$, i.e. as suggested by the term “penalty”, the agent pays the principal each time there is a failure.

Step 5: We next show that any connected clawback contract for type L that solves program [RP2] must use the penalty structure $\bar{l}^L(\cdot)$ of Step 4. The idea is that any slack in the low type's incentive constraint for effort in any period can be used to modify the contract to strictly reduce the high type's expected payoff from taking the low type's contract (without affecting the low type's behavior or expected payoff), based on the fact that the high type succeeds with higher probability in every period when taking the low type's contract.²¹

Although this logic is intuitive, a formal argument must deal with the challenge that modifying

²⁰For the intuition that follows, assume that all penalties being discussed are negative transfers, i.e. transfers from the agent to the principal.

²¹This is because the constraint (Weak-IC^{HL}) in program [RP2] effectively constrains the high type in this way, even though, as previously noted, it may not be optimal for the high type to work in each period when taking an arbitrary contract for the low type.

a transfer in any period to reduce slack in the low type's incentive constraint for effort in that period has feedback on incentives in every prior period — the dynamic agency problem. Our restriction to clawback contracts helps significantly here because clawback contracts have the property that reducing the incentive to exert effort in any period t by decreasing the severity of the penalty in period t has a *positive feedback* of also reducing the incentive for effort in earlier periods, since the continuation value of reaching period t increases. Due to this positive feedback, we are able to show that the low type's incentive for effort in a given period of a connected clawback contract can be modified without affecting his incentives in any other period by solely adjusting the penalties in that period and the previous one. In particular, in an arbitrary connected clawback contract C^L , if type L 's incentive constraint is slack in some period t , we can increase l_t^L and reduce l_{t-1}^L in a way that leaves type L 's incentives for effort unchanged in every period $s \neq t$ while still being satisfied in period t . We then verify that this “local modification” strictly reduces the high type's information rent.

By contrast, bonuses have a *negative feedback*: reducing the bonus in a period t increases the incentive to work in prior periods because the continuation value of reaching period t decreases. Consequently, keeping incentives for effort in earlier periods unchanged after reducing the bonus at period t would require a “global modification” of reducing the bonus in all prior periods, not just the previous period. This makes the analysis with bonus contracts (or any non-clawback contracts) rather intractable.

Step 6: In light of Steps 3–5, an optimal contract for type L in program [RP2] can be found by just choosing the optimal length of connected clawback contracts with the penalty structure $\bar{l}^L(\cdot)$. We first show (Step 6a of Subsection B.6) that the optimal length, \bar{t}^L , cannot be larger than the first-best stopping time: $\bar{t}^L \leq t^L$. This is a monotone comparative statics exercise. The intuition is that incentivizing over-experimentation by type L cannot be optimal because that would not only reduce efficiency but also increase the rent given to type H . The latter point is because $\bar{l}_t^L(T^L) < 0$ for any t and T^L and type H is less likely than type L to reach any period given that both types work in every period.

This comparative statics exercise also allows us establish (in Step 6b of Subsection B.6) that the optimal contract for the low type is generically unique.

Step 7: Let \bar{C}^L be the contract for type L identified in Steps 3–6.²² The final step is to show that there is a solution to [RP2] that combines \bar{C}^L with a suitable onetime-clawback contract for the high type and also solves the original program [P]. First, we show that $\alpha^H(\bar{C}^L) = \mathbf{1}$, i.e. if type H were to take contract \bar{C}^L , it would be optimal for him to work in all periods $1, \dots, \bar{t}^L$. The intuition is as follows: under contract \bar{C}^L , type H has a higher expected probability of success from working in any period $t \leq \bar{t}^L$, no matter his prior choices of effort, than does type L in period t given that type L has exerted effort in all prior periods (recall $\mathbf{1} \in \alpha^L(\bar{C}^L)$). The argument relies on Step 6 having established that $\bar{t}^L \leq t^L$, because then the maintained assumption in this section that $t^H > t^L$ implies that for any $t \in \{1, \dots, \bar{t}^L\}$, $\beta_t^H \lambda^H > \bar{\beta}_t^L \lambda^L$ for any history of effort by type H in periods $1, \dots, t-1$. Then, we verify that because \bar{C}^L

²²The initial transfer in \bar{C}^L is set to make the participation constraint for type L bind.

makes type L indifferent between working and shirking in each period up to \bar{t}^L (given that he has worked in all prior periods), type H would find it strictly optimal to work in each period up to \bar{t}^L no matter his prior history of effort, and hence $\alpha^H(\bar{\mathbf{C}}^L) = 1$. It follows that if type H 's contract is chosen to satisfy (Weak-IC^{HL}) then it will also satisfy (IC^{HL}) and (IR^H); the latter holds because $U_0^H(\bar{\mathbf{C}}^L, 1) \geq U_0^L(\bar{\mathbf{C}}^L, 1)$.

Lastly, we show that by choosing a onetime-clawback contract for type H that imposes a sufficiently severe penalty in period t^H and compensating type H through the initial transfer W_0^H , the principal maximizes the social surplus from the high type, satisfies (Weak-IC^{HL}), and also satisfies (IC^{LH}). In particular, (IC^{LH}) is satisfied because the principal can exploit the two types' differing probabilities of success by making the onetime-clawback contract for type H "risky enough" to deter type L from taking it while still satisfying (Weak-IC^{HL}) and hence (IR^H). This is analogous to the argument used for the case of no moral hazard in the proof of Theorem 1.

4.3 Bonus contracts

As explained earlier, it is analytically convenient to work with clawback contracts to derive the second-best solution. A weakness of clawback contracts, however, is that while we have imposed an ex-ante participation constraint for the agent, they will not satisfy *interim participation constraints*. In other words, in the implementation of Theorem 3, the agent of either type θ would "walk away" from his contract in any period $t \in \{1, \dots, \bar{t}^\theta\}$ if he could. This can be remedied by using the equivalence result of Proposition 1, as follows:

Theorem 4. Assume $t^H > t^L$. The second-best can also be implemented using a menu of bonus contracts. Specifically, the principal offers the low type the bonus contract $\mathbf{C}^L = (\bar{t}^L, W_0^L, \mathbf{b}^L)$ wherein for any $t \in \{1, \dots, \bar{t}^L\}$,

$$b_t^L = \sum_{s=t}^{\bar{t}^L-1} \delta^{s-t}(-l_s^L) + \delta^{\bar{t}^L-t}(-l_{\bar{t}^L}^L), \quad (7)$$

where l_t^L is the optimal clawback-contract penalty given in (6), and W_0^L is chosen to make the participation constraint, (IR^L), bind. For the high type, the principal can use a constant-bonus contract $\mathbf{C}^H = (t^H, W_0^H, b^H)$ with a suitably chosen W_0^H and $b^H > 0$.

Generically, the above contract is the unique optimal contract for type L within the class of bonus contracts. This implementation satisfies interim participation constraints in each period for each type, i.e. each type θ 's continuation utility at the beginning of any period $t \in \{1, \dots, \bar{t}^\theta\}$ in \mathbf{C}^θ is non-negative.

A proof is omitted because the proof of Proposition 1 can be used to verify that each bonus contract in Theorem 4 is equivalent to the corresponding clawback contract in Theorem 3, and hence the (second-best) optimality of those clawback contracts implies the optimality of these bonus contracts. Note from expression (7) that the optimal bonus contract for type L has $b_{\bar{t}^L}^L = -l_{\bar{t}^L}^L$, where $l_{\bar{t}^L}^L$ is the termination-date

penalty in the optimal clawback contract for type L . This is intuitive because the incentive to work in the final period depends only on the difference between the bonus and the penalty in that period. In earlier periods, the relationship between the bonuses in the optimal bonus contract and the penalties in the optimal clawback contract is more complex because future bonuses and future penalties affect present incentive considerations differently, for reasons previously noted. Using (6), it is also readily verified that in the bonus sequence (7),

$$b_t^L = \frac{(1-\delta)c}{\bar{\beta}_t^L \lambda^L} + \delta b_{t+1}^L \quad \text{for any } t \in \{1, \dots, \bar{t}^L - 1\}, \quad (8)$$

and hence the reward for success increases over time. Notice that when $\delta = 1$, type L 's bonus contract becomes a constant-bonus contract, analogous to the clawback contract in Theorem 3 becoming a onetime-clawback contract.

5 Optimal Contracts when $t^H \leq t^L$

We now turn to the case where the first-best stopping times are ordered $t^H \leq t^L$. The principal's maximization program is the same as that defined at the beginning of Section 4, but solving the program is now substantially more difficult. To understand why, consider Figure 3, which depicts the two types' "no-shirk expected marginal product" curves, $\bar{\beta}_t^\theta \lambda^\theta$, as a function of time.²³ For any parameters, these curves cross exactly once as shown in the figure, with the crossing point t^* defined by

$$\bar{\beta}_{t^*}^H \lambda^H - \bar{\beta}_{t^*}^L \lambda^L \geq 0 > \bar{\beta}_{t^*+1}^H \lambda^H - \bar{\beta}_{t^*+1}^L \lambda^L. \quad (9)$$

When the first-best stopping times are ordered $t^H > t^L$, it follows that $t^L < t^*$, as is the case for the high effort cost in Figure 3. Then, for any $t \leq t^L$, the high type always has a higher expected marginal product than the low type conditional on the agent working in all prior periods. It is this fact that allowed us to prove Theorem 3 by conjecturing that the high type would work in every period when taking the low type's contract.

By contrast, when the first-best stopping times are ordered $t^H \leq t^L$, it follows that $t^L \geq t^*$, as is the case for the low effort cost in Figure 3. Since the second-best stopping time for the low type can be arbitrarily close to his first-best stopping time (e.g. if the prior on the low type, $1 - \mu_0$, is sufficiently large), it is no longer valid to conjecture that the high type will work in every period when taking the low type's optimal contract. The reason is that at some period after t^* , given that both types have worked in each prior period, the high type can be sufficiently more pessimistic than the low type that the high type finds it optimal to shirk in some or all of the remaining periods, even though $\lambda^H > \lambda^L$ and the low type would

²³For convenience, the figure is drawn ignoring integer constraints.

be willing to work for the contract's duration.²⁴ Indeed, this will necessarily be true in the last period of the low type's contract if this period is larger than t^* and the contract makes the low type just indifferent between working and shirking in this period as in the characterization of [Theorem 3](#).

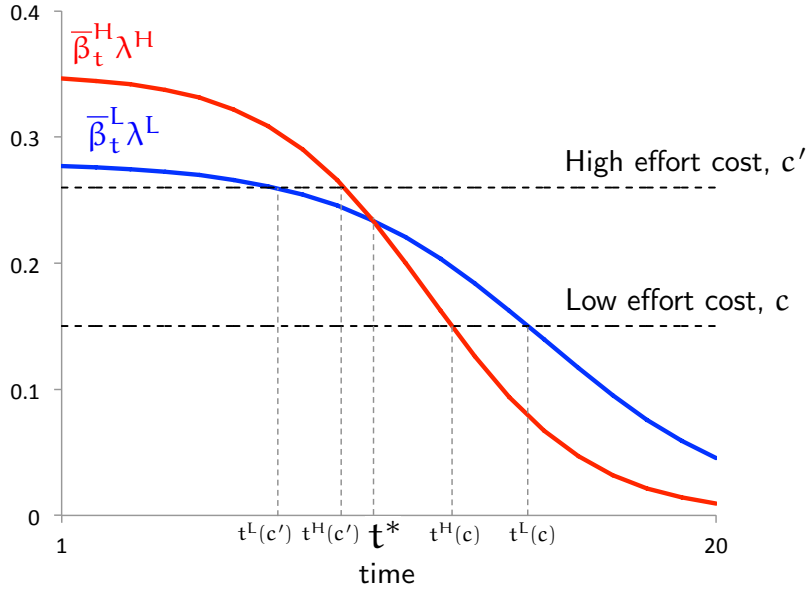


Figure 3 – No-shirk expected marginal product curves with $\beta_0 = 0.99$, $\lambda^L = 0.28$, $\lambda^H = 0.35$.

Solving the principal's program without being able to restrict attention to some suitable subset of action plans for the high type when he takes the low type's contract is daunting. For an arbitrary δ , we have been unable to find a valid restriction. The difficulty is illustrated by the following example:

Example 1. Consider any set of parameters $\{\beta_0, c, \lambda^L, \lambda^H\}$ satisfying the following four conditions:

1. The first-best stopping time for type L is $t^L = 3$: $\bar{\beta}_3^L \lambda^L > c > \bar{\beta}_4^L \lambda^L$.
2. The expected marginal product for type H after one period of work is less than that of type L after one period of work, but larger than that of type L after two periods of work: $\bar{\beta}_3^L \lambda^L < \bar{\beta}_2^H \lambda^H < \bar{\beta}_2^L \lambda^L$.
3. Ex-ante, type H is more likely to have succeeded by working in one period than type L is after working in two periods: $(1 - \lambda^H) < (1 - \lambda^L)^2$.
4. The following equality holds for some $\delta^* \in (0, 1)$:

$$\frac{1}{\beta_0 \lambda^L} - \frac{1}{\beta_0 \lambda^H} = \delta^* (1 - \lambda^L) \left(\frac{1}{\bar{\beta}_2^H \lambda^H} - \frac{1}{\bar{\beta}_2^L \lambda^L} \right).$$

²⁴More precisely, the relaxed program, [RP1], described in Step 2 of the proof sketch of [Theorem 3](#) can yield a solution which is not feasible in the original program, because the constraint (IC^{HL}) is violated; the high type would deviate from accepting his contract to accepting the low type's contract and then shirk in some periods.

(There is an open and dense set of parameters satisfying these conditions.) Further, fix μ_0 sufficiently small so that it is never optimal to distort the stopping time of the low type, i.e. $\bar{t}^L = t^L = 3$.

For any such parameter constellation, the optimal clawback contract for the low type as a function of δ , $\mathbf{C}^L(\delta) = (3, W_0^L(\delta), \mathbf{l}^L(\delta))$, is such that

$$\alpha^H(\mathbf{C}^L(\delta)) = \begin{cases} \{(1, 1, 0), (1, 0, 1)\} & \text{if } \delta \in (0, \delta^*) \\ \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\} & \text{if } \delta = \delta^* \\ \{(1, 0, 1), (0, 1, 1)\} & \text{if } \delta \in (\delta^*, 1) \\ \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\} & \text{if } \delta = 1. \end{cases}$$

Figure 4 illustrates this graphically and also shows the penalty sequence in the optimal clawback contract as a function of δ for a particular set of parameters.²⁵ Notice that the only action plan that is in $\alpha^H(\mathbf{C}^L(\delta))$ for all δ is the non-consecutive-work plan $(1, 0, 1)$, but for each value of δ at least one other plan is also optimal, whose identity varies with δ . Interestingly, $(1, 1, 0) \notin \alpha^H(\mathbf{C}^L(\delta))$ for $\delta \in (\delta^*, 1)$.²⁶

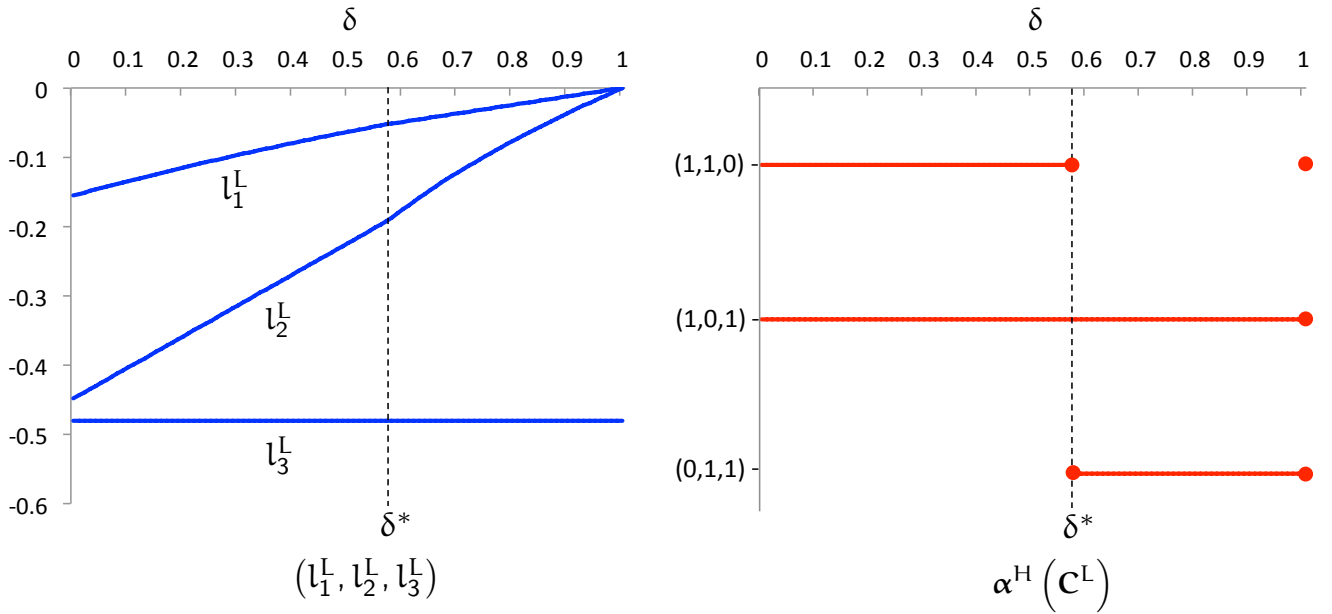


Figure 4 – The optimal clawback contract for the low type in Example 1 with $\beta_0 = 0.86$, $c = 0.1$, $\lambda^L = 0.75$, $\lambda^H = 0.95$ (left graph) and the optimal action profiles for the high type under this contract (right graph).

Nevertheless, we are able to solve the problem when $\delta = 1$. To state the result, it is useful to say that a contract for type $\theta \in \{L, H\}$ is *essentially unique* when $\delta = 1$ if it is unique up to payoff-irrelevant

²⁵The initial transfer W_0^L in each case is determined by making the participation constraint of the low type bind.

²⁶The lack of lower semi-continuity of $\alpha^H(\mathbf{C}^L(\delta))$ at $\delta = 1$ is not an accident, as we will discuss subsequently.

modifications that add delays where type θ would shirk in some periods of the contract; such multiplicity is unavoidable when both the principal and the agent do not discount the future.

Theorem 5. Assume $\delta = 1$ and $t^H \leq t^L$. There is an optimal menu in which the principal separates the two types using onetime-clawback contracts, $\mathbf{C}^H = (t^H, W_0^H, l_{t^H}^H)$ with $l_{t^H}^H < 0 < W_0^H$ for the high type and $\mathbf{C}^L = (\bar{t}^L, W_0^L, l_{\bar{t}^L}^L)$ for the low type, such that:

1. $\bar{t}^L \leq t^L$;
2. $l_{\bar{t}^L}^L = \min \left\{ -\frac{c}{\beta_{\bar{t}^L}^L \lambda^L}, -\frac{c}{\beta_{t^H}^H \lambda^H} \right\}$, where $t^{HL} := \max_{\mathbf{a} \in \alpha^H(\mathbf{C}^L)} \# \{n : a_n = 1, \}$;
3. $W_0^L > 0$ is such that the participation constraint, (\mathbf{IR}^L) , binds;
4. Type H gets an information rent: $U_0^H(\mathbf{C}^H, \alpha^H(\mathbf{C}^H)) > 0$;
5. $\mathbf{1} \in \alpha^H(\mathbf{C}^H)$, $\mathbf{1} \in \alpha^L(\mathbf{C}^L)$.

Generically, the above contract is the essentially-unique optimal contract for type L within the class of clawback contracts.

Proof. See [Appendix D](#).

Q.E.D.

For $\delta = 1$, the optimal menus of clawbacks contracts characterized in [Theorem 5](#) for $t^H \leq t^L$ share some common properties with those characterized in [Theorem 3](#) for $t^H > t^L$: in both cases, a onetime-clawback contract is used for the low type, there is no distortion in the stopping time of the high type whereas the low type is induced to (weakly) under-experiment, and the high type earns an information rent. On the other hand, part 2 of [Theorem 5](#) highlights two differences: (i) it will generally be the case in the optimal \mathbf{C}^L that when $t^H \leq t^L$, $\mathbf{1} \notin \alpha^H(\mathbf{C}^L)$, whereas for $t^H > t^L$, $\alpha^H(\mathbf{C}^L) = \mathbf{1}$; and (ii) when $t^H \leq t^L$, it can be optimal for the principal to induce the low type to work in each period by satisfying the low type's incentive constraint for effort with slack (i.e. with strict inequality), whereas when $t^H > t^L$, the penalty sequence makes this effort constraint bind in each period.

The intuition for these differences derives from information-rent considerations. The high type earns an information rent because by following the same effort profile as the low type he is less likely to incur any penalty for failure, and hence has a higher utility from any clawback contract than the low type.²⁷ Minimizing the rent through this channel suggests minimizing the magnitude of the penalties that are used to incentivize the low type's effort; it is this logic that drives [Theorem 3](#) and for $\delta = 1$ leads to a onetime-clawback contract with

$$l_{\bar{t}^L}^L = -\frac{c}{\beta_{\bar{t}^L}^L \lambda^L}. \quad (10)$$

²⁷Strictly speaking, this logic applies so long as $l_t \leq 0$ for all t in the clawback contract.

However, when $\bar{t}^L > t^*$ (which is only possible when $t^H \leq t^L$), the high type would find it optimal under this contract to work only for some $T < \bar{t}^L$ number of periods, where T is given by

$$\bar{\beta}_T^H \lambda^H l_{\bar{t}^L}^L \leq -c < \bar{\beta}_{T+1}^H \lambda^H l_{\bar{t}^L}^L. \quad (11)$$

Note that the latter inequality in (11) combined with (10) implies $\bar{\beta}_{T+1}^H \lambda^H < \bar{\beta}_{\bar{t}^L}^L \lambda^L$, which in turn through Bayes rule implies

$$(1 - \lambda^H)^T < (1 - \lambda^L)^{\bar{t}^L - 1}. \quad (12)$$

However, because $T < \bar{t}^L$, it is possible — and will be the case for an open and dense set of parameters — that T is such that the high type would incur the onetime penalty with a higher probability than the low type, i.e. that

$$(1 - \lambda^H)^T > (1 - \lambda^L)^{\bar{t}^L}. \quad (13)$$

But in such a case, the penalty given in (10) would not be optimal because the principal can lower $l_{\bar{t}^L}^L$ to reduce the information rent, which she can keep doing until the high type finds it optimal to work for $T + 1$ periods, i.e. the principal would set $l_{\bar{t}^L}^L = -\frac{c}{\bar{\beta}_{T+1}^H \lambda^H}$. At this point, because (12) implies $(1 - \lambda^H)^{T+1} < (1 - \lambda^L)^{\bar{t}^L}$, the high type becomes less likely to incur the onetime penalty than the low type, and lowering $l_{\bar{t}^L}^L$ any further would increase the information rent. This explains part 2 of [Theorem 5](#).

Intuitively, inequalities (12) and (13) can simultaneously hold because time is discrete. It can be shown that when the length of time intervals vanishes, in real-time the t^{HL} and \bar{t}^L in the statement of [Theorem 5](#) are such that $\bar{\beta}_{\bar{t}^L}^L \lambda^L = \bar{\beta}_{t^{HL}}^H \lambda^H$, and hence $l_{\bar{t}^L}^L = -\frac{c}{\bar{\beta}_{\bar{t}^L}^L \lambda^L}$ is always optimal, just as in [Theorem 3](#) when $\delta = 1$.

Remark 2. The proof of [Theorem 5](#) provides an algorithm to solve for an optimal menu of contracts when $t^H \leq t^L$ and $\delta = 1$. For each pair of integers (s, t) such that $0 \leq s \leq t \leq t^L$, one can compute the principal's payoff from using the onetime-clawback contract for type L given by [Theorem 5](#) when \bar{t}^L is replaced by t and t^{HL} is replaced by s in part 2. One then just optimizes over (s, t) .

How do we prove [Theorem 5](#) in light of the difficulties described earlier of finding a suitable restriction on the high type's behavior when taking the low-type's contract? The answer is that when $\delta = 1$, one can conjecture that the optimal contract for the low type must be a onetime-clawback contract (as was also true when $t^H > t^L$). Notice that because of no discounting, any onetime-clawback contract would make the agent of either type indifferent among all action plans that involve the same number of periods of work. In particular, a *stopping strategy* — an action plan that involves consecutive work for

²⁸Given that he has worked in $T - 1$ periods, the high type prefers working in period T and shirking thereafter to shirking in all periods starting at T if and only if $l_{\bar{t}^L}^L \leq -c + l_{\bar{t}^L}^L \left[\bar{\beta}_T^H (1 - \lambda^H) + 1 - \bar{\beta}_T^H \right]$.

²⁹The ex-ante probability for the agent of type θ of incurring the onetime penalty if he works for $t(\theta)$ periods is $\beta_0(1 - \lambda^H)^{t(\theta)} + 1 - \beta_0$, which implies that the comparison between the two types turns on the ranking of $(1 - \lambda^\theta)^{t(\theta)}$.

some number of periods followed by shirking thereafter — is always optimal for either type in a onetime-clawback contract. The heart of the proof of [Theorem 5](#) establishes that it is without loss of generality to restrict attention to clawback contracts for the low type under which the high type would find it optimal to use a stopping strategy (see [Subsection D.5](#) in [Appendix D](#)). With this in hand, we are then able to show that a onetime-clawback contract for the low type is indeed optimal (see [Subsection D.6](#)). Finally, the rent-minimization considerations described above are used to complete the argument. Observe that optimality of a onetime-clawback contract for the low type and that of a stopping strategy for the high type under such a contract is consistent with the solution in [Example 1](#) for $\delta = 1$, as seen in [Figure 4](#). Moreover, the example plainly shows that such a strategy space restriction will not generally be valid when $\delta < 1$.³⁰

We end this section by providing the bonus-contracts implementation of [Theorem 5](#):

Theorem 6. *Assume $\delta = 1$ and $t^H \leq t^L$. The second-best can also be implemented using a menu of constant-bonus contracts: $\mathbf{C}^L = (\bar{t}^L, W_0^L, b^L)$ with $b^L = -l_{\bar{t}^L}^L > 0 > W_0^L$ where $l_{\bar{t}^L}^L$ is given in [Theorem 5](#), and $\mathbf{C}^H = (t^H, W_0^H, b^H)$ with a suitably chosen W_0^H and $b^H > 0$. Generically, this \mathbf{C}^L is the essentially-unique optimal contract for type L within the class of bonus contracts.*

A proof is omitted since this follows directly from [Theorem 5](#) and the proof of [Proposition 1](#) (using $\delta = 1$). Analogous to our discussion around [Theorem 4](#), the implementation in [Theorem 6](#) satisfies interim participation constraints whereas that of [Theorem 5](#) does not.

