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The Cross Nests Flexible Substitution Logit:
Uncovering Category Expansion and Share Impacts of Marketing Instruments

Abstract

Different instruments are relevant for different marketing objectives (category demand expansion or market share stealing). To help brand managers make informed marketing mix decisions, it is essential that marketing mix models appropriately measure the different effects of marketing instruments. Randomly Utility Maximizing (RUM) discrete choice models that have been applied to this problem might not be adequate because they possess the Invariant Proportion of Substitution (IPS) property, which in some situations imposes counter-intuitive restrictions on individual choice behavior. We discuss the theoretical roots of the IPS property, namely, the weak complementarity assumption that underlies RUM models. With the recognition that a set of goods, e.g., goods in a category, can be weak complementary to some common attributes, e.g., total expenditure of marketing instruments in that category, we then derive an alternative choice model specification that relaxes the IPS property at an appropriate level– the “cross nests flexible substitution” logit (CNFSL) model. Our empirical application to prescription writing choices of physicians in the hyperlipidemia category shows that the random coefficient CNFSL model predicts that sales gains from Direct-to-Consumer Advertising (DTCA) and Meeting and Events (M&E) come primarily from the non-drug treatment (82.3% and 59.7% respectively), whereas gains from detailing come at the expense of competing drugs (82.1%). By contrast, the (random coefficient) logit and nested logit model predicts that gains from DTCA, M&E and detailing all would come largely from competing drugs – a counter-intuitive estimate of the sources of demand gains due to different marketing instruments.
1. Introduction

Brand managers carefully build their marketing portfolios to meet the sales and market share goals of the brand. Available marketing instruments differ by industry, but typically include pricing, advertising, trade promotions, consumer promotions such as coupons and sweepstakes, in-store merchandising, and sales force efforts, as well as longer-term choices such as product-line depth and breadth. Since different instruments affect consumer behavior in different ways, the brand manager has the responsibility of mixing the marketing instruments optimally to achieve the brand’s goals. This may require, for instance, that in early stages of the product life cycle, brand managers emphasize category-expanding investments while in later, more mature stages of the life cycle, they emphasize investments that steal market share from competitors.

To illustrate using advertising as an instrument, the “Got Milk” campaign is clearly intended to grow primary demand for the category, milk. Similarly, a campaign that encourages use of a brand in a situation typically associated with a different category is intended to draw new category buyers to the brand (Wansink 1994). In contrast, comparative advertising that persuades the consumer about superiority of a brand’s features over a competing brand is aimed at encouraging within-category brand switching. Temporary reductions in the price of a brand on the retail shelf typically have a similar brand switching goal. (Nijs et al. 2001) reports that such price promotions rarely have persistent category expanding effects, while new product introductions do expand the category.
Given these differences, some marketing actions are more threatening to competitors than others. At one extreme, marketing actions that primarily grow the category by attracting new buyers may even benefit competitors’ sales. On the other hand, actions that primarily induce buyers to switch from competing brands in the category clearly hurt competing brands’ sales and share. Accordingly, a brand manager may expect different degrees of competitive retaliation to different marketing instruments; an instrument that inflicts greater damage on a competing brand is more likely to elicit a reaction. Leeﬂang and Wittink (Leeﬂang and Wittink 2001) ﬁnd empirically that managers’ competitive reactions do take into account consumer response; the greater the cross-brand demand elasticity, the greater the competitive reaction elasticity. Steenkamp, Nijs et al. (Steenkamp et al. 2005) ﬁnd that competitors’ response to price promotions is considerably stronger than competitors’ response to advertising. This is consistent with the conventional wisdom that sales gains from advertising are derived more from category expansion than are sales gains from price promotions. Considerations of likely competitive response naturally affect the manager’s choice of the optimal marketing mix.

To help the brand manager make informed marketing mix decisions, it is essential that marketing mix models accurately measure the different effects of marketing instruments. The key argument of this paper is that extant discrete choice models are insuﬃcient in this regard and can in fact misinform and misguide the manager. Classical models such as the logit, nested logit, and probit model make it appear that all marketing instruments are
identical in terms of the source of share gains (Steenburgh 2008), whereas our previous examples have illustrated that in fact differences between instruments could be substantial.

Discrete choice models are commonly used to analyze how consumers respond to marketing actions in terms of whether or not to buy (purchase incidence) and which brand to buy (brand choice) (Bell et al. 1999; Bucklin et al. 1998). Thus, these models allow measurement of the proportion of increase in a brand’s choice share due to a given marketing action that is attributable to market expansion versus brand switching. Recent work, however, shows that a large class of existing discrete choice models, including ones that have been used to address this problem, possess the Invariant Proportion of Substitution (IPS) property, which implies that the proportion of demand generated by substitution away from a given competing alternative is the same, no matter which marketing instrument is employed (Steenburgh 2008). This is troubling because it implies that the proportion of growth due to new consumers purchasing in the category is the same no matter which marketing action is taken.

We discuss the theoretical roots of the IPS property, namely, the weak complementarity assumption that underlies Randomly Utility Maximizing (RUM) models, including the logit, nested logit and probit models. Weak complementarity, a relatively little known property of RUM models, holds that a consumer’s utility is unaffected by changes of an attribute, $x$, of a particular good unless she consumes that good. Whenever an individual brand’s marketing instrument spills-over to competing brands—either increase consumers’ utility for competing brands or decrease consumers’ utility for competing brands, weak complementarity
condition is violated and the application of conventional RUM model gives counter-intuitive results about the sources of demand gains due to different marketing instruments.

We propose such a model -- the Cross Nests Flexible Substitution Logit (CNFSL) model -- that relaxes the weak complementarity assumption and IPS property. We derive the CNFSL model from a direct utility function and show that it provides a better fit to the data than extant models. Furthermore, we show that its conclusions vary substantively as well. The CNFSL allows greater agreement in substitution patterns between individual and population-level models than extant models because it does not unnecessarily impose IPS on individual choice behavior.

We demonstrate our arguments empirically in the context of the marketing of prescription drugs, a context in which these issues are of central concern not only for brand managers but also for public policy makers. We show that patient-directed marketing instruments such as Direct to Consumer Advertising (DTCA) often work quite differently than physician-directed marketing actions such as detailing in terms of sources of demand gains. As a result, we might expect that models that possess the IPS property will provide an overly restricted representation of the effects of these activities. Indeed, our empirical application to prescription writing choices of physicians in the hyperlipidemia category shows this to be the case.

We find that three commonly used models that all suffer from the IPS restriction -- the homogeneous logit model, the nested logit model, and the random coefficient logit model -- lead to counter-intuitive estimates of the sources of demand gains due to increased
marketing investments in DTCA, detailing, and professional Meetings and Events (M&E). This is not true for the CNFSL. In particular, the CNFSL model provides the important insight that while most of the gains from detailing investments come at the expense of competing brands in the category (82.1%), most of the gains from DTCA and M&E are realized from patients who are not prescribed any drug treatment (82.3% and 59.7% respectively). In other words, competitor brands should be much less threatened by DTCA and M&E actions than by detailing. This key distinction in how the marketing instruments work is disguised by extant models.

The rest of this paper is organized as follows. In Section 2 we describe the data. In Section 3 we discuss the theoretical roots of the IPS property, namely, the weak complementarity assumption that underlies RUM models and derive the CNFSL model by following a direct utility approach. In Section 4 we discuss results of our empirical analysis. Section 5 concludes with a discussion of results and implications for managerial actions and future research.

2. Data

We have chosen to examine differences in share stealing across marketing instruments in the context of pharmaceutical marketing. While there are multiple constituencies that determine demand for a brand drug, pharmaceutical firms in the US devote most of their marketing resources primarily to influence two groups -- physicians and patients. Pharmaceutical manufacturers spent at least $20.5 billion on promotional activities in 2008,
excluding sampling. Of that, $12 billion went to detailing to physicians, $4.7 billion to DTCA, and $3.4 billion to M&E (CBO 2009).

We expect to find different competitive impacts when firms invest in detailing, M&E and DTCA because these marketing instruments work in very different ways. Detailing is personal selling to physicians by pharmaceutical firms’ representatives. The representatives inform physicians about drug efficacy and safety, answer physicians’ questions, and establish and maintain goodwill of the brand. During the detailing visits, sales representatives also provide physicians with drug samples. Firms have full control on what to communicate with physicians as long as messages conform to FDA regulations and these communications take place behind closed doors.

