Volatility, the Macroeconomy and Asset Prices

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ABSTRACT

How important are volatility fluctuations for asset prices and the macroeconomy? We find that a rise in macroeconomic volatility is associated with a rise in discount rates and a decline in consumption. To study the impact of volatility we provide a framework in which cash-flow, discount-rate, and volatility risks determine risk premia. We show that volatility plays a significant role in jointly accounting for returns to human capital and equity. Volatility risks carry a sizeable positive risk premium and help explain the cross-section of expected returns. Our evidence shows that volatility is important for understanding expected returns and macroeconomic fluctuations.

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Recent economic analysis has emphasized the important role of macroeconomic volatility movements in determining asset prices and macro quantities. In the asset pricing model of Bansal and Yaron (2004), an increase in aggregate volatility lowers asset prices and, importantly, shocks to volatility carry a separate risk premium. A growing literature in macroeconomics also highlights the effect of volatility on macro quantities.[1] In this paper, we show that variation in macroeconomic volatility is indeed an important and separate risk that significantly affects the macro economy (aggregate consumption) and asset prices. To guide our analysis we develop a dynamic asset-pricing framework in which the stochastic discount factor and, therefore, the risk premium are determined by three sources of risks: cash-flow, discount rate, and volatility risks. Our empirical work yields three central findings: (i) an increase in volatility is associated with a rise in discount rates and a decline in future consumption; (ii) volatility risks play a significant role in accounting for the joint dynamics of returns to human capital and equity; (iii) volatility risks carry a sizeable positive risk premium and help explain the level and the cross-sectional dispersion of expected returns. In all, our evidence suggests that volatility risk is important for understanding the dynamics of asset prices and macroeconomic fluctuations.

We document that, in the data, both macroeconomic- and return-based volatility measures feature persistent predictable variation, which makes volatility news potentially an important source of economic risks. The earlier works of Bollerslev and Mikkelsen (1996) in the context of market-return volatility, and Kandel and Stambaugh (1991), McConnell and Quiros (2000), Stock and Watson (2002), and Bansal, Khatchatrian, and Yaron (2005) in the context of macroeconomic volatility provide supporting evidence of low-frequency predictable variation in volatility. We incorporate this evidence in our theoretical framework and evaluate the implications of volatility risks for consumption, returns to human capital and equity, and the cross-sectional dispersion in risk premia.

In our dynamic asset-pricing model with time-varying macroeconomic volatility (which we refer to as Macro-DCAPM-SV model), the stochastic discount factor and, therefore, the risk premium are determined by three sources of risks: cash-flow, discount-rate and volatility risks. To identify the underlying economic risks using the standard VAR-based methodology, we model the aggregate wealth return as a weighted average of returns on human capital and financial wealth and assume that the expected return on the human component of wealth is linear in economic states.[2] We estimate the model using the observed macro and financial market data and find that, empirically, high macro-volatility states are high-risk states associated with significant consumption declines, high risk premia and high discount rates. The documented positive relationship between ex-ante volatility and discount rates results in a positive correlation between returns to human capital and financial wealth. This implication is consistent with the standard economic theory, in which the two assets are positively correlated as they both represent claims to aggregate cash flows. In contrast, in a constant volatility setting, Lustig and Van Nieuwerburgh (2008) find that returns to human capital and equity are strongly negatively correlated. Our evidence suggests that their puzzling finding can be resolved once time-variation in economic volatility is taken into account.

Specifically, in the model with constant volatility under benchmark preferences of risk aversion of five and intertemporal elasticity of substitution of two, the correlation between realized returns to human capital and equity is -0.6, and the correlation in their five-year expected returns is -0.5. In our Macro-DCAPM-SV model that incorporates volatility risks, the two assets tend to move together: the correlation in realized returns on human capital and financial wealth is about 0.2, and the correlation in their five-year expected returns is 0.4. We show that the inclusion of volatility risks has important implications for the time-series dynamics of the underlying economic shocks. In particular, in our volatility risk-based model, discount rates are high and positive in recent recessions of 2001 and 2008, which is consistent with a sharp increase in economic volatility and risk premia during those times. In contrast, the constant-volatility specification generates negative discount rate news in the two recessions. We also show that model specifications that ignore volatility risks imply a counter-factual positive correlation between expected consumption and discount rates.

We document that volatility risks carry positive and economically significant risk premia, and help explain the level and the cross-sectional variation in expected returns. The model-implied market price of volatility risk is -1. We show that in the data, all equity portfolios as well as returns on human capital have negative exposure to aggregate volatility risks. Thus, compensation for volatility risks in equity markets is positive. Quantitatively, the model-implied risk premia of the wealth portfolio, human capital and equity are 2.6%, 1.4% and 7.4%, respectively. Volatility risks account for about one-third of the total risk premium of human capital, and about one-half of risk premia of the aggregate and financial wealth portfolios. We show that the Macro-DCAPM-SV model is able to account for the observed dispersion in risk premia across book-to-market and size sorted portfolios. The value spread in the model is 5.5% compared with 5.9% in the data; the size spread is 7.1% and 7.4% in the model and in the data, respectively.

The key time-series and cross-sectional implications of our volatility-based model continue to hold if the aggregate wealth portfolio is measured simply by the return on the stock market.[3] Consistent with the findings in our benchmark macro model, we find that in the market-based (Market-DCAPM-SV) specification all equity portfolios have negative volatility betas, i.e., equity prices fall on positive news about volatility. Given that investors attach a negative price to volatility shocks, volatility risks carry positive premia. We also document a strong co-movement between the risk premium and ex-ante market volatility at both short and more so long horizons, which reflects a positive correlation between discount-rate and volatility risks. Further, we find that in periods of recessions and those with significant economic stress, such as the Great Recession, both discount-rate news and volatility news are large and positive. On average, volatility and discount-rate risks account for about 35% of the overall risk premia in the cross-section, and almost 50% of the total premium of the market portfolio. Our evidence based on the Macro-DCAPM-SV model and the Market-DCAPM-SV specification is consistent in that volatility and discount rates in both cases are strongly positively correlated and volatility risks contribute positively to equity premia.

We show that ignoring volatility risks may result in quantitatively large biases in state prices and misleading inference about underlying sources of risks. The dynamics of consumption, discount-rate and volatility news are intimately linked in equilibrium. Ignoring time-variation in volatility leads to distortions in this equilibrium relationship and, consequently, distortions in the joint dynamics of the other two risks. Using our benchmark Macro-DCAPM-SV model, we find that the dynamics of the stochastic discount factor extracted by relying solely on financial market data and ignoring volatility risks, as in Campbell (1996), is significantly biased. In general, volatility of the implied stochastic discount factor and, hence, the implied risk premia are substantially biased downwards.

The rest of the paper is organized as follows. In Section I we present a theoretical framework for the analysis of volatility risks and their implications for consumption dynamics and the stochastic discount factor. In Section II we empirically implement the Macro-DCAPM-SV model, quantify the role of volatility risks in the data and discuss the model implications for the market, human capital and wealth portfolios, as well as a cross section of equity returns. Section III discusses the implications of the Market-DCAPM-SV specification and the role of volatility risks for explaining the cross-section of assets. Section IV provides concluding remarks.

I. Theoretical Framework

In this section we consider a general economic framework with recursive utility and time-varying economic uncertainty and derive the implications for the innovations into the current and future consumption growth, returns, and the stochastic discount factor. We show that accounting for fluctuations in economic uncertainty is important for a correct inference about economic news, and ignoring volatility risks can alter the implications for the financial markets.

A. *Consumption and Volatility*

We adopt a discrete-time specification of the endowment economy where the agent's preferences are described by a Kreps and Porteus (1978) recursive utility function of Epstein and Zin (1989) and Weil (1989). The life-time utility of the agent U_t satisfies

$$U_t = \left[(1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left(E_t U_{t+1}^{1-\gamma} \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{1}$$

where C_t is the aggregate consumption level, δ is a subjective discount factor, γ is a risk aversion coefficient, ψ is the intertemporal elasticity of substitution (IES), and for notational ease we denote $\theta = (1-\gamma)/(1-\frac{1}{\psi})$. When $\gamma = 1/\psi$, the preferences collapse to a standard expected power utility.

As shown in Epstein and Zin (1989), the stochastic discount discount factor M_{t+1} can be written in terms of the log consumption growth rate, $\Delta c_{t+1} \equiv \log C_{t+1} - \log C_t$, and the log return to the consumption asset (wealth portfolio), $r_{c,t+1}$. In logs,

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}. \tag{2}$$

A standard Euler condition

$$E_t \left[M_{t+1} R_{t+1} \right] = 1 \tag{3}$$

allows us to price any asset in the economy. Assuming that the stochastic discount factor and the consumption asset return are jointly log-normal, the Euler equation for the consumption asset leads to:

$$E_t \Delta c_{t+1} = \psi \log \delta + \psi E_t r_{c,t+1} - \frac{\psi - 1}{\gamma - 1} V_t, \tag{4}$$

where we define V_t to be the conditional variance of the stochastic discount factor plus the consumption asset return:

$$\begin{aligned} V_t &= \frac{1}{2} Var_t(m_{t+1} + r_{c,t+1}) \\ &= \frac{1}{2} Var_t m_{t+1} + Cov_t(m_{t+1}, r_{c,t+1}) + \frac{1}{2} Var_t r_{c,t+1}. \end{aligned} \tag{5}$$

The volatility component V_t is equal to the sum of the conditional variances of the discount factor and the consumption return and the conditional covariance between the two, which are directly related to the movements in aggregate volatility and risk premia in the economy. Hence, V_t is a measure of aggregate economic volatility. In our subsequent discussion we show that, under further model restrictions, V_t is proportional to the conditional variance of future aggregate consumption, and the proportionality coefficient is always positive and depends only on the risk aversion coefficient. As can be seen from Equation (4), economic volatility shocks are not separately reflected in expected consumption when there is no stochastic volatility in the economy (i.e., V_t is a constant), or when the IES parameter is one, $\psi = 1$. The case without variation in volatility has been entertained in Campbell (1996), Campbell and Vuolteenaho (2004), and Lustig and Van Nieuwerburgh (2008). In this paper, we argue that variation in volatility is important for interpreting movements in consumption and asset markets.

We use the equilibrium restriction in the Equation (4) to derive the immediate consumption news. The return to the consumption asset $r_{c,t+1}$ which enters the equilibrium condition in Equation (4) satisfies the usual budget constraint:

$$W_{t+1} = (W_t - C_t) R_{c,t+1}. \tag{6}$$

A standard log-linearization of the budget constraint yields:

$$r_{c,t+1} = \kappa_0 + wc_{t+1} - \frac{1}{\kappa_1} wc_t + \Delta c_{t+1}, \tag{7}$$

where $wc_t \equiv \log(W_t/C_t)$ is the log wealth-to-consumption ratio, and κ_0 and κ_1 are the linearization parameters. Solving the recursive equation forward, we obtain that the immediate consumption innovation can be written as the revision in expectation of future returns on consumption asset minus the revision in expectation of future cash flows:

$$c_{t+1} - E_t c_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j r_{c,t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j \Delta c_{t+j+1}. \tag{8}$$

Using the expected consumption relation in Equation (4), we can further express the consumption shock in terms of the immediate news in consumption return, $N_{R,t+1}$, revisions of expectation of future returns (discount rate news), $N_{DR,t+1}$, as well as news about future volatility $N_{V,t+1}$:

$$N_{C,t+1} = N_{R,t+1} + (1-\psi) N_{DR,t+1} + \frac{\psi - 1}{\gamma - 1} N_{V,t+1}, \tag{9}$$

where for convenience we denote

$$\begin{aligned}
&N_{C,t+1} \equiv c_{t+1} - E_t c_{t+1}, && N_{R,t+1} \equiv r_{c,t+1} - E_t r_{c,t+1}, \\
&N_{DR,t+1} \equiv (E_{t+1} - E_t) \left(\sum_{j=1}^{\infty} \kappa_1^j r_{c,t+j+1} \right), && N_{V,t+1} \equiv (E_{t+1} - E_t) \left(\sum_{j=1}^{\infty} \kappa_1^j V_{t+j} \right), \\
&N_{CF,t+1} \equiv (E_{t+1} - E_t) \left(\sum_{j=0}^{\infty} \kappa_1^j \Delta c_{t+j+1} \right) = N_{DR,t+1} + N_{R,t+1} &&
\end{aligned} \tag{10}$$

To highlight the intuition for the relationship between consumption, asset prices and volatility, let us define news in future expected consumption, $N_{ECF,t+1}$:

$$N_{ECF,t+1} = (E_{t+1} - E_t) \left(\sum_{j=1}^{\infty} \kappa_1^j \Delta c_{t+j+1} \right). \tag{11}$$

Note that the consumption innovation in Equation (4) implies that news in future expected consumption can be decomposed into discount-rate news to the wealth portfolio and news in economic volatility:

$$N_{ECF,t+1} = \psi N_{DR,t+1} - \frac{\psi - 1}{\gamma - 1} N_{V,t+1}. \tag{12}$$

In a similar way, we can decompose the shock in the wealth-to-consumption ratio into expected consumption and volatility news:

$$\begin{aligned}(E_{t+1} - E_t) wc_{t+1} &= N_{ECF,t+1} - N_{DR,t+1} \\ &= \left(1 - \frac{1}{\psi}\right)\left(N_{ECF,t+1} - \frac{1}{\gamma - 1} N_{V,t+1}\right).\end{aligned} \tag{13}$$

When the IES is equal to one, the substitution effect is equal to the income effect, so future expected consumption news moves one-to-one with the discount-rate news. As the two news exactly offset each other, the wealth-to-consumption ratio is constant so that the agent consumes a constant fraction of total wealth. On the other hand, when the IES is not equal to one and aggregate volatility is time-varying, movements in expected consumption no longer correspond to movements in discount rates.[4] In Sections II and III we show that in the data, “bad” economic times are associated with low future expected growth, high risk premia and high uncertainty; that is, volatility news co-move significantly positively with discount-rate news and negatively with cash-flow news. This evidence is consistent with the economic restriction in Equation (12) in the presence of volatility risks and IES above one. Note that when volatility news is ignored, the structural Equation (12) would imply that news to future consumption and discount rates are perfectly positively correlated, so that bad times of high volatility and high discount rates would correspond to good times of positive news to future consumption. This stands in a stark contrast to the empirical observations and economic intuition, and highlights the importance of volatility risks to correctly interpret the movements in consumption and asset prices.