6 Discussion

6.1 Private observability and disclosure

Suppose that project success is privately observed by the agent but can be verifiably disclosed. We assume in this *private-observability* setting that the principal's payoff from project success obtains only when the agent discloses it and contracts are conditioned not on project success but rather the disclosure of project success. Private observability introduces additional constraints for the principal because the agent must also now be incentivized to not withhold project success. For example, in a bonus contract where $\delta b_{t+1} > b_t$, an agent who obtains success in period t would strictly prefer to withhold it and continue to period $t + 1$, shirk in that period, and then reveal the success at the end of period $t + 1$.

³⁰Due to the agent's indifference over all action plans that involve the same number of periods of work in a onetime-clawback contract when $\delta = 1$, the correspondence $\alpha^H(\mathbf{C}^L(\delta))$ will generally fail lower semi-continuity at $\delta = 1$. In particular, the low type's optimal contract for δ close to 1 may be such that a stopping strategy is not optimal for the high type under this contract. However, the correspondence $\alpha^H(\mathbf{C}^L(\delta))$ is upper semi-continuous and the optimal contract is continuous at $\delta = 1$. All these points can be seen in [Figure 4](#).

Theorem 7. *Even if project success is privately observed by the agent, the menus of contracts identified in Theorems 3–6 remain optimal and implement the same outcome as when project success is publicly observable.*

Proof. It suffices to show that in each of the menus, each of the contracts would induce an agent (of either type) to reveal project success immediately when it is obtained.

Consider first the menus of Theorem 3 and Theorem 5: for each $\theta \in \{L, H\}$, the contract for type θ , C^θ , is a clawback contract in which $l_t^\theta \leq 0$ for all t . Hence, no matter which contract the agent takes and no matter his type, it is optimal to reveal a success when obtained.

For the implementation in Theorem 4, observe from (8) that type L 's bonus contract has the property that $\delta b_{t+1}^L \leq b_t^L$ for all $t \in \{1, \dots, \bar{t}^L - 1\}$; moreover, this property also holds in type L 's bonus contract in Theorem 6 and in type H 's bonus contracts in both Theorem 4 and Theorem 6, as these contracts are constant-bonus contracts. Hence, under all these contracts, it is optimal for the agent of either type to disclose success immediately when obtained. Q.E.D.

Therefore, project success being privately observed by the agent does not reduce the principal's payoff compared to the baseline setting where project success is publicly observable (and contractible), so long as the agent can verifiably disclose project success. However, unlike the menus of Theorems 3–6, not every optimal menu under public observability is optimal under private observability.³¹ In this sense, these optimal menus have a desirable robustness property that other optimal menus need not.

6.2 The role of learning

We have assumed that $\beta_0 \in (0, 1)$. If instead $\beta_0 = 1$ then there would be no learning about the project quality, and the first best would entail both types working until project success has been obtained.³² How would the absence of learning change our results?

To simplify the discussion, suppose that there is some (possibly large) exogenous date \bar{T} at which the game ends,³³ so that the first-best stopping times are $t^L = t^H = \bar{T}$. The principal's program can be solved here just as in Section 4, because $\bar{\beta}_t^H \lambda^H = \lambda^H > \bar{\beta}_t^L \lambda^L = \lambda^L$ for all $t \leq \bar{T}$. In the absence of learning, the social surplus from the low type working is constant over time. So long as parameters are

³¹In particular, there are optimal menus of bonus contracts under public observability that are suboptimal under private observability because the contract given to type H is such that he would have an incentive to delay disclosure of project success. Formally, the dynamic incentive constraint for effort under public observability requires $b_t^H \geq \frac{c(1-\delta\beta_t^H\lambda^H)}{\beta_t^H\lambda^H} + \delta\lambda^H b_{t+1}^H$, which can be satisfied with $b_t^H < \delta b_{t+1}^H$, in which case the contract would be suboptimal under private observability as noted earlier. An analogous point applies to menus of clawback contracts.

³²In a setting without adverse selection but with limited liability, Biais et al. (2010) study a continuous-time principal-agent problem where large losses arrive according to a Poisson process whose intensity is determined by the agent's effort, so that the first best involves always exerting effort.

³³It should be clear that nothing changes in the earlier analysis if we had assumed existence of such a suitably large end date, in particular so long as $\bar{T} \geq \max\{t^H, t^L\}$.

such that it is not optimal for the principal to exclude the low type (i.e. $\bar{t}^L > 0$), then it turns out that there is no distortion: $\bar{t}^L = \bar{t}^H = \bar{T}$. We provide a more complete argument in [Appendix E](#), but to see the intuition consider a large \bar{T} . Then, even though both types are likely to succeed prior to \bar{T} , the probability of reaching \bar{T} without a success is an order of magnitude higher for the low type because $\left(\frac{1-\lambda^L}{1-\lambda^H}\right)^t \rightarrow \infty$ as $t \rightarrow \infty$. Therefore, it would not be optimal to locally distort the length of experimentation from \bar{T} because such a distortion would generate a larger efficiency loss from the low type than a gain from reducing the high type's information rent. By contrast, when $\beta_0 < 1$ and there is learning, this logic fails because the incremental social surplus from the low type working vanishes over time. Therefore, learning from experimentation plays an important role in our results: for any parameters with $\beta_0 < 1$ under which there is distortion of the low type's length of experimentation without entirely excluding him, there would instead be no distortion were $\beta_0 = 1$.

6.3 Adverse selection on other dimensions

Another important modeling assumption in this paper is that pre-contractual hidden information is about the agent's ability. Let us briefly comment on two alternatives.

First, suppose the agent has hidden information about his cost of effort but his ability is commonly known; specifically, the low type's cost of working in any period is $c^L > 0$ whereas the high type's cost is $c^H \in (0, c^L)$. It is immediate that the first-best stopping time for the high type would always be larger than that of the low type because there is no speed-of-learning effect. Hence, the problem can be solved following our approach in [Section 4](#) for $t^H > t^L$.³⁴ However, not only would this alternative model miss the considerations involved with $t^H \leq t^L$, but furthermore, it also obviates interesting features of the problem even when $t^H > t^L$. For example, in this setting it would be optimal for the high type to work in all periods in any contract in which it is optimal for the low type to work in all periods; recall that this is not true in our model even when $t^H > t^L$ (cf. [fn. 18](#)).

Another source of adverse selection would be about project quality. Suppose that the agent's ability is commonly known but he has a private noisy signal about project quality: specifically, there is a high type whose belief about the state being good is $\beta_0^H \in (0, 1)$ and a low type whose belief is $\beta_0^L \in (0, \beta_0^H)$.³⁵ Again, the first-best stopping times here would always have $t^H > t^L$ and the problem can be studied following our approach to this case.

³⁴This applies to binary effort choices. Another alternative would be for the agent to choose effort from a richer set, e.g. \mathbb{R}_+ , and effort costs be convex with one type having a lower marginal cost than the other. The speed-of-learning effect would emerge in this setting because the two types would generally choose different effort levels in any period. Analyzing such a problem is beyond the scope of this paper.

³⁵[Gerardi and Maestri \(2012\)](#) consider a similar source of adverse selection albeit in a different setting. We emphasize that we have in mind a setting with a common prior and a private signal for the agent, as opposed to a setting with non-common priors. The latter would involve quite distinct considerations and may be interesting in its own right.

Appendices: Notation and Terminology

It is expositionally convenient in proving our results to work with an apparently larger set of contracts than that defined in the main text. Specifically, in the Appendices, we assume that the principal can stipulate binding “lockout” periods in which the agent is prohibited from working. As discussed in [fn. 10](#) of the main text, this instrument cannot ultimately yield any benefit to the principal because the agent can be induced to shirk in a period regardless of his type and history of work by just setting a sufficiently negative bonus for success in that period. Nevertheless, stipulating lockout periods helps with simplifying notation and statements in the arguments we make.

Accordingly, we denote a general contract by $\mathbf{C} = (\Gamma, W_0, \mathbf{b}, \mathbf{l})$, where all the elements are as introduced in the main text, except that instead of having the termination date of the contract in the first component, we now have a set of periods, $\Gamma \subseteq \mathbb{N}$, at which the agent is not locked out, i.e. at which he is allowed to choose whether to work or shirk. Note that, without loss, $\mathbf{b} = (b_t)_{t \in \Gamma}$ and $\mathbf{l} = (l_t)_{t \in \Gamma}$,³⁶ and the agent’s actions are denoted by $\mathbf{a} = (a_t)_{t \in \Gamma}$, where $a_t = 1$ if the agent works in period $t \in \Gamma$ and $a_t = 0$ if the agent shirks. The termination date of the contract is $\max\{t : t \in \Gamma\}$, which we require to be finite.³⁷ Without loss, we assume that $\max\{t : t \in \Gamma\} = \max\{t : \text{either } b_t \neq 0 \text{ or } l_t \neq 0\}$. We say that a contract is *connected* if $\Gamma = \{1, \dots, T\}$ for some T ; in this case we refer to T as the *length* of the contract, T is also the termination date, and we write $\mathbf{C} = (T, W_0, \mathbf{b}, \mathbf{l})$.

A Proofs of Benchmark Results

A.1 Proposition 1

We prove the result more generally for contracts with lockouts. Fix a contract $\mathbf{C} = (\Gamma, W_0, \mathbf{b}, \mathbf{l})$. The result is trivial if $\Gamma = \emptyset$, so assume $\Gamma \neq \emptyset$. Let $T = \max \Gamma$. For any period $t \in \Gamma$ with $t < T$, define the smallest successor period in Γ as $\sigma(t) = \min\{t' : t' > t, t' \in \Gamma\}$; moreover, let $\sigma(0) = \min \Gamma$.

Given any action profile for the agent, the agent’s time-zero expected discounted payoff when his type is $\theta \in \{L, H\}$ and the principal’s time-zero expected discounted payoff only depend upon a contract’s induced vector of discounted transfers, say $(\tau_t)_{t \in \Gamma}$ when success is obtained in period t and on the discounted transfer when there is no success. Hence, it suffices to construct a clawback contract, $\widehat{\mathbf{C}}$, and bonus contract, $\widetilde{\mathbf{C}}$, that induce the same such vector of transfers as \mathbf{C} .

To this end, define the clawback contract $\widehat{\mathbf{C}} = (\Gamma, \widehat{W}_0, \widehat{\mathbf{l}})$ as follows:

- (a) For any t such that $t < T$ and $t \in \Gamma$, $\widehat{l}_t = l_t - b_t + \delta^{\sigma(t)-t} b_{\sigma(t)}$.
- (b) $\widehat{l}_T = l_T - b_T$.
- (c) $\widehat{W}_0 = W_0 + \delta^{\sigma(0)} b_{\sigma(0)}$.

Define the bonus contract $\widetilde{\mathbf{C}} = (\Gamma, \widetilde{W}_0, \widetilde{\mathbf{b}})$ as follows:

³⁶There is no loss in not allowing for transfers in lockout periods.

³⁷One can show that this restriction does not hurt the principal.

(a) For any $t \in \Gamma$, $\tilde{b}_t = b_t - \sum_{s \geq t, s \in \Gamma} \delta^{s-t} l_s$.

(b) $\tilde{W}_0 = W_0 + \sum_{t \in \Gamma} \delta^t l_t$.

Consider first the discounted transfer induced by each of these three contracts if success is not obtained. For \mathbf{C} , it is $W_0 + \sum_{t \in \Gamma} \delta^t l_t$. For $\hat{\mathbf{C}}$, it is

$$\widehat{W}_0 + \sum_{t \in \Gamma} \delta^t \widehat{l}_t = W_0 + \delta^{\sigma(0)} b_{\sigma(0)} + \sum_{t \in \Gamma, t < T} \delta^t \left(l_t - b_t + \delta^{\sigma(t)-t} b_{\sigma(t)} \right) + \delta^T (l_T - b_T) = W_0 + \sum_{t \in \Gamma} \delta^t l_t,$$

where the first equality follows from the definition of $\hat{\mathbf{C}}$ and the second from algebraic simplification. For $\tilde{\mathbf{C}}$, since there are no penalties, the corresponding discounted transfer is just $\tilde{W}_0 = W_0 + \sum_{t \in \Gamma} \delta^t l_t$. Hence, all three contracts induce the same transfer in the event of no success.

Next, for any $s \in \Gamma$, consider a success obtained in period s . The discounted transfer in this event in \mathbf{C} is $W_0 + \sum_{t \in \Gamma, t < s} \delta^t l_t + \delta^s b_s$. For $\hat{\mathbf{C}}$, since there are no bonuses, it is

$$\widehat{W}_0 + \sum_{t \in \Gamma, t < s} \delta^t \widehat{l}_t = W_0 + \delta^{\sigma(0)} b_{\sigma(0)} + \sum_{t \in \Gamma, t < s} \delta^t \left(l_t - b_t + \delta^{\sigma(t)-t} b_{\sigma(t)} \right) = W_0 + \sum_{t \in \Gamma, t < s} \delta^t l_t + \delta^s b^s,$$

where again the first equality uses the definition of $\hat{\mathbf{C}}$ and the second follows from simplification. For $\tilde{\mathbf{C}}$, since there are no penalties, the corresponding discounted transfer is

$$\tilde{W}_0 + \delta^s \tilde{b}_s = W_0 + \sum_{t \in \Gamma} \delta^t l_t + \delta^s \left(b_s - \sum_{t \geq s, t \in \Gamma} \delta^{t-s} l_s \right) = W_0 + \sum_{t \in \Gamma, t < s} \delta^t l_t + \delta^s b^s,$$

where again the first equality is by definition of $\tilde{\mathbf{C}}$ and the second from simplification. Hence, all three contracts induce the same transfer in the event of success in any period $s \in \Gamma$.

A.2 Theorem 1

We consider the cases of no adverse selection and no moral hazard separately.

No adverse selection

The claim that the principal can implement the first best and extract all the surplus with constant-bonus contracts follows from the analysis in the text. We now show that the principal can also do this with onetime-clawback contracts. Since the agent's type is observable, suppose the principal offers a type θ agent a onetime-clawback contract $\mathbf{C}^\theta = (t^\theta, W_0^\theta, l_{t^\theta}^\theta)$ where W_0^θ is chosen such that, conditional on the agent exerting effort in all periods $t = 1, \dots, t^\theta$, the agent's participation constraint at time 0 binds:

$$U_0^\theta(\mathbf{C}^\theta, 1) = W_0^\theta - c \sum_{t=1}^{t^\theta} \delta^t \left[\beta_0 (1 - \lambda^\theta)^{t-1} + (1 - \beta_0) \right] + l_{t^\theta}^\theta \delta^{t^\theta} \left[\beta_0 (1 - \lambda^\theta)^{t^\theta} + (1 - \beta_0) \right] = 0.$$

First best requires the agent to work in all periods until t^θ so long as success has not been obtained. From the one-step deviation principle, the incentive compatibility conditions for effort are summarized by the following inequality: for any $t \in \{1, \dots, t^\theta\}$,

$$\lambda^\theta \bar{\beta}_t^\theta \left[\delta^{t^\theta-t} (1 - \lambda^\theta)^{t^\theta-t} (-l_{t^\theta}^\theta) + \sum_{s=t+1}^{t^\theta} \delta^{s-t} (1 - \lambda^\theta)^{s-t-1} c \right] \geq c. \quad (\text{A.1})$$

The right-hand side of (A.1) is the constant cost of effort. The left-hand side is the benefit of effort at any time t (given that effort has been exerted and success not obtained at all prior periods): with probability $\lambda^\theta \bar{\beta}_t^\theta$ there will be a success in period t and the agent saves both the expected discounted penalty, $\delta^{t^\theta-t} (1 - \lambda^\theta)^{t^\theta-t} (-l_{t^\theta}^\theta)$, and the expected discounted cost of future effort, $\sum_{s=t+1}^{t^\theta} \delta^{s-t} (1 - \lambda^\theta)^{s-t-1} c$. It follows that the agent will work in all periods if $l_{t^\theta}^\theta$ is chosen low enough, i.e., the clawback penalty is severe enough. In this case, the first best is implemented and the principal extracts all the surplus.

No moral hazard

Now consider the case without moral hazard, i.e. the agent's effort in any period still costs him $c > 0$ but is observable and contractible. We can thus ignore the agent's incentive compatibility constraints for effort by, for example, assuming that the principal stipulates a penalty for shirking (as long as success has not been obtained and up to the termination date) that is severe enough so that the agent would indeed find it optimal to work in every period regardless of his type and regardless of which contract he accepts.

Given that the agent always works, we show that the principal can implement the first best and extract all the surplus with a menu of onetime-clawback contracts, $\mathbf{C}^H = (t^H, W_0^H, l_{t^H}^H)$ and $\mathbf{C}^L = (t^L, W_0^L, l_{t^L}^L)$, where $l_{t^L}^L > 0 > l_{t^H}^H$, and for each $\theta \in \{L, H\}$, W_0^θ is set so that type θ 's participation constraint binds:

$$U_0^\theta(\mathbf{C}^\theta, \mathbf{1}) = W_0^\theta - c \sum_{t=1}^{t^\theta} \delta^t \left[\beta_0 (1 - \lambda^\theta)^{t-1} + (1 - \beta_0) \right] + l_{t^\theta}^\theta \delta^{t^\theta} \left[\beta_0 (1 - \lambda^\theta)^{t^\theta} + (1 - \beta_0) \right] = 0. \quad (\text{A.2})$$

Using (A.2), note that if type L were to choose type H 's contract, his time-zero expected discounted payoff would be

$$\begin{aligned} U_0^L(\mathbf{C}^H, \mathbf{1}) &= W_0^H - c \sum_{t=1}^{t^H} \delta^t \left[\beta_0 (1 - \lambda^L)^{t-1} + (1 - \beta_0) \right] + l_{t^H}^H \delta^{t^H} \left[\beta_0 (1 - \lambda^L)^{t^H} + (1 - \beta_0) \right] \\ &= c \beta_0 \sum_{t=1}^{t^H} \delta^t \left[(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1} \right] + \beta_0 l_{t^H}^H \delta^{t^H} \left[(1 - \lambda^L)^{t^H} - (1 - \lambda^H)^{t^H} \right]. \end{aligned} \quad (\text{A.3})$$

Similarly, if type H were to choose type L 's contract, his time-zero expected discounted payoff would be

$$U_0^H(\mathbf{C}^L, \mathbf{1}) = W_0^L - c \sum_{t=1}^{t^L} \delta^t \left[\beta_0 (1 - \lambda^H)^{t-1} + (1 - \beta_0) \right] + l_{t^L}^L \delta^{t^L} \left[\beta_0 (1 - \lambda^H)^{t^L} + (1 - \beta_0) \right]$$

$$= c\beta_0 \sum_{t=1}^{t^L} \delta^t \left[(1 - \lambda^L)^{t-1} - (1 - \lambda^H)^{t-1} \right] + \beta_0 l_{t^L}^L \delta^{t^L} \left[(1 - \lambda^H)^{t^L} - (1 - \lambda^L)^{t^L} \right]. \quad (\text{A.4})$$

Expression (A.3) is an affine function of $l_{t^H}^H$ with a strictly positive coefficient on $l_{t^H}^H$ (since $\lambda^H > \lambda^L$), and expression (A.4) is an affine function of $l_{t^L}^L$ with a strictly negative coefficient. Hence, we can choose $l_{t^H}^H < 0$ sufficiently low and $l_{t^L}^L > 0$ sufficiently large so that (A.3) and (A.4) are negative, in which case each type prefers to take his own contract over the other's. Thus, the first best is implemented and the principal extracts all the surplus.

The proof that the principal can also achieve this using a menu of constant-bonus contracts is analogous and thus omitted.

B Proof of Theorem 3

The proof below for Theorem 3 also proves Theorem 2. We remind the reader that Subsection 4.2 provides an outline and intuition for the proof.

Without loss of generality by Proposition 1, we focus on clawback contracts through the proof.

B.1 Step 1: Low type always works

We first show that it is without loss to focus on contracts for the low type in which he is prescribed to work in every period. To prove this, denote the set of clawback contracts by \mathcal{C} , and recall that the principal's program with the restriction to clawback contracts is:

$$\max_{(\mathbf{C}^H \in \mathcal{C}, \mathbf{C}^L \in \mathcal{C}, \mathbf{a}^H, \mathbf{a}^L)} \mu_0 \Pi_0^H(\mathbf{C}^H, \mathbf{a}^H) + (1 - \mu_0) \Pi_0^L(\mathbf{C}^L, \mathbf{a}^L)$$

subject to, for all $\theta, \theta' \in \{L, H\}$,

$$\mathbf{a}^\theta \in \alpha^\theta(\mathbf{C}^\theta), \quad (\text{IC}_a^\theta)$$

$$U_0^\theta(\mathbf{C}^\theta, \mathbf{a}^\theta) \geq 0, \quad (\text{IR}^\theta)$$

$$U_0^\theta(\mathbf{C}^\theta, \mathbf{a}^\theta) \geq U_0^\theta(\mathbf{C}^{\theta'}, \alpha^\theta(\mathbf{C}^{\theta'})). \quad (\text{IC}^{\theta, \theta'})$$

Suppose there is a solution to this program, $(\mathbf{C}^H, \mathbf{C}^L, \mathbf{a}^H, \mathbf{a}^L)$, with $\mathbf{a}^L \neq \mathbf{1}$ and $\mathbf{C}^L = (\Gamma^L, W_0^L, l^L)$. It suffices to show that there is another solution to the program, $(\mathbf{C}^H, \widehat{\mathbf{C}}^L, \mathbf{a}^H, \mathbf{1})$, where $\widehat{\mathbf{C}}^L = (\widehat{\Gamma}^L, \widehat{W}_0^L, \widehat{l}^L)$ is such that:

- (i) $\mathbf{1} \in \alpha^L(\widehat{\mathbf{C}}^L)$;
- (ii) $U_0^L(\mathbf{C}^L, \mathbf{a}^L) = U_0^L(\widehat{\mathbf{C}}^L, \mathbf{1})$;
- (iii) $\Pi_0^L(\mathbf{C}^L, \mathbf{a}^L) = \Pi_0^L(\widehat{\mathbf{C}}^L, \mathbf{1})$; and
- (iv) $U_0^H(\mathbf{C}^L, \alpha^H(\mathbf{C}^L)) \geq U_0^H(\widehat{\mathbf{C}}^L, \alpha^H(\widehat{\mathbf{C}}^L))$.

To this end, let $t = \min\{s : a_s = 0\}$ and denote the largest preceding period in Γ^L as

$$p(t) = \begin{cases} \max \Gamma^L \setminus \{t, t+1, \dots\} & \text{if } \exists s \in \Gamma^L \text{ s.t. } s < t, \\ 0 & \text{otherwise.} \end{cases}$$

Construct $\widehat{\mathbf{C}}^L = (\widehat{\Gamma}^L, \widehat{W}_0^L, \widehat{l}^L)$ as follows:

$$\begin{aligned} \widehat{\Gamma}^L &= \Gamma^L \setminus \{t\}; \\ \widehat{l}_s^L &= \begin{cases} l_s^L & \text{if } s \neq p(t) \text{ and } s \in \widehat{\Gamma}^L, \\ l_s^L + \delta^{t-p(t)} l_t^L & \text{if } s = p(t) > 0; \end{cases} \\ \widehat{W}_0^L &= \begin{cases} W_0^L & \text{if } p(t) > 0, \\ W_0^L + \delta^t l_t^L & \text{if } p(t) = 0. \end{cases} \end{aligned}$$

Notice that under contract \mathbf{C}^L , the profile \mathbf{a}^L has type L shirking in period t and thus receiving l_t^L with probability one conditional on not succeeding before this period; the new contract $\widehat{\mathbf{C}}^L$ just locks the agent out in period t and shifts the payment l_t^L up to the preceding non-lockout period, suitably discounted. It follows that the incentives for effort for type L remain unchanged in any other period, i.e. that $\mathbf{a}_{-t}^L \in \alpha^L(\widehat{\mathbf{C}}^L)$; moreover, since $a_t^L = 0$, both the principal's payoff from type L under this contract and type L 's payoff do not change. Finally, observe that for type H , no matter which action he would take at t in any optimal action plan under \mathbf{C}^L (whether it is work or shirk), his payoff from $\widehat{\mathbf{C}}^L$ must be weakly lower because the lockout in period t is effectively as though he has been forced to shirk in period t and receive l_t^L .

Performing this procedure repeatedly for each period in which the original profile \mathbf{a}^L prescribes shirking yields a final contract $\widehat{\mathbf{C}}^L$ which satisfies all the desired properties.