In contrast, pharmaceutical firms also sponsor professional meetings and events, including some that offer physicians credit for continuing medical education. Firms may help fund, organize and advertise M&E, and may also subsidize attendance of physicians. Unlike detailing, firms can only influence the topics that are discussed in M&E indirectly through M&E organizers like medical education companies or professional societies. As a consequence, the content of M&Es tends to be disease-oriented, different from the brand-oriented communications in detailing. In addition, discussion and interaction among attendees makes M&E attendance a different experience for physicians relative to detailing.

Traditionally, a negligible part of the overall marketing budget was spent on influencing patients. However, in the last decade this component has been growing rapidly in the form of direct-to-consumer advertising. DTCA can expand the category via the informational and
educational roles of advertising. Advertising can inform potential patients of the existence of a health condition, possible symptoms and consequences, as well as the availability of a treatment. Better-informed under-diagnosed or under-treated patients, in turn, will be able to understand their health conditions better, and may be prompted to seek medical consultation by visiting a physician. This perspective suggests that an important source of sales gains due to DTCA is newly diagnosed patients, who expand overall category demand and this potentially benefits all competing firms. Another role of DTCA is to persuade patients to ask their physicians for specific brand name drugs. The literature suggests that patient requests do influence physicians’ prescription behavior. As a consequence, sales gains occur due to physicians’ switching from competing brands, but also due to switching from “non-drug prescriptions.” The latter is a source that expands the category (this is discussed further subsequently).

The therapeutical class that we use in this study is statins (or HMG-CoA reductase inhibitors). Statins are drugs used to lower cholesterol levels in people at risk for cardiovascular disease because of hyperlipidemia. Statins are the most potent anti-hyperlipidemia agents and have dominated the anti-hyperlipidemia market. Statins sales surpassed $14.3 billion in 2009, making them one of the biggest selling drugs in the United States\(^1\). During the period spanned by our data (2002–2004), there are four major statins available for prescription: Lipitor produced by Pfizer, Zocor by Merck, Pravachol by Bristol-Myers Squibb (BMS) and Crestor by AstraZeneca. “Non-drug only treatment” is also

\(^1\) Source: IMS National Prescription Audit PLUS.
a common prescription issued by physicians if patients’ diagnosed condition is not severe enough for drug treatment. Non-drug treatment methods include: eating healthy, quitting smoking, increasing physical activity, moderating alcohol intake and maintaining an ideal body weight.

Data on patient visits, prescriptions written by physicians, and detailing and M & E to which the physicians are exposed, are available for a sample of 247 physicians in the U.S. over a 24-month period, June 2002 to May 2004. The data were made available by a marketing research firm, ImpactRx Inc. The firm runs a panel consisting of a representative sample of the universe of physicians in the US, balanced across geographic regions, physician specialties and prescription volumes. Data on monthly DTCA expenditures come from Kantar Media Intelligence. We link each patient visit to Designated Media Area (DMA) level DTCA expenditures through physician-level zip codes. DTCA is measured as $ expenditure per capita based on the population of the DMA.

In Table 1 we present summary statistics of the data. Taking the unit of analysis to be physician-month for each of the four brands and for non-drug treatment, we show the number of prescriptions and market share of prescriptions, and levels of each of the marketing mix instruments. As shown in Table 1, on average, there are more detailing visits and M & E for Crestor than for the other three brands. However, DTCA expenditure on Lipitor is the largest among the four brands. The prescriptions shares show that about one-quarter of visits receive prescription for non-drug treatment instead of a drug treatment.
Among the four drugs, Lipitor is the market leader, followed by Crestor, Zocor, and Pravachol.

The impacts of marketing variables considered in this study are expected to carry over from one period to the next with deteriorating effectiveness. To capture the long term effect, we follow the advertising model of (Nerlove and Arrow 1962) and introduce a vector of stock variables for marketing instruments:

\[ x_{pjt} = \lambda x_{pjt-1} + \{DET_{pjt}, ME_{pjt}, DTC_{pjt}\} \]

where \( DET_{pjt} \) is the number of detailing visits by drug \( j \) to physician \( p \) in month \( t \); \( ME_{pjt} \) is the number of M&E sponsored by drug \( j \) that received participation by physician \( p \) in month \( t \); \( DTC_{pjt} \) is drug \( j \)'s DTCA per capita $ expenditure in physician \( p \)'s DMA area in month \( t \); \( \lambda \) is the carry-over parameter with a value between 0 to 1.

To fix the carryover parameters for marketing instruments, we conducted a grid search and used maximized log-likelihood for a standard logit model to make our final selection of parameter values. This led to the following values of the carryover parameter: 0.75 for detailing, 0.90 for M&E, and 0.75 for DTCA. These are quite close to findings reported in previous literature, for example Narayanan et al. (2004) reports carryover values of detailing and M&E at 0.86 each, and that of DTCA at 0.75. We use the first 14 months of data to

\[ \text{We also fixed the carry over parameters based on nested logit and CNFSL models and conducted the other studies separately. We found no significant difference in results.} \]
calculate the value of initial stock of each marketing instrument. All models are fitted on the remaining 10 months of data.

3. Model

In this section, we discuss the types of restrictions that standard random utility maximization (or RUM) models impose on individual substitution patterns. We then propose a model that allows for greater flexibility in substitution patterns. We also discuss the flexibility that taste heterogeneity adds to these models.

3.1 RUM and IPS

The utility theoretic framework for discrete choice demand analysis has followed two related but distinct approaches. The first approach involves the specification of a particular indirect utility function including a deterministic utility component and a random term. One can obtain the Marshallian demand function from this indirect utility function via Roy’s identity (Roy 1947). This approach is attractive because the maximization of a random indirect utility function yields the choice probability directly and avoids having to derive the choice probability from a complex nonlinear optimization problem. The second approach involves the specification of a direct utility function, and derives the choice probability from the optimality or Karush-Kuhn-Tucker (KKT) conditions associated with the maximization problem. This approach has drawn more attention recently because it imposes fewer restrictions on preferences than does the indirect utility approach.

The most popular discrete choice models based on the indirect utility function approach belong to a class known as random utility maximization (RUM) models, which
includes the conditional multinomial logit model. McFadden (1974) developed the
conditional multinomial logit model from Luce’s choice axiom (Luce 1959). Letting \( P_B(i) \)
denote the probability that a decision maker will choose i from a set of mutually exclusive
and collectively exhaustive alternatives B, Luce’s axiom states that the ratio of choice
probabilities for i and j is the same for every choice set B that includes i and j, i.e.,

\[
\frac{P_B(i)}{P_B(j)} = \frac{P_{i,j}(i)}{P_{i,j}(j)}
\]

Luce called this property Independence of Irrelevant Alternatives (IIA). If this property
holds, there exists for every option j a strictly positive number, so that the probability of
choosing j from B is

\[
P(j, B) = \frac{V_j}{\sum_{k=1}^{n} V_k}
\]

McFadden (1974) taking this strict utility for alternative j to be a parametric
exponential function of its attributes \( x_j, V_j = \exp(x_j\beta) \), gave a practical empirical model.

However, the empirical literature on random choice has documented systematic violations of
the Luce model, best known as the red bus-blue bus problem first noticed by Debreu (1960).
Specifically, IIA implies that demand must be drawn from competing alternatives in
proportion to their market shares, an undesirable assumption about how decision makers
substitute among alternatives.