B. *Asset Prices and Volatility*

The innovation into the stochastic discount factor implied by the representation in Equation (2) is given by,

$$N_{m,t+1} \equiv m_{t+1} - E_t m_{t+1} = -\frac{\theta}{\psi}(\Delta c_{t+1} - E_t \Delta c_{t+1}) + (\theta - 1)(r_{c,t+1} - E_t r_{c,t+1}). \tag{14}$$

Substituting out consumption shock using Equation (9), we obtain that the stochastic discount factor is driven by future cash flow news, $N_{CF,t+1}$, future discount rate news, $N_{DR,t+1}$, and volatility news, $N_{V,t+1}$:

$$m_{t+1} - E_t m_{t+1} = -\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1}. \tag{15}$$

As shown in the above equation, the market price of cash-flow risk is γ, and the market prices of volatility and discount rate news are equal to negative 1. Notably, volatility risks are present at any values of the IES. Thus, even though with IES equal to one, volatility news do not directly affect consumption innovation as shown in Equation (9), the stochastic discount factor still carries volatility risks. Ignoring them will lead to incorrect inference and can significantly affect the interpretation of the asset markets.[5]

Given this decomposition for the stochastic discount factor, we can rewrite the expression for the ex-ante economic volatility V_t in (5) in the following way:

$$\begin{aligned} V_t &= \frac{1}{2} Var_t(m_{t+1} + r_{c,t+1}) \\ &= \frac{1}{2} Var_t\left(-\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1} + N_{R,t+1}\right) \\ &= \frac{1}{2} Var_t\left((1-\gamma) N_{CF,t+1} + N_{V,t+1}\right), \end{aligned} \tag{16}$$

where in the last equation we use the identity that the sum of the immediate and discount rate news on the wealth portfolio is equal to the cash-flow news. Consider the case when variance of volatility news $N_{V,t+1}$ and its covariance with cash-flow news are constant (i.e., volatility shocks are homoscedastic). In this case, V_t is driven by variance of current and future consumption news, where the proportionality coefficient is determined only by the coefficient of risk-aversion:

$$V_t = const + \frac{1}{2}(1-\gamma)^2 Var_t(N_{CF,t+1}). \tag{17}$$

Hence, news in V_t corresponds to news in the future variance of long-run consumption shocks; in this sense, V_t is the measure of the ex-ante economic volatility. Further, note that when there is a single consumption volatility factor, we can identify V_t from the rescaled volatility of immediate consumption news, $V_t = const + \frac{1}{2}(1-\gamma)^2 \chi Var_t(\Delta c_{t+1})$, where χ is the scaling factor equal to the ratio of variance of long-run consumption growth to variance of current consumption growth,

$$\chi = Var(N_{CF})/Var(N_C). \tag{18}$$

We impose this structural restriction to identify economic volatility shocks in our empirical work.

Using Euler equation, we obtain that the risk premium on any asset is equal to the negative covariance of asset return $r_{i,t+1}$ with the stochastic discount factor:

$$E_t r_{i,t+1} - r_{ft} + \frac{1}{2} Var_t r_{i,t+1} = Cov_t(-m_{t+1}, r_{i,t+1}). \tag{19}$$

Hence, knowing exposure (betas) of a return to the fundamental sources of risk, we can calculate the risk premium on the asset, and decompose it into risk compensation for cash-flow, discount rate, and volatility news:

$$\begin{aligned} &E_t r_{i,t+1} - r_{ft} + \frac{1}{2} Var_t r_{i,t+1} \\ &= \gamma Cov_t(r_{i,t+1}, N_{CF,t+1}) - Cov_t(r_{i,t+1}, N_{DR,t+1}) - Cov_t(r_{i,t+1}, N_{V,t+1}). \end{aligned} \tag{20}$$

This risk premia restriction is an asset pricing model with three distinct sources of risk. Cash-flow news corresponds to news about future consumption, discount-rate news corresponds to news about expected return on aggregate wealth, and volatility news corresponds to news about macroeconomic volatility. As stated above, we denote this specification as the Macro-DCAPM-SV model. This specification emphasizes the importance of measuring news using macro economic data. In Section II we provide a way to estimate such a model and evaluate its empirical implications. It is important to note that the measurement of the return to aggregate wealth entails measuring the return to human capital along with the return to financial assets. Consequently, our Macro-DCAPM-SV approach will have implications for the joint dynamics of the return to human capital, the market return, and the entire cross section of asset returns. Through the Macro-DCAPM-SV model we can evaluate the puzzling negative correlation between human capital and market return as highlighted in Lustig and Van Nieuwerburgh (2008), as well the implications for the cross section of asset returns.

A significant body of empirical work in finance replaces the return to aggregate wealth with the financial market return, essentially assuming that the two returns are the same (see, for example, the empirical work based on the static CAPM).[6] Epstein and Zin (1991) use this approach in the context of recursive preferences. While returns to aggregate and financial wealth may not entirely coincide, the ready availability of the extensive stock market data makes it a convenient and easy-to-implement approach. To provide a comprehensive account of volatility risks and their importance for asset prices, we also entertain this approach in Section III. In this specification of the model, cash-flow, discount-rate and volatility news are measured using the available data on the value-weighted stock market return and its realized volatility. We refer to this reduced-form specification as the Market DCAPM with Stochastic Volatility and denote it as Market-DCAPM-SV. By its very nature, this approach cannot be used to address issues related to the co-movement of human capital and financial market returns, yet we can still exploit it to highlight the implications of time-varying volatility for the cross-section of equity returns.

II. Macro-DCAPM-SV Model

In this section we develop and implement a Macro-DCAPM-SV model to quantify the role of the volatility channel for asset markets. As the aggregate consumption return (i.e., aggregate wealth return) is not directly observed in the data we assume that it is a weighted combination of the return to the stock market and human capital. This allows us to adopt a standard VAR-based methodology to extract the innovations to consumption return, volatility, SDF, and assess the importance of the volatility channel for returns to human capital and equity.

A. Econometric Specification

Denote X_t a vector of state variables which include annual real consumption growth Δc_t, real labor income growth Δy_t, real market return $r_{d,t}$, market price-dividend ratio z_t, and the realized variance measure RV_t :

$$X_t = \begin{bmatrix} \Delta c_t & \Delta y_t & r_{d,t} & z_t & RV_t \end{bmatrix}'. \tag{21}$$

For parsimony, we focus on a minimal set of economic variables in our benchmark empirical analysis, and in Section II.E we confirm that our main results are robust to the choice of predictive variables, volatility measurements and estimation strategy.

The vector of state variables X_t follows a VAR(1) specification, which we refer to as Macro VAR:

$$X_{t+1} = \mu_X + \Phi X_t + u_{t+1}, \tag{22}$$

where Φ is a persistence matrix and μ_X is an intercept. Shocks u_{t+1} are assumed to be conditionally Normal with a time-varying variance-covariance matrix Ω_t.

To identify the fluctuations in the aggregate economic volatility, we include as one of the state variables a realized variance measure based on the sum of squares of monthly industrial production growth over the year:

$$RV_{t+1} = \sum_{j=1}^{12} \Delta ip^2_{t+j/12}. \tag{23}$$

Constructing the realized variance from the monthly data helps us capture more accurately the fluctuations in the aggregate macroeconomic volatility in the data, and we use industrial production because high-frequency real consumption data is not available for a long sample. For robustness, we checked that our results do not materially change if we instead construct the measure based on the realized variance of annual consumption growth. To ensure consistency, we re-scale industrial production based realized variance to match the average level of consumption variance.

The expectations of RV_{t+1} implied by the dynamics of the state vector capture the ex-ante macroeconomic volatility in the economy. Following the derivations in Section I, the economic volatility V_t then becomes proportional to the ex-ante expectation of the realized variance RV_{t+1} based on the VAR(1):

$$\begin{aligned} V_t &= V_0 + \frac{1}{2}\chi(1-\gamma)^2 E_t RV_{t+1} \\ &= V_0 + \frac{1}{2}\chi(1-\gamma)^2 i_v' \Phi X_t, \end{aligned} \tag{24}$$

where V_0 is an unimportant constant which disappears in the expressions for shocks, i_v is a column vector which picks out the realized variance measure from X_t, and χ is a scale parameter which captures the link between the aggregate consumption volatility and V_t. In the model with volatility risks, we fix the value of χ to the ratio of the variances of the cash-flow to immediate consumption news, consistent with the theoretical restriction in Section I. In the specification where volatility risks are absent, the parameter χ is set to zero. Then, following standard VAR-based derivations, the revisions in future expectations of the economic volatility can be calculated in the following way:

$$N_{V,t+1} = \frac{1}{2}\chi(1-\gamma)^2 i_v'(I+Q)u_{t+1}, \tag{25}$$

where Q is the matrix of the long-run responses, $Q = \kappa_1 \Phi \left(I - \kappa_1 \Phi\right)^{-1}$.

The VAR specification implies that shocks into immediate market return, $N^d_{R,t+1}$, and future market discount rate news, $N^d_{DR,t+1}$, are given by[7]

$$N^d_{R,t+1} = i_r' u_{t+1}, \quad N^d_{DR,t+1} = i_r' Q u_{t+1}, \tag{26}$$

where i_r is a column vector which picks out market return component from the set of state variables X_t. While the market return is directly observed and the market return news can be extracted directly from the VAR(1), in the data we can only observe the labor income but not the total return to human capital. We make the following identifying assumption, identical to Lustig and Van Nieuwerburgh (2008), that expected labor income return is linear in the state variables:

$$E_t r_{y,t+1} = \alpha + b' X_t, \tag{27}$$

where b captures the loadings of expected human capital return to the economic state variables. Given this restriction, the news into future discounted human capital returns, $N^y_{DR,t+1}$, is given by,

$$N^y_{DR,t+1} = b' \Phi^{-1} Q u_{t+1}, \tag{28}$$

and the immediate shock to labor income return, $N^y_{R,t+1}$, can be computed as follows:

$$\begin{aligned} N^y_{R,t+1} &= (E_{t+1} - E_t)\left(\sum_{j=0}^{\infty} \kappa_1^j \Delta y_{t+j+1}\right) - N^y_{DR,t+1} \\ &= i_y'(I+Q)u_{t+1} - b'\Phi^{-1}Qu_{t+1}, \end{aligned} \tag{29}$$

where the column vector i_y picks out labor income growth from the state vector X_t.

To construct the aggregate consumption return (i.e.,aggregate wealth return), we follow Jagannathan and Wang (1996), Campbell (1996), Lettau and Ludvigson (2001) and Lustig and Van Nieuwerburgh (2008) and assume that it is a portfolio of the returns to the stock market and returns to human capital:

$$r_{c,t} = (1-\omega)r_{d,t} + \omega r_{y,t}. \tag{30}$$

The share of human wealth in total wealth ω is assumed to be constant. It immediately follows that the immediate and future discount rate news on the consumption asset are equal to the weighted average of the corresponding news to the human capital and market return, with a weight parameter ω :

$$\begin{aligned} N_{R,t+1} &= (1-\omega)N^d_{R,t+1} + \omega N^y_{R,t+1}, \\ N_{DR,t+1} &= (1-\omega)N^d_{DR,t+1} + \omega N^y_{DR,t+1}. \end{aligned} \tag{31}$$

These consumption return innovations can be expressed in terms of the VAR(1) parameters and shocks and the vector of the expected labor return loadings b following Equations (26)-(29).

Finally, we can combine the expressions for the volatility news and return news on the consumption asset to back out the implied consumption shock following the Equation (9):

$$\begin{aligned} c_{t+1} - E_t c_{t+1} &= N_{R,t+1} + (1-\psi) N_{DR,t+1} + \frac{\psi - 1}{\gamma - 1} N_{V,t+1} \\ &= \underbrace{\left[(1-\omega) i_r' Q + \omega (i_y'(I+Q) - b'\Phi^{-1} Q)\right] u_{t+1}}_{N_{R,t+1}} \\ &+ (1-\psi) \underbrace{\left[(1-\omega) i_r' Q + \omega b' \Phi^{-1} Q\right] u_{t+1}}_{N_{DR,t+1}} + \left(\frac{\psi - 1}{\gamma - 1}\right) \underbrace{\frac{1}{2} \chi (1-\gamma)^2 i_v' Q u_{t+1}}_{N_{V,t+1}} \\ &\equiv q(b)' u_{t+1}. \end{aligned} \tag{32}$$

The vector $q(b)$ defined above depends on the model parameters, and in particular, it depends linearly on the expected labor return loadings b. On the other hand, as consumption growth itself is one of the state variables in X_t, it follows that the consumption innovation from Macro VAR satisfies,

$$c_{t+1} - E_t c_{t+1} = i_c' u_{t+1}, \tag{33}$$

where i_c is a column vector which picks out consumption growth out of the state vector X_t. We impose this important consistency requirement that the model-implied consumption shock in Equation (32) matches the VAR consumption shock in (33), so that

$$q(b) \equiv i_c, \tag{34}$$

and solve the above equation, which is linear in b, to back out the unique expected human capital return loadings b. That is, in our approach the specification for the expected labor return ensures that the consumption innovation implied by the model is identical to the consumption innovation in the data.

B. Data and Estimation

[Place Table I about here]

In our empirical analysis, we use an annual sample from 1930 to 2010. Real consumption corresponds to real per capita expenditures on non-durable goods and services, and real income is the real per capita disposable personal income; both series are taken from the Bureau of Economic Analysis. Market return data is for the value-weighted stock market portfolio from CRSP. The summary statistics for these variables are presented in Table I. The average labor income and consumption growth rates are about 2%. The labor income is more volatile than consumption growth, but the two series co-move quite closely in the data with the correlation coefficient of 0.80. The average log market return is 5.7%, and its volatility is 20%. The realized variance of industrial production is quite persistent and volatile in the data. It is strongly countercyclical: in recessions, the realized variance is on average 40% above its unconditional mean. Further, the realized variance co-moves negatively with the market price-dividend ratio: the correlation coefficient between the two series is -0.25, so that asset prices fall at times of high macroeconomic volatility, consistent with findings in Bansal, Khatchatrian, and Yaron (2005).

[Place Table II about here]

We estimate the Macro VAR specification in (22) using equation-by-equation OLS. For robustness, we also consider a GLS approach in which we incorporate the information in the conditional variance of the residuals; the results are very similar, and are discussed in the robustness section. The Macro VAR estimation results are reported in Table II. The magnitudes of the R^2s in the regressions vary from 10% for the market return to 80% for the price-dividend ratio. Notably, the consumption and labor income growth rates are quite predictable in this rich setting, with the R^2 of 60% and nearly 40%, respectively. Because of the correlation between the variables, it is hard to interpret individual slope coefficients in the regression. Note, however, that the ex-ante consumption volatility is quite persistent with an autocorrelation coefficient of 0.63 on annual frequency, and it loads significantly and negatively on the market price-dividend ratio. Notably, the ex-ante volatility process is less volatile and more persistent than the realized volatility.