B.2 Step 2: Relaxing the program

By Step 1, we can restrict our attention to clawback contracts $\mathbf{C}^\theta = (\Gamma^\theta, W_0^\theta, l^\theta)$ with type L 's contract inducing type L to exert effort in all periods in Γ^L . Then principal then faces the following program [P]:

$$\max_{(\mathbf{C}^H \in \mathcal{C}, \mathbf{C}^L \in \mathcal{C}, \mathbf{a}^H)} \mu_0 \Pi_0^H(\mathbf{C}^H, \mathbf{a}^H) + (1 - \mu_0) \Pi_0^L(\mathbf{C}^L, \mathbf{1}) \quad (\text{P})$$

subject to

$$\begin{aligned} \mathbf{1} &\in \alpha^L(\mathbf{C}^L) & (\text{IC}_a^L) \\ \mathbf{a}^H &\in \alpha^H(\mathbf{C}^H) & (\text{IC}_a^H) \\ U_0^L(\mathbf{C}^L, \mathbf{1}) &\geq 0 & (\text{IR}^L) \\ U_0^H(\mathbf{C}^H, \mathbf{a}^H) &\geq 0 & (\text{IR}^H) \\ U_0^L(\mathbf{C}^L, \mathbf{1}) &\geq U_0^L(\mathbf{C}^H, \alpha^L(\mathbf{C}^H)) & (\text{IC}^{LH}) \\ U_0^H(\mathbf{C}^H, \mathbf{a}^H) &\geq U_0^H(\mathbf{C}^L, \alpha^H(\mathbf{C}^L)) & (\text{IC}^{HL}) \end{aligned}$$

To solve program [P], we solve a relaxed program and later verify that the solution is feasible in (and hence is a solution to) [P]. Specifically, we relax three constraints in [P]: (i) we ignore (\mathbf{IC}^{LH}) and (\mathbf{IR}^H) , and (ii) we consider a weak version of (\mathbf{IC}^{HL}) in which type H is assumed to exert effort in all periods in Γ^L if he chooses \mathbf{C}^L . The relaxed program, [RP1], is therefore:

$$\max_{(\mathbf{C}^H \in \mathcal{C}, \mathbf{C}^L \in \mathcal{C}, \mathbf{a}^H)} \mu_0 \Pi_0^H(\mathbf{C}^H, \mathbf{a}^H) + (1 - \mu_0) \Pi_0^L(\mathbf{C}^L, \mathbf{1}) \quad (\text{RP1})$$

subject to

$$\mathbf{1} \in \alpha^L(\mathbf{C}^L) \quad (\mathbf{IC}_a^L)$$

$$\mathbf{a}^H \in \alpha^H(\mathbf{C}^H) \quad (\mathbf{IC}_a^H)$$

$$U_0^L(\mathbf{C}^L, \mathbf{1}) \geq 0 \quad (\mathbf{IR}^L)$$

$$U_0^H(\mathbf{C}^H, \mathbf{a}^H) \geq U_0^H(\mathbf{C}^L, \mathbf{1}). \quad (\text{Weak-IC}^{HL})$$

It is clear that in any solution to program [RP1], (\mathbf{IR}^L) must be binding: otherwise, the initial time-zero transfer from the principal to the agent in the contract \mathbf{C}^L can be reduced slightly to strictly improve the second term of the objective function while not violating any of the constraints. Similarly, (Weak-IC^{HL}) must also bind because otherwise the time-zero transfer in the contract \mathbf{C}^H can be reduced to improve the first term of the objective function without violating any of the constraints.

Using these two binding constraints and substituting in the formulae from equations (1) and (2), we can rewrite the objective function (RP1) as the sum of expected total surplus less type H 's "information rent", obtaining the following explicit version of the relaxed program which we call [RP2]:

$$\max_{(\mathbf{C}^H \in \mathcal{C}, \mathbf{C}^L \in \mathcal{C}, \mathbf{a}^H)} \left\{ \begin{aligned} & \mu_0 \left\{ \beta_0 \sum_{t \in \Gamma^H} \delta^t \left[\prod_{s \in \Gamma^H, s \leq t-1} (1 - a_s^H \lambda^H) \right] a_t^H (\lambda^H - c) - (1 - \beta_0) \sum_{t \in \Gamma^H} \delta^t a_t^H c \right\} \\ & + (1 - \mu_0) \left\{ \beta_0 \sum_{t \in \Gamma^L} \delta^t \left[\prod_{s \in \Gamma^L, s \leq t-1} (1 - \lambda^L) \right] (\lambda^L - c) - (1 - \beta_0) \sum_{t \in \Gamma^L} \delta^t c \right\} \\ & - \underbrace{\mu_0 \beta_0 \left\{ \sum_{t \in \Gamma^L} \delta^t l_t^L \left[\prod_{s \in \Gamma^L, s \leq t} (1 - \lambda^H) - \prod_{s \in \Gamma^L, s \leq t} (1 - \lambda^L) \right] \right.}_{\text{Information rent of type H}} \\ & \quad \left. - \sum_{t \in \Gamma^L} \delta^t c \left[\prod_{s \in \Gamma^L, s \leq t-1} (1 - \lambda^H) - \prod_{s \in \Gamma^L, s \leq t-1} (1 - \lambda^L) \right] \right\} \end{aligned} \right\} \quad (\text{RP2})$$

subject to

$$\mathbf{1} \in \arg \max_{(a_t)_{t \in \Gamma^L}} \left\{ \begin{aligned} & \beta_0 \sum_{t \in \Gamma^L} \delta^t \left[\prod_{s \in \Gamma^L, s \leq t-1} (1 - a_s \lambda^L) \right] [(1 - a_t \lambda^L) l_t^L - a_t c] \\ & + (1 - \beta_0) \sum_{t \in \Gamma^L} \delta^t (l_t^L - a_t c) + W_0^L \end{aligned} \right\} \quad (\mathbf{IC}_a^L)$$

$$\mathbf{a}^H \in \arg \max_{(a_t)_{t \in \Gamma^H}} \left\{ \begin{aligned} & \beta_0 \sum_{t \in \Gamma^H} \delta^t \left[\prod_{s \in \Gamma^H, s \leq t-1} (1 - a_s \lambda^H) \right] [(1 - a_t \lambda^H) l_t^H - a_t c] \\ & + (1 - \beta_0) \sum_{t \in \Gamma^H} \delta^t (l_t^H - a_t c) + W_0^H \end{aligned} \right\}. \quad (\mathbf{IC}_a^H)$$

A key observation is that this program [RP2] is separable, i.e. it can be solved by separately maximizing (RP2) with respect to \mathbf{C}^L subject to (\mathbf{IC}_a^L) and separately maximizing (RP2) with respect to $(\mathbf{C}^H, \mathbf{a}^H)$ subject to (\mathbf{IC}_a^H) .

B.3 Step 3: Connected contracts for the low type

We now claim that in program [RP2], it is without loss to consider solutions in which the low type's contract is a connected clawback contract, i.e. solutions \mathbf{C}^L in which $\Gamma^L = \{1, \dots, T^L\}$ for some T^L .

To prove this, observe that the optimal \mathbf{C}^L is a solution of

$$\max_{\mathbf{C}^L} \left\{ \begin{array}{l} (1 - \mu_0) \left\{ \beta_0 \sum_{t \in \Gamma^L} \delta^t \left[\prod_{s \in \Gamma^L, s \leq t-1} (1 - \lambda^L) \right] (\lambda^L - c) - (1 - \beta_0) \sum_{t \in \Gamma^L} \delta^t c \right\} \\ - \mu_0 \beta_0 \left\{ \begin{array}{l} \sum_{t \in \Gamma^L} \delta^t l_t^L \left[\prod_{s \in \Gamma^L, s \leq t} (1 - \lambda^H) - \prod_{s \in \Gamma^L, s \leq t} (1 - \lambda^L) \right] \\ - \sum_{t \in \Gamma^L} \delta^t c \left[\prod_{s \in \Gamma^L, s \leq t-1} (1 - \lambda^H) - \prod_{s \in \Gamma^L, s \leq t-1} (1 - \lambda^L) \right] \end{array} \right\} \end{array} \right\} \quad (\text{B.1})$$

subject to (\mathbf{IC}_a^L) ,

$$\mathbf{1} \in \arg \max_{(a_t)_{t \in \Gamma^L}} \left\{ \begin{array}{l} \beta_0 \sum_{t \in \Gamma^L} \delta^t \left[\prod_{s \in \Gamma^L, s \leq t-1} (1 - a_s \lambda^L) \right] [(1 - a_t \lambda^L) l_t^L - a_t c] \\ + (1 - \beta_0) \sum_{t \in \Gamma^L} \delta^t (l_t^L - a_t c) + W_0^L \end{array} \right\}. \quad (\text{B.2})$$

To avoid trivialities, consider any optimal \mathbf{C}^L with $\Gamma^L \neq \emptyset$. First consider the possibility $1 \notin \Gamma^L$. In this case, construct a new clawback contract $\hat{\mathbf{C}}^L$ that is “shifted up by one period”:

$$\begin{aligned} \hat{\Gamma}^L &= \{s : s + 1 \in \Gamma^L\}, \\ \hat{l}_s^L &= l_{s+1}^L \text{ for all } s \in \hat{\Gamma}^L, \\ \hat{W}_0^L &= W_0^L. \end{aligned}$$

Clearly it remains optimal for the agent to work in every period in $\hat{\Gamma}^L$, and since the value of (B.1) must have been weakly positive under \mathbf{C}^L , it is now weakly higher since the modification has just multiplied it by $\delta^{-1} > 1$. This procedure can be repeated for all lockout periods at the beginning of the contract, so that without loss, we hereafter assume that $1 \in \Gamma^L$. We are of course done if Γ^L is now connected, so also assume that Γ^L is not connected.

Let t° be the earliest lockout period in Γ^L , i.e. $t^\circ = \min\{t : t \notin \Gamma^L \text{ and } t^\circ - 1 \in \Gamma^L\}$. (Such a $t^\circ > 1$ exists given the preceding discussion.) We will argue that one of two possible modifications preserves the agent's incentive to work in all periods in the modified contract and weakly improves the principal's payoff. This suffices because the procedure can then be applied iteratively to produce a connected contract.

Modification 1: Consider first a modified clawback contract $\hat{\mathbf{C}}^L$ that removes the lockout period

t° and shortens the contract by one period as follows:

$$\begin{aligned}\widehat{\Gamma}^L &= \{1, \dots, t^\circ - 1\} \cup \{s : s \geq t^\circ \text{ and } s + 1 \in \Gamma^L\}, \\ \widehat{l}_s^L &= \begin{cases} l_s^L & \text{if } s < t^\circ - 1, \\ l_s^L + \Delta_1 & \text{if } s = t^\circ - 1, \\ l_{s+1}^L & \text{if } s \geq t^\circ \text{ and } s \in \widehat{\Gamma}^L, \end{cases} \\ \widehat{W}_0^L &= W_0^L.\end{aligned}$$

Note that in the above construction, Δ_1 is a free parameter. We will find conditions on Δ_1 such that type L 's incentives for effort are unchanged and the principal is weakly better off.

For an arbitrary t , define

$$\begin{aligned}S(t) &= (\lambda^L - c) \prod_{s \in \Gamma^L, s \leq t-1} (1 - \lambda^L), \\ R(t) &= \prod_{s \in \Gamma^L, s \leq t} (1 - \lambda^H) - \prod_{s \in \Gamma^L, s \leq t} (1 - \lambda^L).\end{aligned}$$

The value of (B.1) under \mathbf{C}^L is

$$V(\mathbf{C}^L) = (1 - \mu_0) \left[\beta_0 \sum_{t \in \Gamma^L} \delta^t S(t) - (1 - \beta_0) \sum_{t \in \Gamma^L} \delta^t c \right] - \mu_0 \beta_0 \left[\sum_{t \in \Gamma^L} \delta^t l_t^L R(t) - \sum_{t \in \Gamma^L} \delta^t c R(t-1) \right].$$

The value of (B.1) after the modification to $\widehat{\mathbf{C}}^L$ is

$$\begin{aligned}V(\widehat{\mathbf{C}}^L) &= (1 - \mu_0) \left[\beta_0 \left(\sum_{t \in \Gamma^L, t < t^\circ} \delta^t S(t) + \delta^{-1} \sum_{t \in \Gamma^L, t > t^\circ} \delta^t S(t) \right) \right. \\ &\quad \left. - (1 - \beta_0) \left(\sum_{t \in \Gamma^L, t < t^\circ} \delta^t c + \delta^{-1} \sum_{t \in \Gamma^L, t > t^\circ} \delta^t c \right) \right] \\ &\quad - \mu_0 \beta_0 \left[\sum_{t \in \Gamma^L, t < t^\circ - 1} \delta^t l_t^L R(t) + \delta^{-1} \sum_{t \in \Gamma^L, t > t^\circ} \delta^t l_t^L R(t) \right. \\ &\quad \left. + \delta^{t^\circ - 1} (l_{t^\circ - 1}^L + \Delta_1) R(t^\circ - 1) \right. \\ &\quad \left. - \sum_{t \in \Gamma^L, t < t^\circ} \delta^t c R(t-1) - \delta^{-1} \sum_{t \in \Gamma^L, t > t^\circ} \delta^t c R(t-1) \right].\end{aligned}$$

Therefore, the modification benefits the principal if and only if

$$0 \leq V(\widehat{\mathbf{C}}^L) - V(\mathbf{C}^L) = (1 - \mu_0) \left[\beta_0 (\delta^{-1} - 1) \sum_{t \in \Gamma^L, t > t^\circ} \delta^t S(t) - (1 - \beta_0) (\delta^{-1} - 1) \sum_{t \in \Gamma^L, t > t^\circ} \delta^t c \right]$$

$$-\mu_0\beta_0 \left[\begin{array}{c} (\delta^{-1}-1) \sum_{t \in \Gamma^L, t > t^\circ} \delta^t l_t^L R(t) + \delta^{t^\circ-1} \Delta_1 R(t^\circ-1) \\ - (\delta^{-1}-1) \sum_{t \in \Gamma^L, t > t^\circ} \delta^t c R(t-1) \end{array} \right].$$

The above inequality is satisfied for any Δ_1 if $\delta = 1$, and if $\delta < 1$, then after rearranging terms, the above inequality is equivalent to

$$\begin{aligned} & (1-\mu_0) \left[\beta_0 \sum_{t \in \Gamma^L, t > t^\circ} \delta^t S(t) - (1-\beta_0) \sum_{t \in \Gamma^L, t > t^\circ} \delta^t c \right] \\ \geq & \mu_0\beta_0 \left[\sum_{t \in \Gamma^L, t > t^\circ} \delta^t l_t^L R(t) - \delta^{t^\circ-1} \frac{\Delta_1}{1-\delta^{-1}} R(t^\circ-1) - \sum_{t \in \Gamma^L, t > t^\circ} \delta^t c R(t-1) \right]. \end{aligned} \quad (\text{B.3})$$

Now turn to the incentives for effort for the agent of type L . Clearly, since \mathbf{C}^L induces the agent to work in all periods, it remains optimal for the agent to work under $\hat{\mathbf{C}}^L$ in all periods beginning with t° . Consider the incentive constraint for effort in period $t^\circ - 1$ under $\hat{\mathbf{C}}^L$. Using (B.2), this is given by:

$$-\bar{\beta}_{t^\circ-1}^L \lambda^L \left\{ l_{t^\circ-1}^L + \Delta_1 + \delta^{-1} \sum_{t \in \Gamma^L, t > t^\circ} \delta^{t-(t^\circ-1)} \left[\prod_{s \in \Gamma^L, t^\circ-1 < s \leq t-1} (1-\lambda^L) \right] [(1-\lambda^L) l_t^L - c] \right\} \geq c. \quad (\text{B.4})$$

Analogously, the incentive constraint in period $t^\circ - 1$ under the original contract \mathbf{C}^L is:

$$-\bar{\beta}_{t^\circ-1}^L \lambda^L \left\{ l_{t^\circ-1}^L + \sum_{t \in \Gamma^L, t > t^\circ-1} \delta^{t-(t^\circ-1)} \left[\prod_{s \in \Gamma^L, t^\circ-1 < s \leq t-1} (1-\lambda^L) \right] [(1-\lambda^L) l_t^L - c] \right\} \geq c. \quad (\text{B.5})$$

If we choose Δ_1 such that the left-hand side of (B.4) is equal to the left-hand side of (B.5), then since it is optimal to work under the original contract in period $t^\circ - 1$, it will also be optimal to work under the new contract in period $t^\circ - 1$. Accordingly, we choose Δ_1 such that:

$$\begin{aligned} \Delta_1 &= \sum_{t \in \Gamma^L, t > t^\circ-1} \delta^{t-(t^\circ-1)} \left[\prod_{s \in \Gamma^L, t^\circ-1 < s \leq t-1} (1-\lambda^L) \right] [(1-\lambda^L) l_t^L - c] \\ &\quad - \delta^{-1} \sum_{t \in \Gamma^L, t > t^\circ} \delta^{t-(t^\circ-1)} \left[\prod_{s \in \Gamma^L, t^\circ-1 < s \leq t-1} (1-\lambda^L) \right] [(1-\lambda^L) l_t^L - c] \\ &= (1-\delta^{-1}) \sum_{t \in \Gamma^L, t > t^\circ-1} \delta^{t-(t^\circ-1)} \left[\prod_{s \in \Gamma^L, t^\circ-1 < s \leq t-1} (1-\lambda^L) \right] [(1-\lambda^L) l_t^L - c], \end{aligned} \quad (\text{B.6})$$

where the second equality is because $\{t : t \in \Gamma^L, t > t^\circ - 1\} = \{t : t \in \Gamma^L, t > t^\circ\}$, since $t^\circ \notin \Gamma^L$. Note that (B.6) implies $\Delta_1 = 0$ if $\delta = 1$.

Now consider the incentive constraint for effort in any period $\tau < t^\circ - 1$. We will show that because Δ_1 is such that the left-hand side of (B.4) is equal to the left-hand side of (B.5), the fact that it

was optimal to work in period τ under contract \mathbf{C}^L implies that it is optimal to work in period τ under contract $\widehat{\mathbf{C}}^L$. Formally, the incentive constraint for effort in period τ under \mathbf{C}^L is

$$-\bar{\beta}_\tau^L \lambda^L \left\{ l_\tau^L + \sum_{t \in \Gamma^L, t > \tau} \delta^{t-\tau} \left[\prod_{s \in \Gamma^L, \tau < s \leq t-1} (1 - \lambda^L) \right] [(1 - \lambda^L) l_t^L - c] \right\} \geq c, \quad (\text{B.7})$$

which is satisfied since \mathbf{C}^L induces the agent to work in all periods. The incentive constraint for effort in period τ under $\widehat{\mathbf{C}}^L$ can be written as:

$$\begin{aligned} c &\leq -\bar{\beta}_\tau^L \lambda^L \left\{ \widehat{l}_\tau^L + \sum_{t \in \widehat{\Gamma}^L, t > \tau} \delta^{t-\tau} \left[\prod_{s \in \widehat{\Gamma}^L, \tau < s \leq t-1} (1 - \lambda^L) \right] [(1 - \lambda^L) \widehat{l}_t^L - c] \right\} \\ &= -\bar{\beta}_\tau^L \lambda^L \left\{ l_\tau^L + \sum_{t \in \Gamma^L, \tau < t < t^\circ-1} \delta^{t-\tau} \left[\prod_{s \in \Gamma^L, \tau < s \leq t-1} (1 - \lambda^L) \right] [(1 - \lambda^L) l_t^L - c] \right. \\ &\quad \left. + \delta^{t^\circ-1-\tau} \left[\prod_{s \in \Gamma^L, \tau < s \leq t^\circ-2} (1 - \lambda^L) \right] [(1 - \lambda^L) (l_{t^\circ-1}^L + \Delta_1) - c] \right. \\ &\quad \left. + \delta^{-1} \sum_{t \in \Gamma^L, t > t^\circ-1} \delta^{t-\tau} \left[\prod_{s \in \Gamma^L, \tau < s \leq t-1} (1 - \lambda^L) \right] [(1 - \lambda^L) l_t^L - c] \right\} \\ &= -\bar{\beta}_\tau^L \lambda^L \left\{ l_\tau^L + \sum_{t \in \Gamma^L, \tau < t < t^\circ-1} \delta^{t-\tau} \left[\prod_{s \in \Gamma^L, \tau < s \leq t-1} (1 - \lambda^L) \right] [(1 - \lambda^L) l_t^L - c] \right. \\ &\quad \left. + \delta^{t^\circ-1-\tau} \left[\prod_{s \in \Gamma^L, \tau < s \leq t^\circ-2} (1 - \lambda^L) \right] [(1 - \lambda^L) l_{t^\circ-1}^L - c] \right. \\ &\quad \left. + (1 - \delta^{-1}) \left[\sum_{t \in \Gamma^L, t > t^\circ-1} \delta^{t-\tau} \left[\prod_{s \in \Gamma^L, \tau < s \leq t-1} (1 - \lambda^L) \right] [(1 - \lambda^L) l_t^L - c] \right] \right. \\ &\quad \left. + \delta^{-1} \sum_{t \in \Gamma^L, t > t^\circ-1} \delta^{t-\tau} \left[\prod_{s \in \Gamma^L, \tau < s \leq t-1} (1 - \lambda^L) \right] [(1 - \lambda^L) l_t^L - c] \right\} \\ &= -\bar{\beta}_\tau^L \lambda^L \left\{ l_\tau^L + \sum_{t \in \Gamma^L, t > \tau} \delta^{t-\tau} \left[\prod_{s \in \Gamma^L, \tau < s \leq t-1} (1 - \lambda^L) \right] [(1 - \lambda^L) l_t^L - c] \right\} \end{aligned}$$

where the first equality is from the construction of $\widehat{\mathbf{C}}^L$, the second equality uses (B.6), and the third equality follows from algebraic simplification. Since the above constraint is identical to (B.7), it is satisfied.

Consequently, if $\delta = 1$, this modification with $\Delta_1 = 0$ weakly benefits the principal while preserving the agent's incentives, and we are done. So hereafter assume $\delta < 1$, which requires us to also consider another modification.

Modification 2: Now we consider a modified contract $\widetilde{\mathbf{C}}^L$ that eliminates all periods after t° , de-

defined as follows:

$$\begin{aligned}\tilde{\Gamma}^L &= \{1, \dots, t^\circ - 1\}, \\ \tilde{l}_s^L &= \begin{cases} l_s^L & \text{if } s < t^\circ - 1, \\ l_s^L + \Delta_2 & \text{if } s = t^\circ - 1, \end{cases} \\ \tilde{W}_0^L &= W_0^L.\end{aligned}$$

Again, Δ_2 is a free parameter above. We find conditions on Δ_2 such that type L 's incentives are unchanged and the principal is weakly better off.

The value of (B.1) under the modification $\tilde{\mathbf{C}}^L$ is

$$\begin{aligned}V(\tilde{\mathbf{C}}^L) &= (1 - \mu_0) \left[\beta_0 \sum_{t \in \Gamma^L, t < t^\circ} \delta^t S(t) - (1 - \beta_0) \sum_{t \in \Gamma^L, t < t^\circ} \delta^t c \right] \\ &\quad - \mu_0 \beta_0 \left[\sum_{t \in \Gamma^L, t < t^\circ - 1} \delta^t l_t^L R(t) + \delta^{t^\circ - 1} (l_{t^\circ - 1}^L + \Delta_2) R(t^\circ - 1) - \sum_{t \in \Gamma^L, t < t^\circ} \delta^t c R(t - 1) \right].\end{aligned}$$

Therefore, using the previous formula for $V(\mathbf{C}^L)$, this modification benefits the principal if and only if

$$\begin{aligned}0 \leq V(\tilde{\mathbf{C}}^L) - V(\mathbf{C}^L) &= -(1 - \mu_0) \left[\beta_0 \sum_{t \in \Gamma^L, t > t^\circ} \delta^t S(t) - (1 - \beta_0) \sum_{t \in \Gamma^L, t > t^\circ} \delta^t c \right] \\ &\quad - \mu_0 \beta_0 \left[- \sum_{t \in \Gamma^L, t > t^\circ} \delta^t l_t^L R(t) + \delta^{t^\circ - 1} \Delta_2 R(t^\circ - 1) + \sum_{t \in \Gamma^L, t > t^\circ} \delta^t c R(t - 1) \right],\end{aligned}$$

or equivalently after rearranging terms, if and only if

$$\begin{aligned}& (1 - \mu_0) \left[\beta_0 \sum_{t \in \Gamma^L, t > t^\circ} \delta^t S(t) - (1 - \beta_0) \sum_{t \in \Gamma^L, t > t^\circ} \delta^t c \right] \\ & \leq \mu_0 \beta_0 \left[\sum_{t \in \Gamma^L, t > t^\circ} \delta^t l_t^L R(t) - \delta^{t^\circ - 1} \Delta_2 R(t^\circ - 1) - \sum_{t \in \Gamma^L, t > t^\circ} \delta^t c R(t - 1) \right].\end{aligned}\tag{B.8}$$

As with the previous modification, the only incentive constraint for effort that needs to be verified in $\tilde{\mathbf{C}}^L$ is that of period $t^\circ - 1$, which since it is the last period of the contract is simply:

$$-\bar{\beta}_{t^\circ - 1}^L \lambda^L (l_{t^\circ - 1}^L + \Delta_2) \geq c.\tag{B.9}$$

We choose Δ_2 so that the left-hand side of (B.9) is equal to the left-hand side of (B.5):

$$\Delta_2 = \sum_{t \in \Gamma^L, t > t^\circ - 1} \delta^{t - (t^\circ - 1)} \left[\prod_{s \in \Gamma^L, t^\circ - 1 < s \leq t - 1} (1 - \lambda^L) \right] [(1 - \lambda^L) l_t^L - c] = \frac{\Delta_1}{1 - \delta^{-1}},\tag{B.10}$$

where the second equality follows from (B.6). But now, observe that (B.10) implies that either (B.3) or (B.8)

is guaranteed to hold, and hence either the modification to $\widehat{\mathbf{C}}^L$ or to $\widetilde{\mathbf{C}}^L$ weakly benefits the principal while preserving the agent's effort incentives.

Remark 3. Given $\delta < 1$, the choice of Δ_2 in (B.10) implies that if inequality (B.3) holds with equality then so does inequality (B.8), and vice-versa. In other words, if neither of the modifications strictly benefits the principal (while preserving the agent's effort incentives), then it must be that both modifications leave the principal's payoff unchanged (while preserving the agent's effort incentives).

B.4 Step 4: Defining the critical contract for the low type

Take any connected clawback contract, $\mathbf{C}^L = (T^L, W_0^L, l^L)$ that induces effort from the low type in each period $t \in \{1, \dots, T^L\}$. We claim that the low type's incentive constraint for effort binds at all periods if and only if $l^L = \bar{l}^L(T^L)$, where $\bar{l}^L(T^L)$ is defined as follows:

$$\bar{l}_t^L = \begin{cases} -(1-\delta) \frac{c}{\bar{\beta}_t^L \lambda^L} & \text{if } t < T^L, \\ -\frac{c}{\bar{\beta}_{T^L}^L \lambda^L} & \text{if } t = T^L. \end{cases} \quad (\text{B.11})$$

The proof of this claim is via three sub-steps; for the remainder of this step, since T^L is given and held fixed, we ease notation by just writing \bar{l}^L instead of $\bar{l}^L(T^L)$.