A less well known restriction invoked in RUM models is the weak complementarity
property originally defined by Mäler (1974) in establishing the welfare basis of environmental
valuation. The weak complementarity property holds that a consumer’s utility is unaffected by changes of an attribute, $x$, of a particular good unless she consumes that good. When the weak complementarity property holds, we say $x$ is a weak complement to the good and the good weakly complements $x$ as well. The concept of weak complementarity and the related formal mathematical condition have grown beyond application to environmental valuation and come to be used much more broadly in practice (Just et al. 2004). Let $U$ represent the utility function of a decision maker who does not consume good $j$, $Z_{-j}$ her consumption of commodities other than commodity $j$, and $x_j$ be an attribute of good $j$. Then weak complementarity can formally be defined as a property of preferences, such that

$$\frac{\partial U(0, Z_{-j}, x_j)}{\partial x_j} = 0$$

In the RUM specification, weak complementarity can be easily seen since indirect utility derived from any alternative depends on the attributes of that alternative alone. Weak complementarity is necessary for RUM-consistent choice models because it constitutes the basic assumption of measurement of social surplus from which McFadden (1981) derives the models. When the weak complementarity condition holds, the change in the area under the Hicksian demand function caused by a change in an attribute equals the compensating variation for the weak complement if the weak complement to the attribute is a non-essential

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3. In Mäler (1974)'s original definition, weak complementarity means that the demand for a public environmental good (such as the air at a tourist site) is zero if the demand for the related private good (such as the tourist site itself) is zero.

4. The attribute of a good here is defined in a broad sense. It includes not only the quality of the good but also the amount or the quality of the publicly provided good that affects the utility of the focal good.
good5 (Bockstael and McConnell 2007). Further, together with the non-essential condition, weak complementarity ensures that the area under the demand curve yields a closed form integral for choice probabilities that satisfy McFadden’s condition for a probability choice system consistent with RUM.

The weak complementarity property seems intuitively appealing in many situations. However, there are circumstances in which this property appears to be a restriction. The most often mentioned case in environmental economics is where publicly determined and provided environmental goods may act as a symbol of something inherently valued by the decision maker – generating an “existence value” even when use is absent (Bockstael and McConnell 2007; Krutilla 1967). For example, some environmentally conscious individuals derive satisfaction from the fact that endangered species are protected in a remote area even though they will most likely never travel to the area to see those species. Altruism has often been given as a rationale for existence value.

Second, and more related with our work here, there may be more than one good that forms the set of goods that is weakly complementary to one or more attributes of these goods that are common. An example is that the quality of water (common attribute) may affect the utility of drinking, swimming, and fishing (Just et al. 2004). In this case, water quality is not weakly complementary to each of the individual alternatives: drinking, swimming, and fishing. To illustrate this, consider how weak complementarity fails between

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5 A good is non-essential if a bundle of other goods can be found that will compensate the consumer for its complete absence Willig, R. D. (1978), "Incremental consumer's surplus and hedonic price adjustments," Journal of Economic Theory, 17, 227-53.
water quality and drinking. The utility of a consumer who does not drink this water at all is nevertheless affected because this consumer goes fishing or swimming. This kind of violation of the weak complementarity property at the individual alternative level is especially important in marketing because non-price promotion of a product has the potential to affect the utilities of competing goods in the same category. For instance, the information provided by certain promotions may convince consumers that all products in a category have certain superior characteristics, and as a consequence, boost utility of all products in that category.

On the other hand, comparative advertising by a brand is sometimes intended to convince consumers that competing brands have inferior characteristics, and therefore, decrease the utility of those goods (Just et al. 2004). This failure of weak complementarity implies that RUM models may not be appropriate in modeling category expansion and share stealing situations.

To maintain the weak complementarity condition in RUM, the indirect utility derived from any good must be specified to depend on the quality of that alternative alone. This restriction in indirect utility specification leads to restrictions on substitution patterns among alternatives, defined as the IPS property by Steenburgh (2008). IPS implies that the proportion of demand generated by substitution away from a given competing alternative is the same, no matter which marketing instrument is employed. The IPS property is especially troubling in our context because the point of our study is to determine whether specific marketing investments steal demand from competing drugs or from the non-drug treatment. We might expect detailing to draw a greater proportion of demand from competing goods
than M&E and DTCA do. The standard RUM would not let us find this out because it requires demand to be drawn from each competing alternative in proportion to its market share. Returning to the example, the model requires 60% of the incremental demand to be drawn from competing drugs and 40% to be drawn from the non-drug treatment no matter which investment is made.

3.2 Cross Nests Flexible Substitution Logit (CNFSL)

Given that weak complementarity is an inherent assumption of RUM consistent discrete choice model, relaxing the IPS property within the RUM framework seems too challenging. By contrast, the direct utility representation provides a utility theoretical framework that allows a more general representation of consumer preferences, within which weak complementarity need not be assumed a priori. In this section, we follow the direct utility approach to propose a utility maximization model called the cross nests flexible substitution logit (CNFSL); this model allows weak complementarity to apply to a group of goods.

Unlike a standard RUM-consistent discrete choice model, it accommodates the possibility of non-price promotions on one product to have spillover effects on the utility of other products.

Consider an economy with J brands and one consumer making purchases from multiple occasions. Let the choice set be grouped into N non-overlapping subsets (or nests) denoted by $B_1, B_2, \ldots, B_N$. The consumer is assumed to follow a two-step sequential choice process when selecting her most preferred alternative. In the first step, the consumer must decide which subset she wants to choose from. In the second step, the consumer chooses a
particular alternative \( j \) from subset \( n \) given that she chooses subset \( n \) in the first step. We assume that the consumer makes her two-step decision by maximizing her total direct utility \( U \). This is equivalent to the consumer maximizing utility by solving a two-stage dynamic programming problem. In the first step, the consumer chooses the subset with the highest utility taking into account the current utility associated with subsets and the expected maximum utility from choosing an alternative from the chosen subset. Given her decision in the first step, the consumer chooses the alternative with highest utility from the chosen subset.

We next derive the choice probability based on demand constraints and the direct utility specification. Since the demand for subset \( n, Z_n \), is split among \( J_n \) alternatives options, this gives us the first constraint:

\[
Z_n = \sum_{j=1}^{J_n} Z_{jn} \quad (1)
\]

where \( Z_{jn} \) is the demand for alternative \( j \) in subset \( n \).

The consumer buys a given amount \( T \) of these differentiated goods across multiple purchase occasions, leading to the following constraint:

\[
\sum_{n=1}^{N} Z_n = \sum_{n=1}^{N} \sum_{j=1}^{J_n} Z_{jn} = T \quad (2)
\]

If \( Y \) is the income of the representative consumer, we have the budget constraint,

\[
Y = \sum_{n=1}^{N} \sum_{j=1}^{J_n} p_{jn} Z_{jn} \quad (3)
\]

where \( p_{jn} \) is the price of commodity \( j \) from subset \( n \).
Following Anderson et al. (1988), we specify a consumer’s utility function as

\[
U = \begin{cases} 
\sum_{n=1}^{N} w_n Z_n + \sum_{n=1}^{N} \sum_{j=1}^{J_n} \mu_{jn} Z_{jn} - \sum_{n=1}^{N} Z_n \log \frac{Z_n}{\sum_{n=1}^{N} Z_n} - \sum_{n=1}^{N} \left( \sum_{j=1}^{J_n} \mu_{jn} \log \frac{Z_{jn}}{\sum_{j'=1}^{J_n} Z_{jn}} \right) & \text{if } (1) \text{ and } (2) \\
-\infty, & \text{otherwise}
\end{cases}
\]

The utility specified here includes four different effects. The first term expresses the utility from consumption of nests \(Z_1, \ldots, Z_N\), and the second term expresses the utility from consumption of \(Z_{11}, \ldots, Z_{JN}\). \(w_n\) measures marginal quality of one unit of nest \(n\), \(\mu_{jn}\) measures marginal quality of one unit of alternative \(j\) in nest \(n\), and the last two terms represent this consumer’s preference for diversity. These preferences for diversity are added to explain the intrapersonal or interpersonal variation in preference.

The nest structure above allows us to accommodate situations where weak complementarity is unreasonably restrictive. Let \(x_j\) be non-price promotion of alternative \(j\) that may spill-over across alternatives within a nest. We define \(w_n = \theta_n + \theta \left( \sum_{j \in J_n} x_j \right)\) and \(\mu_{jn} = \alpha + \beta x_j\), where \(x_j\) affects the decision maker’s utility in two ways:

1. It increases preference for the focal alternative over competing alternative in the nest by \(\beta\).
2. It increases preference for the focal alternative and other alternatives in the same nest over alternatives outside the nest by \(\beta + \theta\).