[Place Table III about here]

[Place Figure 1 about here]

We plot the ex-ante consumption volatility and the expected consumption growth rate on Figure 1. Our evidence underscores persistent fluctuations in ex-ante macroeconomic volatility and a gradual decline in the volatility over time, which is similar to the findings in McConnell and Quiros (2000) and Stock and Watson (2002).[8] Notably, the volatility process is strongly counter-cyclical: its correlation with the NBER recession indicator is -40%, and it is -30% with the expected real consumption growth from the Macro VAR. Consistent with this evidence, the news in future expected consumption implied by the Macro VAR, N_{ECF}, is sharply negative at times of high volatility. As shown in Table III, future expected consumption news is on average -1.70% at times of high (top 25%) versus 2.2% in low (bottom 25%) volatility times. To show the dynamic impact of volatility news on consumption, we compute an impulse response of consumption growth to one standard deviation shock in ex-ante consumption volatility, $Var_t \Delta c_{t+1}$ (see Appendix for the details of the computations). Based on our estimation results, one standard deviation volatility shock corresponds to a $(1.95\%)^2$ increase in ex-ante consumption variance. As shown in Figure 2, consumption growth declines by almost 1% on the impact of positive volatility news and remains negative up to five years in the future. The response of the labor income growth is similar: labor income growth drops by 2% on the impact of positive volatility news, and the response remains negative up to five years in the future.

[Place Figure 2 about here]

Given the estimates of our Macro VAR specification, we can compute the volatility news and the stock market return news following the derivations in Section II. To derive the implications for the human capital and wealth portfolio returns, we set the risk aversion coefficient γ to 5 and the IES parameter ψ to 2. The share of human wealth in the overall wealth ω is set to 0.8, the average value used in Lustig and Van Nieuwerburgh (2008). In the full model specification featuring volatility risks we fix the volatility parameter χ to the ratio of the volatilities of long-run to immediate consumption news, according to the restriction in Equation (18). To discuss the model implications in the absence of volatility risks, we set χ equal to zero.

[Place Figure 3 about here]

In the full model with volatility risks, the volatility news is strongly correlated with the discount rate news in the data. As documented in Table III, the correlation between the volatility news and the discount rate news on the market reaches nearly 90%, and the correlations of the volatility news with the discount rate news to labor return and the wealth portfolio are 30% and 80%, respectively. A high correlation between the volatility news and the discount rate news to the wealth return is evident on Figure 3. These findings are consistent with the intuition of the economic long-run risks model where a significant component of the discount rate news is driven by shocks to consumption volatility. On the other hand, when the volatility risks are absent, the discount rate news no longer reflects the fluctuations in the volatility, but rather tracks the revisions in future expectations of consumption. As a result, the correlation of the implied discount rate news with our earlier measure of the volatility news becomes -0.85 for the labor return, and -0.30 for the wealth portfolio. The implied discount rate news ignoring volatility risks is very different from the discount rate news when volatility risks are taken into account. For example, when volatility risks are accounted for, the measured discount rate news is 5.3% in the latest recession of 2008 and 1% in 2001. Without the volatility channel, however, it would appear that the discount rate news is negative at those times: the measured discount rate shock is -2.8%

in 2008 and -0.7% in 2001. Thus, ignoring the volatility channel, the discount rate on the consumption asset can be significantly mis-specified due to the omission of the volatility risk component, which would alter the dynamics of the wealth returns as we discuss in subsequent section.

C. Macro-DCAPM-SV: Labor and Market Returns

[Place Table IV about here]

Table IV reports the evidence on the correlations between immediate and future returns on the market, human capital and wealth portfolio. Without the volatility risk channel, shocks to the market and human capital returns are significantly negatively correlated, which is consistent with the evidence in Lustig and Van Nieuwerburgh (2008). Indeed, as shown in the top panel of the Table, the correlations between immediate stock market and labor income return news, $N^d_{R,t+1}$ and $N^y_{R,t+1}$, the discount rate news, $N^d_{DR,t+1}$ and $N^y_{DR,t+1}$, and the expected 5-year returns, $E_t r^d_{t\to t+5}$ and $E_t r^y_{t\to t+5}$, range between -0.50 and -0.72. All these correlations turn positive when the volatility channel is present: the correlation of immediate return news increases to 0.19; and for discount rates and the expected 5-year returns it goes up to 0.25 and 0.39, respectively. Figure 4 plots the implied time-series of long-term expected returns on the market and human capital. A negative correlation between the two series is evident in the model specification which ignores volatility risks. The evidence for the co-movements of returns is similar for the wealth and human capital, and the market and wealth portfolios, as shown in the middle and lower panels of Table IV. Because the wealth return is a weighted average of the market and human capital returns, these correlations are in fact positive without the volatility channel. These correlations increase considerably and become closer to one once volatility risks are introduced. For example, the correlation between market return and wealth return news is 80% with the volatility risk channel, while without volatility risks the correlation is 46%. Similarly, the correlations between five-year expected returns of the market and aggregate wealth portfolios are 79% and 27% with and without volatility risks, respectively.

[Place Figure 4 about here]

To understand the role of the volatility risks for the properties of the wealth portfolio, consider again the consumption equation in (12), which explicitly accounts for the different discount rates for human capital and the market:

$$\begin{aligned}(E_{t+1}-E_t)\left(\sum_{j=1}^{\infty}\kappa_1^j\Delta c_{t+j+1}\right) &= \psi N_{DR,t+1}-\frac{\psi-1}{\gamma-1}N_{V,t+1}\\ &= \psi\left(\omega N^y_{DR,t+1}+(1-\omega)N^d_{DR,t+1}\right)-\frac{\psi-1}{\gamma-1}N_{V,t+1}\end{aligned} \tag{35}$$

When the volatility risks are not accounted for, $N_V = 0$ and all the variation in the future cash flows is driven by news to discount rates on the market and the human capital. However, as shown in Table III, in the data consumption growth is much smoother than asset returns: the volatility of cash-flow news is about 5% relative to 15% for the discount rate news on the market. Hence, to explain relatively smooth variation in cash flows in the absence of volatility news, the discount rate news to human capital must offset a large portion of the discount rate news on the market, which manifests into a large negative correlation between the two returns documented in Table IV. On the other hand, when volatility news is accounted for, they remove the risk premia fluctuations from the discount rates and isolate the news in expected cash flows. Indeed, a strong positive correlation between volatility news and discount rate news in the data is evident in Table III. This allows the model to explain the link between consumption and asset markets without forcing a negative correlation between labor and market returns.

[Place Table V about here]

We use the extracted news components to identify the innovation into the stochastic discount factor, according to Equation (15), and document the implications for the risk premia in Table V. At our calibrated preference parameters, in the model with volatility risks the risk premium on the market is 7.4%; it is 2.6% for the wealth portfolio, and 1.4% for the labor return. Interestingly the risk premium for the wealth return is very similar to estimates in Lustig, Nieuwerburgh, and Verdelhan (2011) who use a different empirical approach to measure the wealth consumption ratio. Most of the risk premium comes from the cash-flow and volatility risks. The contribution of the volatility risk ranges from about a third of the total risk premium on the labor return, to a half for the wealth portfolio and about 60% for the market. Cash-flow news contribute about one-third to the risk premium on the market, and 60% to the risk premium on the labor return. Both cash-flow and volatility risks contribute about equally to the risk premium on the consumption asset. The discount rate shocks contribute virtually nothing to the risk premia across all the assets. Without the volatility channel, the risk premia are 2.4% for the market, 0.9% for the wealth return and 0.5% for the labor return, and essentially all of the risk premium reflects the compensation for the cash-flow risk.

[Place Table VI about here]

The main results in the paper are obtained with the benchmark preference parameters $\gamma = 5$ and $\psi = 2$ and share of human capital $\omega = 0.80$. In Table VI we document the model implications for a range of the IES parameter from 0.5 to 3.0, and for a value of $\omega = 0.85$. Without the volatility channel, the correlations between labor and market returns are negative and large at all considered values for the preference parameters, which is consistent with the evidence in Lustig and Van Nieuwerburgh (2008). In the model with volatility risks, one requires IES sufficiently above one to generate a positive link between labor and market returns – with IES below one, the volatility component no longer offsets risk premia variation in the consumption equation, which makes the labor-market return correlations even lower than in the case without volatility risks. At our benchmark value of human capital share $\omega = 0.80$, the IES parameter of 2 enables us to achieve positive correlations. For higher values of human capital share, $\omega = 0.85$, the correlations turn positive at lower values of IES. The evidence is similar for other values of risk aversion parameter. Higher values for risk aversion lead to higher risk premium, that is why we chose a moderate values of γ in our analysis.

D. Macro-DCAPM-SV: The Cross-Section of Assets

In addition to the market, human capital and wealth returns, we consider asset-pricing implications for a broader cross-section of assets which includes the five size and five value (B/M) sorted portfolios.

To evaluate the model implication we use the risk premia restriction as in equation (20). Specifically, we use the extracted news to construct the innovation in the stochastic discount factor and price a cross section of equity returns by exploiting the Euler equation, i.e.,

$$\begin{aligned} E_t\left[R_{i,t+1} - R_{ft}\right] &= -Cov_t\left(m_{t+1} - E_t m_{t+1}, r_{i,t+1} - E_t r_{i,t+1}\right) \\ &= \gamma Cov_t\left(N_{CF,t+1}, \epsilon_{i,t+1}\right) - Cov_t\left(N_{DR,t+1}, \epsilon_{i,t+1}\right) - Cov_t\left(N_{V,t+1}, \epsilon_{i,t+1}\right) \end{aligned} \tag{36}$$

where $E_t\left[R_{i,t+1} - R_{ft}\right]$ is the arithmetic risk premium of asset i, and $\epsilon_{i,t+1} \equiv r_{i,t+1} - E_t r_{i,t+1}$ is the innovation into asset-i return. It is important to emphasize that we evaluate the model implications under the estimated Macro VAR parameters and for the benchmark risk aversion and IES of 5 and 2 respectively, and impose the model's restrictions on the market prices of discount-rate and volatility risks.

From the log-linear approximation of return:

$$r_{i,t+1} = \kappa_{i,0} + \Delta d_{i,t+1} + \kappa_{i,1} z_{i,t+1} - z_{i,t} \ , \tag{37}$$

it follows that the return innovation depends on the dividend innovation and that to the price-dividend ratio. To measure cash-flow risks and dividend shocks we use an econometric approach that is similar to the one in Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2008) and Bansal, Dittmar, and Kiku (2009). This economically structured approach allows for sharper identification of low-frequency aggregate risks in cash-flows (dividends). For each equity portfolio, we estimate its cash-flow exposure (ϕ_i) by regressing portfolio's dividend growth rate, $\Delta d_{i,t}$, on the two-year moving average of consumption growth, $\overline{\Delta c}_{t-1\to t}$, i.e.,

$$\Delta d_{i,t} = \mu_i + \phi_i \overline{\Delta c}_{t-1\to t} + \epsilon^d_{i,t} \ , \tag{38}$$

The innovation in the price-dividend ratio, $\epsilon^z_{i,t+1}$, is obtained by regressing $z_{i,t}$ on the VAR predictive variables. It then follows that the return innovation of portfolio i is given by:

$$r_{i,t+1} - E_t r_{i,t+1} = \phi_i(\Delta c_{t+1} - E_t \Delta c_{t+1}) + \epsilon^d_{i,t+1} + \kappa_i \epsilon^z_{i,t+1} \ , \tag{39}$$

where $\Delta c_{t+1} - E_t \Delta c_{t+1}$ is the VAR-based innovation in consumption growth. We find that this return innovation, as should be the case, is not predicted by any of the VAR predictive variables we use.[9]

[Place Table VII about here]

According to Equation (20), the model risk premium on any portfolio can be decomposed into the contribution of cash-flow, discount rate and volatility news. The risk compensation for each of the three risk is given by the product of the covariance of return innovation with the corresponding news (return beta) and the market price of risk; in our Macro-DCAPM-SV specification, the market price of cash-flow risk is equal to the risk aversion coefficient $\gamma = 5$, and it is -1 for both the discount and volatility risks. Table VII documents the risk premia in the data and in the model, as well as the beta of each portfolio to cash-flow, discount rate and volatility risk. As shown in the Table, our Macro-DCAPM-SV model can account very well for the cross-sectional spread in the equity premium across book-to-market and size portfolios: the value spread is 5.5% in the model relative to 5.9% in the data, and the size spread is 7.1% and 7.4% in the model and in the data, respectively. Most of the risk premium comes from the cash-flow and volatility news, while the discount rate shocks contribute quite little to the overall risk premium. Specifically, the cash-flow betas increase monotonically from low to high book-to-market portfolios, and from large to small size portfolios, so both the level and the relative contribution of the cash-flow risk increases with the overall risk premium in the cross-section. Indeed, for the book-to-market portfolios, the cash-flow risk premium goes up monotonically from 2% for growth firms to 11.5% for value firms, and in relative terms, the contribution of the cash-flow risk increases from one-quarter to almost 90% of the total premium, respectively. Similarly, for the size portfolios, the relative contribution of the cash-flow risk decreases from two-thirds of the total premium for small firms, to about one-third for the large firms. The volatility risk accounts for the remaining part of the risk premium. The contribution of the volatility risk is highest for growth portfolio and the portfolio of large firms, where it accounts for 60% and 70% of the total premium, respectively, and it goes down to about 30% for the value firms and small firms. On average, both cash-flow and volatility risks account for about 50% of the total risk premium each, while discount rates contribute virtually nothing to the risk premium. The magnitude of the contributions of each source of risks in the cross-section of book-to-market and size portfolios is consistent with our findings for the labor, market, and consumption return.

Without the volatility risks, the magnitudes of the risk premia are much smaller: the risk aversion has to be increased to about 15 to match the risk premia levels in the data. Without volatility risks, nearly all the risk premium is attributed to the cash-flow risk.

E. Robustness

We conduct a number of robustness checks to ensure that our main results are not sensitive to volatility measurements, choice of the predictive variables, and sample period.