Step 4a: First, we argue that with the above penalty sequence, the low type is indifferent between working and shirking in each period $t \in \{1, \dots, T^L\}$ given that he has worked in all prior periods and will do in all subsequent periods no matter his action at period t . In other words, we need to show that for all $t \in \{1, \dots, T^L\}$:

$$-\bar{\beta}_t^L \lambda^L \left\{ \bar{l}_t^L + \sum_{s=t+1}^{T^L} \delta^{s-t} (1-\lambda^L)^{s-(t+1)} \left[(1-\lambda^L) \bar{l}_s^L - c \right] \right\} = c. \quad (\text{B.12})$$

We prove that (B.12) is indeed satisfied for all t by induction. First, it is immediate from (B.11) that (B.12) holds for $t = T^L$. Next, for any $t < T^L$, assume (B.12) holds for $t+1$. This is equivalent to

$$\sum_{s=t+2}^{T^L} \delta^{s-(t+1)} (1-\lambda^L)^{s-(t+2)} \left[(1-\lambda^L) \bar{l}_s^L - c \right] = -\frac{c}{\bar{\beta}_{t+1}^L \lambda^L} - \bar{l}_{t+1}^L. \quad (\text{B.13})$$

³⁸To derive this equality, observe that under the hypotheses, the payoff for type L from working at t is

$$-c + (1 - \bar{\beta}_t^L) \bar{l}_t^L + (1 - \bar{\beta}_t^L) \sum_{s=t+1}^{T^L} \delta^{s-t} (\bar{l}_s^L - c) + \bar{\beta}_t^L \left\{ (1 - \lambda^L) \bar{l}_t^L + \sum_{s=t+1}^{T^L} \delta^{s-t} (1 - \lambda^L)^{s-t} \left[(1 - \lambda^L) \bar{l}_s^L - c \right] \right\},$$

while the payoff from shirking at time t is

$$\bar{l}_t^L + (1 - \bar{\beta}_t^L) \sum_{s=t+1}^{T^L} \delta^{s-t} (\bar{l}_s^L - c) + \bar{\beta}_t^L \left\{ \sum_{s=t+1}^{T^L} \delta^{s-t} (1 - \lambda^L)^{s-(t+1)} \left[(1 - \lambda^L) \bar{l}_s^L - c \right] \right\}.$$

Setting these payoffs from working and shirking equal to each other and manipulating terms yields (B.12).

To show that (B.12) holds for t , it suffices to show that

$$-\bar{\beta}_t^L \lambda^L \left\{ \bar{l}_t^L + \delta \left[(1 - \lambda^L) \bar{l}_{t+1}^L - c \right] + \delta (1 - \lambda^L) \sum_{s=t+2}^{T^L} \delta^{s-(t+1)} (1 - \lambda^L)^{s-(t+2)} \left[(1 - \lambda^L) \bar{l}_s^L - c \right] \right\} = c.$$

Using (B.13), the above equality is equivalent to

$$-\bar{\beta}_t^L \lambda^L \left\{ \bar{l}_t^L + \delta \left[(1 - \lambda^L) \bar{l}_{t+1}^L - c \right] + \delta (1 - \lambda^L) \left[-\frac{c}{\bar{\beta}_{t+1}^L \lambda^L} - \bar{l}_{t+1}^L \right] \right\} = c,$$

which simplifies to

$$\bar{l}_t^L = -\frac{c}{\bar{\beta}_t^L \lambda^L} + \delta c + \delta (1 - \lambda^L) \frac{c}{\bar{\beta}_{t+1}^L \lambda^L}. \quad (\text{B.14})$$

Since $\bar{\beta}_{t+1}^L = \frac{\bar{\beta}_t^L (1 - \lambda^L)}{1 - \bar{\beta}_t^L \lambda^L}$, (B.14) is in turn equivalent to $\bar{l}_t^L = -(1 - \delta) \frac{c}{\bar{\beta}_t^L \lambda^L}$, which is true by the definition of \bar{l}^L in (B.11).

Step 4b: Next, we show that given the sequence \bar{l}^L , it would be optimal for the low type to work in any period no matter the prior history of effort. The argument is by induction. Consider first the last period, T^L . Since no matter the history of prior effort, the current belief is some $\beta_{T^L}^L \geq \bar{\beta}_{T^L}^L$, and hence

$$-\beta_{T^L}^L \lambda^L \bar{l}_t^L \geq \bar{\beta}_{T^L}^L \lambda^L \bar{l}_t^L = c,$$

it is optimal to work in the last period (note that the equality above is by definition).

Now assume inductively that the assertion is true for period $t + 1 \leq T^L$, and consider period $t < T^L$ after any history of prior effort, with current belief β_t^L . Since we already showed that Equation (B.12) holds, it follows from $\beta_t^L \geq \bar{\beta}_t^L$ that

$$-\beta_t^L \lambda^L \left\{ \bar{l}_t^L + \sum_{s=t+1}^{T^L} \delta^{s-t} (1 - \lambda^L)^{s-(t+1)} \left[(1 - \lambda^L) \bar{l}_s^L - c \right] \right\} \geq c,$$

and hence it is optimal for the agent to work in period t .

Step 4c: Finally, we argue that any profile of penalties, l^L , that makes the low type's incentive constraint for effort bind at every period $t \in \{1, \dots, T^L\}$ must coincide with \bar{l}^L , given that the clawback contract must induce work from the low type in each period up to T^L . Again, we use induction. Since $\bar{l}_{T^L}^L$ is the unique penalty that makes the agent indifferent between working and shirking at period T^L given that he has worked in all prior periods, it follows that $l_{T^L}^L = \bar{l}_{T^L}^L$. Note from Step 4b that it would remain optimal for the agent to work in period T^L given any profile of effort in prior periods.

For the inductive step, pick some period $t < T^L$ and assume that in every period $x \in \{t, \dots, T^L\}$, the agent is indifferent between working and shirking given that he has worked in all prior periods, and would also find it optimal to work at x following any other profile of effort prior to x . Under these

hypotheses, the indifference at period $t + 1$ implies that

$$-\bar{\beta}_{t+1}^L \lambda^L \left\{ l_{t+1}^L + \sum_{s=t+2}^{T^L} \delta^{s-(t+1)} (1 - \lambda^L)^{s-(t+2)} [(1 - \lambda^L) l_s^L - c] \right\} = c. \quad (\text{B.15})$$

Given the inductive hypothesis, the incentive constraint for effort at period t is

$$-\bar{\beta}_t^L \lambda^L \left\{ l_t^L + \sum_{s=t+1}^{T^L} \delta^{s-t} (1 - \lambda^L)^{s-(t+1)} [(1 - \lambda^L) l_s^L - c] \right\} \geq c,$$

which, when set to bind, can be written as

$$-\bar{\beta}_t^L \lambda^L \left\{ l_t^L + \delta [(1 - \lambda^L) l_{t+1}^L - c] + \delta (1 - \lambda^L) \sum_{s=t+2}^{T^L} \delta^{s-(t+1)} (1 - \lambda^L)^{s-(t+2)} [(1 - \lambda^L) l_s^L - c] \right\} = c. \quad (\text{B.16})$$

Substituting (B.15) into (B.16), using the fact that $\bar{\beta}_{t+1}^L = \frac{\bar{\beta}_t^L (1 - \lambda)}{\bar{\beta}_t^L (1 - \lambda) + 1 - \bar{\beta}_t^L}$, and performing some algebra shows that $l_t^L = \bar{l}_t^L$. Moreover, by the reasoning in Step 4b, this also ensures that the agent would find it optimal to work in period t for any other history of actions prior to period t .

B.5 Step 5: The critical contract is optimal

By Step 3, we can restrict attention in solving program [RP2] to connected clawback contracts for the low type. For any T^L , Step 4 identified a particular sequence of penalties, $\bar{l}^L(T^L)$. We now show that any connected clawback contract for the low type that solves [RP2] must have precisely this penalty structure.

The proof involves two sub-steps; throughout, we hold an arbitrary T^L fixed and, to ease notation, drop the dependence of $\bar{l}^L(\cdot)$ on T^L .

Step 5a: We first show that any connected clawback contract for the low type of length T^L that satisfies (IC_a^L) and has $l_t^L > \bar{l}_t^L$ in some period $t \leq T^L$ is not optimal. To prove this, consider any connected clawback contract of length T^L that satisfies (IC_a^L) and specifies a penalty $l_{t'}^L > \bar{l}_{t'}^L$ in some period $t' \leq T^L$. Define

$$\hat{t} = \max \left\{ t : t \leq T^L \text{ and } l_t^L > \bar{l}_t^L \right\}.$$

Observe that we must have $\hat{t} < T^L$ because otherwise (IC_a^L) would be violated in period T^L . Furthermore, by definition of \hat{t} , $l_t^L \leq \bar{l}_t^L$ for all $T^L \geq t > \hat{t}$. We will prove that we can change the penalty structure by lowering $l_{\hat{t}}^L$ and raising some subsequent l_s^L for $s \in \{\hat{t} + 1, \dots, T^L\}$ in a way that keeps type L 's incentives for effort unchanged, and yet increase the value of the objective function (RP2).

Claim: There exists $\tilde{t} \in \{\hat{t} + 1, \dots, T^L\}$ such that (IC_a^L) at \tilde{t} is slack and $l_{\tilde{t}}^L < \bar{l}_{\tilde{t}}^L$.

Proof: Suppose not, then for each $T^L \geq t > \hat{t}$, either $l_t^L = \bar{l}_t^L$, or $l_t^L < \bar{l}_t^L$ and (IC_a^L) binds. Then since

whenever $l_t^L < \bar{l}_t^L$, (IC_a^L) binds by supposition, it must be that in all $t > \hat{t}$, (IC_a^L) binds (this follows from Step 4). But then (IC_a^L) at \hat{t} is violated since $l_{\hat{t}}^L > \bar{l}_{\hat{t}}^L$. \parallel

Claim: There exists $\bar{t} \in \{\hat{t} + 1, \dots, T^L\}$ such that $l_{\bar{t}}^L < \bar{l}_{\bar{t}}^L$ and for any $t \in \{\hat{t} + 1, \dots, \bar{t}\}$, (IC_a^L) at t is slack. In particular, we can take \bar{t} to be the first such period after \hat{t} .

Proof: Fix \tilde{t} in the previous claim. Note that (IC_a^L) at $\hat{t} + 1$ must be slack because otherwise (IC_a^L) at \hat{t} is violated by $l_{\hat{t}}^L > \bar{l}_{\hat{t}}^L$ and Step 4. There are two cases. (1) $l_{\hat{t}+1}^L < \bar{l}_{\hat{t}+1}^L$; then $\hat{t} + 1$ is the \bar{t} we want. (2) $l_{\hat{t}+1}^L = \bar{l}_{\hat{t}+1}^L$ — in this case, since (IC_a^L) is slack at $\hat{t} + 1$, it must be that (IC_a^L) at $\hat{t} + 2$ is slack (otherwise, the claim in Step 4 is violated); now if $l_{\hat{t}+2}^L < \bar{l}_{\hat{t}+2}^L$, we are done because $\hat{t} + 2$ is the \bar{t} we are looking for; if $l_{\hat{t}+2}^L = \bar{l}_{\hat{t}+2}^L$, then we continue to $\hat{t} + 3$ and so on until we reach \tilde{t} which we know gives us a slack (IC_a^L) , $l_{\tilde{t}}^L < \bar{l}_{\tilde{t}}^L$, and we are sure that (IC_a^L) is slack in all periods of this process before reaching \tilde{t} . \parallel

Now we shall show that we can slightly reduce $l_{\hat{t}}^L > \bar{l}_{\hat{t}}^L$ and slightly increase $l_{\bar{t}}^L < \bar{l}_{\bar{t}}^L$ and meanwhile keep the incentives for effort of type L satisfied for all periods. We know that we do not violate (IC_a^L) for $t \in \{\hat{t} + 1, \dots, \bar{t}\}$ because (IC_a^L) is slack there; plainly, incentives are not affected after \bar{t} . We shall show that the modification strictly reduces type H 's rent and meanwhile does not violate (IC_a^L) at \hat{t} nor any previous period. Therefore, the modified contract strictly dominates the original contract.

We first want to guarantee that (IC_a^L) at \hat{t} is unchanged. By the same reasoning as used in Step 3, the incentive constraint for effort in period \hat{t} (given that the agent will work in all subsequent periods no matter his behavior at period t) can be written as

$$-\beta_{\hat{t}}^L \lambda^L \left\{ l_{\hat{t}}^L + \sum_{t>\hat{t}} \delta^{t-\hat{t}} (1-\lambda^L)^{t-(\hat{t}+1)} [(1-\lambda^L) l_t^L - c] \right\} \geq c. \quad (\text{B.17})$$

Observe that if we reduce $l_{\hat{t}}^L$ by $\Delta > 0$ and increase $l_{\bar{t}}^L$ by $\frac{\Delta}{\delta^{\bar{t}-\hat{t}}(1-\lambda^L)^{\bar{t}-(\hat{t}+1)}(1-\lambda^L)} = \frac{\Delta}{\delta^{\bar{t}-\hat{t}}(1-\lambda^L)^{\bar{t}-\hat{t}}}$, then the left-hand side of (B.17) does not change. Moreover, it follows that incentives for effort at $t < \hat{t}$ are also unchanged (see Step 3), and the incentive condition at \bar{t} will be satisfied if Δ is small enough because the original (IC_a^L) at \bar{t} is slack.

We now show that the modification above leads to a reduction of the rent of type H in (RP2), i.e. raises the value of the objective. The rent is given by

$$\mu_0 \beta_0 \left\{ \sum_{t=1}^{T^L} \delta^t l_t^L [(1-\lambda^H)^t - (1-\lambda^L)^t] - \sum_{t=1}^{T^L} \delta^t c [(1-\lambda^H)^{t-1} - (1-\lambda^L)^{t-1}] \right\}.$$

Hence, the change in the rent from reducing $l_{\hat{t}}^L$ by Δ and increasing $l_{\bar{t}}^L$ by $\frac{\Delta}{\delta^{\bar{t}-\hat{t}}(1-\lambda^L)^{\bar{t}-\hat{t}}}$ is

$$\begin{aligned} & \mu_0 \beta_0 \delta^{\hat{t}} \Delta \left\{ - \left[(1-\lambda^H)^{\hat{t}} - (1-\lambda^L)^{\hat{t}} \right] + \frac{1}{(1-\lambda^L)^{\bar{t}-\hat{t}}} \left[(1-\lambda^H)^{\bar{t}} - (1-\lambda^L)^{\bar{t}} \right] \right\} \\ &= \mu_0 \beta_0 \delta^{\hat{t}} \Delta \frac{(1-\lambda^H)^{\hat{t}}}{(1-\lambda^L)^{\bar{t}-\hat{t}}} \left[(1-\lambda^H)^{\bar{t}-\hat{t}} - (1-\lambda^L)^{\bar{t}-\hat{t}} \right] < 0, \end{aligned}$$

where the inequality is because $\bar{t} > \hat{t}$ and $1 - \lambda^H < 1 - \lambda^L$.

Step 5b: By Step 5a, we can restrict attention to penalty sequences l^L such that $l_t^L \leq \bar{l}_t^L$ for all $t \leq T^L$. Now we show that unless $l^L(\cdot) = \bar{l}^L(\cdot)$, the value of the objective (RP2) can be improved while satisfying the incentive constraint for effort, (IC_a^L). To show this, recall that the high type's rent is

$$\beta_0 \sum_{t=1}^{T^L} \delta^t l_t^L \left[(1 - \lambda^H)^t - (1 - \lambda^L)^t \right] - \beta_0 c \sum_{t=1}^{T^L} \delta^t \left[(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1} \right].$$

By Step 4a, (IC_a^L) is satisfied in all periods $t = 1, \dots, T^L$ whenever $l_t^L = \bar{l}_t^L$. Now, if $l_t^L < \bar{l}_t^L$ for any period, we can replace l_t^L by \bar{l}_t^L without affecting the effort incentives for type L , and by doing this we reduce the rent of type H , thereby raising the value of (RP2).

B.6 Step 6: Under-experimentation by the low type

By Step 5, an optimal contract for the low type that solves program [RP2] can be found by optimizing over T^L , i.e. the length of connected clawback contracts with the penalty structure $\bar{l}^L(T^L)$. In this step, we first argue that the optimal length is no larger than t^L (recall that t^L is the first-best stopping time), and then establish generic uniqueness of the optimal contract for the low type.

Step 6a: The portion of the objective (RP2) that involves T^L is

$$(1 - \mu_0) \left[\beta_0 \sum_{t=1}^{T^L} \delta^t (1 - \lambda^L)^{t-1} (\lambda^L - c) - (1 - \beta_0) \sum_{t=1}^{T^L} \delta^t c \right] - \mu_0 \beta_0 \left\{ \sum_{t=1}^{T^L} \delta^t \bar{l}_t^L(T^L) \left[(1 - \lambda^H)^t - (1 - \lambda^L)^t \right] - \sum_{t=1}^{T^L} \delta^t c \left[(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1} \right] \right\}, \quad (\text{B.18})$$

where we have used the desired penalty sequence. Now consider the following definition:

$$\begin{aligned} \Pi(z, T^L) &= z(-\mu_0 \beta_0) \left\{ \sum_{t=1}^{T^L} \delta^t \bar{l}_t^L(T^L) \left[(1 - \lambda^H)^t - (1 - \lambda^L)^t \right] - \sum_{t=1}^{T^L} \delta^t c \left[(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1} \right] \right\} \\ &\quad + (1 - \mu_0) \left[\beta_0 \sum_{t=1}^{T^L} \delta^t (1 - \lambda^L)^{t-1} (\lambda^L - c) - (1 - \beta_0) \sum_{t=1}^{T^L} \delta^t c \right]. \end{aligned}$$

If $z = 0$, the expression above corresponds to surplus maximization; if $z = 1$, the expression corresponds to the principal's objective (B.18). Consider the term in $\Pi(\cdot)$ that is multiplied by z modulo a negative constant:

$$K(T^L) = \sum_{t=1}^{T^L} \delta^t \bar{l}_t^L(T^L) \left[(1 - \lambda^H)^t - (1 - \lambda^L)^t \right] - \sum_{t=1}^{T^L} \delta^t c \left[(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1} \right].$$

If $K(\cdot)$ is shown to be increasing, then $\Pi(z, T^L)$ has decreasing differences, which implies that the optimal T^L when $z = 0$ is no smaller than the optimal T^L when $z = 1$, as desired. To see that $K(\cdot)$ is indeed increasing, observe that

$$\begin{aligned}
K(T+1) - K(T) &= \sum_{t=1}^{T+1} \delta^t \bar{l}_t^L(T+1) \left[(1-\lambda^H)^t - (1-\lambda^L)^t \right] - \sum_{t=1}^{T+1} \delta^t c \left[(1-\lambda^H)^{t-1} - (1-\lambda^L)^{t-1} \right] \\
&\quad - \sum_{t=1}^T \delta^t \bar{l}_t^L(T) \left[(1-\lambda^H)^t - (1-\lambda^L)^t \right] + \sum_{t=1}^T \delta^t c \left[(1-\lambda^H)^{t-1} - (1-\lambda^L)^{t-1} \right] \\
&= \delta^T \left[\frac{-(1-\delta)c}{\bar{\beta}_T^L \lambda^L} \right] \left[(1-\lambda^H)^T - (1-\lambda^L)^T \right] \\
&\quad + \delta^{T+1} \left(\frac{-c}{\bar{\beta}_{T+1}^L \lambda^L} \right) \left[(1-\lambda^H)^{T+1} - (1-\lambda^L)^{T+1} \right] \\
&\quad - \delta^{T+1} c \left[(1-\lambda^H)^T - (1-\lambda^L)^T \right] - \delta^T \left(\frac{-c}{\bar{\beta}_T^L \lambda^L} \right) \left[(1-\lambda^H)^T - (1-\lambda^L)^T \right] \\
&= \delta^{T+1} c \left(\frac{1}{\bar{\beta}_T^L \lambda^L} - 1 \right) \left(\frac{\lambda^H - \lambda^L}{1 - \lambda^L} \right) (1-\lambda^H)^T > 0,
\end{aligned}$$

where the second equality uses the definition of $\bar{l}^L(\cdot)$ and the final inequality is because $\bar{\beta}_T^L \lambda^L < 1$.

It is also clear that there is generically a unique T^L that maximizes $\Pi(1, T^L)$; hereafter we denote this solution \bar{t}^L . In the non-generic cases where multiple maximizers exist, we select the largest one.

Step 6b: We have shown so far that among connected clawback contracts, there is generically a unique contract for type L that solves [RP2] (or, more precisely, the portion of the program involving the low type's contract). We now claim that there generically cannot be any other clawback contract for type L that solves [RP2]. Suppose, to contradiction, that this is false: there is an optimal non-connected clawback contract $\mathbf{C}^L = (\Gamma^L, W_0^L, \mathbf{l}^L)$ in which $1 \in \alpha^L(\mathbf{C}^L)$. Let $t^\circ < \max \Gamma^L$ be the earliest lockout period in \mathbf{C}^L . Without loss, owing to genericity, we take $\delta < 1$. Following the arguments of Step 3, in particular Remark 3, the optimality of \mathbf{C}^L implies that there are two *connected* clawback contracts that are also optimal: $\hat{\mathbf{C}}^L = (\hat{T}^L, \hat{W}_0^L, \hat{\mathbf{l}}^L)$ obtained from \mathbf{C}^L by applying Modification 1 of Step 3 as many times as needed to eliminate all lockout periods, and $\tilde{\mathbf{C}}^L = (\tilde{T}^L, \tilde{W}_0^L, \tilde{\mathbf{l}}^L)$ obtained from \mathbf{C}^L by applying Modification 2 of Step 3 to shorten the contract by just eliminating all periods from t° on. Note that the modifications ensure that $1 \in \alpha^L(\hat{\mathbf{C}}^L)$ and $1 \in \alpha^L(\tilde{\mathbf{C}}^L)$. But now, the fact that $\hat{T}^L > \tilde{T}^L$ contradicts the generic uniqueness of connected clawback contracts for the low type that solve [RP2].

Remark 4. Now consider any low-type contract and prescribed action profile that is optimal with regards to the principal's original program defined at the outset of Subsection 4.1. By Proposition 1 and Step 1 of the current proof, there is a corresponding (possibly non-connected) clawback contract that solves the relevant portion of [RP2] and has the low type working in all the same periods. The arguments above then show that, generically, the original solution must have prescribed the low type to work in periods $1, \dots, \bar{t}^L$. This explains Theorem 2.

B.7 Step 7: Back to the original program

We have shown so far that there is a solution to program [RP2] in which the low type's contract is a connected clawback contract of length $\bar{t}^L \leq t^L$ and in which the penalty sequence is given by $\bar{l}^L(\bar{t}^L)$. In terms of optimizing over the high type's contract, note that any solution must induce the high type to work in each period up to t^H and no longer: this follows from the fact that the objective in (RP2) involving the high type's contract is social surplus from the high type, and that there is clearly a sequence of (sufficiently low) penalties l^H to ensure that (IC_a^H) is satisfied.

Recall that solutions to [RP2] produce solutions to [RP1] by choosing W_0^L to make (IR^L) bind and W_0^H to make (Weak-IC^{HL}) bind, which can always be done. Accordingly, let $\bar{C}^L = (\bar{t}^L, \bar{W}_0^L, \bar{l}^L(\bar{t}^L))$ be the connected clawback contract where \bar{W}_0^L is set to make (IR^L) bind, and consider the solutions to program [RP1] in which the low type's contract is \bar{C}^L . We will argue that some of these solutions to [RP1], namely \bar{C}^L combined with a suitable onetime-clawback contract for the high type, also solve the original program [P]. Recall that [RP1] differs from [P] in three ways:

1. it imposes (Weak-IC^{HL}) rather than (IC^{HL});
2. it ignores (IR^H);
3. it ignores (IC^{LH}).

We address each of these constraints in order.

Step 7a: First, we argue that given any connected clawback contract of length $T^L \leq t^L$ with penalty sequence $\bar{l}^L(T^L)$, it would be optimal for type H to work in every period $1 \dots, T^L$, no matter the history of prior effort. Consequently, any solution to [RP1] using \bar{C}^L satisfies (IC^{HL}).

To prove the claim, we fix any $T^L \leq t^L$ and write \bar{l}^L as shorthand for $\bar{l}^L(T^L)$. The argument is by induction. Consider first the last period, T^L . Since

$$-\bar{\beta}_{T^L}^L \lambda^L \bar{l}_{T^L}^L = c,$$

it follows from the fact that $t^H > t^L$ (hence $\bar{\beta}_t^H \lambda^H > \bar{\beta}_t^L \lambda^L$ for all $t < t^H$) that no matter the history of effort,

$$-\beta_{T^L}^H \lambda^H \bar{l}_{T^L}^L \geq c,$$

i.e., regardless of the history, type H will work in period T^L .