In this specification, the summation of non-price promotion for alternatives in nest \(n\) accounts for the possible spill-over effect and acts as a nest-level attribute. As a consequence, within nest \(n\) weak complementarity for individual good does not hold; a
change in the quality of good \( j \) due to non-price promotion affects the utility of a consumer who does not consume good \( j \), but consumes a different good in the same nest. The inclusion of term, \( \theta \left( \sum_{x_i} x_i \right) \), in the model specification gives us a general framework for empirical work. If \( \theta \) is zero, it indicates that the spill-over effect is zero and weak complementarity condition is not violated. If \( \theta \) is positive, it indicates that the focal marketing instrument positively affect consumers’ utility for all brands in the choice set and suggests a strong category expansion effect. If \( \theta \) is negative, it indicates a negative spill-over effect and suggests that the focal marketing instrument has strong business stealing effect.

The Lagrangian function is written as,

\[
L = \sum_{n=1}^{N} w_n Z_n + \sum_{n=1}^{N} J_n \mu_n Z_{jn} - \sum_{n=1}^{N} Z_n (\log Z_n - \log T) - \sum_{n=1}^{N} \left( \sum_{j=1}^{J_n} Z_{jn} (\log Z_{jn} - \log(\sum_{i=1}^{J_n} Z_{in})) \right)
\]

\[
+ \lambda (Y - \sum_{n=1}^{N} \sum_{j=1}^{J_n} P_{jn} Z_{jn}) + \gamma_0 (T - \sum_{n=1}^{N} Z_n) + \sum_{n=1}^{N} \gamma_n (Z_n - \sum_{j=1}^{J_n} Z_{jn})
\]

Taking the first-order condition with respect to \( Z_{jn} \), we get:

\[
\frac{\partial L}{\partial Z_{jn}} = \mu_n - \lambda P_{jn} - \log Z_{jn} + \log(\sum_{i \in B_n} Z_{in}) + \gamma_n = 0 \quad \forall n, \forall j \in B_n
\]

Solving this we have the choice probability of alternative \( j \) given that nest \( n \) is chosen:

\[
P_{jn} = \frac{Z_{in}}{\sum_{i \in B_n} Z_{in}} = e^{\mu_n - \lambda P_{jn} + \gamma_n}
\]

Since \( \sum_{i \in B_n} P_{jn} = 1 \), we can easily see that \( \sum_{i \in B_n} e^{\mu_n - \lambda P_{jn} + \gamma_n} = 1 \). Thus, it follows that

\[
P_{jn} = \frac{e^{\mu_n - \lambda P_{jn} + \gamma_n}}{\sum_{i \in B_n} e^{\mu_i - \lambda P_{in} + \gamma_i}} = \frac{e^{\mu_n - \lambda P_{jn}}}{\sum_{i \in B_n} e^{\mu_i - \lambda P_{in}}}
\]
This is consistent with a standard multinomial logit choice probability. Let \( \phi_n \) be the expected maximum utility associated with alternatives in nest \( n \). We have

\[
\phi_n = E(\max_{i \in B_n} \{\mu_{in} - \lambda p_{in} + \varepsilon_{in}\}) = \log(\sum_{i \in B_n} e^{\mu_{in} - \lambda p_{in}})
\]

Taking the first-order condition with respect to \( Z_n \), we get:

\[
\frac{\partial L}{\partial Z_n} = w_n - \log Z_n + \log T + \gamma_n - \gamma_0 = 0 \quad \forall n
\]

Solving this we have

\[
P_n = \frac{Z_n}{T} = e^{w_n + \gamma_n - \gamma}
\]

Given that \( \sum P_n = 1 \), we have \( \sum e^{w_n + \gamma_n - \gamma} = 1 \). This gives the following result:

\[
P_n = \frac{\sum e^{w_n + \gamma_n - \gamma}}{\sum e^{w_n + \gamma_n - \gamma}} = \frac{e^{w_n + \gamma_n}}{\sum e^{w_n + \gamma_n}}
\]

Taking \( \gamma_n = \phi_n \), we end up with the choice probability of nest \( n \) as:

\[
P_n = \frac{e^{w_n + \phi_n}}{\sum m e^{w_m + \phi_m}} = \frac{e^{w_n + \log(\sum_{i \in B_n} e^{\mu_{in} - \lambda p_{in}})}}{\sum m e^{w_m + \log(\sum_{i \in B_m} e^{\mu_{im} - \lambda p_{im}})}}
\]

The choice probability for alternative \( j \) in nest \( n \) is then given as:

\[
P_{jn} = P_n P_{jn|n} = \frac{e^{w_n + \log(\sum_{i \in B_n} e^{\mu_{in} - \lambda p_{in}})}}{\sum m e^{w_m + \log(\sum_{i \in B_m} e^{\mu_{im} - \lambda p_{im}})}} \frac{e^{\mu_{jn} - \lambda p_{jn}}}{\sum_{i \in B_n} e^{\mu_{jn} - \lambda p_{jn}}}
\]

Plug in \( w_n = \vartheta_n + \theta \left( \sum x_i \right) \) and \( \mu_{jn} = \alpha + \beta x_j \), we have
\[ P_{jn} = \frac{\theta_n + \Theta \left( \sum_{i \in B_n} x_i \right) + \log( \sum_{i \in B_n} e^{\alpha_i + \beta x_i - \lambda p_{jn}} )}{\sum_m e^{\theta_n + \Theta \left( \sum_{i \in B_m} x_i \right) + \log( \sum_{i \in B_m} e^{\alpha_i + \beta x_i - \lambda p_{jm}} )}} \]

\[ = \frac{e^{\theta_n + \Theta \left( \sum_{i \in B_n} x_i \right) + \alpha_j + \beta x_j - \lambda p_{jn}}}{\sum_m \sum_{B_m} e^{\theta_n + \Theta \left( \sum_{i \in B_m} x_i \right) + \alpha_i + \beta x_i - \lambda p_{jm}}}. \]

(Proof is provided in the appendix.) The above choice probability coincides with that derived from a special form of the universal logit model (Koppelman and Sethi 2000; McFadden 1975) because it allows the attributes of competing alternatives to enter the utility function of the focal brand via a summation term. If \( \theta = 0 \), then the choice probability is equal to that derived from a standard logit model. If the utility function does not contain the two terms capturing preferences for diversity, a consumer will buy only the alternative that gives the largest

\[ \Theta \left( \sum_{i \in B_n} x_i \right) + \alpha_j + \beta x_j - \lambda p_{jn} \quad \forall j, n \]

On the other hand, if the utility function only includes the last two terms, the consumer’s purchasing will be equally divided among available subsets and alternatives.

Unlike the RUM framework, the demand system derived here does not impose the weak complementarity condition, and therefore relaxes the IPS restriction across nests. This can

---

6 The universal logit has not been used much in practice. A notable exception is Krishnamurthi et al. (1995).
be seen by the substitution ratio \((-\partial P_k / \partial x_{ja}) / (\partial P_j / \partial x_{ja})\) derived below, which represents the proportion of the increase in expected demand for alternative \(j\) that is generated by substitution away from alternative \(k\) following an improvement to attribute \(x_{ja}\).

\[
-\frac{\partial P_k}{\partial x_{ja}} = \begin{cases} 
\frac{P_k P_j \beta_a - P_k (1 - P_n) \theta_a}{P_j (1 - P_j) \beta_a + (1 - P_n) \theta_a} & \text{for } k \neq j, \text{ and } k, j \in B_n \\
1 & \text{for } k = j, \text{ and } k, j \in B_n \\
\frac{P_j (P_k \beta_a + P_n \theta_a)}{P_j (1 - P_j) \beta_a + (1 - P_n) \theta_a} & \text{for } k \neq j, \text{ and } k \in B_m, j \in B_n
\end{cases}
\]

(Proof is provided in the appendix.)

No matter whether \(k\) is from the same nest or a different nest than \(j\), the substitution ratio does depend on marketing instrument specific parameters. The flexibility is achieved because \(\beta_a\) and \(\theta_a\) can vary across marketing instruments.