In our benchmark Macro-DCAPM-SV, we measure the realized variance using squared monthly industrial production growth rates, scaled to match the overall consumption volatility. First, we check that our results remain broadly similar if we instead compute the realized variance based on the square of the annual consumption growth. Adjusting the risk aversion to target the market risk premium, we obtain that the implied correlations between immediate news to the stock market and labor return is 0.20; it is 0.01 for the discount rate news and 0.26 for the 5-year expected returns. The model fit for the cross sectional evidence is materially unchanged.

To evaluate the robustness to alternative VAR predictive variables we augment the VAR to include (i) risk free rate, (ii) risk free rate and term spread, (iii) risk free rate, term spread, and default spread. In all cases our result do not materially change. The correlation between the labor return and the market return as well as the cross section of return are similar to the benchmark results. For example, if we include the data on interest rate, slope of the term structure and default spread into our benchmark Macro VAR specification, the implied correlation between the immediate returns to human capital and stock market is 0.13, it is 0.07 for the discount rates and 0.33 for the 5-year returns.

Our empirical evidence remains quantitatively similar if we relax the assumption that volatility of volatility shocks is constant and, in estimation, explicitly account for variation in conditional second moments. In a more general set-up that we consider, we allow for time-variation in the variance of volatility shocks and estimate the model using generalized least squares (GLS). To keep estimation tractable, here we assume that the dynamics of the variance-covariance matrix of the VAR innovations in Equation (22) are governed by the conditional variance of consumption innovation, i.e., $\Omega_t = \sigma_t^2 \Omega$, where $\sigma_t^2 = Var_t(\Delta c_{t+1})$. Similar to our benchmark specification, variation in V_t in this case is proportional to σ_t^2, i.e.,

$$V_t = \xi \, \sigma_t^2, \tag{40}$$

where ξ is a non-linear function of the underlying preference and time-series parameters and is provided in Appendix C. The estimation is carried out by imposing restrictions that guarantee positivity of the estimate of the conditional variance, and under constraints that limit variation in GLS weights to ensure sensible time-series estimates. Consistent with the evidence presented and discussed above, we find that volatility and discount-rate news in this generalized specification are strongly positively correlated. Likewise, the correlation between market and human-capital returns is positive and is equal to 0.31 and 0.45 for realized and discount-rate news, respectively. Allowing for time-varying volatility of volatility shocks increases the relative contribution of volatility risks to risk premia but the increase is fairly marginal. On average across portfolios, volatility risks account for about 53% of the overall risk premia compared with 50% in the benchmark case.

Finally, we check that our results are robust across the sub-periods. Using the post war subsample based on our benchmark VAR results in a correlation between the immediate market return and human capital return of 0.27, 0.26 for the discount rate news, and 0.52 for the expected returns. The cross sectional evidence, again, remains similar.

F. *The Importance of Volatility and Model Misspecification*

[Place Table VIII about here]

To gain further understanding of how volatility affects inference about consumption and the stochastic discount factor we utilize our VAR estimates to compare implied consumption, discount rates, and the stochastic discount factor when one ignores the volatility term. To that end, we reconstruct the consumption and SDF innovation series based on the estimated components of N_R, N_{DR} and N_V via the right hand side of equations (9) and (15) respectively. In Table VIII we show what would the consumption and SDF dynamics be if the term N_V is not accounted for–that is the implications of ignoring the volatility component. As the table shows, relative to the true dynamics of consumption the implied consumption innovations are larger and have a correlation of about 0.16 with the true consumption dynamics. It can be shown that this larger volatility and low correlation is true whenever IES is different from one. Table VIII also shows the resulting SDF that would arise based on these misspecified consumption dynamics that ignore the volatility component. We can see that now for all IES values the implied SDF is misspecified and in general is not as volatile as the true one and the risk premia are substantially lower. Moreover, the SDF has a low correlation with the true SDF. These findings have important implications for researchers that use financial data to extract cashflow and discount rate news to infer economically the market and cross-sectional movements of equity returns.

In Bansal, Kiku, Shaliastovich, and Yaron (2012) we show that the misspecification bias for the consumption dynamics and SDF analyzed within our estimated VAR also arise within a plausibly calibrated economy. The calibrated economy highlights the theory outlined in Section I and allows us to analytically account for the 'true' counterparts for each component in Equation (9). For brevity we defer details regarding the calibration to an online appendix but point out that we calibrate a Long Run Risks (LRR) model similar to Bansal, Kiku, and Yaron (2011) which includes time variation in uncertainty. This model captures many salient features of macroeconomic and asset market data and importantly ascribes a prominent role for the volatility risk.[10] We document that the model matches key moments of consumption and asset-market data, and show it thus provides a realistic laboratory for our analysis. Notably, the model produces a significant positive correlation between the discount rate news and the volatility news: it is 83% for the consumption asset and similarly for the market. Further, for both consumption and market return, most of the risk compensation comes from the cash-flow and volatility news, while the contribution of the discount rate news is quite small.

Using the calibrated model, we evaluate the consumption innovations implied by asset market data via the right-hand-side of equation (9) when the volatility term is and is not accounted for. We show that when the volatility component is ignored, for all values of IES different from one, the true and implied consumption have a correlation of about 50% and the implied consumption is much more volatile than the true one. Further it is also shown that the N_V and N_{DR} news are negatively correlated with the innovation to consumption while the analytical 'true' correlations are zero. Finally, as in the VAR case, the SDF is misspecified relative to the true one for *all* values of the IES. Specifically, the market risk premium is about 60% that of the true one, and the correlations of the SDF with the return, discount rate, and cash-flow news are distorted. In all, the calibration evidence, consistent with the VAR findings, clearly demonstrates the potential pitfalls that might arise in interpreting asset pricing models and the risk sources driving asset markets if the volatility channel is ignored.

The analysis in Table VIII assumed the researcher has access to the return on wealth, $r_{c,t+1}$. In many instances, however, that is not the case (e.g., Campbell and Vuolteenaho (2004), Campbell (1996)) and the return on the market $r_{d,t+1}$ is utilized instead. The fact the market return is a levered asset relative to the consumption/wealth return exacerbate the inference problems shown earlier. This is indeed the case – in Bansal, Kiku, Shaliastovich, and Yaron (2012) we show that when the IES is equal to two, the volatility of the implied consumption shocks is about 15%, relative to the true volatility of only 2.5%.[11]

Finally, we show that one has to be cautious in empirically assuming volatility is constant while appealing to discount rate variation. To be specific, consider a case when the volatility is constant and all the economic shocks are homoscedastic. First, it immediately implies that the revision in expected future volatility news is zero, $N_{V,t+1} = 0$. Further, when all the economic shocks are homoscedastic, all the variances and covariances are constant, which implies that the risk premium on the consumption asset is constant as well. Thus, the discount rate shocks just capture the innovations into the future expected risk-free rates.

Hence, under homoscedasticity, the economic sources of risks include the revisions in future expected cash flow, and the revisions in future expected risk-free rates:

$$m_{t+1}^{NoVol} - E_t m_{t+1}^{NoVol} = -\gamma N_{CF,t+1} + N_{RF,t+1}, \tag{41}$$

for $N_{RF,t+1} = (E_{t+1} - E_t)\left(\sum_{j=1}^{\infty} \kappa_1^j r_{f,t+j}\right)$. When volatility is constant, the risk premia are constant and are determined by the unconditional covariances of asset returns with future risk-free rate news and future cash-flow news. Further, the beta of returns with respect to discount rate shocks, $N_{DR,t+1}$, should just be equal to the return beta to the future expected risk-free shocks, $N_{RF,t+1}$. In several empirical studies in the literature (see e.g., Campbell and Vuolteenaho (2004)), the risk-free rates are assumed to be constant. Following the above analysis, it implies, then, that the news about future discount rates is exactly zero, and so is the discount-rate beta, and all the risk premium in the economy is captured just by risks in future cash-flows. Thus, ignoring volatility risks can significantly alter the interpretation of the risk and return in financial markets.

III. Market-DCAPM-SV Model

To further highlight the importance of volatility risks for understanding the dynamics of asset prices, we use a market-based VAR approach to news decomposition. As frequently done in the literature, here, we assume that the wealth portfolio corresponds to the aggregate stock market and, therefore, is observable. This assumption allows us to measure cash-flow, discount-rate and volatility news directly from the available stock market data. As shown in Section II, in a more general setting that explicitly makes a distinction between aggregate and financial wealth and accounts for time-variation in volatility, realized and expected returns on wealth and stock market are highly correlated. This evidence suggests that we should be able to learn about time-series dynamics of fundamental risks and their prices from the observed equity data. Furthermore, to sharpen identification of underlying risks, we will extract them by exploiting both time-series and cross-sectional moment restrictions.

The theoretical framework here is same as the one in Section I with the return on the consumption asset equal to the return on the market portfolio. Hence, the equilibrium risk premium on any asset is determined by its exposure to the innovation in the market return and news about future discount rates and future volatility. The multi-beta implication of our model is similar to the multi-beta pricing of the intertemporal CAPM of Merton (1973) where the risk premium depends on the market beta and asset exposure to state variables that capture changes in future investment opportunities. What distinguishes our market volatility-based dynamic capital asset pricing model (Market-DCAPM-SV) from the Merton's framework is that, in our model, both relevant risk factors and their prices are identified and pinned down by the underlying model primitives and preferences. This is important from an empirical perspective as it provides us with testable implications that can be taken to the data. Note also that in our Market-DCAPM-SV model, derived from recursive preferences, the relevant economic risks comprise not only short-run fluctuations (as in the equilibrium C-CAPM of Breeden (1979)) but also risks that matter in the long run.

A. Market-Based Setup

As derived above, the stochastic discount rate of the economy is given by:

$$m_{t+1} - E_t m_{t+1} = -\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1}. \tag{42}$$

In order to measure news components from equity data, we assume that the state of the economy is described by vector:

$$X_t \equiv \left(RV_{r,t},\ z_t,\ \Delta d_t,\ ts_t,\ ds_t,\ i_t\right)' \ , \tag{43}$$

where $RV_{r,t}$ is the realized variance of the aggregate market portfolio; z_t is the log of the market price-dividend ratio; Δd_t is the continuously compounded dividend growth of the aggregate market; ts_t is the term spread defined as a difference in yields on the 10-year Treasury bond and three-month T-bill; ds_t is the yield differential between Moody's BAA- and AAA-rated corporate bonds; and i_t is the log of the real long-term interest rate. The data are real, sampled on an annual frequency and span the period from 1930 till 2010. The realized variance is constructed by summing up squared monthly rates of market return within a year. The real long-term rate is measured by the yield on the 10-year Treasury bond adjusted by inflation expectations. The latter are estimated using a first-order autoregression specification for inflation. Excess returns on the market and a cross section are constructed by subtracting the annualized rate on the three-month Treasury bill from annual, nominal equity returns. Our state vector comprises variables that are often used in the return- and volatility-forecasting literature (see Fama and French (1988, 1989), Kandel and Stambaugh (1990), and Hodrick (1992)). We discuss the robustness of our evidence to the state specification below.

We model the dynamics of X_t via a first-order vector-autoregression:

$$X_{t+1} = \mu_X + \Phi X_t + u_{t+1}, \tag{44}$$

where Φ is a (6×6)-matrix of VAR coefficients, μ_X is a (6×1)-vector of intercepts, and u_{t+1} is a (6×1)-vector of zero-mean, conditionally normal VAR innovations. Note that the dynamics of the log return on the aggregate market portfolio (r) are implied by the dynamics of its price-dividend ratio and dividend growth:

$$r_{t+1} = \kappa_0 + \Delta d_{t+1} + \kappa_1 z_{t+1} - z_t \ , \tag{45}$$

where κ_0 and κ_1 are constants of log-linearization. To construct cash-flow, discount-rate and volatility news we iterate on the VAR, using the same algebra as in Section II.A with a simplification that all news components are now directly read from the VAR since the return on the market is assumed to represent the return on the overall wealth. For example, cash-flow news and discount-rate news are computed as:

$$N_{CF,t+1} = i'_{\Delta d}(I+Q)u_{t+1} \ , \quad (46)$$

$$N_{DR,t+1} = \Big(i_{\Delta d} + \kappa_1 i_z\Big)' Q u_{t+1} \ , \quad (47)$$

where i_z and $i_{\Delta d}$ are (6×1) indicator vectors with an entry of one in the second and third positions, respectively, and $Q = \kappa_1 \Phi \left(I - \kappa_1 \Phi\right)^{-1}$. Volatility news is computed in a similar way.

We use the extracted news to construct the innovation in the stochastic discount factor and price a cross section of equity returns by exploiting the Euler equation in Equation (36). The procedure for extracting the return news follows the discussion in Section II.D using aggregate dividend instead of consumption. It is important to emphasize that we carry out estimation under the null of the model. In particular, we restrict the premium of a zero-beta asset and impose the model's restrictions on the market prices of discount-rate and volatility risks (both are equal to -1). The price of cash-flow risks is estimated along with time-series parameters of the model.

To extract news and construct the innovation in the stochastic discount factor, we estimate time-series parameters and the market price of cash-flow risks using GMM by exploiting two sets of moment restrictions. The first set of moments comprises the VAR orthogonality moments; the second set contains the Euler equation restrictions for the market portfolio and a cross-section of five book-to-market and five size sorted portfolios. To ensure that the moment conditions are scaled appropriately, we weight each moment by the inverse of its variance and allow the weights to be continuously up-dated throughout estimation. Further details of the GMM estimation are provided in Appendix B.

B. Ex-Ante Volatility and Discount-Rate Dynamics

[Place Table IX about here]

The GMM estimates of the market-based VAR dynamics are presented in Table IX. As shown in the first row of the Table, the realized variance of the market return is highly predictable with an R^2 of more than 60%. Time-variation in the one-year ahead expected variance is coming mostly from variation in realized variance, term and default spreads, and the risk-free rate, all of which are quite persistent in the data. The conditional variance, therefore, features persistent time-series dynamics with a first-order autocorrelation coefficient of about 0.69. These persistent dynamics are consistent with empirical evidence of low-frequency fluctuations in market volatility documented in the literature (see, for example, Bollerslev and Mikkelsen (1996) among others).