Now assume inductively that it is optimal for type H to work in period $t+1 \leq T^L$ no matter the history of effort, and consider period t with belief β_t^H . The inductive hypothesis implies that

$$-\beta_{t+1}^H \lambda^H \left\{ \bar{l}_{t+1}^L + \sum_{s=t+2}^{T^L} \delta^{s-(t+1)} (1 - \lambda^H)^{s-(t+2)} \left[(1 - \lambda^H) \bar{l}_s^L - c \right] \right\} \geq c,$$

or equivalently,

$$\sum_{s=t+2}^{T^L} \delta^{s-(t+1)} (1 - \lambda^H)^{s-(t+2)} \left[(1 - \lambda^H) \bar{l}_s^L - c \right] \leq -\frac{c}{\beta_{t+1}^H \lambda^H} - \bar{l}_{t+1}^L. \quad (\text{B.19})$$

Therefore, at period $t < T^L$:

$$\begin{aligned} & -\beta_t^H \lambda^H \left\{ \bar{l}_t^L + \delta \left[(1 - \lambda^H) \bar{l}_{t+1}^L - c \right] + \delta (1 - \lambda^H) \sum_{s=t+2}^{T^L} \delta^{s-(t+1)} (1 - \lambda^H)^{s-(t+2)} \left[(1 - \lambda^H) \bar{l}_s^L - c \right] \right\} \\ \geq & -\beta_t^H \lambda^H \left\{ \bar{l}_t^L + \delta \left[(1 - \lambda^H) \bar{l}_{t+1}^L - c \right] + \delta (1 - \lambda^H) \left(-\frac{c}{\beta_{t+1}^H \lambda^H} - \bar{l}_{t+1}^L \right) \right\} \\ = & -\beta_t^H \lambda^H \left(\bar{l}_t^L - \delta c \right) + \delta (1 - \lambda^H) \frac{\beta_t^H c}{\beta_{t+1}^H} \\ = & -\beta_t^H \lambda^H \bar{l}_t^L + \delta c \\ \geq & -\bar{\beta}_t^L \lambda^L \bar{l}_t^L + \delta c \\ = & c, \end{aligned}$$

where the first inequality uses (B.19), the second equality uses $\beta_{t+1}^H = \frac{\beta_t^H (1 - \lambda^H)}{1 - \beta_t^H + \beta_t^H (1 - \lambda^H)}$, and the final equality uses the fact that $\bar{l}_t^L = -\frac{(1 - \delta)c}{\bar{\beta}_t^L \lambda^L}$.

Step 7b: Next, we show that any solution to [RP1] using $\bar{\mathbf{C}}^L$ also satisfies (IR^H). To show this, observe first that

$$\begin{aligned} U_0^H(\bar{\mathbf{C}}^L, \mathbf{1}) &= \beta_0 \sum_{t=1}^{\bar{t}^L} \delta^t (1 - \lambda^H)^{t-1} \left[(1 - \lambda^H) \bar{l}_t^L (\bar{t}^L) - c \right] + (1 - \beta_0) \sum_{t \in \Gamma} \delta^t \left(\bar{l}_t^L (\bar{t}^L) - c \right) + \bar{W}_0^L \\ &\geq \beta_0 \sum_{t=1}^{\bar{t}^L} \delta^t (1 - \lambda^L)^{t-1} \left[(1 - \lambda^L) \bar{l}_t^L (\bar{t}^L) - c \right] + (1 - \beta_0) \sum_{t \in \Gamma} \delta^t \left(\bar{l}_t^L (\bar{t}^L) - c \right) + \bar{W}_0^L \\ &= U_0^L(\bar{\mathbf{C}}^L, \mathbf{1}), \end{aligned} \quad (\text{B.20})$$

where the inequality follows from the fact that for all $t \in \{1, \dots, \bar{t}^L\}$, $\bar{l}_t^L \leq 0$.

Consequently, in any solution to [RP1] using $\bar{\mathbf{C}}^L$,

$$\begin{aligned} U_0^H(\mathbf{C}^H, \boldsymbol{\alpha}^H(\mathbf{C}^H)) &\geq U_0^H(\bar{\mathbf{C}}^L, \mathbf{1}) \text{ (by (IC}_a^H \text{) and (Weak-IC}^{HL} \text{))} \\ &\geq U_0^L(\bar{\mathbf{C}}^L, \mathbf{1}) \text{ (by inequality (B.20))} \\ &\geq 0 \text{ (by (IR}^L \text{))}, \end{aligned}$$

and hence (IR^H) is satisfied.

Step 7c: Finally, we show that there is a solution to [RP1] using $\overline{\mathbf{C}}^L$ that also satisfies (IC^{LH}) in [P], which completes the proof. As previously noted, any optimal contract for the high type in [RP1] must induce effort from this type in periods $1, \dots, t^H$ and make (Weak-IC^{HL}) bind. We will construct such a *onetime*-clawback contract, $\mathbf{C}^H = (t^H, W_0^H, l_{t^H}^H)$, where given the penalty $l_{t^H}^H$ (a free parameter at this point) and that the high type works in all periods, W_0^H is chosen to make (Weak-IC^{HL}) bind, i.e. by the equation:

$$\left[(1 - \lambda^H)^{t^H} \beta_0 + (1 - \beta_0) \right] \delta^{t^H} l_{t^H}^H - \beta_0 \sum_{t=1}^{t^H} \delta^t (1 - \lambda^H)^{t-1} c - (1 - \beta_0) \sum_{t=1}^{t^H} \delta^t c + W_0^H = \rho, \quad (\text{B.21})$$

where

$$\rho = \beta_0 \sum_{t=1}^{\bar{t}^L} \delta^t \bar{l}_t^L (\bar{t}^L) \left[(1 - \lambda^H)^t - (1 - \lambda^L)^t \right] - \beta_0 c \sum_{t=1}^{\bar{t}^L} \delta^t \left[(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1} \right]$$

is the rent earned by type H given type L 's contract $\overline{\mathbf{C}}^L$.

Plainly, the penalty $l_{t^H}^H$ can be chosen to be severe enough (i.e. sufficiently negative) to ensure that it is optimal for an agent of *either type*, H or L , to work in all periods after accepting such a contract \mathbf{C}^H , i.e. that for all $\theta \in \{L, H\}$, $\alpha^\theta(\mathbf{C}^H) = 1$. All that remains is to show that a sufficiently severe $l_{t^H}^H$ and its corresponding W_0^H (determined by (B.21)) also satisfy (IC^{LH}) given that $\alpha^L(\mathbf{C}^H) = 1$.

Using (B.21), we compute

$$\begin{aligned} U_0^L(\mathbf{C}^H, 1) &= W_0^H - c \left[\beta_0 \sum_{t=1}^{t^H} \delta^t (1 - \lambda^L)^{t-1} + (1 - \beta_0) \sum_{t=1}^{t^H} \delta^t \right] + l_{t^H}^H \delta^{t^H} \left[\beta_0 (1 - \lambda^L)^{t^H} + (1 - \beta_0) \right] \\ &= - \left\{ \left[(1 - \lambda^H)^{t^H} \beta_0 + (1 - \beta_0) \right] \delta^{t^H} l_{t^H}^H - \beta_0 \sum_{t=1}^{t^H} \delta^t (1 - \lambda^H)^{t-1} c - (1 - \beta_0) \sum_{t=1}^{t^H} \delta^t c \right\} \\ &\quad + \rho - c \left[\beta_0 \sum_{t=1}^{t^H} \delta^t (1 - \lambda^L)^{t-1} + (1 - \beta_0) \sum_{t=1}^{t^H} \delta^t \right] + l_{t^H}^H \delta^{t^H} \left[\beta_0 (1 - \lambda^L)^{t^H} + (1 - \beta_0) \right] \\ &= \beta_0 \delta^{t^H} \left[(1 - \lambda^L)^{t^H} - (1 - \lambda^H)^{t^H} \right] l_{t^H}^H + k, \end{aligned} \quad (\text{B.22})$$

where $k = \rho + c \left\{ \beta_0 \sum_{t=1}^{t^H} \delta^t \left[(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1} \right] \right\}$ is independent of $l_{t^H}^H$.

The expression (B.22) is an affine function of $l_{t^H}^H$, with a strictly positive coefficient on $l_{t^H}^H$, since $\lambda^H > \lambda^L$. Hence, we can choose $l_{t^H}^H$ sufficiently low so that (B.22) is negative, in which case (IC^{LH}) is satisfied because $U_0^L(\overline{\mathbf{C}}^L, 1) = 0$.

C Proof of Proposition 2

We use a similar monotone comparative statics argument as that employed in [Step 6](#) of the proof of [Theorem 3](#). Recall expression [\(B.18\)](#), which was the portion of the principal's objective that involves a stopping time for the low type, T :

$$V(T, \beta_0, \mu_0, c, \delta) := (1 - \mu_0) \left[\beta_0 \sum_{t=1}^T \delta^t (1 - \lambda^L)^{t-1} (\lambda^L - c) - (1 - \beta_0) \sum_{t=1}^T \delta^t c \right] \\ - \mu_0 \beta_0 \left\{ \sum_{t=1}^T \delta^t \bar{l}_t^L(T) \left[(1 - \lambda^H)^t - (1 - \lambda^L)^t \right] - \sum_{t=1}^T \delta^t c \left[(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1} \right] \right\},$$

where $\bar{l}_t^L(T)$ is given by [\(6\)](#) in [Theorem 3](#). The second-best stopping time, \bar{t}^L , is the T that maximizes $V(T, \cdot)$.³⁹ To establish the comparative statics of \bar{t}^L with respect to the parameters, we show that $V(T, \cdot)$ has increasing or decreasing differences in T and the relevant parameter.

Substituting $\bar{l}_t^L(T)$ from [\(6\)](#) into $V(\cdot)$ above yields

$$V(T, \beta_0, \mu_0, c, \delta) = (1 - \mu_0) \left[\beta_0 \sum_{t=1}^T \delta^t (1 - \lambda^L)^{t-1} (\lambda^L - c) - (1 - \beta_0) \sum_{t=1}^T \delta^t c \right] \\ - \mu_0 \left\{ \begin{aligned} & -c \sum_{t=1}^{T-1} \delta^t (1 - \delta) \frac{\beta_0 (1 - \lambda^L)^{t-1} + 1 - \beta_0}{\lambda^L (1 - \lambda^L)^{t-1}} \left[(1 - \lambda^H)^t - (1 - \lambda^L)^t \right] \\ & - c \delta^T \frac{\beta_0 (1 - \lambda^L)^{T-1} + 1 - \beta_0}{\lambda^L (1 - \lambda^L)^{T-1}} \left[(1 - \lambda^H)^T - (1 - \lambda^L)^T \right] \\ & - \beta_0 \sum_{t=1}^T \delta^t c \left[(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1} \right] \end{aligned} \right\}. \quad (\text{C.1})$$

After some algebraic manipulation, we obtain

$$V(T+1, \beta_0, \mu_0, c, \delta) - V(T, \beta_0, \mu_0, c, \delta) = \delta^{T+1} \left\{ \begin{aligned} & (1 - \mu_0) \left[\beta_0 (1 - \lambda^L)^T (\lambda^L - c) - (1 - \beta_0) c \right] \\ & - \mu_0 c \frac{\beta_0 (1 - \lambda^L)^T + 1 - \beta_0}{(1 - \lambda^L)^T \lambda^L} (1 - \lambda^H)^T (\lambda^H - \lambda^L) \end{aligned} \right\}. \quad (\text{C.2})$$

[\(C.2\)](#) implies that $V(T, \beta_0, \mu_0, c, \delta)$ has increasing differences in (T, β_0) , because

$$\frac{\partial}{\partial \beta_0} [V(T+1, \beta_0, \cdot) - V(T, \beta_0, \cdot)] = \delta^{T+1} \left\{ \begin{aligned} & (1 - \mu_0) \left[(1 - \lambda^L)^T (\lambda^L - c) + c \right] \\ & + \mu_0 c \frac{1 - (1 - \lambda^L)^T}{(1 - \lambda^L)^T \lambda^L} (1 - \lambda^H)^T (\lambda^H - \lambda^L) \end{aligned} \right\} > 0.$$

It thus follows that \bar{t}^L is increasing in β_0 . Similarly, [\(C.2\)](#) also implies

$$\frac{\partial}{\partial c} [V(T+1, c, \cdot) - V(T, c, \cdot)] = \delta^{T+1} \left\{ \begin{aligned} & -(1 - \mu_0) \left[\beta_0 (1 - \lambda^L)^T + (1 - \beta_0) \right] \\ & - \mu_0 \frac{\beta_0 (1 - \lambda^L)^T + 1 - \beta_0}{(1 - \lambda^L)^T \lambda^L} (1 - \lambda^H)^T (\lambda^H - \lambda^L) \end{aligned} \right\} < 0,$$

³⁹While the maximizer is generically unique, recall that if multiple maximizers exist we select the largest one.

and hence \bar{t}^L is decreasing in c .

Finally, consider the comparative static with respect to μ_0 . From (C.2), we obtain

$$\frac{\partial}{\partial \mu_0} [V(T+1, \mu_0, \cdot) - V(T, \mu_0, \cdot)] = \delta^{T+1} \left\{ \begin{array}{l} -[\beta_0 (1 - \lambda^L)^T (\lambda^L - c) - (1 - \beta_0)c] \\ -c \frac{\beta_0 (1 - \lambda^L)^T + 1 - \beta_0}{(1 - \lambda^L)^T \lambda^L} (1 - \lambda^H)^T (\lambda^H - \lambda^L) \end{array} \right\}. \quad (\text{C.3})$$

Recall that the first-best stopping time t^L is such that $\frac{\beta_0 (1 - \lambda^L)^{t^L - 1}}{\beta_0 (1 - \lambda^L)^{t^L - 1} + 1 - \beta_0} \lambda^L \geq c$, which is equivalent to $\beta_0 (1 - \lambda^L)^{t^L - 1} (\lambda^L - c) - (1 - \beta_0)c \geq 0$. Thus, for $T+1 \leq t^L$,

$$\beta_0 (1 - \lambda^L)^T (\lambda^L - c) - (1 - \beta_0)c \geq 0. \quad (\text{C.4})$$

Combining (C.3) and (C.4) implies

$$\frac{\partial}{\partial \mu_0} [V(T+1, \beta_0, \mu_0, c, \delta) - V(T, \beta_0, \mu_0, c, \delta)] \leq -\delta^{T+1} c \frac{\beta_0 (1 - \lambda^L)^T + 1 - \beta_0}{(1 - \lambda^L)^T \lambda^L} (1 - \lambda^H)^T (\lambda^H - \lambda^L) < 0.$$

It follows that \bar{t}^L is decreasing in μ_0 .

D Proof of Theorem 5

We assume throughout this appendix that $\delta = 1$. Without loss of generality by Proposition 1, we focus on menus of clawback contracts. In this appendix, we will introduce programs and constraints that have analogies with those used in Appendix B for the case of $t^H > t^L$. Accordingly, we often use the same labels for equations as before, but the reader should bear in mind that all references in this appendix to such equations are to those defined in this appendix.

Outline. Since this is a long proof, let us describe the pieces involved. We begin by showing in Step 1 that it is without loss to focus on contracts for type L that induce him to work in every non-lockout period; this is identical to the first step in the proof of Theorem 3. In Step 2, we relax the principal's program [P] (which already takes into account Step 1) into a relaxed program, [RP1], that ignores the high type's participation constraint, (IR^H) , and the low type's self-selection constraint, (IC^{LH}) , and show that the resulting program can be further simplified into a program called [RP2] whose only constraints are the dynamic moral hazard constraints (IC_a^H) and (IC_a^L) . A critical difference here relative to the analogous relaxed program in the proof of Theorem 3 is that the current program [RP2] does not constrain what the high type must do when taking the low type's contract.

Focusing thereafter on [RP2], we show in Step 3 that there is an optimal clawback contract for type L that is connected. In Step 4, we develop three lemmas pertaining to properties of the set $\alpha^H(\mathbf{C}^L)$ in any \mathbf{C}^L that is an optimal contract for type L . We then use these lemmas in Step 5 to show that in solving [RP2], we can restrict attention to connected clawback contracts \mathbf{C}^L for type L such that $\alpha^H(\mathbf{C}^L)$ includes a *stopping strategy with the most work property*, i.e., an action plan that involves consecutive work for some number of periods followed by shirking thereafter, and where the number of work periods is larger than in any action plan in $\alpha^H(\mathbf{C}^L)$. Building on the restriction to stopping strategies, we then show in Step 6

that there is always an optimal contract for type L that is a onetime-clawback contract.

For an arbitrary time T^L , [Step 7](#) first defines a particular last-period penalty $l_{T^L}^L(T^L)$ and an associated time $T^{HL}(T^L) \leq T^L$, and then establishes that if T^L is the optimal length of experimentation for type L , there is an optimal onetime-clawback contract for type L with penalty $l_{T^L}^L(T^L)$ and in which type H 's most-work optimal stopping strategy involves $T^{HL}(T^L)$ periods of work. Hence, using $l_{T^L}^L(T^L)$ and $T^{HL}(T^L)$, an optimal contract for type L that solves [\[RP2\]](#) can be found by optimizing over the length T^L . In [Step 8](#), we show that the optimal length, \bar{t}^L , is no larger than the first-best stopping time, t^L . Finally, in [Step 9](#), we show that there is a solution to [\[RP2\]](#) that combines type L 's contract with a suitable onetime-clawback contract for type H and also solves the original program [\[P\]](#).

D.1 Step 1: Low type always works

Given any contract for type L , $\mathbf{C}^L = (\Gamma^L, W_0^L, l^L)$, we claim that there is a contract $\hat{\mathbf{C}}^L = (\hat{\Gamma}^L, \hat{W}_0^L, \hat{l}^L)$ such that:

- (i) $\mathbf{1} \in \alpha^L(\hat{\mathbf{C}}^L)$;
- (ii) $U_0^L(\mathbf{C}^L, \alpha^L(\mathbf{C}^L)) = U_0^L(\hat{\mathbf{C}}^L, \mathbf{1})$;
- (iii) $\Pi_0^L(\mathbf{C}^L, \alpha^L(\mathbf{C}^L)) = \Pi_0^L(\hat{\mathbf{C}}^L, \mathbf{1})$; and
- (iv) $U_0^H(\mathbf{C}^L, \alpha^H(\mathbf{C}^L)) \geq U_0^H(\hat{\mathbf{C}}^L, \alpha^H(\hat{\mathbf{C}}^L))$.

The proof is identical to [Step 1](#) in the proof of [Theorem 3](#) and thus omitted.

D.2 Step 2: Simplifying the program

By Step 1, we can restrict our attention to clawback contracts $\mathbf{C}^\theta = (\Gamma^\theta, W_0^\theta, l^\theta)$, with type L 's contract inducing type L to exert effort in all periods in Γ^L . Denoting the set of clawback contracts by \mathcal{C} , the principal faces the following program [\[P\]](#):

$$\max_{(\mathbf{C}^H \in \mathcal{C}, \mathbf{C}^L \in \mathcal{C}, \mathbf{a}^H)} \mu_0 \Pi_0^H(\mathbf{C}^H, \mathbf{a}^H) + (1 - \mu_0) \Pi_0^L(\mathbf{C}^L, \mathbf{1}) \quad (\text{P})$$

subject to

$$\begin{aligned} \mathbf{1} &\in \alpha^L(\mathbf{C}^L) & (\text{IC}_a^L) \\ \mathbf{a}^H &\in \alpha^H(\mathbf{C}^H) & (\text{IC}_a^H) \\ U_0^L(\mathbf{C}^L, \mathbf{1}) &\geq 0 & (\text{IR}^L) \\ U_0^H(\mathbf{C}^H, \mathbf{a}^H) &\geq 0 & (\text{IR}^H) \\ U_0^L(\mathbf{C}^L, \mathbf{1}) &\geq U_0^L(\mathbf{C}^H, \alpha^L(\mathbf{C}^H)) & (\text{IC}^{LH}) \\ U_0^H(\mathbf{C}^H, \mathbf{a}^H) &\geq U_0^H(\mathbf{C}^L, \alpha^H(\mathbf{C}^L)) & (\text{IC}^{HL}) \end{aligned}$$

To solve program [P], we solve a relaxed program and later verify that the solution is feasible in (and hence is a solution to) [P]. Specifically, we ignore (IC^{LH}) and (IR^H). The relaxed program, [RP1], is therefore:

$$\max_{(\mathbf{C}^H \in \mathcal{C}, \mathbf{C}^L \in \mathcal{C}, \mathbf{a}^H)} \mu_0 \Pi_0^H(\mathbf{C}^H, \mathbf{a}^H) + (1 - \mu_0) \Pi_0^L(\mathbf{C}^L, \mathbf{1}) \quad (\text{RP1})$$

subject to

$$\mathbf{1} \in \alpha^L(\mathbf{C}^L) \quad (\text{IC}_a^L)$$

$$\mathbf{a}^H \in \alpha^H(\mathbf{C}^H) \quad (\text{IC}_a^H)$$

$$U_0^L(\mathbf{C}^L, \mathbf{1}) \geq 0 \quad (\text{IR}^L)$$

$$U_0^H(\mathbf{C}^H, \mathbf{a}^H) \geq U_0^H(\mathbf{C}^L, \alpha^H(\mathbf{C}^L)). \quad (\text{IC}^{HL})$$

It is clear that in any solution to program [RP1], (IR^L) must be binding: otherwise, the initial time-zero transfer from the principal to the agent in the contract \mathbf{C}^L can be reduced slightly to strictly improve the second term of the objective function while not violating any of the constraints. Similarly, (IC^{HL}) must also bind because otherwise the time-zero transfer in the contract \mathbf{C}^H can be reduced to improve the first term of the objective function without violating any of the constraints.

Using these two binding constraints, substituting in the formulae from equations (1) and (2), and letting the principal select which optimal action plan the high type should use when taking the low type's contract ($\mathbf{a}^{HL} \in \alpha^H(\mathbf{C}^L)$), we can rewrite the objective function (RP1) as the expected total surplus less type H 's "information rent", obtaining the following explicit version of the relaxed program which we call [RP2]:

$$\max_{(\mathbf{C}^H \in \mathcal{C}, \mathbf{C}^L \in \mathcal{C}, \mathbf{a}^H, \mathbf{a}^{HL} \in \alpha^H(\mathbf{C}^L))} \left\{ \begin{aligned} & \mu_0 \left\{ \beta_0 \sum_{t \in \Gamma^H} \left[\prod_{s \in \Gamma^H, s \leq t-1} (1 - a_s^H \lambda^H) \right] a_t^H (\lambda^H - c) - (1 - \beta_0) \sum_{t \in \Gamma^H} a_t^H c \right\} \\ & + (1 - \mu_0) \left\{ \beta_0 \sum_{t \in \Gamma^L} \left[\prod_{s \in \Gamma^L, s \leq t-1} (1 - \lambda^L) \right] (\lambda^L - c) - (1 - \beta_0) \sum_{t \in \Gamma^L} c \right\} \\ & - \mu_0 \left\{ \underbrace{\beta_0 \sum_{t \in \Gamma^L} l_t^L \left[\prod_{s \in \Gamma^L, s \leq t} (1 - a_s^{HL} \lambda^H) - \prod_{s \in \Gamma^L, s \leq t} (1 - \lambda^L) \right]}_{\text{Information rent of type H}} \right. \\ & \quad \left. - \beta_0 c \sum_{t \in \Gamma^L} a_t^{HL} \left[\prod_{s \in \Gamma^L, s \leq t-1} (1 - a_s^{HL} \lambda^H) - \prod_{s \in \Gamma^L, s \leq t-1} (1 - \lambda^L) \right] \right. \\ & \quad \left. + c \sum_{t \in \Gamma^L} (1 - a_t^{HL}) \left[(1 - \beta_0) + \beta_0 \prod_{s \in \Gamma^L, s \leq t-1} (1 - \lambda^L) \right] \right\} \end{aligned} \right\} \quad (\text{RP2})$$

subject to

$$\mathbf{1} \in \arg \max_{(a_t)_{t \in \Gamma^L}} \left\{ \begin{aligned} & \beta_0 \sum_{t \in \Gamma^L} \left[\prod_{s \in \Gamma^L, s \leq t-1} (1 - a_s \lambda^L) \right] [(1 - a_t \lambda^L) l_t^L - a_t c] \\ & + (1 - \beta_0) \sum_{t \in \Gamma^L} (l_t^L - a_t c) + W_0^L \end{aligned} \right\}, \quad (\text{IC}_a^L)$$

$$\mathbf{a}^H \in \arg \max_{(a_t)_{t \in \Gamma^H}} \left\{ \begin{aligned} & \beta_0 \sum_{t \in \Gamma^H} \left[\prod_{s \in \Gamma^H, s \leq t-1} (1 - a_s \lambda^H) \right] [(1 - a_t \lambda^H) l_t^H - a_t c] \\ & + (1 - \beta_0) \sum_{t \in \Gamma^H} (l_t^H - a_t c) + W_0^H \end{aligned} \right\}. \quad (\text{IC}_a^H)$$

Program [RP2] is separable, i.e. it can be solved by separately maximizing (RP2) with respect to $(\mathbf{C}^L, \mathbf{a}^{HL})$ subject to (IC_a^L) and separately maximizing (RP2) with respect to $(\mathbf{C}^H, \mathbf{a}^H)$ subject to (IC_a^H) .

We denote the information rent of type H by $R(\mathbf{C}^L, \mathbf{a}^{HL})$. Note that given any action plan \mathbf{a} that type H uses when taking type L 's contract,

$$R(\mathbf{C}^L, \mathbf{a}) = U_0^H(\mathbf{C}^L, \mathbf{a}) - U_0^L(\mathbf{C}^L, \mathbf{1}).$$

Hence, $R(\mathbf{C}^L, \mathbf{a}) = R(\mathbf{C}^L, \hat{\mathbf{a}})$ whenever $\mathbf{a}, \hat{\mathbf{a}} \in \alpha^H(\mathbf{C}^L)$.

It will be convenient at various places to consider the difference in information rents under contracts $\hat{\mathbf{C}}^L$ and \mathbf{C}^L and corresponding action plans $\hat{\mathbf{a}}$ and \mathbf{a} :

$$\begin{aligned} R(\hat{\mathbf{C}}^L, \hat{\mathbf{a}}) - R(\mathbf{C}^L, \mathbf{a}) &= U_0^H(\hat{\mathbf{C}}^L, \hat{\mathbf{a}}) - U_0^L(\hat{\mathbf{C}}^L, \mathbf{1}) - (U_0^H(\mathbf{C}^L, \mathbf{a}) - U_0^L(\mathbf{C}^L, \mathbf{1})) \\ &= (U_0^H(\hat{\mathbf{C}}^L, \hat{\mathbf{a}}) - U_0^H(\mathbf{C}^L, \mathbf{a})) - (U_0^L(\hat{\mathbf{C}}^L, \mathbf{1}) - U_0^L(\mathbf{C}^L, \mathbf{1})). \end{aligned} \quad (\text{D.1})$$

Moreover, when the action plan does not change across contracts (i.e. $\mathbf{a} = \hat{\mathbf{a}}$ above), (D.1) specializes to

$$R(\hat{\mathbf{C}}^L, \mathbf{a}) - R(\mathbf{C}^L, \mathbf{a}) = \beta_0 \sum_{t \in \Gamma^L} (\hat{l}_t^L - l_t^L) \left[\prod_{s \in \Gamma^L, s \leq t} (1 - a_s \lambda^H) - \prod_{s \in \Gamma^L, s \leq t} (1 - \lambda^L) \right]. \quad (\text{D.2})$$

D.3 Step 3: Connected contracts for the low type

We now claim that in program [RP2], it is without loss to consider solutions in which the low type's contract is a connected clawback contract, i.e. solutions \mathbf{C}^L in which $\Gamma^L = \{1, \dots, T^L\}$ for some T^L .