Following the work of Bucklin, Gupta et al. (1998) and Bell, Chiang, et al (1999), we divide the choice alternatives into two nests: one \((B_0)\) containing the non-drug treatment and the other \((B_1)\) containing the four drug brands. However, unlike the previous two studies, we use summation of all non-price promotions at the nest level to allow for spill-over effects across brands within nest \(B_1\). The physician’s decision tree is as follows:
Unlike either of the previous models, the CNFSL model allows the proportion of demand drawn from both competing drugs and the non-drug treatment to vary across marketing instruments. For example, 80% of the incremental demand could be created by market expansion if the brand manager were to invest in DTCA, but only 15% of the incremental demand could be created by market expansion if the manager were to invest in detailing. It seems reasonable to allow for this possibility given our prior expectations of how the two marketing instruments work.

It is worthwhile to note that the proposed demand has some limitation in terms of relaxing the IPS property across alternatives. As shown below, the ratio of the proportions of demand drawn from two brands within nest \( n \) between them, or two brands outside nest \( n \) does not depend on the instrument used by alternative \( j \). In other words, for subset of brands within nest \( n \) and outside nest \( n \), IPS property still applies.

\[
\frac{\partial P_{k \mid j}}{\partial x_{klj}} = \begin{cases} 
\frac{P_k}{P_l} & \text{for } k, l \neq j, \text{ and } k, l, j \in B_n \\
\frac{P_k}{P_l} & \text{for } k, l \notin B_n, \text{ and } j \in B_n \\
\frac{-P_k \left( P_j \beta_a + P_m \theta_a \right)}{P_l \left( 1 - P_n \theta_a - P_j \beta_a \right)} & \text{for } k \notin B_n, \text{ and } l, j \in B_n 
\end{cases}
\]

It is not so troubling for the former case given that we can always separate brands into different nests if there are enough reasons to believe the non-price promotion affects these brands in a different way. To address the limitation that applies to brands outside \( n \), but in different nest, we can expand the proposed model to multi-levels. Taking a pharmaceutical
example, we can create a choice decision tree for anti-diabetic medication category as shown below:

```
Non-drug treatment

Drug treatment

Oral hypoglycemic agents

Januvia

Squibb

Onglyza

Injected insulin

Novolog

Humulog
```

In this three-level structure, summations of non-price promotion of all subsidiary options enter the utility function as nest-level quality for each nest in different level. With this specification, we can relax IPS for alternative across all nests.

3.4 Flexibility Provided by Taste Heterogeneity

Heterogeneous choice models allow a wider variety of substitution patterns to occur among market shares than their homogeneous counterparts do. This does not mean, however, that allowing for heterogeneous tastes solves the problems associated with IIA and IPS. Adding taste heterogeneity to a choice model does not change individual substitution patterns, and models such as the random coefficient logit and random coefficient nested
logit preclude individual choice behavior that is reasonable (Steenburgh, 2008). By contrast, the CNFSL allows a wider variety of individual-level choice behavior to be recovered from the data.

Even though individual decision maker’s choice behavior follows IPS, the aggregate demand can break IPS if decision makers are heterogeneous. To illustrate this, in the following table we give an example of two physicians with different tastes each making drug choices from the set \{B1, B2, B3\}. Brand B1 employs two different marketing instruments I1 and I2. The proportion of demand drawn by brand B1 from brands B2 and B3 is the same for both instruments, for each physician. This implies that choices of each physician conform to IPS. However, for the market as a whole (i.e., the aggregation of the two physicians’ choices), instrument I1 draws more market share from B2, while instrument I2 draws more market share from B3. This indicates that we cannot exclude the possibility that non-conformity with IPS at the market level may be only a mathematical outcome instead of a result of non-conformity at the individual level.

<table>
<thead>
<tr>
<th>Physician</th>
<th>Brand</th>
<th>Original Choice Probability</th>
<th>Choice Probability after Instrument I1 Improves</th>
<th>Proportion Drawn from B2 and B3</th>
<th>Choice Probability after Instrument I2 Improves</th>
<th>Proportion Drawn from B2 and B3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physician 1</td>
<td>B1</td>
<td>8/32</td>
<td>14/32</td>
<td>20/32</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>10/32</td>
<td>8/32</td>
<td>1/3</td>
<td>6/32</td>
<td>1/3</td>
</tr>
<tr>
<td></td>
<td>B3</td>
<td>14/32</td>
<td>10/32</td>
<td>2/3</td>
<td>6/32</td>
<td>2/3</td>
</tr>
<tr>
<td>Physician 2</td>
<td>B1</td>
<td>10/32</td>
<td>22/32</td>
<td>6/32</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>16/32</td>
<td>8/32</td>
<td>2/3</td>
<td>12/32</td>
<td>2/3</td>
</tr>
<tr>
<td></td>
<td>B3</td>
<td>6/32</td>
<td>2/32</td>
<td>1/3</td>
<td>4/32</td>
<td>1/3</td>
</tr>
<tr>
<td>Total</td>
<td>B1</td>
<td>9/32</td>
<td>18/32</td>
<td>18/32</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
An important aspect of marketing practice is the targeting of consumer segments for differential marketing instruments. Rossi et al. (1996) has shown that individual level targeting can achieve substantial gain even with rather short purchase histories. The increased availability of individual consumer panel data and the development of hierarchical Bayesian estimation techniques have facilitated direct targeting of consumer at individual level. As individual-level targeting becomes more popular in practice, it becomes increasingly important that individual-level models be correctly and flexibly specified.

We create heterogeneous versions of all three models (logit model, nested logit model, and CNFSL model) through random coefficients specifications. For example, the random coefficients CNFSL is specified as

$$U_{ij} = X_{ij} \beta_i + \epsilon_{ij}$$

where

$$\beta_i \sim N(\beta, \Sigma)$$

Since the random coefficients CNFSL nests the random coefficients logit, we can empirically test whether adding flexibility at the individual-level of the model matters. Furthermore, we will use the estimates to compare the substitution patterns of all three models.

---

7 In pharmaceutical market, individual-level targeting is mainly at physician level since physicians are main decision makers for prescriptions. In addition, physician-level prescription data are made available by marketing research firms such as impact Rx or IMS health, while patient-level data are often hard to get because of the protection of privacy.
models at both the individual and population levels, showing that the patterns of the CNFSL are logically consistent at both levels.

4. Results

To begin with we assumed parameter homogeneity across physicians and estimated a standard logit, a nested logit, and a CNFSL model. We then incorporated physician heterogeneity and estimated a random coefficient logit, a random coefficient nested logit, and a random coefficient CNFSL model on the data.

4.1 Homogeneous Case

We show model fit statistics of the three homogeneous models in Table 2 but do not show parameter estimates for reasons of space. Instead, we present the own elasticities for Lipitor (as an illustration) and the substitution matrices. Both AIC and BIC indicate that the CNFSL model fits the data best, followed by the nested logit model and then the logit.

Table 2 about here

All the models find positive effects of detailing, DTCA, and M&E on physicians’ probability of prescribing the marketed drug. Notice that each model comes to roughly the same conclusion about the ability of marketing instruments to generate demand. The elasticity of demand is greatest from detailing (the own-elasticity is 0.257 in the logit, 0.284 in the nested logit, and 0.307 in the CNFSL model). This is followed by the elasticity of demand from DTCA (0.107 in the logit, 0.088 in the nested logit, and 0.082 in the CNFSL). The elasticity of demand is smallest from M&E (0.064 in the logit, 0.061 in the nested logit, and 0.060 in the CNFSL). Although the models come to roughly the same conclusion about the ability of
the marketing instruments to generate demand, they predict very different substitution patterns among the drugs. Due to the IIA property, the logit model predicts that demand will be drawn from each of the alternatives in proportion to their market share. Thus, for every marketing instrument, the logit model implies that 67.6% of the incremental demand for Lipitor is drawn from competing drugs (21.1% from Zocor, 14.8% from Pravachol, and 31.7% from Crestor) and 32.4% is drawn from the non-drug treatment. This approach to decomposition is consistent with the unit-based decomposition proposed by van Heerde, Gupta et al. (Van Heerde et al. 2003) and Steenburgh (Steenburgh 2007). By comparison, the market shares in the raw data, excluding Lipitor, are 21.0% for Zocor, 14.7% for Pravachol, 31.9% for Crestor and 32.4% for the non-drug treatment.