[Place Figure 5 about here]

We find that the extracted discount-rate and volatility risks are strongly counter-cyclical and positively correlated. Both news tend to increase during recessions and decline during economic expansions. At the one-year horizon, the correlation between discount-rate and volatility news is 0.47. This evidence aligns well with economic intuition. As the contribution of risk-free rate news is generally small, discount-rate risks are mostly driven by news about future risk premia, and the latter is tied to expectations about future economic uncertainty. Consequently, discount rates and conditional volatility of the market portfolio share common time-series dynamics, especially at low frequencies. We illustrate their co-movement in Figure 5 by plotting the 5-year expected market return and the 5-year conditional variance implied by the estimated VAR. As the figure shows, both discount rates and the conditional variance feature counter-cyclical dynamics and closely follow one another. The correlation between the two time series is 0.75.

C. Pricing Implications of the Market-DCAPM-SV model

[Place Table X about here]

The cross-sectional implications of the Market-DCAPM-SV model are given in Table X. The table presents sample average excess returns of the market portfolio and the cross section, risk premia implied by the market-based model, and asset exposure to cash-flow, discount-rate and volatility risks. The bottom panel of the table shows the estimate of the market price of cash-flow risks. The evidence reported in the table yields several important insights. First, we find that cash-flow risks play a dominant role in explaining both the level and the cross-sectional variation in risk premia. At the aggregate market level, cash-flow risks account for 4.8% or, in relative terms, for about 60% of the total risk premium. Cash-flow betas are monotonically increasing in book-to-market characteristics and monotonically declining with size. Value and small stocks in the data are more sensitive to persistent cash-flow risks than are growth and large firms, which is consistent with the evidence in Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2008) and Bansal, Dittmar, and Kiku (2009).

Second, we find that all assets have negative exposure to discount-rate and volatility risks. That is, in the data, prices of all equity portfolios tend to fall when discount rates or volatility are expected to be high. Because the prices of discount-rate and volatility risks, according to the model, are equal to negative one, both risks carry positive premia. The negative market price of volatility risk is consistent with evidence reported in Drechsler and Yaron (2011), and Bollerslev, Tauchen, and Zhou (2009). These papers show that the estimate of the variance risk premium defined as a difference between expected variances under the risk-neutral and physical measure is not only positive on average, it is almost always positive in time series.[12] Our findings are also consistent with the option-based evidence in Coval and Shumway (2001) who show that, in the data, average returns on zero-market-beta straddles are significantly negative.

In a recent paper, Campbell, Giglio, Polk, and Turley (2012; CGPT, henceforth) also consider the contribution of volatility risks to the cross-section of expected returns. Both papers agree that the market price of volatility risks is negative. Our paper shows that equity exposure to volatility risks is also negative (i.e., equities have negative volatility betas), hence, equities carry positive volatility risk premia. In contrast, for the post-1963 sample, CGPT argue that equities have positive volatility betas and, therefore, volatility risk premia in all equities are negative. Their evidence implies that agents may hold equities because they provide insurance against high volatility states, that is, equities are a volatility hedge. This implication is quite hard to justify from the perspective of economic models and from the evidence of high market volatility and concurrent equity price declines (e.g., during the recession of 2008). Indeed, it is hard to understand as to why high volatility raises aggregate wealth (i.e., volatility betas are positive) and at the same time the return on aggregate wealth carries a negative risk premium in equilibrium. To provide an independent confirmation of our evidence regarding volatility betas, we compute equity exposure to the return on the zero-market-beta S&P100 straddle considered in Coval and Shumway (2001).[13] We find that straddle-betas of the aggregate market portfolio and the cross-section of size/book-to-market portfolios (either single or double sorted) are all significantly negative.[14] The evidence of negative exposure holds even when we consider shorter sub-samples; for example, if we restrict the sample to the 1990-2000 period, straddle-betas of all equity portfolios including the aggregate market remain significantly negative. This evidence suggests that equity exposure to volatility risks is reliably negative.

[Place Figure 6 about here]

The evidence in Table X also shows that discount-rate and volatility risks, each, account for about 20% of the overall market risk premium, and seem to affect the cross section of book-to-market sorted portfolios in a similar way. Both discount-rate and volatility risks matter more for the valuation of growth firms than that of value firms. Overall, our Market-DCAPM-SV model accounts for about 96% of the cross-sectional variation in risk premia, and implies a value premium of 6.1% and a size premium of about 6.8%. The cross-sectional fit of the model is illustrated in Figure 6. The estimate of the market price of cash-flow risks is statistically significant: $\widehat{\gamma} = 2.64$ (SE=0.41), and the model is not rejected by the overidentifying restrictions: the χ^2 test statistic is equal to 7.74 with a p-value of 0.65.[15]

D. Robustness of the Market-Based Evidence

Our empirical evidence is fairly robust to economically reasonable changes in the VAR specification, sample period or frequency of the data. For example, omitting the long-term bond and term spread from the VAR yields a p-value of 0.19. The estimation of the model using post-1963 quarterly-sampled data results in the cross-sectional R-squared of 89%, χ^2 statistic of 15.6 with a corresponding p-value of 0.11. Across these alternative specifications, the estimates of the market price of cash-flow risks continue to be significant, and the extracted discount-rate and volatility risks remain strongly positively correlated. Equity exposure to cash-flow risks remains positive, and discount-rate and volatility betas are consistently negative.

[Place Table XI about here]

In Sections III.A–III.C we assume that volatility of volatility shocks is constant. To confirm robustness of our Market-DCAPM-SV evidence, we estimate a more general set-up that allows for time-variation in the variance of volatility shocks. The dynamics of the aggregate volatility component V_t under this specification are provided in Appendix C. Table XI presents the asset pricing implications of this generalized Market-DCAPM-SV set-up. Consistent with the evidence from our benchmark specification, cash-flow risks remain the key determinant of the level of the risk premium and its dispersion in the cross section. The contribution of volatility risks remains significant and, in fact, is slightly larger relative to the case when volatility shocks are homoscedastic. On average, volatility and discount-rate risks account for about 35% of the overall risk premia in the cross-section, and for almost 50% of the total premium of the market portfolio. Similar to the implications of our benchmark model, growth stocks seem to be more sensitive to volatility (and discount rate) variation compared with value stocks.

IV. Conclusion

In this paper we show that volatility news is an important source of risk that affects the measurement and interpretation of underlying risks in the economy and financial markets. Our theoretical analysis yields a dynamics asset pricing model with three sources of risks: cash-flow, discount-rate and volatility risks, each carrying a separate risk premium. We show that ignoring volatility risks may lead to sizable mis-specifications of the dynamics of the stochastic discount factor and equilibrium consumption, and distorted inferences about risk and return. We find that potential distortions caused by neglecting time-variation in economic volatility are indeed significant and result in upward biases in the volatility of consumption news and large downward biases in the volatility of the implied stochastic discount factor and, consequently, risk premia.

Consistent with the existing empirical evidence, we document that both macro-economic and return-based measures of volatility are highly persistent. Importantly, we also find that, in the data, a rise in volatility is typically accompanied by a significant decline in realized and expected consumption and an increase in risk premia. That is, high volatility states are states of high risk reinforced by low economic growth and high discount rates. This evidence is consistent with the equilibrium relationship among volatility, consumption and asset prices implied by our model. A specification that ignores time-variation in volatility, in contrast to the data, would imply an upward revision in expected lifetime consumption following an increase in discount rates and would clearly fail to account for a strong positive correlation between volatility and discount-rate risks.

The empirical evidence we present highlights the importance of volatility risks for the joint dynamics of human capital and equity returns, and the cross-sectional risk-risk tradeoff. Our dynamic volatility-based asset pricing model is able to reverse the puzzling negative correlation between equity and human-capital returns documented previously in the literature in the context of a homoscedastic economy. By incorporating empirically robust positive relationship between ex-ante volatility and discount rates (a missing link in the homoscedastic case), our model implies a positive correlation between returns to human capital and equity while, simultaneously, matching time-series dynamics of aggregate consumption. We also show that quantitatively, volatility risks help explain both the level and variation in risk premia across portfolios sorted on size and book-to-market characteristics. We find that during times of high volatility in financial markets (hence, high marginal utility), equity portfolios tend to realize low returns. Therefore, equity markets carry a positive premium for volatility risk exposure.

Appendix A. Impulse Response Computations

The VAR(1) dynamics for the state variables follows,

$$X_{t+1} = \mu + \Phi X_t + u_{t+1}, \tag{A1}$$

where the unconditional variance-covariance matrix of shocks is $\Omega = \Sigma\Sigma'$.

The ex-ante consumption variance is $Var\Delta c_t = \upsilon_0 + \upsilon_1' X_t$, for $\upsilon_1 = i_{rv}'\Phi$. Hence, ex-ante volatility shocks are $\upsilon_1' u_{t+1}$. To generate a one-standard deviation ex-ante volatility shock, we choose a combination of primitive shocks $\tilde{u}_{t+1}$ proportional to their impact on the volatility:

$$\tilde{u}_{t+1} = \frac{(\upsilon_1'\Sigma)'}{\sqrt{\upsilon_1'\Sigma\Sigma'\upsilon_1}}. \tag{A2}$$

Based on the VAR, we can compute impulse responses for consumption growth, labor income growth, price-dividend ratio and expected market return in the data. Using the structure of the model and the solution to the labor return sensitivity b, we can also compute the impulse response of model-implied consumption return and price-consumption ratio to the volatility shocks.

Appendix B. GMM Estimation

The dynamics of the state vector are described by a first-order VAR:

$$X_t = \mu_X + \Phi X_{t-1} + u_t \tag{B1}$$

where X_t is a (6×1)-vector of the state variables, μ_X is a (6×1)-vector of intercepts, Φ is a (6×6)-matrix, and u_t is a (6×1)-vector of gaussian shocks. The VAR orthogonality moments compose the first set of moment restrictions in our GMM estimation:

$$E\big[h_t^{VAR}\big] = E\begin{bmatrix} u_t \\ u_t \otimes X_{t-1} \end{bmatrix} = 0. \tag{B2}$$

The second set of moments comprises the Euler conditions for 11 portfolios (the aggregate market and the cross section of five size and five book-to-market sorted portfolios):

$$E\big[h_t^{CS}\big] = E\Big[R_{i,t}^e - \text{RiskPrem}_i\Big]_{i=1}^{11} = 0, \tag{B3}$$

where $R_{i,t}^e$ is the excess return of assets i, and $\text{RiskPrem}_i \equiv -Cov\big(m_{t+1}–E_t m_{t+1},\ r_{i,t+1}– E_t r_{i,t+1}\big)$ is the model-implied risk premium of asset i.

Let $\widehat{h}$ denote the sample counterpart of the combined set of moment restrictions, i.e.,

$$\widehat{h} = \frac{1}{T}\sum_{t=1}^{T}\begin{bmatrix} h_t^{VAR} \\ h_t^{CS} \end{bmatrix}. \tag{B4}$$

The parameters of the VAR dynamics and the market price of cash-flow risks are estimated by minimizing a quadratic form of the sample moments:

$$\{\widehat{\mu_X, \Phi, \gamma}\} = \underset{\Phi_0,\Phi,\gamma}{\operatorname{argmin}}\ \widehat{h}'\, W\, \widehat{h}\ , \tag{B5}$$

where W is a weight matrix. The moments are weighted by the inverse of their corresponding variances; the off-diagonal elements of matrix W are set at zero. We allow the weights to be updated throughout estimation (as in a continuously up-dated GMM). The reported standard errors are based on the New-West variance-covariance estimator.

Appendix C. Generalized Specification

In a more generalized specification of the model, we allow for variation in volatility of volatility shocks. For tractability, we assume that time-variation in conditional second moments of all innovations (including the innovation to the variance component) is driven by a single state variable. In particular, we assume the state vector follows the first-order dynamics:

$$X_{t+1} = \mu_X + \Phi X_t + u_{t+1}, \tag{C1}$$

where $u_{t+1} \sim N(0, \sigma_t^2 \Omega)$. In the macro-model discussed in Section II.E, $\sigma_t^2 \equiv Var_t(\Delta c_{t+1})$, and in the market-based model presented in Section III.D, $\sigma_t^2 \equiv Var_t(N_{R,t+1})$. The dynamics of economic volatility in this case are proportional to σ_t^2:

$$V_t = \xi \, \sigma_t^2, \tag{C2}$$

where ξ can be found by exploiting the definition of V_t:

$$V_t = \xi \, \sigma_t^2 = \frac{1}{2} Var_t\Big((1-\gamma) N_{CF,t+1} + N_{V,t+1}\Big). \tag{C3}$$

It follows then that

$$\xi = 0.5\,(1-\gamma)^2 \left[\iota_{CF}\,\Omega\,\iota_{CF}'\right] + (1-\gamma)\,\xi \left[\iota_{CF}\,\Omega\,\iota_{\sigma^2}'\right] + 0.5\,\xi^2 \left[\iota_{\sigma^2}\,\Omega\,\iota_{\sigma^2}'\right], \tag{C4}$$

where ι_{CF} and ι_{σ^2} correspond to cash-flow and volatility news functions, respectively. In the macro-model:

$$\begin{aligned} \iota_{CF} &= \Big[(1-\omega) i_r' + \omega i_y'\Big]\Big(I - \kappa_1 \Phi\Big)^{-1} \qquad (C5) \\ \iota_{\sigma^2} &= i_v' \kappa_1 \Phi \Big(I - \kappa_1 \Phi\Big)^{-1}, \end{aligned}$$

and in the market-based model:

$$\begin{aligned} \iota_{CF} &= i'_{\Delta d}\big(I - \kappa_1 \Phi\big)^{-1} \\ \iota_{\sigma^2} &= i'_{v}\kappa_1 \Phi\big(I - \kappa_1 \Phi\big)^{-1}. \end{aligned} \tag{C6}$$

Equation (C4) is quadratic in ξ but the admissible solution is unique. However, because of log-linearization of the model, the solution is not guaranteed to be real. To resolve this issue, in the empirical implementation, we rely on the linearized solution derived by using a first-order Taylor series approximation around $\xi = 0$, which is given by:

$$\xi \approx \frac{0.5\left(1-\gamma\right)^2 \iota_{CF}\,\Omega\,\iota'_{CF}}{1-(1-\gamma)\,\iota_{CF}\,\Omega\,\iota'_{\sigma^2}}\,. \tag{C7}$$

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Notes

[1]Bloom (2009) analyzes the effect of time-varying volatility for investment; Ramey and Ramey (1995), Gilchrist and Williams (2005), Gilchrist, Sim, and Zakrajsek (2010), Basu and Bundick (2012), Fernandez-Villaverde and Rubio-Ramrez (2011), and Justiniano and Primiceri (2008) discuss the relationship between volatility and output; Caldara, Fernandez-Villaverde, Rubio-Ramrez, and Wen (2012) consider volatility risks in the context of a production-based asset pricing model.