To avoid trivialities, consider any optimal \mathbf{C}^L with $\Gamma^L \neq \emptyset$. Let t° be the earliest lockout period in Γ^L , i.e. $t^\circ = \min\{t : t \notin \Gamma^L\}$. Consider a modified clawback contract $\hat{\mathbf{C}}^L$ that removes the lockout period t° and shortens the contract by one period as follows:

$$\begin{aligned} \hat{\Gamma}^L &= \{1, \dots, t^\circ - 1\} \cup \{s : s \geq t^\circ \text{ and } s + 1 \in \Gamma^L\}, \\ \hat{l}_s^L &= \begin{cases} l_s^L & \text{if } s \leq t^\circ - 1, \\ l_{s+1}^L & \text{if } s \geq t^\circ \text{ and } s \in \hat{\Gamma}^L, \end{cases} \\ \hat{W}_0^L &= W_0^L. \end{aligned}$$

Given no discounting, it is straightforward that it remains optimal for type L to work in every period in $\hat{\Gamma}^L$, and given any optimal action plan for type H under the original contract, $\mathbf{a}^{HL} \in \alpha^H(\mathbf{C}^L)$, the action plan

$$\hat{\mathbf{a}}^{HL} = (\hat{a}_s^{HL})_{s \in \hat{\Gamma}^L} = \begin{cases} a_s^{HL} & \text{if } s \leq t^\circ - 1, \\ a_{s+1}^{HL} & \text{if } s \geq t^\circ \text{ and } s \in \hat{\Gamma}^L, \end{cases}$$

is optimal for type H under the modified contract, i.e. $\hat{\mathbf{a}}^{HL} \in \alpha^H(\hat{\mathbf{C}}^L)$. Given no discounting, it is also immediate that the surplus generated by type L is unchanged by the modification. It thus follows that the value of (RP2) is unchanged by the modification. This procedure can be applied iteratively to all lockout periods to produce a connected contract.

D.4 Step 4: Optimal deviation action plans for the high type

By the previous steps, we can restrict our attention to connected clawback contracts $\mathbf{C}^L = (T^L, W_0^L, \mathbf{l}^L)$ that induce effort from the low type in each period $t \in \{1, \dots, T^L\}$. We now describe properties of an optimal connected clawback contract for the low type (Step 4a) and an optimal action plan for the high type when taking the low type's contract (Step 4b).

Step 4a: Consider an optimal connected clawback contract for type L , $\mathbf{C}^L = (T^L, W_0^L, \mathbf{l}^L)$. The next two lemmas describe properties of such a contract.

Lemma 1. Suppose that $\mathbf{C}^L = (T^L, W_0^L, \mathbf{l}^L)$ is an optimal contract for type L . Then for any $t = 1, \dots, T^L$, there exists an optimal action plan $\mathbf{a} \in \alpha^H(\mathbf{C}^L)$ such that $a_t = 1$.

Proof. Suppose to the contrary that for some $\tau \in \{1, \dots, T^L\}$, $a_\tau = 0$ for all $\mathbf{a} \in \alpha^H(\mathbf{C}^L)$. For any $\varepsilon > 0$, define a contract $\mathbf{C}^L(\varepsilon) = (T^L, W_0^L, \mathbf{l}^L(\varepsilon))$ modified from $\mathbf{C}^L = (T^L, W_0^L, \mathbf{l}^L)$ as follows: (i) $l_\tau^L(\varepsilon) = l_\tau^L - \varepsilon$; (ii) $l_{\tau-1}^L(\varepsilon) = l_{\tau-1}^L + \varepsilon(1 - \lambda^L)$; and (iii) $l_t^L(\varepsilon) = l_t^L$ if $t \notin \{\tau - 1, \tau\}$. We derive a contradiction by showing that for small enough $\varepsilon > 0$, $\mathbf{C}^L(\varepsilon)$ together with an original optimal contract for type H , \mathbf{C}^H , is feasible in [RP2] and strictly improves the objective. Note that by construction, $(\mathbf{C}^L(\varepsilon), \mathbf{C}^H)$ satisfy (\mathbf{IC}_a^L) and (\mathbf{IC}_a^H) . To evaluate how the objective changes when $\mathbf{C}^L(\varepsilon)$ is used instead of \mathbf{C}^L , we thus only need to consider the difference in the information rents associated with these contracts, $R(\mathbf{C}^L(\varepsilon)) - R(\mathbf{C}^L)$.

We first claim that $\alpha^H(\mathbf{C}^L(\varepsilon)) \subseteq \alpha^H(\mathbf{C}^L)$ when ε is small enough. To see this, fix any $\mathbf{a} \in \alpha^H(\mathbf{C}^L)$. Since the set of action plans is discrete, the optimality of \mathbf{a} implies that there is some $\eta > 0$ such that $U_0^H(\mathbf{C}^L, \mathbf{a}) > U_0^H(\mathbf{C}^L, \mathbf{a}') + \eta$ for any $\mathbf{a}' \notin \alpha^H(\mathbf{C}^L)$. Since $U_0^H(\mathbf{C}^L(\varepsilon), \mathbf{a}')$ is continuous in ε , it follows immediately that for all ε small enough and all $\mathbf{a}' \notin \alpha^H(\mathbf{C}^L)$:

$$U_0^H(\mathbf{C}^L(\varepsilon), \mathbf{a}) > U_0^H(\mathbf{C}^L(\varepsilon), \mathbf{a}') + \eta.$$

Thus, $\mathbf{a}' \notin \alpha^H(\mathbf{C}^L(\varepsilon))$. It follows that $\alpha^H(\mathbf{C}^L(\varepsilon)) \subseteq \alpha^H(\mathbf{C}^L)$.

Next, for small enough ε , take $\mathbf{a} \in \alpha^H(\mathbf{C}^L(\varepsilon)) \subseteq \alpha^H(\mathbf{C}^L)$. Recall that by assumption $a_\tau = 0$. Using (D.2),

$$\begin{aligned} R(\mathbf{C}^L(\varepsilon), \mathbf{a}) - R(\mathbf{C}^L, \mathbf{a}) &= \beta_0 \varepsilon (1 - \lambda^L) \left[\prod_{s=1}^{\tau-1} (1 - a_s \lambda^H) - (1 - \lambda^L)^{\tau-1} \right] \\ &\quad - \beta_0 \varepsilon \left[\prod_{s=1}^{\tau} (1 - a_s \lambda^H) - (1 - \lambda^L)^\tau \right] \\ &= -\lambda^L \beta_0 \varepsilon \prod_{s=1}^{\tau-1} (1 - a_s \lambda^H) < 0. \end{aligned}$$

Hence, $\mathbf{C}^L(\varepsilon)$ strictly improves the objective relative to \mathbf{C}^L .

Q.E.D.

Lemma 2. Suppose that $\mathbf{C}^L = (T^L, W_0^L, \mathbf{l}^L)$ is an optimal contract for type L and there is some $\tau \in \{1, \dots, T^L\}$ such that $a_\tau = 1$ for all $\mathbf{a} \in \alpha^H(\mathbf{C}^L)$. Then (\mathbf{IC}_a^L) binds at τ .

Proof. Recall from (\mathbf{IC}_a^L) that $\mathbf{a}^L = 1$. Suppose to the contrary that (\mathbf{IC}_a^L) is *not* binding at some τ but $a_\tau = 1$ for all $\mathbf{a} \in \alpha^H(\mathbf{C}^L)$. For any $\varepsilon > 0$, define a contract $\mathbf{C}^L(\varepsilon) = (T^L, W_0^L, \mathbf{l}^L(\varepsilon))$ modified from $\mathbf{C}^L = (T^L, W_0^L, \mathbf{l}^L)$ as follows: (i) $l_\tau^L(\varepsilon) = l_\tau^L + \varepsilon$; (ii) $l_{\tau-1}^L(\varepsilon) = l_{\tau-1}^L - \varepsilon(1 - \lambda^L)$; and (iii) $l_t^L(\varepsilon) = l_t^L$ if $t \notin \{\tau - 1, \tau\}$. We derive a contradiction by showing that for small enough $\varepsilon > 0$, $\mathbf{C}^L(\varepsilon)$ together with an original optimal contract for type H , \mathbf{C}^H , is feasible in [RP2] and strictly improves the objective. Note that by construction (\mathbf{IC}_a^L) is still satisfied under $\mathbf{C}^L(\varepsilon)$ at $t = 1, \dots, \tau - 1, \tau + 1, \dots, T^L$. Moreover, since (\mathbf{IC}_a^L) is slack at τ under contract \mathbf{C}^L , it continues to be slack at τ under $\mathbf{C}^L(\varepsilon)$ for ε small enough.

Now for small enough ε , take any $\mathbf{a} \in \alpha^H(\mathbf{C}^L(\varepsilon)) \subseteq \alpha^H(\mathbf{C}^L)$, where the subset inequality follows from the arguments in the proof of Lemma 1. Recall that by assumption $a_\tau = 1$. Using (D.2),

$$\begin{aligned} R(\mathbf{C}^L(\varepsilon), \mathbf{a}) - R(\mathbf{C}^L, \mathbf{a}) &= -\beta_0 \varepsilon (1 - \lambda^L) \left[\prod_{s=1}^{\tau-1} (1 - a_s \lambda^H) - (1 - \lambda^L)^{\tau-1} \right] \\ &\quad + \beta_0 \varepsilon \left[\prod_{s=1}^{\tau} (1 - a_s \lambda^H) - (1 - \lambda^L)^\tau \right] \\ &= \beta_0 \varepsilon \left\{ \prod_{s=1}^{\tau-1} (1 - a_s \lambda^H) [-(1 - \lambda^L) + (1 - \lambda^H)] \right\} \\ &= -(\lambda^H - \lambda^L) \beta_0 \varepsilon \prod_{s=1}^{\tau-1} (1 - a_s \lambda^H) < 0. \end{aligned}$$

Hence, $\mathbf{C}^L(\varepsilon)$ strictly improves the objective relative to \mathbf{C}^L .

Q.E.D.

Step 4b: For any optimal action plan for the high type under the low type's contract $\mathbf{a} \in \alpha^H(\mathbf{C}^L)$ and $s < t$, define

$$D(s, t, \mathbf{a}) = \sum_{\tau=s}^{t-1} l_\tau^L (1 - \lambda^H)^{\sum_{n=s+1}^{\tau} a_n}.$$

The next lemma describes properties of any action plan $\mathbf{a} \in \alpha^H(\mathbf{C}^L)$.

Lemma 3. Suppose $\mathbf{a} \in \alpha^H(\mathbf{C}^L)$ and $s < t$.

- (1) If $D(s, t, \mathbf{a}) > 0$ and $a_s = 1$, then $a_t = 1$.
- (2) If $D(s, t, \mathbf{a}) < 0$ and $a_s = 0$, then $a_t = 0$.
- (3) If $D(s, t, \mathbf{a}) = 0$, then $\mathbf{a}' \in \alpha^H(\mathbf{C}^L)$ where $a'_s = a_t$, $a'_t = a_s$, and $a'_\tau = a_\tau$ if $\tau \neq s, t$.

Proof. Consider the first case of $D(s, t, \mathbf{a}) > 0$. Suppose to the contrary that for some optimal action plan \mathbf{a} and two periods $s < t$, we have $D(s, t, \mathbf{a}) > 0$ and $a_s = 1$ but $a_t = 0$. Consider an action plan \mathbf{a}' such

that \mathbf{a}' and \mathbf{a} agree except that $a'_s = 0$ and $a'_t = 1$. That is,

$$\begin{aligned}\mathbf{a} &= (\dots, \underbrace{1}_{\text{period } s}, \dots, \underbrace{0}_{\text{period } t}, \dots), \\ \mathbf{a}' &= (\dots, \underbrace{0}_{\text{period } s}, \dots, \underbrace{1}_{\text{period } t}, \dots).\end{aligned}$$

Let $U_s^H(\mathbf{C}^L, \mathbf{a})$ be type H 's payoff evaluated at the beginning of period s . Then,

$$U_s^H(\mathbf{C}^L, \mathbf{a}) - U_s^H(\mathbf{C}^L, \mathbf{a}') = -\beta_s^H \lambda^H \sum_{\tau=s}^{t-1} l_\tau^L (1 - \lambda^H)^{\sum_{n=s+1}^{\tau} a_n} = -\beta_s^H \lambda^H D(s, t, \mathbf{a}).$$

The intuition for this expression is as follows. Since action plans \mathbf{a} and \mathbf{a}' have the same number of working periods, the assumption of no discounting implies that neither the effort costs nor the penalty sequence matters for the difference in utilities conditional on the bad state. Conditional on the good state, the effort costs again do not affect the difference in utilities; however, the probability with which the agent receives l_t^L for any $\tau \in \{s, \dots, t-1\}$ is “shifted up” in \mathbf{a}' as compared to \mathbf{a} .

Therefore, $U_s^H(\mathbf{C}^L, \mathbf{a}) - U_s^H(\mathbf{C}^L, \mathbf{a}') < 0$ if $D(s, t, \mathbf{a}) > 0$. But this contradicts the assumption that \mathbf{a} is optimal; hence, the claim in part (1) follows. The proof of part (2) is analogous.

Finally, consider part (3). This claim is immediate if $a_s = a_t$. If $a_s = 1$ and $a_t = 0$, from the argument above, $U_s^H(\mathbf{C}^L, \mathbf{a}) - U_s^H(\mathbf{C}^L, \mathbf{a}') = 0$; hence, both \mathbf{a} and \mathbf{a}' are optimal. The case of $a_s = 0$ and $a_t = 1$ is analogous. *Q.E.D.*

D.5 Step 5: Stopping strategies for the high type

We use the following concepts to characterize the solution to [RP2]:

Definition 4. An action plan \mathbf{a} is a *stopping strategy* (that stops at t) if there exists $t \geq 1$ such that $a_s = 1$ for $s \leq t$ and $a_s = 0$ for $s > t$.

Definition 5. An optimal action plan for type θ under contract \mathbf{C} , $\mathbf{a} \in \alpha^\theta(\mathbf{C})$, has the *most-work* property (or is a *most-work optimal strategy*) if no other optimal action plan under the contract has more work periods; that is, for all $\mathbf{a}' \in \alpha^\theta(\mathbf{C})$, $\#\{n : a_n = 1\} \geq \#\{n : a'_n = 1\}$.

Step 4 described properties of optimal contracts for the low type and optimal action plans for the high type under the low type's contract. We now use these properties to show that in solving program [RP2], we can restrict attention to connected clawback contracts for the low type $\mathbf{C}^L = (T^L, W_0^L, l^L)$ such that there is an optimal action plan for the high type under the contract $\mathbf{a} \in \alpha^H(\mathbf{C}^L)$ that is a stopping strategy with the most work property.

Let $N = \min_{\mathbf{a} \in \alpha^H(\mathbf{C}^L)} \#\{n : a_n = 0\}$. That is, among all action plans that are optimal for type H under contract \mathbf{C}^L , the action plan in which type H works the largest number of periods involves type H shirking in N periods. Let A_N be the set of optimal action plans that involve type H shirking in exactly N periods. Let

$$k = \max \{ \kappa : \exists \mathbf{a} \in A_N, a_t = 0 \text{ for all } t > T^L - \kappa \}$$

and

$$A_{N,k} = \{ \mathbf{a} \in A_N : a_t = 0 \text{ for all } t > T^L - k \}.$$

That is, $\mathbf{a} \in A_{N,k}$ contains a total of N shirking periods, k of which are in the tail. We want to show that $k = N$ and thus $A_{N,N} \neq \emptyset$. Suppose to contradiction that this is not true; i.e., $k < N$. Then there exists

$$\hat{t} = \min \{t : a_t = 0, \mathbf{a} \in A_{N,k}, t < T^L - k\}. \quad (\text{D.3})$$

In words, \hat{t} is the smallest shirking period preceding a working period such that there is an optimal action plan $\mathbf{a} \in A_{N,k}$ with $k+1$ shirking periods from (including) \hat{t} . Now take $\hat{t}_0 = \hat{t}$. For $n = 0, 1, \dots$, whenever $\{t : a_t = 0, \mathbf{a} \in A_{N,k}, t < \hat{t}_n\} \neq \emptyset$, define

$$\hat{t}_{n+1} = \min \{t : a_t = 0, \mathbf{a} \in A_{N,k}, t < \hat{t}_n\}. \quad (\text{D.4})$$

The sequence $\{\hat{t}_n\}$ uniquely pins down $\hat{\mathbf{a}} \in A_{N,k}$. Note that $\hat{\mathbf{a}}$ takes the following form:

period:	\hat{t}	$\hat{t} + 1$	\dots	$T^L - k$	$T^L - k + 1$	\dots
$\hat{\mathbf{a}}$:	0	1	\dots	1	0	\dots

Our goal is to show that $\bigcup_{n=k+1}^N A_{N,n} \neq \emptyset$; that is, there exists an action plan $\mathbf{a} \in \alpha^H(\mathbf{C}^L)$ with a total of N shirk periods and at least $k+1$ consecutive shirk periods in the tail. To prove this, we will show that we can “move” the shirking in period \hat{t} of $\hat{\mathbf{a}}$ to the end. This is done via three lemmas.

Lemma 4. Suppose $\bigcup_{n=k+1}^N A_{N,n} = \emptyset$. Then $l_t^L = 0$ for any $t = \hat{t} + 1, \dots, T^L - k - 1$.

Proof. We proceed by induction. Take any $t \in \{\hat{t} + 1, \dots, T^L - k - 1\}$ and assume that $l_s^L = 0$ for $s = t + 1, \dots, T^L - k - 1$. We show that $l_t^L = 0$.

Step 1: $l_t^L \geq 0$.

Proof of Step 1: Suppose not, i.e., $l_t^L < 0$. Then the hypotheses that (IC_a^L) is satisfied at period $t+1$ and $l^t < 0$ imply that (IC_a^L) is slack at period t .⁴⁰ Hence, by Lemma 2, there exists an action plan $\mathbf{a}' \in \alpha^H(\mathbf{C}^L)$ such that $a'_t = 0$. Now, by the assumption that $l_t^L < 0$ together with the induction hypothesis, we obtain $\sum_{s=t}^m l_s^L < 0$ for $m \in \{t, \dots, T^L - k - 1\}$. By Lemma 3, part (2), $a'_s = 0$ for any $s = t, \dots, T^L - k$. Thus, $\mathbf{a}' \in \alpha^H(\mathbf{C}^L)$ is as follows:

period:	\hat{t}	$\hat{t} + 1$	\dots	t	$t + 1$	\dots	$T^L - k - 1$	$T^L - k$	$T^L - k + 1$	\dots
$\hat{\mathbf{a}}$:	0	1	\dots	1	1	\dots	1	1	0	\dots
\mathbf{a}' :				0	0	\dots	0	0		

Claim 1: There exists $s^* \geq T^L - k$ such that $a'_{s^*} = 1$.

Proof: Suppose the claim is false, i.e., $a'_s = 0$ for all $s \geq T^L - k$. Then $\#\{n : a'_n = 0\} > N$. To see this, note that $\#\{n : a'_n = 0\} \geq N$ by assumption. Moreover, if $\#\{n : a'_n = 0\} = N$, then \mathbf{a}' contains $k+1$ shirking periods in its tail and $\mathbf{a}' \in A_N$, so $\mathbf{a}' \in A_{N,k+1}$, contradicting the assumption that $\bigcup_{n=k+1}^N A_{N,n} = \emptyset$. Given that $\#\{n : a'_n = 0\} > N$ and $a'_s = 0$ for $s \geq T^L - k - 1$, it follows that $\beta_{T^L-k}^H(\mathbf{a}') \geq \beta_{T^L-k}^H(\hat{\mathbf{a}})$ and taking $a'_{T^L-k} = 1$ is optimal, a contradiction. \parallel

⁴⁰This can be proved along very similar lines to part (2) of Lemma 3.

Now let s^* be the first such working period after $T^L - k$. Then,

period:	\hat{t}	$\hat{t} + 1$	\dots	t	$t + 1$	\dots	$T^L - k - 1$	$T^L - k$	$T^L - k + 1$	\dots	s^*	\dots
$\hat{\mathbf{a}}$:	0	1	\dots	1	1	\dots	1	1	0	\dots	0	\dots
\mathbf{a}' :				0	0	\dots	0	0	0	\dots	1	

Applying parts (1) and (2) of [Lemma 3](#) to $\hat{\mathbf{a}}$ and \mathbf{a}' , we obtain $\sum_{s=T^L-k}^{s^*-1} l_s^L = 0$. Now applying part (3) of [Lemma 3](#), we obtain that the agent is indifferent between \mathbf{a}' and \mathbf{a}'' where \mathbf{a}'' differs from \mathbf{a}' only by switching the actions in period $T^L - k$ and period s^* . But since $\sum_{s=t}^{T^L-k-1} l_s^L < 0$, the optimality of $a_t'' = 0, a_{T^L-k}'' = 1$ contradicts part (2) of [Lemma 3](#).

Step 2: $l_t^L \leq 0$.

Proof of Step 2: Assume to the contrary that $l_t^L > 0$. We have two cases to consider.

Case 1: $l_{T^L-k}^L \geq 0$.

By the induction hypothesis and the assumption that $l_t^L > 0$, we have $\sum_{s=t}^{T^L-k} l_s^L (1 - \lambda^H)^{\sum_{n=t+1}^s \hat{a}_n} > 0$. Therefore, by part (1) of [Lemma 3](#), $\hat{a}_{T^L-k+1} = 1$. But this contradicts the definition of $\hat{\mathbf{a}}$.

Case 2: $l_{T^L-k}^L < 0$.

In this case, (IC_a^L) must be slack in period $T^L - k$ (since it is satisfied in the next period and $l_{T^L-k}^L < 0$). Hence by [Lemma 2](#), there exists $\tilde{\mathbf{a}}$ such that $\tilde{a}_{T^L-k} = 0$.

period:	\hat{t}	$\hat{t} + 1$	\dots	t	$t + 1$	\dots	$T^L - k - 1$	$T^L - k$	\dots
$\hat{\mathbf{a}}$:	0	1	\dots	1	1	\dots	1	1	\dots
$\tilde{\mathbf{a}}$:								0	

Claim 2: $\tilde{a}_s = 0$ for any $s > T^L - k$.

Proof: Suppose the claim is not true. Then define

$$\tau = \min \{s : s > T^L - k \text{ and } \tilde{a}_s = 1\}.$$

This is shown in the following table:

period:	\hat{t}	$\hat{t} + 1$	\dots	t	$t + 1$	\dots	$T^L - k - 1$	$T^L - k$	\dots	$\tau - 1$	τ	\dots
$\hat{\mathbf{a}}$:	0	1	\dots	1	1	\dots	1	1	\dots	0	0	\dots
$\tilde{\mathbf{a}}$:								0	\dots	0	1	

Applying parts (1) and (2) of [Lemma 3](#) to $\hat{\mathbf{a}}$ and $\tilde{\mathbf{a}}$ respectively, we obtain

$$\sum_{s=T^L-k}^{\tau-1} l_s^L = 0. \quad (\text{D.5})$$

But then, by the induction hypothesis and the assumption that $l_t^L > 0$, we obtain

$$\sum_{s=t}^{T^L-k-1} l_s^L (1 - \lambda^H)^{\sum_{n=t+1}^s \hat{a}_n} > 0. \quad (\text{D.6})$$

Notice that $\tilde{a}_s = 0$ for $s > T^L - k$ by definition. Hence, (D.5) and (D.6) imply $\sum_{s=t}^{\tau-1} l_s^L (1 - \lambda^H)^{\sum_{n=t+1}^s \hat{a}_n} > 0$.

Now applying part (1) of [Lemma 3](#) to $\hat{\mathbf{a}}$, we reach the conclusion that $\hat{a}_\tau = 1$, a contradiction. \parallel

Hence, we have established the claim that $\tilde{a}_s = 0$ for all $s > T^L - k$, as depicted below:

period:	\hat{t}	$\hat{t} + 1$	\dots	t	$t + 1$	\dots	$T^L - k - 1$	$T^L - k$	\dots	$\tau - 1$	τ	\dots
$\hat{\mathbf{a}}$:	0	1	\dots	1	1	\dots	1	1	\dots	0	0	\dots
$\tilde{\mathbf{a}}$:								0	\dots	0	0	\dots

Claim 3: $\#\{n : \tilde{a}_n = 0\} = N + 1$ and $\beta_{T^L-k}^H(\tilde{\mathbf{a}}) = \beta_{T^L-k}^H(\hat{\mathbf{a}})$.

Proof: By definition of N , $\#\{n : \tilde{a}_n = 0\} \geq N$. If $\#\{n : \tilde{a}_n = 0\} = N$, then $\tilde{\mathbf{a}}$ contains $k + 1$ shirking periods in its tail, contradicting the assumption that $A_{N,k+1} = \emptyset$. Moreover, if $\#\{n : \tilde{a}_n = 0\} > N + 1$, then $\beta_{T^L-k}^H(\tilde{\mathbf{a}}) > \beta_{T^L-k}^H(\hat{\mathbf{a}})$. But then since $\hat{a}_{T^L-k} = 1$, we should have $\tilde{a}_{T^L-k} = 1$, a contradiction. Therefore, it must be $\#\{n : \tilde{a}_n = 0\} = N + 1$. \parallel

By Claim 3, we can choose $\tilde{\mathbf{a}}$ such that $\tilde{\mathbf{a}}$ differs from $\hat{\mathbf{a}}$ only in period $T^L - k$. This is shown in the following table:

period:	\hat{t}	$\hat{t} + 1$	\dots	t	$t + 1$	\dots	$T^L - k - 1$	$T^L - k$	\dots	$\tau - 1$	τ	\dots
$\hat{\mathbf{a}}$:	0	1	\dots	1	1	\dots	1	1	\dots	0	0	\dots
$\tilde{\mathbf{a}}$:	0	1	\dots	1	1	\dots	1	0	\dots	0	0	\dots

But by assumption $l_t^L > 0$, and by the induction hypothesis $l_s^L = 0$ for $s = t + 1, \dots, T^L - k - 1$. Therefore,

$$\sum_{s=t}^{T^L-k-1} l_s^L (1 - \lambda^H)^{\sum_{n=t+1}^s \tilde{a}_n} > 0.$$

Applying part (1) of [Lemma 3](#), we must conclude that $\tilde{a}_{T^L-k} = 1$, a contradiction. Q.E.D.