The logit model imposes overly restrictive substitution patterns on the data. First, there is no reason to believe that demand will be drawn from the competing alternatives in proportion to their market share. Second, there is no reason to believe that the substitution patterns will be the same across the marketing instruments. The nested logit model has been used in many previous decomposition studies because it cures the first problem. However, it does not cure the second problem which is due to the IPS property. Regardless of the marketing instrument being used, 78.5% of the incremental demand for Lipitor is drawn from competing drugs (24.5% from Zocor, 17.1% from Pravachol, and 36.9% from Crestor) and only 21.5% is drawn from the non-drug treatment. There is no reason to believe that the proportion of demand created by market expansion is the same for detailing, DCTA and M&E.
The CNFSL model allows a much richer set of substitution patterns to be recovered from the data because it is not subject to the IPS property. Most of the incremental demand created by detailing, 83.9%, is stolen from competing drugs (26.3% from Zocor, 18.3% from Pravachol, and 39.3% from Crestor) and only 16.1% is drawn from the non-drug treatment. These results suggest that salespeople may be selling the benefits of Lipitor against the benefits of competing drugs behind the closed doors of a doctor’s office. In stark contrast, the opposite occurs with the other marketing instruments. Most of the incremental demand created by DTCA -- 87.3% -- is drawn from the non-drug treatment, with only 12.7% being drawn from the competing drugs (4.0% from Zocor, 2.8% from Pravachol, and 6.0% from Crestor). Similarly, most of the incremental demand created by M&E, 74.0%, is drawn from the non-drug treatment, with only 26.0% being drawn from the competing drugs (8.1% from Zocor, 3.7% from Pravachol, and 12.2% from Crestor). These results suggest that DTCA and M&E have spillover effects not found in detailing.

These results seem reasonable because some pharmaceutical advertisements create awareness of a drug option and may also generate patient requests for medication. Donohue, Berndt et al. (Donohue et al. 2004) studied how DTCA works for antidepressant drugs and observed that “for conditions like depression, which are associated with social stigma, advertising may reduce negative views associated with treatment” thereby making it easier for patients to request medication. Furthermore, meetings and events are disease oriented communications in nature and allow physicians to speak to one another, which may make the drug companies less willing to draw comparisons between the drugs.
These results have important managerial implications too. Suppose a brand manager is trying to decide whether to invest marketing dollars in detailing or DTCA. An investment in detailing will lead to a greater immediate increase in demand. The estimated elasticity implies that a 10% increase in the level of detailing will yield a 2.69% increase in demand, whereas a 10% increase in the level of DTCA will yield only a .95% increase in demand. Given these numbers, it seems like we would much rather invest in detailing than in DTCA. Nevertheless, 84.0% of the demand created by detailing is stolen from competing drugs, meaning that the demand for Lipitor increases by 2.26% by stealing demand away from other drugs and 0.43% comes at the expense of the non-drug option. By comparison, 87.4% of the demand created by DTCA comes from the non-drug option. This means that the demand for Lipitor increases by 0.12% by stealing demand away from other brands and .83% comes at the expense of the non-drug option. Thus, it would seem that competing drugs would have a greater incentive to retaliate if the Lipitor brand manager invests in detailing than if she invests in DTCA.

One may wonder whether there are factors that drive increases over time in both drug usage and marketing instruments (like DTCA and M&E). If so, the category expanding effect we observe in the model results might only be a spurious effect. To examine whether there are such missing factors, we try to add a time trend to the model. We tried a linear time trend and a log transformation of a linear time trend but neither was statistically significant, alleviating our concern.

4.2 Heterogeneous Case
Although the CNFSL is the most flexible of the three homogeneous models we considered, we may wonder whether allowing for heterogeneity across physicians increases the flexibility of the logit and nested logit models and allows them to recover more realistic substitution patterns. To answer this empirical question we estimate heterogeneous versions of the three previously presented models – a random coefficient logit, a random coefficient nested logit, and a random coefficient CNFSL model. Estimation results are presented in Table 3. In all models, we find evidence of significant heterogeneity across physicians in their responsiveness to marketing instruments (standard deviations are not shown for reasons of space). Furthermore, we find that the random coefficient CNFSL fits the data best, followed by the random coefficient logit, and then the random coefficient nested logit.

In Table 4, we present the own elasticities for Lipitor (as an illustration) and the substitution matrices. All three models imply that detailing, DTCA, and M&E have positive effects on a physician’s probability of prescribing the marketed drug. As before, notice that all three models come to roughly the same conclusion about the ability of the marketing instruments to generate demand for Lipitor. The elasticity of demand is greatest for detailing (the own-elasticity is 0.334 in the random coefficient logit, 0.319 in the random coefficient nested logit, and 0.275 in the random coefficient FSL). The elasticities of demand for DTCA and M&E for all three models are considerably smaller than the elasticities for detailing.
Nevertheless, the models again come to very different conclusions about the substitution patterns among drugs. Unlike the homogeneous case, the random coefficient logit does allow for some variation in the substitution patterns across marketing instruments. The proportion of demand drawn from the non-drug treatment is 20.8% from detailing, 34.8% from DTCA, 33.9% from M&E. Similarly, the random coefficient nested logit implies that the proportion of demand drawn from the non-drug treatment is 15.5% from detailing, 27.4% from DTCA, and 27.2% from M&E. The direction of these results is consistent with what we found with the homogeneous CNFSL model. The proportion of demand that is stolen from competing drugs is greater for detailing than it is for DTCA and M&E. The magnitude of these differences, however, is much smaller, suggesting that the model is not as flexible as might be desired.

In contrast, the random coefficient CNFSL allows a richer set of substitution patterns to be recovered from the data. As we found in the homogeneous case, most of the incremental demand created by detailing -- 80.1% -- is stolen from competing drugs. Yet, the opposite occurs for the other marketing instruments. Most of the incremental demand created by DTCA and M&E is drawn from the non-drug treatment, 82.3% and 59.7% respectively. Allowing for heterogeneity adds only a small amount of flexibility. Depending on the question being addressed, it may be more important to allow for flexibility across marketing instruments than across individuals.

We explore this issue further by examining the flexibility allowed and restrictions imposed by the models on the substitution patterns of individual physicians. In Tables 5a
and 5b, we report the own elasticities and the substitution patterns for two systematically selected physicians in our data set. The random coefficient logit and random coefficient nested logit do provide more flexibility than their homogeneous counterparts because they allow the own elasticites to vary across physicians. For example, the random coefficient (nested) logit implies that the own elasticity from detailing is 0.219 (0.125) for physician A and 0.387 (0.200) for physician B. Furthermore, these models allow the substitution patterns to vary across physicians. The random coefficient (nested) logit implies that 27.8% (20.1%) of the incremental likelihood of Physician A prescribing Lipitor is drawn from the non-drug alternative whereas Physician B draws 33.3% (31.8%) from the non-drug treatment.

Nevertheless, both the random coefficient logit and the random coefficient nested logit impose the IPS property on individual physicians’ choice behavior. This means that the substitution patterns for a given physician must be the same across marketing instruments. For example, regardless of the instrument being used, the random coefficient (nested) logit implies that 72.2% (79.9%) of the incremental demand for Lipitor attributable to physician A is drawn from competing drugs and 27.8% (20.1%) from the non-drug treatment. Yet, there is no reason to believe that the Physician A will behave the same way regardless of the marketing investment being made. The same pattern can be seen in Physician B’s choice behavior. Given that the focus of the study is to make statements about differences in the substitution patterns across marketing instruments, it seems especially hard to justify requiring them to be the same at the individual level.
In contrast, the random coefficient CNFSL allows the substitution patterns to vary across marketing instruments at both the individual and aggregate levels. For example, the random coefficient CNFSL model implies that Physician A substitutes among the drugs in different ways depending on the marketing action being taken. Most of the incremental demand for Lipitor, 69.1%, is drawn from the competing drugs when detailing is used. Yet, most of the demand is drawn from the non-drug treatment when the other marketing instruments are used, 89.9% for DTCA and 85.0% for M&E. Unlike the other models, the random coefficient CNFSL can recover more realistic substitution patterns at both levels of the model.