[2]The importance of human capital component of wealth for explaining equity prices has been illustrated in earlier work by Jagannathan and Wang (1996), Campbell (1996), and Santos and Veronesi (2006).

[3]Substituting the return to aggregate wealth with the return on the stock market has a long tradition in the static CAPM literature. It has also been practiced in empirical work based on recursive preferences (e.g., Epstein and Zin (1991) among others). The issue of unobservability of return to aggregate wealth is discussed in Roll (1977).

[4]Time-varying risk aversion as in Campbell and Cochrane (1999), as well as incomplete markets and/or market segmentation as in Basak and Cuoco (1998), Guvenen (2009) and Povala (2012) would also induce time-varying risk premia. The ability of these alternative models to answer questions that are explored in this paper is left for future research.

[5]It is easily seen that when $\theta = 1$, Equation (14) reduces to the familiar power utility case, and it follows that the decomposition of the SDF in (15) in that case reduces to $-\gamma N_C$.

[6]Bansal, Kiku, and Yaron (2007) show that the gap between aggregate wealth return and the market return can be quite substantial. Hence, this approximation can significantly alter risk-premia implications.

[7]In what follows, we use superscript "d" to denote shocks to the market return, and superscript "y" to identify shocks to the human capital return. Shocks without the superscript refer to the consumption asset, consistent with the notations in Section I.

[8]Time-series dynamics of conditional volatility of consumption growth are also discussed in Duffee (2005) and Bansal, Kiku, and Yaron (2007).

[9]The cross-sectional evidence that we present and discuss below, including evidence on equity exposure to cash-flow, discount-rate and volatility risks is robust if instead we directly use returns to measure return innovations.

[10]See Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2011) for a discussion of the long-run risks channels for the asset markets and specifically the role of volatility risks, Bansal, Khatchatrian, and Yaron (2005) for an early extensive empirical evidence on the role of volatility risks, Eraker and Shaliastovich (2008), Bansal and Shaliastovich (2010), and Drechsler and Yaron (2011) for the importance of volatility risks for derivative markets, and Bansal and Shaliastovich (2013) for the importance of volatility risks for the bond and currency markets.

[11]Campbell (1996) (Table 9) reports the implied consumption innovations based on equation (9) when volatility is ignored and the return and discount rate shocks are read off a VAR using observed financial data. The volatility of the consumption innovations when the IES is assumed to be 2 is about 22%, not far from the quantity displayed in our simulated model. As in our case, lower IES values lead to somewhat smoother implied consumption innovations. While Campbell (1996) concludes that this evidence is more consistent with a low IES, the analysis here suggests that in fact this evidence is consistent with an environment in which the IES is greater than one and the innovation structure contains a volatility component.

[12]Theoretically, under the models highlighted here, a negative market price of volatility risk implies a negative volatility beta of the aggregate market portfolio and, hence, a positive

volatility premium of market equity. Bali and Zhou (2012), Han and Zhou (2012), Tedongap (2010), Todorov and Tauchen (2010) provide additional evidence of negative exposure of equity returns to alternative measures of volatility risks, and Sohn (2009) shows empirically that the market price of volatility risk is negative.

[13]We thank Joshua Coval and Tyler Shumway for sharing the up-to-date straddle return series with us.

[14]For example, the straddle-beta of the market portfolio is -0.067 (t-stat=-7.5); in the cross-section of size/book-to-market portfolios, straddle-betas vary between -0.05 and -0.11 and are *all* significantly negative (t-statistics are all below -6).

[15]It is worth noting that interpreting γ as risk aversion is valid only *if* the market return is indeed equal to the return on the wealth portfolio. In a general environment in which financial markets are a levered claim on the aggregate economy as described in Section II and in the LRR literature, γ will be a mixture of leverage and true risk aversion.

Table I

Data Summary Statistics

The Table reports summary statistics for real consumption growth, real labor income growth, real market return, stock market price-dividend ratio and the realized variance. Realized variance is based on the sum of squared monthly industrial production growth rates over the year, re-scaled to match the unconditional variance of consumption growth. Annual observations from 1930 to 2010. Consumption growth, labor income growth and market return statistics are in percent; realized variance is multiplied by 10000.

	Mean	Std. Dev.	AR(1)
Consumption growth	1.86	2.18	0.48
Labor income growth	2.01	3.91	0.39
Market return	5.70	19.64	-0.01
Price-dividend ratio	3.38	0.45	0.88
Realized variance	4.76	11.13	0.44

Table II

Macro VAR Estimates

The Table reports the OLS estimates of the persistence matrix, and the predictive R^2 for each of the variable in the Macro VAR. The economic variables in Macro VAR include real consumption growth Δc_t, real labor income growth Δy_t, real market return $r_{d,t}$, market price-dividend ratio z_t and the realized variance RV_t. Realized variance is based on the sum of squared monthly industrial production growth rates over the year, re-scaled to match the unconditional variance of consumption growth. Annual observations from 1930 to 2010. Robust standard errors are in the brackets.

	Δc_t	Δy_t	r_{dt}	z_t	RV_t	R^2
Δc_{t+1}	0.221	0.148	0.057	0.002	2.660	0.61
	[0.090]	[0.036]	[0.012]	[0.003]	[1.092]	
Δy_{t+1}	-0.272	0.506	0.078	0.005	3.014	0.37
	[0.271]	[0.146]	[0.026]	[0.007]	[3.430]	
$r_{d,t+1}$	-3.095	1.126	0.068	-0.076	-10.906	0.08
	[0.899]	[0.377]	[0.086]	[0.038]	[10.557]	
z_{t+1}	-3.588	0.978	-0.230	0.921	-9.029	0.80
	[0.937]	[0.644]	[0.144]	[0.042]	[10.955]	
RV_{t+1}	-0.007	-0.006	0.001	-0.001	0.310	0.35
	[0.005]	[0.004]	[0.0004]	[0.0003]	[0.091]	

Table III

Role of Volatility for Economic News

The Table reports the standard deviations and the correlations of economic news with the volatility news, and the average magnitude of the economic news in lowest 25% and highest 25% volatility news periods. Economic news include future expected consumption news N_{ECF}, volatility news N_V, news in stochastic discount factor N_M, and news in discount rates on the market return N^d_{DR}, labor return N^y_{DR} and wealth return N_{DR}. The news are constructed based on the Macro-DCAPM-SV model. "With Vol Risk" columns show the economic news in the model specification with volatility risk, while "No Vol Risk" columns document the implications when volatility risks are absent. The risk aversion coefficient is set at $\gamma = 5$, and the elasticity of inter-temporal substitution parameter is $\psi = 2$.

				With Vol Risk			No Vol Risk		
	N_{ECF}	N_V	N^d_{DR}	N^y_{DR}	N_{DR}	N_M	N^y_{DR}	N_{DR}	N_M
Std. Dev.	5.27	30.06	14.56	2.58	3.97	47.79	4.70	2.63	26.71
Corr. with N_V	-0.27	1.00	0.86	0.27	0.77	0.84	-0.85	-0.27	0.24
Volatility News Periods:									
Lowest 25%	2.16	-33.01	-12.72	-0.63	-3.05	-44.99	4.53	1.08	-7.86
Highest 25%	-1.69	37.37	16.41	0.68	3.82	50.21	-5.16	-0.85	8.16

Table IV

Labor, Market and Aggregate Wealth Return Correlations

The Table reports the correlations between immediate return shocks, discount rate shocks and 5-year expected returns to the market, labor, and aggregate wealth. N_R, N_{DR} and $E_t r_{t\to t+5}$ stand for the immediate return news, discount rate news, and 5 year expected return to the wealth portfolio, respectively. Subscripts "y" and "d" denote the corresponding news for the labor return and stock market return. The news are constructed based on the Macro-DCAPM-SV model. "With Vol Risk" columns show the correlations in the model specification with volatility risk, while "No Vol Risk" columns document the correlations when volatility risks are absent. The risk aversion coefficient is set at $\gamma = 5$, and the elasticity of inter-temporal substitution parameter is $\psi = 2$.

		No Vol Risk	With Vol Risk
Market and Labor Return:			
Immediate Shocks	$Corr(N_R^d, N_R^y)$	-0.61	0.19
Discount Shocks	$Corr(N_{DR}^d, N_{DR}^y)$	-0.72	0.25
5-year Expectations	$Corr(E_t r_{t\to t+5}^d, E_t r_{t\to t+5}^y)$	-0.50	0.39
Market and Wealth Return:			
Immediate Shocks	$Corr(N_R^d, N_R)$	0.46	0.80
Discount Shocks	$Corr(N_{DR}^d, N_{DR})$	0.08	0.86
5-year Expectations	$Corr(E_t r_{t\to t+5}^d, E_t r_{t\to t+5})$	0.27	0.79
Wealth and Labor Return:			
Immediate Shocks	$Corr(N_R, N_R^y)$	0.42	0.74
Discount Shocks	$Corr(N_{DR}, N_{DR}^y)$	0.64	0.70
5-year Expectations	$Corr(E_t r_{t\to t+5}, E_t r_{t\to t+5}^y)$	0.70	0.87

Table V

Labor, Market and Aggregate Wealth Return Risk Premium

The Table reports the total risk premium on labor, market and wealth return, and its decomposition into cash-flow, discount rate and volatility risk premium components. N_M denotes the stochastic discount factor shock, and N_{CF}, N_{DR} and N_V stand for the cash-flow news, discount rate news on the wealth portfolio and the volatility news, respectively. N_R, N_R^y and N_R^m denote the immediate return news to wealth, labor and stock market. The news are constructed based on the Macro-DCAPM-SV model. "With Vol Risk" columns show the risk premia in the model specification with volatility risk, while "No Vol Risk" columns document the risk premia when volatility risks are absent. The risk aversion coefficient is set at $\gamma = 5$, and the elasticity of inter-temporal substitution parameter is $\psi = 2$.

		No Vol Risk	With Vol Risk
Market Return:			
Risk Premium	$Cov(-N_M, N_R^d)$	2.40	7.40
Cash-Flow Risk Premium	$Cov(-\gamma N_{CF}, N_R^d)$	2.64	2.64
Vol Risk Premium	$Cov(-N_V, N_R^d)$	0	4.44
Discount Rate Risk Premium	$Cov(-N_{DR}, N_R^d)$	-0.23	0.32
Wealth Return:			
Risk Premium	$Cov(-N_M, N_R)$	0.88	2.62
Cash-Flow Risk Premium	$Cov(-\gamma N_{CF}, N_R)$	0.96	1.22
Vol Risk Premium	$Cov(-N_V, N_R)$	0	1.34
Discount Rate Risk Premium	$Cov(-N_{DR}, N_R)$	-0.08	0.06
Labor Return:			
Risk Premium	$Cov(-N_M, N_R^y)$	0.50	1.43
Cash-Flow Risk Premium	$Cov(-\gamma N_{CF}, N_R^y)$	0.54	0.87
Vol Risk Premium	$Cov(-N_V, N_R^y)$	0	0.56
Discount Rate Risk Premium	$Cov(-N_{DR}, N_R^y)$	-0.04	0.00

Table VI

Role of Preferences for Aggregate Returns

The Table shows the implied correlations between labor and and market returns (for the immediate news N_R, discount rate news N_{DR} and 5-year expected returns Er), and the risk premia on the market, labor and aggregate wealth returns, for various parameter values for the inter-temporal elasticity of substitution ψ and the human capital share ω. The news are constructed based on the Macro-DCAPM-SV model. "With Vol Risk" columns show the correlations and the risk premia in the model specification with volatility risk, while "No Vol Risk" columns document the correlations and the risk premia when volatility risks are absent. The risk aversion coefficient is set at $\gamma = 5$.

	Lbr-Mkt Corr			Risk Premia			Lbr-Mkt Corr			Risk Premia		
ψ	N_R	N_{DR}	Er	Mkt	Lbr	Wlth	N_R	N_{DR}	Er	Mkt	Lbr	Wlth
			With Vol Risk						**No Vol Risk**			
						$\omega = 0.80$						
0.5	-0.89	-0.54	-0.34	5.02	-5.16	-3.13	-0.81	-0.19	0.07	1.68	-1.42	-0.80
1.0	-0.94	-0.43	-0.15	6.60	-1.43	0.18	-0.94	-0.43	-0.15	2.16	-0.31	0.18
1.5	-0.30	-0.17	0.14	7.13	0.40	1.75	-0.77	-0.60	-0.34	2.32	0.21	0.63
2.0	0.19	0.25	0.39	7.40	1.43	2.62	-0.61	-0.72	-0.50	2.40	0.50	0.88
2.5	0.36	0.52	0.51	7.56	2.08	3.17	-0.52	-0.79	-0.62	2.45	0.68	1.03
3.0	0.43	0.61	0.57	7.66	2.53	3.55	-0.45	-0.85	-0.71	2.48	0.81	1.14
						$\omega = 0.85$						
0.5	-0.86	-0.49	-0.33	4.55	-3.96	-2.69	-0.75	-0.13	0.07	1.50	-1.11	-0.71
1.0	-0.90	-0.33	-0.10	5.98	-0.85	0.17	-0.90	-0.33	-0.10	1.92	-0.13	0.18
1.5	-0.03	0.03	0.23	6.46	0.66	1.53	-0.64	-0.49	-0.27	2.06	0.32	0.58
2.0	0.34	0.47	0.50	6.69	1.51	2.29	-0.46	-0.62	-0.41	2.13	0.58	0.81
2.5	0.45	0.68	0.62	6.84	2.04	2.76	-0.36	-0.71	-0.53	2.17	0.74	0.95
3.0	0.51	0.74	0.66	6.93	2.41	3.09	-0.30	-0.77	-0.63	2.20	0.85	1.05

Table VII

Asset Pricing Implications of Macro-DCAPM-SV Model

The Table shows risk premia implied by the Macro-DCAPM-SV model and risk exposures (betas) of the aggregate market and a cross section of five book-to-market and five size sorted portfolios. The bottom panel presents the market prices of risks. According to the model, the market price of cash-flow risk is equal to the coefficient of risk aversion $\gamma = 5$, and market prices of discount-rate and volatility risks are fixed at -1. "Data" column reports average returns in excess of the three-month Treasury bill rate in the 1930-2010 sample.