Lemma 5. Suppose $\bigcup_{m=k+1}^N A_{N,m} = \emptyset$. Then $l_t^L = 0$.

Proof. Step 1: $l_t^L \geq 0$.

Proof of Step 1: Suppose to the contrary that $l_t^L < 0$. Then by [Lemma 4](#),

$$\sum_{s=\hat{t}}^{T^L-k-1} l_s^L (1 - \lambda^H)^{\sum_{n=\hat{t}+1}^s \hat{a}_n} < 0.$$

Then part (2) of [Lemma 3](#) implies that $\hat{a}_{T^L-k} = 0$, a contradiction.

Step 2: $l_t^L \leq 0$.

Proof of Step 2: Suppose to the contrary that $l_t^L > 0$. Note that by [Lemma 1](#), there exists an action plan $\mathbf{a}' \in \alpha^H(\mathbf{C}^L)$ such that $a'_t = 1$. Then since, by [Lemma 4](#), $l_t^L = 0$ for $t = \hat{t} + 1, \dots, T^L - k - 1$, it follows from part (1) of [Lemma 3](#) that $a'_s = 1$ for $s = \hat{t} + 1, \dots, T^L - k$. Hence, we obtain the following table:

period:	\hat{t}	$\hat{t} + 1$	\dots	$T^L - k - 1$	$T^L - k$	$T^L - k + 1$	\dots
$\hat{\mathbf{a}}$:	0	1	\dots	1	1	0	\dots
\mathbf{a}' :	1	1	\dots	1	1		

Claim: there exists $\tilde{t} < \hat{t}$, such that $a'_{\tilde{t}} = 0$ and $\hat{a}_{\tilde{t}} = 1$.

Proof: since $\#\{t : a'_t = 0\} \geq N = \#\{t : \hat{a}_t = 0\}$, $a'_{\hat{t}} = 1$, $\hat{a}_{\hat{t}} = 0$, and $\hat{a}_t = 0$ for all $t > T^L - k$, we have $\#\{t : a'_t = 0, t < \hat{t}\} > \#\{t : \hat{a}_t = 0, t < \hat{t}\}$. The claim follows immediately.

We can take \tilde{t} to be the largest period that satisfies the above claim. Hence $\hat{\mathbf{a}}$ and \mathbf{a}' are as follows:

period:	\tilde{t}	\dots	\hat{t}	$\hat{t} + 1$	\dots	$T^L - k - 1$	$T^L - k$	$T^L - k + 1$	\dots
$\hat{\mathbf{a}}:$	1		0	1	\dots	1	1	0	\dots
$\mathbf{a}':$	0		1	1	\dots	1	1		

There are two cases to consider.

Case 1: $\hat{a}_t = a'_t$ for each $t = \tilde{t} + 1, \dots, \hat{t} - 1$.

Lemma 3 implies that $\sum_{s=\tilde{t}}^{\hat{t}-1} l_s^L (1 - \lambda^H)^{\sum_{n=\tilde{t}+1}^s \hat{a}_n} = 0$ and the agent is indifferent between $\hat{\mathbf{a}}$ and $\hat{\mathbf{a}}'$ where $\hat{\mathbf{a}}'$ differs from $\hat{\mathbf{a}}$ only in that the actions at periods \tilde{t} and \hat{t} are switched. But this contradicts the definition of \hat{t} (see (D.3)).

Case 2: $\hat{a}_m = 0$ and $a'_m = 1$ for some $m \in \{\tilde{t} + 1, \dots, \hat{t} - 1\}$.

First note that Case 1 and Case 2 are exhaustive because \tilde{t} is taken to be the largest period $t < \hat{t}$ such that $\hat{a}_t = 1$ and $a'_t = 0$. Without loss, we take m to be the smallest possible. Hence $\hat{a}_t = a'_t$ for each $t = \tilde{t} + 1, \dots, m - 1$. Then $\hat{\mathbf{a}}$ and \mathbf{a}' are as follows:

period:	\tilde{t}	\dots	m	\dots	\hat{t}	$\hat{t} + 1$	\dots	$T^L - k - 1$	$T^L - k$	$T^L - k + 1$	\dots
$\hat{\mathbf{a}}:$	1		0		0	1	\dots	1	1	0	\dots
$\mathbf{a}':$	0		1		1	1	\dots	1	1		

But again, by **Lemma 3**, we can switch the actions at periods \tilde{t} and m in $\hat{\mathbf{a}}$, contradicting the definition of $\hat{\mathbf{a}}$ (see (D.4)).

The proof of Step 2 is therefore complete. Q.E.D.

Lemma 6. $\bigcup_{n=k+1}^N A_{N,n} \neq \emptyset$.

Proof. Suppose to the contrary that $\bigcup_{n=k+1}^N A_{N,n} = \emptyset$. Then $l_t^L = 0$ for $t = \hat{t}, \dots, T^L - k - 1$, by **Lemma 4** and **Lemma 5**. Therefore, by part (3) of **Lemma 3**, we can switch $\hat{a}_{\hat{t}}$ with \hat{a}_{T^L-k} to obtain $\hat{\mathbf{a}}'$. However, since $\#\{t : \hat{a}'_t = 0\} = \#\{t : \hat{a}_t = 0\} = N$, it follows immediately that $\hat{\mathbf{a}}' \in A_N$. Since $\hat{a}'_t = 0$ for all $t > T^L - k - 1$, $\hat{\mathbf{a}}' \in \bigcup_{n=k+1}^N A_{N,n}$. Q.E.D.

D.6 Step 6: Onetime-clawback contracts for the low type

In Step 5, we showed that we can restrict attention in solving program [RP2] to connected clawback contracts for the low type $\mathbf{C}^L = (T^L, W_0^L, l^L)$ such that there is an optimal action plan for the high type $\mathbf{a} \in \alpha^H(\mathbf{C}^L)$ that is a stopping strategy with the most work property. We now use this result to show that we can further restrict attention to onetime-clawback contracts for the low type, $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$.

This result is proved via two lemmas.

Lemma 7. Let $\mathbf{C}^L = (T^L, W_0^L, \mathbf{l}^L)$ be an optimal contract for the low type with a most-work optimal stopping strategy for the high type $\hat{\mathbf{a}}$ that stops at \hat{t} , i.e. $\hat{t} = \max\{t \in \{1, \dots, T^L\} : \hat{a}_t = 1\}$. For each $t > \hat{t}$, there is an optimal action plan, $\tilde{\mathbf{a}} \in \alpha^H(\mathbf{C}^L)$, such that for any s , $\hat{a}_s = \tilde{a}_s \iff s \notin \{\hat{t}, t\}$.

Proof. Step 1: First, we show that the Lemma's claim is true for some $t > \hat{t}$ (rather than for all $t > \hat{t}$). Suppose not, to contradiction. Then Lemma 3 implies that

$$\text{for any } n \in \{\hat{t}, \hat{t} + 1, \dots, T^L - 1\}, \sum_{s=\hat{t}}^n l_s^L < 0. \quad (\text{D.7})$$

Hence, (\mathbf{IC}_a^L) is slack at \hat{t} (since it is satisfied in the next period and $l_{\hat{t}}^L < 0$) and, by Lemma 2, there exists an optimal action plan, \mathbf{a}'' , with $a_{\hat{t}}'' = 0$.

Claim 1: $a_s'' = 0$ for all $s > \hat{t}$.

Proof. Suppose to contradiction that there exists $\tau > \hat{t}$ such that $a_\tau'' = 1$. Take the smallest such τ . Then it follows from Lemma 3 applied to $\hat{\mathbf{a}}$ and \mathbf{a}'' that $\sum_{s=\hat{t}}^{\tau-1} l_s^L = 0$, contradicting (D.7). \parallel

Hence, we obtain that $a_s'' = 0$ for all $s \geq \hat{t}$, and it follows from the optimality of $\hat{a}_{\hat{t}} = 1$ and $a_{\hat{t}}'' = 0$ that \mathbf{a}'' is a stopping strategy that stops at $\hat{t} - 1$:

period:	\dots	$\hat{t} - 2$	$\hat{t} - 1$	\hat{t}	$\hat{t} + 1$	$\hat{t} + 2$	\dots
$\hat{\mathbf{a}}$:	\dots	1	1	1	0	0	\dots
\mathbf{a}'' :	\dots	1	1	0	0	0	\dots

Next, note that by Lemma 1, there is an optimal action plan, \mathbf{a}' , with $a'_{T^L} = 1$.

Claim 2: $a'_{\hat{t}} = 1$.

Proof. Suppose to contradiction that $a'_{\hat{t}} = 0$. Then by (D.7) and Lemma 3, $a'_{\hat{t}+1} = 0$. But then again by (D.7) and Lemma 3, $a'_{\hat{t}+2} = 0$, and using induction we arrive at the conclusion that $a'_{T^L} = 0$. Contradiction. \parallel

Since $a'_{T^L} = 1$ and $a'_{\hat{t}} = 1$, by the most work property of $\hat{\mathbf{a}}$, there must exist a period $m < \hat{t}$ such that $a'_m = 0$. Take the largest such period:

period:	\dots	m	$m + 1$	\dots	$\hat{t} - 1$	\hat{t}	$\hat{t} + 1$	$\hat{t} + 2$	\dots	T^L
$\hat{\mathbf{a}}$:	\dots	1	1	\dots	1	1	0	0	\dots	0
\mathbf{a}'' :	\dots	1	1	\dots	1	0	0	0	\dots	0
\mathbf{a}' :		0	1	\dots	1	1				1

Applying Lemma 3 to \mathbf{a}'' and \mathbf{a}' yields $\sum_{s=m}^{\hat{t}-1} l_s^L (1 - \lambda^H)^{\sum_{n=m+1}^s a'_n} = 0$. Hence, there exists an optimal action plan \mathbf{a}''' obtained from \mathbf{a}' by switching a'_m and $a'_{\hat{t}}$. But then the optimality of \mathbf{a}''' contradicts $a'''_{\hat{t}} = 0$, $a'''_{T^L} = 1$, (D.7), and Lemma 3.

Step 2: We now prove the Lemma's claim for $\hat{t} + 1$. That is, we show that there exists an optimal action plan, call it $\hat{\mathbf{a}}^{\hat{t}+1}$, such that for any s , $\hat{a}_s^{\hat{t}+1} = \hat{a}_s \iff s \notin \{\hat{t}, \hat{t} + 1\}$. Suppose, to contradiction, that

the claim is false. Then, by Lemma 3, $l_t^L < 0$. Using Step 1, there is some $\tau > \hat{t}$ that satisfies the Lemma's claim; let \mathbf{a}^τ be the corresponding optimal action plan (which is identical to $\hat{\mathbf{a}}$ in exactly all periods except from \hat{t} and τ). Since by Lemma 1 there exists an optimal action plan, call it \mathbf{a}' , with $a'_{\hat{t}+1} = 1$, Lemma 3 and $l_t^L < 0$ imply $a'_t = 1$. By the most work property of $\hat{\mathbf{a}}$, there must exist a period $m < \hat{t}$ such that $a'_m = 0$. Take the largest such period:

period:	...	m	$m+1$...	$\hat{t}-1$	\hat{t}	$\hat{t}+1$...	τ	$\tau+1$...
$\hat{\mathbf{a}}$:	...	1	1	...	1	1	0	...	0	0	...
\mathbf{a}^τ :	...	1	1	...	1	0	0	...	1	0	...
\mathbf{a}' :		0	1	...	1	1	1				

Applying Lemma 3 to \mathbf{a}^τ and \mathbf{a}' yields $\sum_{s=m}^{\hat{t}-1} l_s^L (1 - \lambda^H)^{\sum_{n=m+1}^s a'_n} = 0$. Hence, there exists an optimal action plan \mathbf{a}'' obtained from \mathbf{a}' by switching a'_m and a'_t . But then the optimality of \mathbf{a}'' contradicts $a''_t = 0$, $a''_{t+1} = 1$, $l_t^L < 0$, and Lemma 3.

Step 3: Finally, we use induction to prove that the Lemma's claim is true for any $s > \hat{t} + 1$. (Note the claim is true for $\hat{t} + 1$ by Step 2.) Take any $t + 1 \in \{\hat{t} + 2, \dots, T^L\}$. Assume the claim is true for $s = \hat{t} + 2, \dots, t$. We show that the claim is true for $t + 1$.

By Step 2 and the induction hypothesis, there exists an optimal action plan, $\hat{\mathbf{a}}^t$, such that for any s , $\hat{a}_s^t = \hat{a}_s \iff s \notin \{\hat{t}, t\}$. We shall show that there exists an optimal action plan, $\hat{\mathbf{a}}^{t+1}$, such that for any s , $\hat{a}_s^{t+1} = \hat{a}_s \iff s \notin \{\hat{t}, t+1\}$. Suppose, to contradiction, that the claim is false. Note that Step 2, the induction hypothesis, and Lemma 3 imply $l_s^L = 0$ for all $s = \hat{t}, \dots, t-1$. It thus follows from Lemma 3 and the claim being false that $l_t^L < 0$. By Lemma 1 there exists an optimal action plan, call it \mathbf{a}' , with $a'_{t+1} = 1$. Then Lemma 3 and $l_t^L < 0$ imply that $a'_t = 1$.

Claim 3: $a'_s = 1$ for all $s = \hat{t}, \dots, t-1$.

Proof: Suppose to contradiction that $a'_{s^*} = 0$ for some $s^* \in \{\hat{t}, \dots, t-1\}$. Then since $l_s^L = 0$ for all $s = \hat{t}, \dots, t-1$, by Lemma 3, there exists an optimal action plan, \mathbf{a}'' , obtained from \mathbf{a}' by switching a'_{s^*} and a'_t . But then the optimality of \mathbf{a}'' contradicts $a''_t = 0$, $a''_{t+1} = 1$, $l_t^L < 0$, and Lemma 3. ||

Hence, we obtain $a'_s = 1$ for all $s = \hat{t}, \dots, t+1$, and by the most work property of $\hat{\mathbf{a}}$, there must exist a period $m < \hat{t}$ such that $a'_m = 0$. Take the largest such period:

period:	...	m	$m+1$...	$\hat{t}-1$	\hat{t}	$\hat{t}+1$...	t	$t+1$...
$\hat{\mathbf{a}}$:	...	1	1	...	1	1	0	...	0	0	...
$\hat{\mathbf{a}}^t$:	...	1	1	...	1	0	0	...	1	0	...
\mathbf{a}' :	...	0	1	...	1	1	1	...	1	1	

Applying Lemma 3 to $\hat{\mathbf{a}}^t$ and \mathbf{a}' yields $\sum_{s=m}^{\hat{t}-1} l_s^L (1 - \lambda^H)^{\sum_{n=m+1}^s a'_n} = 0$. Since $l_s^L = 0$ for all $s = \hat{t}, \dots, t-1$, we obtain $\sum_{s=m}^{t-1} l_s^L (1 - \lambda^H)^{\sum_{n=m+1}^s a'_n} = 0$. Hence, by Lemma 3, there exists an optimal action plan \mathbf{a}'' obtained from \mathbf{a}' by switching a'_m and a'_t . But then the optimality of \mathbf{a}'' contradicts $a''_t = 0$, $a''_{t+1} = 1$, $l_t^L < 0$, and Lemma 3. Q.E.D.

Lemma 8. *If \mathbf{C}^L is an optimal contract for the low type with a most-work optimal stopping strategy for the high type, then \mathbf{C}^L is a onetime-clawback contract.*

Proof. Fix \mathbf{C}^L per the Lemma's assumptions. Let $\hat{\mathbf{a}}$ and \hat{t} be as defined in the statement of Lemma 7. Then, it immediately follows from Lemma 7 and Lemma 3 that $l_t^L = 0$ for all $t \in \{\hat{t}, \hat{t} + 1, \dots, T^L - 1\}$. We use induction to prove that $l_t^L = 0$ for all $t < \hat{t}$.

Assume $l_t^L = 0$ for all $t \in \{m+1, m+2, \dots, T^L - 1\}$ for $m < \hat{t}$. We will show that $l_m^L = 0$. First, $l_m^L > 0$ is not possible because then $\sum_{s=m}^{\hat{t}} l_s^L (1 - \lambda^H)^{\sum_{n=m+1}^s \hat{a}_n} > 0$ (by Lemma 7 and the inductive assumption), contradicting the optimality of $\hat{\mathbf{a}}$ and Lemma 3. Second, we claim $l_m^L < 0$ is not possible. Suppose, to contradiction, that $l_m^L < 0$. Then (\mathbf{IC}_a^L) is slack at m and, by Lemma 2, there exists an optimal plan \mathbf{a}' with $a'_m = 0$. Now by Lemma 3, Lemma 7, and the inductive assumption, $a'_s = 0$ for all $s \geq m$. Hence, $\beta_{\hat{t}}^H(\mathbf{a}') > \beta_{\hat{t}}^H(\hat{\mathbf{a}})$, and thus the optimality of $\hat{\mathbf{a}}$ implies that \mathbf{a}' is suboptimal at \hat{t} , a contradiction. *Q.E.D.*

D.7 Step 7: The optimal penalty in a onetime-clawback contract

By the previous steps, we restrict attention to onetime-clawback contracts for the low type such that the low type works in all periods $t \in \{1, \dots, T^L\}$ and the high type has a most-work optimal stopping strategy. For an arbitrary such contract \mathbf{C}^L , let $\hat{t}(\mathbf{C}^L)$ denote the high type's most-work optimal stopping time, i.e. $\hat{t}(\mathbf{C}^L) := \max\{t \in \{1, \dots, T^L\} : \hat{a}_s = 1 \text{ for all } s = 1, \dots, t, \hat{\mathbf{a}} \in \alpha^H(\mathbf{C}^L)\}$. We now show that given T^L , there exists an optimal onetime-clawback contract for the low type $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$ where $\hat{t}(\mathbf{C}^L)$ is given by

$$T^{HL}(T^L) := \min \left\{ t \in \{1, \dots, T^L\} : \bar{\beta}_{t+1}^H \lambda^H < \bar{\beta}_{T^L}^L \lambda^L \text{ and } (1 - \lambda^H)^t \leq (1 - \lambda^L)^{T^L} \right\},$$

and $l_{T^L}^L$ is given by

$$\bar{l}_{T^L}^L(T^L) := \min \left\{ -\frac{c}{\bar{\beta}_{T^L}^L \lambda^L}, -\frac{c}{\bar{\beta}_{T^{HL}(T^L)}^H \lambda^H} \right\}.$$

When not essential, we suppress the dependence of $\hat{t}(\mathbf{C}^L)$ on \mathbf{C}^L . We proceed by proving five claims.

Claim 1: Given any onetime-clawback contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$, $-\bar{\beta}_{\hat{t}+1}^H \lambda^H l_{T^L}^L < c$.

Proof: Suppose to contradiction that $-\bar{\beta}_{\hat{t}+1}^H \lambda^H l_{T^L}^L \geq c$. Then type H is willing to work one more period after having worked for \hat{t} periods, contradicting the definition of \hat{t} . \parallel

Claim 2: Given an optimal onetime-clawback contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$, $(1 - \lambda^H)^{\hat{t}} \leq (1 - \lambda^L)^{T^L}$.

Proof: Suppose to contradiction that given an optimal contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$, type H 's most-work optimal stopping time \hat{t} is such that $(1 - \lambda^H)^{\hat{t}} > (1 - \lambda^L)^{T^L}$. Then for any strategy $\tilde{\mathbf{a}} \in \alpha^H(\mathbf{C}^L)$ where type H works for a total of \tilde{t} periods, $(1 - \lambda^H)^{\tilde{t}} > (1 - \lambda^L)^{T^L}$. Now note that given \mathbf{C}^L and $\tilde{\mathbf{a}}$, type

H 's information rent is

$$\mu_0 \left\{ \begin{aligned} & \beta_0 l_{T^L}^L \left[(1 - \lambda^H)^{\tilde{t}} - (1 - \lambda^L)^{T^L} \right] \\ & - \beta_0 c \sum_{t=1}^{T^L} \tilde{a}_t \left[\prod_{s=1}^{t-1} (1 - \tilde{a}_s \lambda^H) - (1 - \lambda^L)^{t-1} \right] + c \sum_{t=1}^{T^L} (1 - \tilde{a}_t) \left[(1 - \beta_0) + \beta_0 (1 - \lambda^L)^{t-1} \right] \end{aligned} \right\}.$$

Consider a modification that reduces $l_{T^L}^L$ by $\varepsilon > 0$. By Claim 1, for ε small enough, this modification does not affect incentives, and by $(1 - \lambda^H)^{\tilde{t}} > (1 - \lambda^L)^{T^L}$, the modification strictly reduces type H 's information rent. But then \mathbf{C}^L cannot be optimal. \parallel

Claim 3: In any onetime-clawback contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$, if $\mathbf{1} \in \alpha^L(\mathbf{C}^L)$ then

$$l_{T^L}^L \leq \min \left\{ -\frac{c}{\bar{\beta}_{T^L}^L \lambda^L}, -\frac{c}{\bar{\beta}_{\hat{t}(\mathbf{C}^L)}^H \lambda^H} \right\}. \quad (\text{D.8})$$

Conversely, given any onetime clawback contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$, if $l_{T^L}^L \leq \min \left\{ -\frac{c}{\bar{\beta}_{T^L}^L \lambda^L}, -\frac{c}{\bar{\beta}_{\hat{t}}^H \lambda^H} \right\}$ for some $t \leq T^L$, then $\hat{t}(\mathbf{C}^L) \geq t$ and $\mathbf{1} \in \alpha^L(\mathbf{C}^L)$.

Proof. For the first part of the claim, assume to contradiction that there is $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$ such that $\mathbf{1} \in \alpha^L(\mathbf{C}^L)$ but (D.8) does not hold. Suppose first that $-\frac{c}{\bar{\beta}_{T^L}^L \lambda^L} \leq -\frac{c}{\bar{\beta}_{\hat{t}}^H \lambda^H}$. Then type L is not willing to work for T^L periods; having worked for $T^L - 1$ periods, type L 's incentive compatibility constraint for effort in period T^L is $-\bar{\beta}_{T^L}^L \lambda^L l_{T^L}^L \geq c$, which is not satisfied with $l_{T^L}^L > -\frac{c}{\bar{\beta}_{T^L}^L \lambda^L}$. Suppose next that $-\frac{c}{\bar{\beta}_{T^L}^L \lambda^L} > -\frac{c}{\bar{\beta}_{\hat{t}}^H \lambda^H}$. Then type H is not willing to work for \hat{t} periods; having worked for $\hat{t} - 1$ periods, type H is willing to work one more period only if $-\bar{\beta}_{\hat{t}}^H \lambda^H l_{T^L}^L \geq c$, which is not satisfied with $l_{T^L}^L > -\frac{c}{\bar{\beta}_{\hat{t}}^H \lambda^H}$.

For the second part of the claim, assume $l_{T^L}^L \leq \min \left\{ -\frac{c}{\bar{\beta}_{T^L}^L \lambda^L}, -\frac{c}{\bar{\beta}_{\hat{t}}^H \lambda^H} \right\}$. Consider first type L . The proof is by induction. Consider the last period, T^L . Since no matter the history of effort the current belief is some $\beta_{T^L}^L \geq \bar{\beta}_{T^L}^L$, it is immediate that $-\beta_{T^L}^L \lambda^L l_{T^L}^L \geq c$, and thus it is optimal for type L to work in the last period. Now assume inductively that it is optimal for type L to work in period $t+1 \leq T^L$ no matter the history of effort, and consider period t with belief β_t^L . The inductive hypothesis implies that

$$-\beta_{t+1}^L \lambda^L \left\{ l_{T^L}^L (1 - \lambda^L)^{T^L - (t+1)} - c \sum_{s=t+2}^{T^L} (1 - \lambda^L)^{s - (t+2)} \right\} \geq c. \quad (\text{D.9})$$

Therefore, at period t :

$$\begin{aligned} & -\beta_t^L \lambda^L \left\{ -c + (1 - \lambda^L) \left[l_{T^L}^L (1 - \lambda^L)^{T^L - (t+1)} - c \sum_{s=t+2}^{T^L} (1 - \lambda^L)^{s - (t+2)} \right] \right\} \\ & \geq -\beta_t^L \lambda^L \left[-c + (1 - \lambda^L) \left(-\frac{c}{\beta_{t+1}^L \lambda^L} \right) \right] = c, \end{aligned}$$

where the inequality uses (D.9) and the equality uses $\beta_{t+1}^L = \frac{\beta_t^L(1-\lambda^L)}{1-\beta_t^L+\beta_t^L(1-\lambda^L)}$.