5. Discussion, Conclusions, and Future Research

An essential decision facing any brand manager is the choice of marketing instruments to enhance the sales of the brand. Different instruments are relevant for different marketing objectives (category demand expansion or market share stealing). Discrete choice models that include the logit, the nested logit, and the probit have been used to analyze how consumers respond to marketing actions in terms of whether or not to buy (purchase incidence) and which brand to buy (brand choice). However, these models possess the IPS property. The IPS property implies that the proportion of demand generated by substitution away from a given competing alternative is the same, no matter which marketing instrument is employed. We discuss the theoretical roots of the IPS property, namely, the weak complementarity assumption that underlies RUM models. To avoid the weak complementarity assumption it is necessary to derive utility theoretic discrete choice models.
by beginning with the direct utility function. With the recognition that a set of goods, e.g.,
goods in a category, can be weak complementary to some common attributes, e.g., total
expenditure of marketing instruments in that category, we propose such a model -- the Cross
Nests Flexible Substitution Logit (CNFSL) model -- that relaxes the IPS property. The
CNFSL model, both homogeneous and random coefficient forms, predicts that increases in
DTCA and M&E result in sales gains that come primarily from non-drug treatments rather
than from other cholesterol lowering drugs. By contrast, the random coefficient logit model
predicts for all three marketing instruments – DTCA, detailing, and M&E -- that gains would
come largely at the expense of competing drugs, a counter-intuitive estimation of the sources
of demand gain due to different marketing instruments This empirical result also suggests
that the IPS property cannot be relaxed by adding physician heterogeneity.

With the proposed CNFSL model, a brand manager of prescription drugs can develop a
more nuanced and precise understanding of how different marketing instruments work, and
plan the marketing mix accordingly. For example, the brand manager may place greater
emphasis on category expanding instruments like DTCA or M&E if retaliation by competing
brands is a significant concern. We believe there is considerable room for future research in
this area. For instance, it would be important to identify other contexts in which the IPS
property has important implications. Similarly, alternative models that overcome IPS
should also be explored.
Table 1: Summary Statistics
Unit of analysis is physician-month. N=5928

<table>
<thead>
<tr>
<th>Number of Prescriptions</th>
<th>Brand</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Share of prescriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lipitor</td>
<td>0.557</td>
<td>1.177</td>
<td>0.287</td>
</tr>
<tr>
<td></td>
<td>Zocor</td>
<td>0.291</td>
<td>0.620</td>
<td>0.150</td>
</tr>
<tr>
<td></td>
<td>Pravachol</td>
<td>0.203</td>
<td>0.704</td>
<td>0.105</td>
</tr>
<tr>
<td></td>
<td>Crestor</td>
<td>0.442</td>
<td>1.415</td>
<td>0.228</td>
</tr>
<tr>
<td></td>
<td>Non-drug treatment</td>
<td>0.448</td>
<td>1.076</td>
<td>0.231</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marketing Instrument</th>
<th>Brand</th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detailing (number of visits)</td>
<td>Lipitor</td>
<td>0.634</td>
<td>1.035</td>
</tr>
<tr>
<td></td>
<td>Zocor</td>
<td>0.728</td>
<td>1.128</td>
</tr>
<tr>
<td></td>
<td>Pravachol</td>
<td>0.366</td>
<td>0.746</td>
</tr>
<tr>
<td></td>
<td>Crestor</td>
<td>0.960</td>
<td>1.316</td>
</tr>
<tr>
<td>DTCA ($ per capita)</td>
<td>Lipitor</td>
<td>0.040</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>Zocor</td>
<td>0.028</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>Pravachol</td>
<td>0.008</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>Crestor</td>
<td>0.023</td>
<td>0.039</td>
</tr>
<tr>
<td>M&amp;E (number of meetings &amp; events)</td>
<td>Lipitor</td>
<td>0.031</td>
<td>0.202</td>
</tr>
<tr>
<td></td>
<td>Zocor</td>
<td>0.006</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td>Pravachol</td>
<td>0.004</td>
<td>0.064</td>
</tr>
<tr>
<td></td>
<td>Crestor</td>
<td>0.047</td>
<td>0.227</td>
</tr>
</tbody>
</table>
Table 2: Substitution Matrices and Own Elasticities (for Lipitor) for the Three Homogeneous Models and Fit Statistics

<table>
<thead>
<tr>
<th></th>
<th>Logit Model</th>
<th>Nested Logit Model</th>
<th>CNFSL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Detailing</td>
<td>DTCA</td>
<td>M&amp;E</td>
</tr>
<tr>
<td>Lipitor</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Zocor</td>
<td>21.1%</td>
<td>21.1%</td>
<td>21.1%</td>
</tr>
<tr>
<td>Pravachol</td>
<td>14.8</td>
<td>14.8</td>
<td>14.8</td>
</tr>
<tr>
<td>Crestor</td>
<td>31.7</td>
<td>31.7</td>
<td>31.7</td>
</tr>
<tr>
<td>Nondrug Treatm</td>
<td>32.4</td>
<td>32.4</td>
<td>32.4</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Own Elasticity</td>
<td>0.257</td>
<td>0.107</td>
<td>0.064</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-7751</td>
<td>-7746</td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>15516</td>
<td>15508</td>
<td></td>
</tr>
<tr>
<td>BIC</td>
<td>15562</td>
<td>15560</td>
<td></td>
</tr>
</tbody>
</table>

For each model, cell entries in each column indicate the percentage of sales increase of Lipitor due to a 1% increase in its marketing instrument (e.g. detailing) that is drawn from the alternative indicated in the row. For example, the logit model predicts that if Lipitor increases its detailing by 1%, 21.1% of its incremental sales will come from Zocor.
Table 3: Parameter Estimates and Fit Statistics of Random Coefficient Models

<table>
<thead>
<tr>
<th>Variables</th>
<th>Logit</th>
<th>Nested Logit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Interval (95%)</td>
</tr>
<tr>
<td>Intercept of Lipitor</td>
<td>-0.170</td>
<td>-0.315, -0.018</td>
</tr>
<tr>
<td>Intercept of Zocor</td>
<td>-1.167</td>
<td>-1.386, -1.948</td>
</tr>
<tr>
<td>Intercept of Pravachol</td>
<td>-1.699</td>
<td>-1.961, -1.430</td>
</tr>
<tr>
<td>Intercept of Crestor</td>
<td>-1.008</td>
<td>-1.282, -0.749</td>
</tr>
<tr>
<td>Own - detailing - stock</td>
<td>0.174</td>
<td>0.143, 0.207</td>
</tr>
<tr>
<td>Own - DTCA – stock</td>
<td>0.849</td>
<td>0.646, 1.076</td>
</tr>
<tr>
<td>Own - M&amp;E - stock</td>
<td>0.204</td>
<td>0.156, 0.250</td>
</tr>
<tr>
<td>Inclusive Value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total-detailing-stock*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total-DTCA-stock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total-M&amp;E-stock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log of Integrated Likelihood</td>
<td>-5900</td>
<td></td>
</tr>
</tbody>
</table>

* The effect of total-detailing-stock is not significant and therefore removed from the model.
Table 4: Substitution Matrices and Own Elasticities (for Lipitor) for Random coefficient Logit, Random coefficient Nested Logit and Random coefficient CNFSL Models

<table>
<thead>
<tr>
<th></th>
<th>Random coefficient Logit Model</th>
<th>Random coefficient Nested Logit Model</th>
<th>Random coefficient CNFSL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Detailing</td>
<td>DTCA</td>
<td>M&amp;E</td>
</tr>
<tr>
<td>Lipitor</td>
<td>-</td>
<td>28.0%</td>
<td>23.4%</td>
</tr>
<tr>
<td>Zocor</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Pravachol</td>
<td>15.0</td>
<td>14.2</td>
<td>14.2</td>
</tr>
<tr>
<td>Crestor</td>
<td>36.3</td>
<td>27.5</td>
<td>28.0</td>
</tr>
<tr>
<td>Non-drug Treat</td>
<td>20.8</td>
<td>34.8</td>
<td>33.9</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Own Elasticity</td>
<td>0.334</td>
<td>0.017</td>
<td>0.039</td>
</tr>
</tbody>
</table>
Table 5a: Substitution Matrices for Random coefficient Logit, Random coefficient Nested Logit and Random coefficient CNFSL – Physician A who is relatively insensitive to detailing