	Risk Premia (%)		Betas × 100		
	Data	Model	CF	DR	Vol
Market	7.9	7.4	0.53	-0.32	-4.45
BM1	7.2	7.7	0.39	-0.49	-5.22
BM2	7.6	8.5	0.55	-0.42	-5.32
BM3	9.4	9.1	0.82	-0.23	-4.71
BM4	10.8	10.2	1.22	0.02	-4.11
BM5	13.1	13.2	2.31	0.70	-2.38
Size1	14.8	14.1	1.89	0.21	-4.86
Size2	13.0	11.7	1.14	-0.22	-5.75
Size3	11.6	10.6	0.92	-0.31	-5.72
Size4	10.4	8,8	0.67	-0.33	-5.06
Size5	7.4	7.0	0.51	-0.29	-4.14
			Price(CF)	Price(DR)	Price(Vol)
			5	-1	-1

Table VIII

Mis-Specification of Consumption and SDF Dynamics

The Table compares the implications for the consumption shock, shock in the stochastic discount factor and the market risk premium under the benchmark Macro-DCAPM-SV model (column "Vol") and under the specification which ignores the volatility news component in construction of these shocks ("Ignore Vol"). The Table reports the standard deviations of consumption shock and the stochastic discount factor shock, correlations between consumption shocks under these two specifications, correlations between the stochastic discount factor shocks under these two specifications, and the levels of the market risk premium, under various levels of the inter-temporal elasticity of substitution parameter (IES) of 2, 1 and 0.75. The risk aversion coefficient is set at $\gamma = 5$. Volatility is annualized, in percent.

	IES = 2		IES= 1		IES = 0.75	
	Ignore Vol	Vol	Ignore Vol	Vol	Ignore Vol	Vol
Implied Consumption Shock						
Vol of cons. shock	7.62	1.29	1.29	1.29	2.29	1.29
Corr. with cons. shock	0.16	1.00	1.00	1.00	0.57	1.00
Implied SDF Shock						
Vol of SDF shock	27.84	47.79	24.15	42.71	22.02	39.33
Market Risk Premium	2.96	7.40	2.16	6.60	1.63	6.07
Corr. with SDF shock	0.81	1.00	0.73	1.00	0.65	1.00

Table IX

Market-based VAR Estimates

The Table presents GMM estimates of the persistence matrix, and the predictive R^2 for each of the variable in the market-based VAR. $RV_{r,t}$ denotes the realized variance of the aggregate market portfolio; z_t is the log of the price-dividend ratio; Δd_t is the log dividend growth; ts_t and ds_t are term- and default spreads, respectively; i_t is the log of the interest rate. Robust standard errors are presented in brackets. The data used in estimation are real and cover the period from 1930 to 2010.

	$RV_{r,t}$	z_t	Δd_t	ts_t	ds_t	i_t	R^2
$RV_{r,t+1}$	0.274	-0.017	0.023	-0.696	2.579	0.353	0.63
	[0.167]	[0.012]	[0.035]	[0.336]	[1.031]	[0.202]	
z_{t+1}	-0.961	0.904	-0.584	0.856	4.981	0.593	0.80
	[0.863]	[0.074]	[0.261]	[1.276]	[5.213]	[0.556]	
Δd_{t+1}	0.715	0.050	0.161	2.347	-7.887	-0.407	0.27
	[0.408]	[0.031]	[0.131]	[0.952]	[3.121]	[0.370]	
ts_{t+1}	-0.071	0.004	-0.016	0.410	1.036	-0.029	0.50
	[0.023]	[0.003]	[0.011]	[0.103]	[0.147]	[0.019]	
ds_{t+1}	0.020	-0.001	0.009	-0.127	0.682	0.021	0.56
	[0.017]	[0.002]	[0.006]	[0.034]	[0.146]	[0.024]	
i_{t+1}	-0.352	0.001	-0.050	-0.197	2.075	0.613	0.48
	[0.105]	[0.007]	[0.026]	[0.182]	[0.792]	[0.109]	

Table X

Asset Pricing Implications of the Market-DCAPM-SV Model

The Table shows risk premia implied by the Market-DCAPM-SV model and risk exposures (betas) of the aggregate market and a cross section of five book-to-market and five size sorted portfolios. The bottom panel presents the estimates of the market prices of risks and the corresponding robust standard errors (in brackets). According to the model, prices of discount-rate and volatility risks are fixed at -1. “Data” column reports average returns in excess of the three-month Treasury bill rate from 1930 to 2010.

	Risk Premia (%)		Betas × 100		
	Data	Model	CF	DR	Vol
Market	7.9	7.7	1.80	-1.67	-1.23
BM1	7.2	7.2	1.72	-1.53	-1.09
BM2	7.6	8.3	2.11	-1.59	-1.13
BM3	9.4	10.0	2.66	-1.58	-1.37
BM4	10.8	11.3	3.29	-1.58	-1.04
BM5	13.1	13.2	4.64	-0.91	-0.09
Size1	14.8	14.2	4.57	-1.55	-0.62
Size2	13.0	12.4	3.47	-1.98	-1.29
Size3	11.6	11.2	2.97	-1.94	-1.38
Size4	10.4	9.7	2.43	-1.88	-1.44
Size5	7.4	7.4	1.91	-1.44	-0.98
			Price(CF)	Price(DR)	Price(Vol)
			2.64	-1	-1
			[0.38]	[na]	[na]

Table XI

Asset Pricing Implications of Generalized Market-DCAPM-SV Model

The Table shows risk premia implied by the generalized market-DCAPM-SV model and risk exposures (betas) of the aggregate market and a cross section of five book-to-market and five size sorted portfolios. The bottom panel presents the estimates of the market prices of risks and the corresponding robust standard errors (in brackets). According to the model, prices of discount-rate and volatility risks are fixed at -1. "Data" column reports average returns in excess of the three-month Treasury bill rate in the 1930-2010 sample.

	Risk Premia (%)		Betas $\times$ 100		
	Data	Model	CF	DR	Vol
Market	7.9	7.5	1.61	-1.83	-1.80
BM1	7.2	7.2	1.61	-1.68	-1.68
BM2	7.6	8.3	2.02	-1.74	-1.75
BM3	9.4	10.1	2.64	-1.70	-2.18
BM4	10.8	11.3	3.37	-1.62	-1.73
BM5	13.1	13.1	5.07	-0.54	-0.56
Size1	14.8	14.1	4.82	-1.37	-1.25
Size2	13.0	12.4	3.48	-2.06	-2.12
Size3	11.6	11.3	2.95	-2.05	-2.23
Size4	10.4	9.8	2.33	-2.05	-2.22
Size5	7.4	7.3	1.81	-1.58	-1.46
			Price(CF)	Price(DR)	Price(Vol)
			2.37	-1	-1
			[0.96]	[na]	[na]

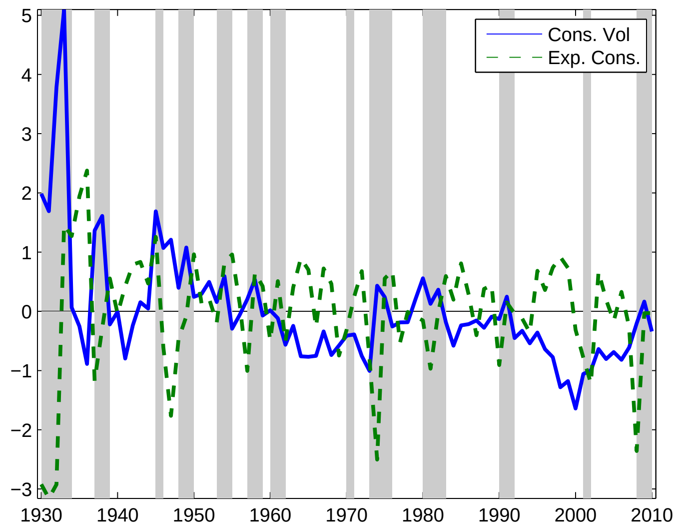

Figure 1. Volatility and expected consumption growth from Macro VAR. The Figure plots time-series of consumption volatility (solid line) and expected consumption growth (dashed line) implied by the Macro VAR estimates. The two lines are normalized to have a zero mean and a unit variance; shaded areas represent the NBER recession dates.

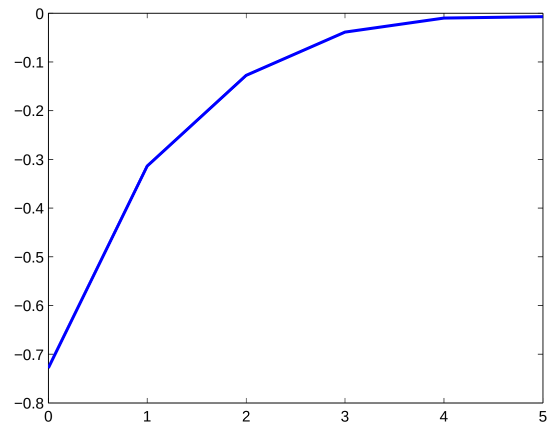

Figure 2. Consumption response to volatility shock from Macro VAR. The Figure shows an impulse response of consumption growth to one standard deviation shock in volatility of consumption, implied by the Macro VAR estimates. Consumption growth is annual, in percent.

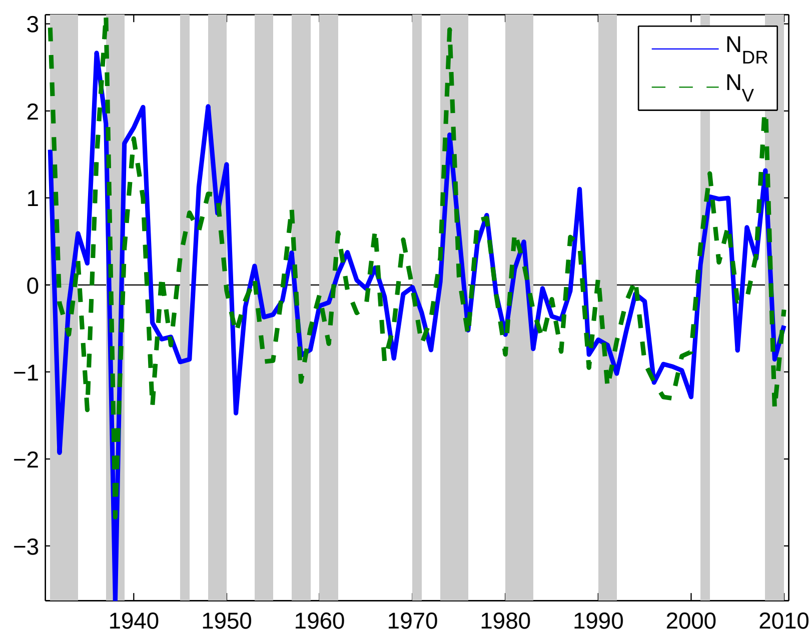

Figure 3. Discount rate news and volatility news. The Figure plots time-series of discount rate news on the wealth portfolio N_{DR} and the volatility news N_V, implied by the Macro-DCAPM-SV model. The two lines are normalized to have a zero mean and a unit variance; shaded areas represent the NBER recession dates.

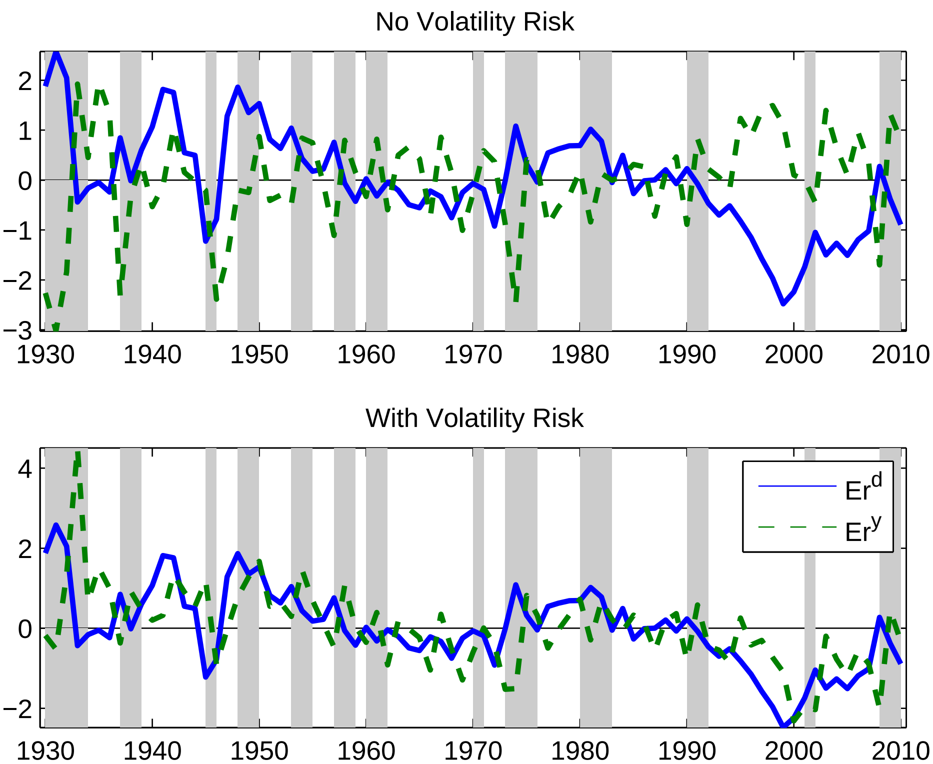

Figure 4. Five-year expected market and labor returns. The Figure plots time-series of five-year expected returns on the market (solid line) and human capital (dashed line) implied by the the Macro-DCAPM-SV model. "With Vol Risk" panel show the implied returns in the model specification with volatility risk, while "No Vol Risk" panel plots the returns in the model specification when volatility risks are absent. The risk aversion coefficient is set at $\gamma = 5$. The two lines are normalized to have a zero mean and a unit variance; shaded areas represent the NBER recession dates.