Finally, consider type H . By Lemma 3 and the fact that $l_t^L = 0$ for all $t = 1, \dots, T^L - 1$, type H is indifferent between any two action plans \mathbf{a} and \mathbf{a}' such that $\#\{t : a_t = 0\} = \#\{t : a'_t = 0\}$. Thus, without loss, we restrict attention to stopping strategies, and we only need to show that it is optimal for type H to stop at $s \geq t$. Note that for any $s < t$, given that type H has worked consecutively until and including period s , $-\bar{\beta}_{s+1}^H \lambda^H l_{T^L}^L \geq c$, and thus type H does not want to stop at s . \parallel

Claim 4: There exists an optimal onetime-clawback contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$ satisfying

$$l_{T^L}^L \geq \min \left\{ -\frac{c}{\bar{\beta}_{T^L}^L \lambda^L}, -\frac{c}{\bar{\beta}_t^H \lambda^H} \right\}.$$

Proof: Suppose to contradiction that the claim is false. Given an optimal onetime-clawback contract for type L , $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$, and type H 's most-work optimal stopping strategy $\hat{\mathbf{a}}$, type H 's information rent is

$$\mu_0 \left\{ \begin{aligned} & \beta_0 l_{T^L}^L [(1-\lambda^H)^{\hat{t}} - (1-\lambda^L)^{T^L}] \\ & - \beta_0 c \sum_{t=1}^{\hat{t}} [(1-\lambda^H)^{t-1} - (1-\lambda^L)^{t-1}] + c \sum_{t=\hat{t}+1}^{T^L} [(1-\beta_0) + \beta_0 (1-\lambda^L)^{t-1}] \end{aligned} \right\}.$$

Consider a modification that increases $l_{T^L}^L$ by $\varepsilon > 0$. By Claim 4 being false and Claim 3, for ε small enough, working in all periods $t = 1, \dots, T^L$ remains optimal for type L , and $\hat{\mathbf{a}}$ remains optimal for type H . But then by Claim 2, type H 's information rent either goes down or remains unchanged with the modification, and thus there exists an optimal contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$ where the claim is true. \parallel

Claim 5: There is an optimal onetime-clawback contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$ with $\hat{t}(\mathbf{C}^L) = T^{HL}(T^L)$.

Proof: Take an arbitrary optimal contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$. By Claims 1 and 5, $\hat{t}(\mathbf{C}^L)$ satisfies $\bar{\beta}_{\hat{t}(\mathbf{C}^L)+1}^H \lambda^H < \bar{\beta}_{T^L}^L \lambda^L$. By Claim 2, $\hat{t}(\mathbf{C}^L)$ satisfies $(1-\lambda^H)^{\hat{t}(\mathbf{C}^L)} \leq (1-\lambda^L)^{T^L}$. Thus, all that remains to be shown is that there exists \mathbf{C}^L where $\hat{t}(\mathbf{C}^L)$ is the smallest period $t \in \{1, \dots, T^L\}$ that satisfies these two conditions. Suppose to contradiction that this claim is false. Then $\hat{t}(\mathbf{C}^L) - 1$ also satisfies the conditions; that is, $\bar{\beta}_{\hat{t}(\mathbf{C}^L)}^H \lambda^H < \bar{\beta}_{T^L}^L \lambda^L$ and $(1-\lambda^H)^{\hat{t}(\mathbf{C}^L)-1} \leq (1-\lambda^L)^{T^L}$. By Claims 4 and 5, $l_{T^L}^L = \min \left\{ -\frac{c}{\bar{\beta}_{T^L}^L \lambda^L}, -\frac{c}{\bar{\beta}_{\hat{t}(\mathbf{C}^L)}^H \lambda^H} \right\}$, and thus since $\bar{\beta}_{\hat{t}(\mathbf{C}^L)}^H \lambda^H < \bar{\beta}_{T^L}^L \lambda^L$, $l_{T^L}^L = -\frac{c}{\bar{\beta}_{\hat{t}(\mathbf{C}^L)}^H \lambda^H} < -\frac{c}{\bar{\beta}_{T^L}^L \lambda^L}$. It follows that type H 's incentive constraint in period $\hat{t}(\mathbf{C}^L)$ binds; i.e., type H is indifferent between working and shirking at $\hat{t}(\mathbf{C}^L)$ given that he has worked in all periods $t = 1, \dots, \hat{t}(\mathbf{C}^L) - 1$ and will shirk in all periods $t = \hat{t}(\mathbf{C}^L) + 1, \dots, T^L$. Hence, both a stopping strategy that stops at $\hat{t}(\mathbf{C}^L)$ and a stopping strategy that stops at $\hat{t}(\mathbf{C}^L) - 1$ are optimal for type H given \mathbf{C}^L , and type H 's information rent is the same for either of these two action plans (see Step 3). Type H 's information rent can thus be written as

$$\mu_0 \left\{ \begin{aligned} & \beta_0 l_{T^L}^L [(1-\lambda^H)^{\hat{t}(\mathbf{C}^L)-1} - (1-\lambda^L)^{T^L}] \\ & - \beta_0 c \sum_{t=1}^{\hat{t}(\mathbf{C}^L)-1} [(1-\lambda^H)^{t-1} - (1-\lambda^L)^{t-1}] + c \sum_{t=\hat{t}(\mathbf{C}^L)}^{T^L} [(1-\beta_0) + \beta_0 (1-\lambda^L)^{t-1}] \end{aligned} \right\}.$$

Now consider a modified contract, $\hat{\mathbf{C}}^L$, obtained from \mathbf{C}^L by increasing $l_{T^L}^L$ by $\varepsilon > 0$. Since $l_{T^L}^L = -\frac{c}{\bar{\beta}_{\hat{t}(\mathbf{C}^L)}^H \lambda^H}$,

a stopping strategy that stops at $\hat{t}(\mathbf{C}^L)$ is no longer optimal for type H under $\hat{\mathbf{C}}^L$. Since $l_{T^L}^L < -\frac{c}{\beta_{T^L}^L \lambda^L}$ and $l_{T^L}^L < -\frac{c}{\beta_{\hat{t}(\mathbf{C}^L)-1}^H \lambda^H}$, for ε small enough, $\mathbf{1} \in \alpha^L(\hat{\mathbf{C}}^L)$ and a stopping strategy that stops at $\hat{t}(\mathbf{C}^L) - 1$ remains optimal for type H under $\hat{\mathbf{C}}^L$. Then $\hat{t}(\hat{\mathbf{C}}^L) = \hat{t}(\mathbf{C}^L) - 1$, and since $(1 - \lambda^H)^{\hat{t}(\mathbf{C}^L)-1} \leq (1 - \lambda^L)^{T^L}$, type H 's information rent either goes down or remains unchanged with the modification, so $\hat{\mathbf{C}}^L$ is optimal. If $\hat{t}(\hat{\mathbf{C}}^L) = T^{HL}(T^L)$, we are done. Otherwise, we can apply the argument to $\hat{t}(\hat{\mathbf{C}}^L)$ and repeat until we eventually arrive at the desired contract \mathbf{C}^L with $\hat{t}(\mathbf{C}^L) = T^{HL}$. \parallel

D.8 Step 8: Under-experimentation by the low type

By Step 7, an optimal contract for the low type that solves program [RP2] can be found by optimizing over T^L , i.e. the length of the onetime-clawback contracts with optimal stopping time for the high type $T^{HL}(T^L)$ and last period penalty $\bar{l}_{T^L}^L(T^L)$. We now argue that the optimal length is no larger than the first-best stopping time, t^L .

The portion of the objective (RP2) that involves T^L is

$$(1 - \mu_0) \left[\beta_0 \sum_{t=1}^{T^L} (1 - \lambda^L)^{t-1} (\lambda^L - c) - (1 - \beta_0) \sum_{t=1}^{T^L} c \right] - \mu_0 \left\{ \begin{array}{l} \beta_0 \bar{l}_{T^L}^L(T^L) [(1 - \lambda^H)^{T^{HL}(T^L)} - (1 - \lambda^L)^{T^L}] \\ - \beta_0 c \sum_{t=1}^{T^{HL}(T^L)} [(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1}] \\ + c \sum_{t=T^{HL}(T^L)+1}^{T^L} [(1 - \beta_0) + \beta_0 (1 - \lambda^L)^{t-1}] \end{array} \right\},$$

where we have used the desired stopping time for type H and the desired last-period penalty. Now consider the following definition:

$$\begin{aligned} \Pi(z, T^L) = & z(-\mu_0) \left\{ \begin{array}{l} \beta_0 \bar{l}_{T^L}^L(T^L) [(1 - \lambda^H)^{T^{HL}(T^L)} - (1 - \lambda^L)^{T^L}] \\ - \beta_0 c \sum_{t=1}^{T^{HL}(T^L)} [(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1}] \\ + c \sum_{t=T^{HL}(T^L)+1}^{T^L} [(1 - \beta_0) + \beta_0 (1 - \lambda^L)^{t-1}] \end{array} \right\} \\ & + (1 - \mu_0) \left[\beta_0 \sum_{t=1}^{T^L} (1 - \lambda^L)^{t-1} (\lambda^L - c) - (1 - \beta_0) \sum_{t=1}^{T^L} c \right]. \end{aligned}$$

If $z = 0$, the expression above corresponds to surplus maximization; if $z = 1$, the expression corresponds to the principal's optimization problem. Consider type H 's information rent modulo a negative constant:

$$\begin{aligned} K(T^L) = & \beta_0 \bar{l}_{T^L}^L(T^L) [(1 - \lambda^H)^{T^{HL}(T^L)} - (1 - \lambda^L)^{T^L}] - \beta_0 c \sum_{t=1}^{T^{HL}(T^L)} [(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1}] \\ & + c \sum_{t=T^{HL}(T^L)+1}^{T^L} [(1 - \beta_0) + \beta_0 (1 - \lambda^L)^{t-1}] \end{aligned}$$

If $K(\cdot)$ is shown to be increasing, then $\Pi(z, T^L)$ has decreasing differences, which implies that the optimal T^L when $z = 0$ is no smaller than the optimal T^L when $z = 1$, as desired. We show that $K(\cdot)$ is indeed increasing through the following claims.

Claim 1: $T^{HL}(T+1) \in \{T^{HL}(T), T^{HL}(T) + 1\}$.

Proof: The fact that $T^{HL}(T+1) \geq T^{HL}(T)$ follows immediately from the definition of $T^{HL}(T)$. To show that $T^{HL}(T+1) \leq T^{HL}(T) + 1$, note that by the definition of $T^{HL}(T)$, $\bar{\beta}_{T^{HL}(T)+1}^H \lambda^H < \bar{\beta}_T^L \lambda^L$ and $(1 - \lambda^H)^{T^{HL}(T)} \leq (1 - \lambda^L)^T$. Thus, since $(1 - \lambda^H) < (1 - \lambda^L)$, it follows that

$$\bar{\beta}_{T^{HL}(T)+2}^H \lambda^H = \frac{\bar{\beta}_{T^{HL}(T)+1}^H \lambda^H (1 - \lambda^H)}{1 - \bar{\beta}_{T^{HL}(T)+1}^H \lambda^H} < \frac{\bar{\beta}_T^L \lambda^L (1 - \lambda^L)}{1 - \bar{\beta}_T^L \lambda^L} = \bar{\beta}_{T+1}^L \lambda^L$$

and

$$(1 - \lambda^H)^{T^{HL}(T)+1} = (1 - \lambda^H)^{T^{HL}(T)} (1 - \lambda^H) \leq (1 - \lambda^L)^T (1 - \lambda^L) = (1 - \lambda^L)^{T+1}.$$

Hence, by the definition of $T^{HL}(T)$, $T^{HL}(T) + 1 \geq T^{HL}(T+1)$. \parallel

Claim 2: $\bar{l}_{T+1}^L(T+1) \leq \bar{l}_T^L(T)$.

Proof: This follows from the definition of $\bar{l}_T^L(T)$ and the fact that $T^{HL}(T+1) \geq T^{HL}(T)$. \parallel

Claim 3: If $T^{HL}(T+1) = T^{HL}(T)$, then $\bar{l}_{T+1}^L(T+1) = -\frac{c}{\bar{\beta}_{T+1}^L \lambda^L} \leq -\frac{c}{\bar{\beta}_{T^{HL}(T)+1}^H \lambda^H} = -\frac{c}{\bar{\beta}_{T^{HL}(T)}^H \lambda^H}$.

Proof: Suppose to contradiction that $\bar{l}_{T+1}^L(T+1) = -\frac{c}{\bar{\beta}_{T^{HL}(T)+1}^H \lambda^H} = -\frac{c}{\bar{\beta}_{T^{HL}(T)}^H \lambda^H} < -\frac{c}{\bar{\beta}_{T+1}^L \lambda^L}$. Then $\bar{\beta}_{T^{HL}(T)}^H \lambda^H < \bar{\beta}_{T+1}^L \lambda^L$ and thus

$$\bar{\beta}_{T^{HL}(T)}^H = \frac{\beta_0 (1 - \lambda^H)^{T^{HL}(T)-1}}{\beta_0 (1 - \lambda^H)^{T^{HL}(T)-1} + 1 - \beta_0} < \frac{\beta_0 (1 - \lambda^L)^T}{\beta_0 (1 - \lambda^L)^T + 1 - \beta_0} = \bar{\beta}_{T+1}^L,$$

which implies $(1 - \lambda^H)^{T^{HL}(T)-1} < (1 - \lambda^L)^T$. But then since $\bar{\beta}_{T^{HL}(T)}^H \lambda^H < \bar{\beta}_{T+1}^L \lambda^L < \bar{\beta}_T^L \lambda^L$, we have that $T^{HL}(T) - 1$ satisfies the two conditions defining $T^{HL}(T)$, namely $\bar{\beta}_{T^{HL}(T)-1+1}^H \lambda^H < \bar{\beta}_T^L \lambda^L$ and $(1 - \lambda^H)^{T^{HL}(T)-1} \leq (1 - \lambda^L)^T$, contradicting the definition of $T^{HL}(T)$ (recall $T^{HL}(T)$ is the smallest period satisfying the two conditions). \parallel

Claim 4: If $T^{HL}(T+1) = T^{HL}(T)$, then $K(T+1) - K(T) \geq 0$.

Proof: Let $T^{HL}(T+1) = T^{HL}(T) \equiv T^{HL}$. Then

$$\begin{aligned} K(T+1) - K(T) &= \beta_0 \bar{l}_{T+1}^L(T+1) \left[(1 - \lambda^H)^{T^{HL}} - (1 - \lambda^L)^{T+1} \right] \\ &\quad - \beta_0 \bar{l}_T^L(T) \left[(1 - \lambda^H)^{T^{HL}} - (1 - \lambda^L)^T \right] + c \left[(1 - \beta_0) + \beta_0 (1 - \lambda^L)^T \right] \\ &= \beta_0 \left(\bar{l}_{T+1}^L(T+1) - \bar{l}_T^L(T) \right) \left[(1 - \lambda^H)^{T^{HL}} - (1 - \lambda^L)^T \right] \end{aligned}$$

$$\begin{aligned}
& +\beta_0\lambda^L(1-\lambda^L)^T\bar{l}_{T+1}^L(T+1)+c\left[(1-\beta_0)+\beta_0(1-\lambda^L)^T\right] \\
& = \beta_0\left(\bar{l}_{T+1}^L(T+1)-\bar{l}_T^L(T)\right)\left[(1-\lambda^H)^{T^{HL}}-(1-\lambda^L)^T\right] \\
& \geq 0,
\end{aligned}$$

where the last equality follows from Claim 3 and the last inequality follows from $(1-\lambda^H)^{T^{HL}} \leq (1-\lambda^L)^T$ and Claim 2. \parallel

Claim 5: If $T^{HL}(T+1) = T^{HL}(T) + 1$, then $K(T+1) - K(T) \geq 0$.

Proof: Assume $T^{HL}(T+1) = T^{HL}(T) + 1$. Note that by Claim 1 this is the only case that remains to be shown. We now have:

$$\begin{aligned}
K(T+1) - K(T) &= \beta_0\bar{l}_{T+1}^L(T+1)\left[(1-\lambda^H)^{T^{HL}(T)+1}-(1-\lambda^L)^{T+1}\right] \\
&\quad -\beta_0\bar{l}_T^L(T)\left[(1-\lambda^H)^{T^{HL}(T)}-(1-\lambda^L)^T\right] -\beta_0c\left[(1-\lambda^H)^{T^{HL}(T)}-(1-\lambda^L)^{T^{HL}(T)}\right] \\
&\quad -c\left[(1-\beta_0)+\beta_0(1-\lambda^L)^{T^{HL}(T)}\right]+c\left[(1-\beta_0)+\beta_0(1-\lambda^L)^T\right] \\
&= \beta_0\bar{l}_{T+1}^L(T+1)\left[(1-\lambda^H)^{T^{HL}(T)+1}-(1-\lambda^L)^{T+1}\right] \\
&\quad -\beta_0\left(\bar{l}_T^L(T)+c\right)\left[(1-\lambda^H)^{T^{HL}(T)}-(1-\lambda^L)^T\right]. \tag{D.10}
\end{aligned}$$

We consider two exhaustive cases.

Case 1: $\bar{l}_T^L(T) = -\frac{c}{\beta_T^L\lambda^L}$. Then, substituting into (D.10) and manipulating terms yields

$$\begin{aligned}
K(T+1) - K(T) &= \beta_0\bar{l}_{T+1}^L(T+1)\left[(1-\lambda^H)^{T^{HL}(T)+1}-(1-\lambda^L)^{T+1}\right] \\
&\quad +\beta_0\frac{c(1-\bar{\beta}_T^L\lambda^L)}{\bar{\beta}_T^L\lambda^L(1-\lambda^L)}\left[(1-\lambda^H)^{T^{HL}(T)}(1-\lambda^L)-(1-\lambda^L)^{T+1}\right] \\
&\geq -\beta_0\frac{c}{\bar{\beta}_{T+1}^L\lambda^L}\left[(1-\lambda^H)^{T^{HL}(T)+1}-(1-\lambda^L)^{T+1}\right] \\
&\quad +\beta_0\frac{c}{\bar{\beta}_{T+1}^L\lambda^L}\left[(1-\lambda^H)^{T^{HL}(T)}(1-\lambda^L)-(1-\lambda^L)^{T+1}\right] > 0,
\end{aligned}$$

where the first inequality follows from $\bar{l}_{T+1}^L(T+1) \leq -\frac{c}{\bar{\beta}_{T+1}^L\lambda^L}$ and $(1-\lambda^H)^{T^{HL}(T)+1} \leq (1-\lambda^L)^{T+1}$, and the last inequality follows from $(1-\lambda^H)^{T^{HL}(T)+1} < (1-\lambda^H)^{T^{HL}(T)}(1-\lambda^L)$.

Case 2: $\bar{l}_T^L(T) = -\frac{c}{\bar{\beta}_{T^{HL}(T)}^H\lambda^H}$. Then, substituting into (D.10) and manipulating terms yields

$$K(T+1) - K(T) = \beta_0\bar{l}_{T+1}^L(T+1)\left[(1-\lambda^H)^{T^{HL}(T)+1}-(1-\lambda^L)^{T+1}\right]$$

$$\begin{aligned}
& + \beta_0 \frac{c(1 - \bar{\beta}_{T^{HL}(T)}^H \lambda^H)}{\bar{\beta}_{T^{HL}(T)}^H \lambda^H (1 - \lambda^H)} \left[(1 - \lambda^H)^{T^{HL}(T)+1} - (1 - \lambda^L)^T (1 - \lambda^H) \right] \\
& \geq -\beta_0 \frac{c}{\bar{\beta}_{T^{HL}(T)+1}^H \lambda^H} \left[(1 - \lambda^H)^{T^{HL}(T)+1} - (1 - \lambda^L)^{T+1} \right] \\
& + \beta_0 \frac{c}{\bar{\beta}_{T^{HL}(T)+1}^H \lambda^H} \left[(1 - \lambda^H)^{T^{HL}(T)+1} - (1 - \lambda^L)^T (1 - \lambda^H) \right] > 0,
\end{aligned}$$

where the first inequality follows from $\bar{l}_{T+1}^L(T+1) \leq -\frac{c}{\bar{\beta}_{T^{HL}(T)+1}^H \lambda^H}$ and $(1 - \lambda^H)^{T^{HL}(T)+1} \leq (1 - \lambda^L)^{T+1}$, and the last inequality follows from $(1 - \lambda^L)^{T+1} > (1 - \lambda^L)^T (1 - \lambda^H)$. \parallel

The claims above establish that $K(\cdot)$ is indeed increasing, and hence we obtain that $T^L \leq t^L$. Note also that it is clear that there is generically a unique T^L that maximizes $\Pi(1, T^L)$; hereafter we denote this solution by \bar{t}^L and the associated optimal stopping time for type H by $t^{HL} := T^{HL}(\bar{t}^L)$.

D.9 Step 9: Back to the original program

We have shown so far that there is a solution to program [RP2] in which the low type's contract is a onetime-clawback contract of length $\bar{t}^L \leq t^L$ and in which the penalty in period \bar{t}^L is given by $\bar{l}_{\bar{t}^L}^L(\bar{t}^L)$. In terms of optimizing over the high type's contract, note that any solution must induce the high type to work in each period up to t^H and no longer: this follows from the fact that the objective in [RP2] involving the high type's contract is social surplus from the high type, and that there is clearly a sequence of (sufficiently low) penalties l^H to ensure that (IC_a^H) is satisfied.

Recall that solutions to [RP2] produce solutions to [RP1] by choosing W_0^L to make (IR^L) bind and W_0^H to make (IC^{HL}) bind, which can always be done. Accordingly, let $\bar{C}^L = (\bar{t}^L, \bar{W}_0^L, \bar{l}_{\bar{t}^L}^L(\bar{t}^L))$ be the onetime-clawback contract where \bar{W}_0^L is set to make (IR^L) bind, and consider the solutions to program [RP1] in which the low type's contract is \bar{C}^L . We will argue that some of these solutions to [RP1], namely \bar{C}^L combined with a suitable onetime-clawback contract for the high type, also solve the original program [P]. Recall that [RP1] differs from [P] in two ways: (1) it ignores (IR^H); and (2) it ignores (IC^{LH}). We address each of these constraints in order.

Step 9a: To see that any solution to [RP1] using \bar{C}^L also satisfies (IR^H), observe that

$$U_0^H(\bar{C}^L, \alpha^H(\bar{C}^L)) \geq U_0^H(\bar{C}^L, \mathbf{1}) > U_0^L(\bar{C}^L, \mathbf{1}), \quad (\text{D.11})$$

where the first inequality follows by definition of $\alpha^H(\bar{C}^L)$ and the second inequality follows from the fact that \bar{C}^L is a onetime-clawback contract with penalty $\bar{l}_{\bar{t}^L}^L < 0$. Hence, in any solution to [RP1] using \bar{C}^L ,

$$U_0^H(\mathbf{C}^H, \alpha^H(\mathbf{C}^H)) \geq U_0^H(\bar{C}^L, \alpha^H(\bar{C}^L)) \geq U_0^L(\bar{C}^L, \mathbf{1}) \geq 0,$$

where the first inequality is by (IC_a^H) and (IC^{HL}), the second by (D.11), and the last by (IR^L).

Step 9b: Finally, we show that there is a solution to [RP1] using \bar{C}^L that also satisfies (IC^{LH}) in [P], which completes the proof. We can show this by the same argument as the one used in Step 7c of the proof of Theorem 3 in Appendix D: we construct a *onetime*-clawback contract for type H , $C^H = (t^H, W_0^H, l_{t^H}^H)$, where given the penalty $l_{t^H}^H$ and that type H works in all periods under this contract, W_0^H is chosen to make (IC^{HL}) bind. We omit the details since the argument is the same.

E Details for No Learning

This appendix provides details for the discussion in Subsection 6.2 of the main text. Assume $\beta_0 = 1$ and for simplicity that there is some finite time, \bar{T} , at which the game ends. Since $\bar{\beta}_t^\theta = 1$ for all $\theta \in \{L, H\}$ and $t \in \{1, \dots, \bar{T}\}$, the high type always has a higher expected marginal product than the low type, i.e. $\bar{\beta}_t^H \lambda^H = \lambda_H > \bar{\beta}_t^L \lambda^L = \lambda_L$ for all t . Consequently, the methodology used in proving Theorem 3 can be applied, with the conclusions that if the optimal length of experimentation for the low type is some T (constrained to be no larger than \bar{T}), the optimal clawback contract for the low type is given by the analog of (6) with $\bar{\beta}_t^L = 1$ for all t :

$$l_t^L = \begin{cases} -(1 - \delta) \frac{c}{\lambda^L} & \text{if } t < T, \\ -\frac{c}{\lambda^L} & \text{if } t = T, \end{cases} \quad (\text{E.1})$$

and the portion of the principal's payoff that depends on T is given by the analog of (C.1) with the simplification of $\beta_0 = 1$:

$$\begin{aligned} \hat{V}(T) &= (1 - \mu_0) \sum_{t=1}^T \delta^t (1 - \lambda^L)^{t-1} (\lambda^L - c) \\ &\quad - \mu_0 \left\{ -\frac{c}{\lambda^L} \sum_{t=1}^{T-1} \delta^t (1 - \delta) \left[(1 - \lambda^H)^t - (1 - \lambda^L)^t \right] - \frac{c}{\lambda^L} \delta^T \left[(1 - \lambda^H)^T - (1 - \lambda^L)^T \right] \right. \\ &\quad \left. - \sum_{t=1}^T \delta^t c \left[(1 - \lambda^H)^{t-1} - (1 - \lambda^L)^{t-1} \right] \right\}. \end{aligned}$$

Hence, for any $T \in \{0, \dots, \bar{T} - 1\}$ we have the following analog of (C.2):

$$\hat{V}(T+1) - \hat{V}(T) = \delta^{T+1} \left[(1 - \mu_0) (1 - \lambda^L)^T (\lambda^L - c) - \mu_0 \frac{c}{\lambda^L} (1 - \lambda^H)^T (\lambda^H - \lambda^L) \right].$$

Clearly, $\hat{V}(T+1) - \hat{V}(T) > (<) 0$ if and only if

$$\left(\frac{1 - \lambda^L}{1 - \lambda^H} \right)^T > (<) \frac{\mu_0 c (\lambda^H - \lambda^L)}{(1 - \mu_0) (\lambda^L - c) \lambda^L}.$$

Since the left-hand side above is strictly increasing in T , it follows that $V(T)$ is maximized by $\bar{t}^L \in \{0, \bar{T}\}$. Hence, whenever it is optimal to have the low type experiment for any positive amount of time, it is optimal to have the low type experiment until \bar{T} , no matter the value of \bar{T} . Note that whenever exclusion is optimal (i.e. $\bar{t}^L = 0$) when $\beta_0 = 1$, it would also be optimal for all $\beta_0 \leq 1$; this follows from the comparative static of \bar{t}^L with respect to β_0 in Proposition 2.

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