<table>
<thead>
<tr>
<th></th>
<th>Random coefficient Logit Model</th>
<th>Random coefficient Nested Logit Model</th>
<th>Random coefficient CNFSL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Detailing</td>
<td>DTCA</td>
<td>M&amp;E</td>
</tr>
<tr>
<td>Lipitor</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Zocor</td>
<td>21.1%</td>
<td>21.1%</td>
<td>21.1%</td>
</tr>
<tr>
<td>Pravachol</td>
<td>19.1</td>
<td>19.1</td>
<td>19.1</td>
</tr>
<tr>
<td>Crestor</td>
<td>31.4</td>
<td>31.4</td>
<td>31.4</td>
</tr>
<tr>
<td>Non-drug Treatment</td>
<td>27.8</td>
<td>27.8</td>
<td>27.8</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Own Elasticity</td>
<td>0.219</td>
<td>0.024</td>
<td>0.036</td>
</tr>
</tbody>
</table>

Table 5b: Substitution Matrices for Random coefficient Logit, Random coefficient Nested Logit and Random coefficient CNFSL – Physician B who is more sensitive to detailing

<table>
<thead>
<tr>
<th></th>
<th>Random coefficient Logit Model</th>
<th>Random coefficient Nested Logit Model</th>
<th>Random coefficient CNFSL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Detailing</td>
<td>DTCA</td>
<td>M&amp;E</td>
</tr>
<tr>
<td>Lipitor</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Zocor</td>
<td>29.0%</td>
<td>29.0%</td>
<td>29.0%</td>
</tr>
<tr>
<td>Pravachol</td>
<td>5.1</td>
<td>5.1</td>
<td>5.1</td>
</tr>
<tr>
<td>Crestor</td>
<td>32.6</td>
<td>32.6</td>
<td>32.6</td>
</tr>
<tr>
<td>Non-drug Treatment</td>
<td>33.3</td>
<td>33.3</td>
<td>33.3</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Own Elasticity</td>
<td>0.3864</td>
<td>0.012</td>
<td>0.029</td>
</tr>
</tbody>
</table>
REFERENCES


CBO (2009), "Promotional Spending for Prescription Drugs," Congressional Budget Office (Ed.).


---- (1975), "On independence, structure, and simultaneity in transportation demand analysis," in Working Paper 7511, Urban Travel Demand Forecasting Project: Institute of Transportation Studies, University of California, Berkeley, CA.


Appendix

Proof:

\[ P_{jn} = P_n P_{jn} \]

\[ = \frac{\sum_m e^{w_m + \log(\sum_{i \in B_m} e^{\mu_{in} - \lambda p_{jn}})}}{\sum_{i \in B_n} e^{\mu_{in} - \lambda p_{jn}}} \frac{\sum_{i \in B_m} e^{\mu_{in} - \lambda p_{jn}}}{\sum_{i \in B_n} e^{\mu_{in} - \lambda p_{jn}}} \]

\[ = \frac{\sum_{i \in B_n} e^{w_n + \mu_{in} - \lambda p_{jn}}}{\sum_{i \in B_n} e^{w_n + \mu_{in} - \lambda p_{jn}}} \frac{\sum_{i \in B_m} e^{w_m + \mu_{in} - \lambda p_{jn}}}{\sum_{i \in B_m} e^{w_m + \mu_{in} - \lambda p_{jn}}} \]

\[ = \sum_m \left( \sum_{i \in B_m} e^{w_m + \mu_{in} - \lambda p_{jn}} \right) \]

Plug in \( w_n = \partial_n + \theta \left( \sum_{i \in B_n} x_i \right) \) and \( \mu_{jn} = \alpha_j + \beta x_j \), we have

\[ P_{jn} = \frac{\partial_n + \theta \left( \sum_{i \in B_n} x_i \right) + \alpha_j + \beta x_j - \lambda p_{jn}}{\sum_m \left( \sum_{i \in B_m} e^{\partial_m + \theta \left( \sum_{i \in B_m} x_i \right) + \alpha_j + \beta x_j - \lambda p_{jn}} \right) } \]

Define \( v_j = \partial_n + \theta \left( \sum_{i \in B_n} x_i \right) + \alpha_j + \beta x_j - \lambda p_{jn} \), we get the derivatives of \( P_{jn} \) with respect to \( v_j \) and \( x_{j_B} \):

\[ \frac{\partial P_{jn}}{\partial v_j} \begin{cases} P_j(1 - P_j) & \text{for } k = j, \text{ and } k, j \in B_n \\ -P_k P_j & \text{for } k \neq j, \text{ and } j \in B_n \end{cases} \]
\[
\frac{\partial v_k}{\partial x_{ja}} = \begin{cases} 
\beta_a + \theta_a & \text{for } k = j \\
\theta_a & \text{for } k \neq j, \text{and } k, j \in B_n \\
0 & \text{for } k \neq j, \text{and } k \in B_m, j \in B_n
\end{cases}
\]

\[
\frac{\partial P_j}{\partial x_{ja}} = \sum_{l=1}^{j} \frac{\partial P_j}{\partial v_l} \cdot \frac{\partial v_l}{\partial x_{ja}} + \sum_{l=j+1}^{j'} \frac{\partial P_j}{\partial v_l} \cdot \frac{\partial v_l}{\partial x_{ja}}
\]

\[
= \frac{\partial P_j}{\partial v_j} \cdot \frac{\partial v_j}{\partial x_{ja}} + \sum_{l=j+1}^{j'} \frac{\partial P_j}{\partial v_l} \cdot \frac{\partial v_l}{\partial x_{ja}}
\]

\[
= P_j \left( 1 - P_j \right) (\beta_a + \theta_a) - \sum_{l \neq j} P_j P_l \theta_a
\]

\[
= P_j \left( (1 - P_j) \beta_a + (1 - P_j) \theta_a \right)
\]

For \( k \neq j \) and \( k, j \in B_n \):

\[
\frac{\partial P_k}{\partial x_{ja}} = \sum_{j=1}^{j'} \frac{\partial P_k}{\partial v_j} \cdot \frac{\partial v_j}{\partial x_{ja}} + \sum_{k \neq j} \frac{\partial P_k}{\partial v_k} \cdot \frac{\partial v_k}{\partial x_{ja}}
\]

\[
= \frac{\partial P_k}{\partial v_j} \cdot \frac{\partial v_j}{\partial x_{ja}} + \sum_{k \neq j} \frac{\partial P_k}{\partial v_k} \cdot \frac{\partial v_k}{\partial x_{ja}}
\]

\[
= -P_j P_k (\beta_a + \theta_a) + P_k (1 - P_k) \theta_a - \sum_{l \neq j,k} P_k P_l \theta_a
\]

\[
= -P_k P_j \beta_a + P_k (1 - P_k) \theta_a
\]

For \( k \neq j \) and \( k \in B_m, j \in B_n \):

\[
\]
\[
\frac{\partial P_k}{\partial x_{ja}} = \sum_{l=1}^{j} \frac{\partial P_k}{\partial v_l} \frac{\partial v_l}{\partial x_{ja}} \\
= \frac{\partial P_k}{\partial v_j} \frac{\partial v_j}{\partial x_{ja}} + \sum_{l \neq j} \frac{\partial P_k}{\partial v_l} \frac{\partial v_l}{\partial x_{ja}} \\
= -P_k P_j (\beta_a + \theta_a) - \sum_{l \neq j} P_k P_l \theta_a \\
= -P_k (P_j \beta_a + P_j \theta_a + (P_n - P_j) \theta_a) \\
= -P_k (P_j \beta_a + P_n \theta_a) \\
\]

\[
\frac{-\partial P_k}{\partial x_{ja}} = \begin{cases} 
\frac{P_k P_j \beta_a - P_k (1 - P_n) \theta_a}{P_j \left((1 - P_j) \beta_a + (1 - P_n) \theta_a\right)} & \text{for } k \neq j, \text{ and } k, j \in B_n \\
\frac{P_k (P_j \beta_a + P_n \theta_a)}{P_j \left((1 - P_j) \beta_a + (1 - P_n) \theta_a\right)} & \text{for } k \neq j, \text{ and } k \in B_m, j \in B_n 
\end{cases}
\]