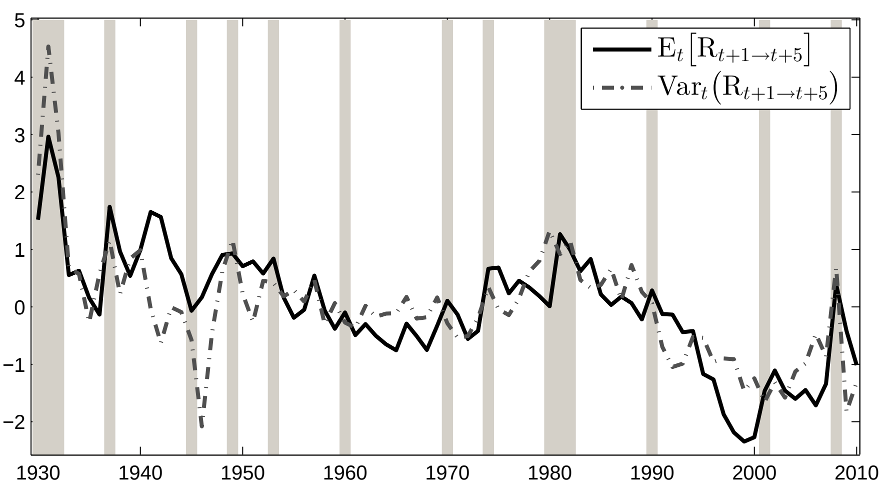

Figure 5. Discount rate and conditional variance of the market portfolio. The Figure plots time-series of the 5-year expected market return and 5-year conditional variance of the market portfolio implied by the unrestricted market-based VAR estimates. The two lines are normalized to have a zero mean and a unit variance; shaded areas represent the NBER recession dates.

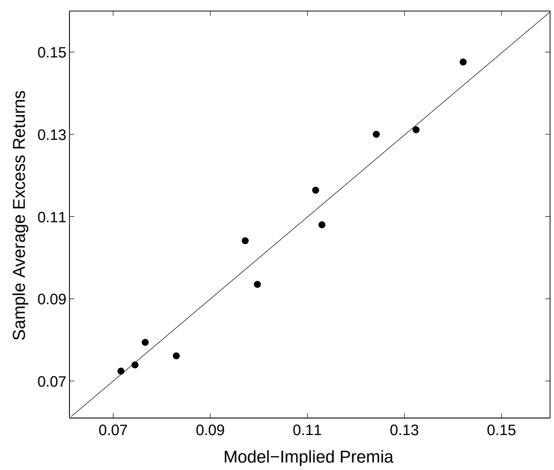

Figure 6. Data and model-implied risk premia. The Figure presents a scatterplot of sample average excess returns versus model-implied risk premia of the aggregate market and a cross section of five book-to-market and five size sorted portfolios in Market-DCAPM-SV model. Sample averages correspond to average returns in excess of the three-month Treasury bill rate from 1930 to 2010.

Internet Appendix to "Volatility, the Macroeconomy and Asset Prices"*

A. Long-Run Risks Model

To gain further understanding on how volatility affects inference about consumption innovations, cash-flow and discount rate variation, and more generally fluctuations in the stochastic discount factor, asset prices and risk premia, we utilize a standard long-run risks model of Bansal and Yaron (2004). This model captures many salient features of the asset market data and importantly ascribes a prominent role to the volatility risk.[1]

In a standard long-run risks model consumption dynamics satisfies

$$\begin{aligned}\Delta c_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1}, && \text{(IA.1)}\\ x_{t+1} &= \rho x_t + \varphi_e \sigma_t \epsilon_{t+1}, && \text{(IA.2)}\\ \sigma_{t+1}^2 &= \sigma_c^2 + \nu(\sigma_t^2 - \sigma_c^2) + \sigma_w w_{t+1}, && \text{(IA.3)}\end{aligned}$$

where ρ governs the persistence of expected consumption growth x_t, and ν determines the persistence of the conditional aggregate volatility σ_t^2. Shock η_t denotes a short-run consumption shock, ϵ_t is the shock to the expected consumption growth, and w_{t+1} is the shock to the conditional volatility of consumption growth; for parsimony, these three shocks are assumed to be *i.i.d* Normal.

Agent's s preferences are described by a Kreps and Porteus (1978) recursive utility function of Epstein and Zin (1989) and Weil (1989). The life-time utility of the agent U_t satisfies

$$U_t = \left[(1-\delta)C_t^{1-\frac{1}{\psi}} + \delta\left(E_t U_{t+1}^{1-\gamma}\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}, \qquad \text{(IA.4)}$$

where C_t is the aggregate consumption level, δ is a subjective discount factor, γ is a risk aversion coefficient, ψ is the intertemporal elasticity of substitution (IES), and for notational ease we denote $\theta = (1-\gamma)/(1-\frac{1}{\psi})$. When $\gamma = 1/\psi$, the preferences collapse to a standard expected power utility.

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The equilibrium model solution is derived in Bansal and Yaron (2004). In particular, the equilibrium solution to the price-consumption ratio, pc_t, is linear in the expected growth and consumption volatility:

$$pc_t = A_0 + A_x x_t + A_\sigma \sigma_t^2, \tag{IA.5}$$

and the innovation into the stochastic discount factor is determined by the short-run, expected consumption and the volatility news:

$$m_{t+1} - E_t m_{t+1} = -\lambda_c \sigma_t \eta_{t+1} - \lambda_x \varphi_e \sigma_t \epsilon_{t+1} - \lambda_\sigma \sigma_w w_{t+1}. \tag{IA.6}$$

The equilibrium loadings for the price-consumption ratio and the market prices of risks depend on the preference parameters and the consumption dynamics, and are provided Bansal and Yaron (2004). In particular, when agents have preference for early resolution of uncertainty ($\gamma > 1/\psi$) and the risk aversion coefficient is above one, the market price of expected consumption risk is positive: $\lambda_x > 0$, and the market price of the volatility risks is negative: $\lambda_\sigma < 0$. Further, when the intertemporal elasticity of substitution is above one, the equilibrium equity prices fall in bad times of low economic growth and high volatility: $A_x < 0$, and $A_\sigma > 0$. Thus, in the long-run risks model, investors dislike positive shocks to aggregate volatility, and the equity beta to volatility risks is negative.

Given the model solution, we can provide explicit expressions for the aggregate volatility V_t which appears in our Macro-DCAPM-SV model in terms of the underlying economic structure of the long-run risks model. The economic volatility component, V_t, is directly related to the conditional variance of consumption growth:

$$V_t = \frac{1}{2} Var_t(r_{c,t+1} + m_{t+1}) = const + \frac{1}{2}\chi(1-\gamma)^2\sigma_t^2, \tag{IA.7}$$

where the proportionality parameter χ satisfies:

$$\chi = \left(\frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho}\right)^2 + 1. \tag{IA.8}$$

Notably, under the model restrictions, the volatility parameter χ is unambiguously positive and is equal to the ratio of variances of the long-run cash flows news, $N_{CF,t+1}$, to the immediate consumption news, $N_{C,t+1}$:

$$\chi = \frac{Var(N_{CF,t+1})}{Var(N_{C,t+1})}. \tag{IA.9}$$

Further, given the underlying structure of the economic model, we can provide an explicit mapping of the future cash-flow news, $N_{CF,t+1}$, the discount rate shocks,

$N_{DR,t+1}$, and the volatility news shocks, $N_{V,t+1}$, into the primitive consumption shocks in the economy:

$$\begin{aligned}
N_{CF,t+1} &= \frac{\kappa_1}{1-\kappa_1\rho}\varphi_e\sigma_t\epsilon_{t+1} + \sigma_t\eta_{t+1}, \\
N_{DR,t+1} &= \frac{1}{\psi}\frac{\kappa_1}{1-\kappa_1\rho}\varphi_e\sigma_t\epsilon_{t+1} - \kappa_1 A_\sigma\sigma_w w_{t+1}, \\
N_{V,t+1} &= \frac{1}{2}\chi(1-\gamma)^2\frac{\kappa_1}{1-\kappa_1\nu}\sigma_w w_{t+1}.
\end{aligned} \tag{IA.10}$$

These mappings based on the economic model can be used to validate the decomposition of the stochastic discount factor into the the cash-flow, discount rate and volatility news implied by the Macro-DCAPM-SV model. Indeed, it easy to verify that,

$$\begin{aligned}
m_{t+1} - E_t m_{t+1} &= \lambda_c\sigma_t\eta_{t+1} - \lambda_x\varphi_e\sigma_t\epsilon_{t+1} - \lambda_\sigma\sigma_w w_{t+1} \\
&\equiv -\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1}.
\end{aligned} \tag{IA.11}$$

Note that while the primitive consumption shocks $\eta_{t+1}, \epsilon_{t+1}$ and w_{t+1} are by construction independent of each other, the three news $N_{CF,t+1}$, $N_{DR,t+1}$ and $N_{V,t+1}$ are correlated with each other because they are linear combinations of the underlying economic shocks. The co-movement between the risks can impact the interpretation of the market prices of the risks. In the long-run risks model, the impact of volatility shocks on the SDF is fully captured by the the market price of volatility risk λ_σ. Its magnitude depends on preference parameters γ and ψ, and it is negative when agents have a preference for early resolution of uncertainty ($\psi > 1/\gamma$). Based on the SDF decomposition in Macro-DCAPM-SV model, the market price of the volatility risks is equal to -1; however, to measure the full impact of volatility risks on the stochastic discount factor one also needs to consider the discount rate news $N_{DR,t+1}$ in addition to the volatility news $N_{V,t+1}$. Indeed, the discount rate news is driven both by the expected consumption shocks ϵ_{t+1} and the shocks in consumption volatility w_{t+1}. As can be seen in (IA.10), for high values of IES ($\psi > 1$), discount rates increase at times of high volatility so the correlation of discount rate and volatility news is positive, while for low IES the correlation between the discount rates and the volatility news is negative. Thus, positive volatility shocks lead to an increase in the stochastic factor for high values of IES, while they can decrease the stochastic discount factor for low IES when the drop in discount rates is large. This is consistent with the economic long-run risks model; of course, the total impact of volatility shock on the SDF in the long-run risks model will be the same as in Macro-DCAPM-SV model taking into account the co-movements of the shocks.

We can get similar decomposition for the consumption shock. Recall that in Macro-DCAPM-SV model, the consumption innovation $N_{C,t+1}$ can be written in terms of the innovations to the consumption return, discount rate and volatility shocks:

$$c_{t+1} - E_t(c_{t+1}) = N_{R,t+1} + (1-\psi)N_{DR,t+1} + \frac{\psi - 1}{\gamma - 1} N_{V,t+1} \tag{IA.12}$$

Under the null of the long-run risks model, the above expression is just equal to the consumption shock $\sigma_t \eta_{t+1}$.

It is important to recognize that volatility innovations are relevant for the correct inference on the consumption return, discount rate, and volatility shocks, as long as ψ is different from 1 (as $A_\sigma \neq 0$). Ignoring the volatility component can distort the measurement of $N_{R,t+1}$, and $N_{DR,t+1}$, and $N_{V,t+1}$. Even if the return news $N_{R,t+1}$ and $N_{DR,t+1}$ could be correctly estimated using flexible specification in the data, it is clear from equation (IA.12) that the economic implications about the consumption innovations and the stochastic discount factor can be very misleading when the volatility channel is ignored.

The distortions into the stochastic discount factor affect the implications for risk premia and the asset sensitivity (beta) to economic sources of risks. To study this effect within the model environment we introduce a generic dividend process,

$$\Delta d_{t+1} = \mu_d + \phi x_t + \pi \sigma_t \eta_{t+1} + \varphi_d \sigma_t u_{d,t+1}. \tag{IA.13}$$

One potential important aspect in conducting empirical work is the fact the consumption return itself is not observed and therefore the market return is often used instead. The discrepancy between these two assets can exacerbate the distortions and economic inference problems described above. In the next section we quantify these various issues in turn.

B. Volatility Risks and Mis-Measurement of Consumption Innovation

In this section we use the calibration of the the long-run risks model to evaluate the extent to which consumption innovations are mis-measured if one ignores the presence of volatility.

[Place Table IA.1 about here]

The parameter configuration for consumption and dividend dynamics used in our model simulation is identical to Bansal, Kiku, and Yaron (2011), and is given in

Table IA.1. To be consistent with the output of Macro-DCAPM-SV model, we set the inter-temporal elasticity of substitution parameter to 2, and recalibrate the subjective discount factor $\delta = 0.9987$ to match the risk-free rate of 1%.

[Place Table IA.2 about here]

The model reproduces key asset market and consumption moments of the data and thus provides a realistic laboratory for our analysis. The implications for the consumption and dividend dynamics are discussed in Bansal, Kiku, and Yaron (2011), and in Table IA.2 we show the model output for the returns on the wealth portfolio and the market. Notice that the model produces a significant positive correlation between the discount rate news and the volatility news: it is 83% for the consumption asset, and 96% for the market. Further, for both consumption and market return, most of the risk compensation comes from the cash-flow and volatility news, while the contribution of the discount rate news is quite small. Indeed, the volatility risk compensation is about 40% of the total risk premium for the market return, and 50% of the premium on the consumption asset, while the risk compensation for discount rate shocks is 0.04%. Alternatively, one can measure the contribution of volatility shocks to the variation of the stochastic discount factor in (IA.6): as the fundamental consumption and volatility shocks are independent, this will determine the role of the volatility risks for the maximum risk premia in the economy. In our calibration, the contribution of the volatility shocks is about 60%.

[Place Table IA.3 about here]

Table IA.3 reports the implied consumption innovations when volatility is ignored, that is when the term N_V is not accounted for in constructing the consumption innovations. In constructing the implied consumption innovations via equation (IA.12) we use the analytical expressions for $N_{CF,t+1}$, $N_{DR,t+1}$, and $N_{V,t+1}$ which are given in (IA.10). In particular, we assume that the consumption return news $N_{R,t+1}$ and $N_{DR,t+1}$ can be identified correctly even if the volatility component is ignored, and we focus only on the mis-specification caused by an omission of the volatility news $N_{V,t+1}$.

[Place Table IA.4 about here]

Table IA.3 shows that when IES is not equal to one, the implied consumption innovations are distorted. In the benchmark model, when IES is equal to two, the volatility of consumption innovations is more than twice that of the true consumption innovations. Furthermore, when volatility is ignored, the correlation between the true consumption shock and the implied consumption shock is only 0.4. In addition, the correlation of the implied consumption innovation and the discount rate and volatility news are very negative while in the model they should be zero when volatility is correctly accounted for. Similar distortions are present when the IES is less than one albeit by a smaller magnitude. In Table IA.4 we report the implications of ignoring volatility for the stochastic discount factor. When volatility is ignored, for all values of the IES the SDF's volatility is downward biased by about one-third, and the market risk premium is almost half that of the true one. Finally, it is important to note that even when the IES is equal to one, the SDF is still misspecified. In all, the evidence clearly demonstrates the potential pitfalls that might arise in interpreting asset pricing models and the asset markets sources of risks if the volatility channel is ignored.

[Place Table IA.5 about here]

The analysis above assumed the researcher has access to the return on wealth, $r_{c,t+1}$. In many instances, however, that is not the case (e.g., Campbell and Vuolteenaho (2004), Campbell (1996)) and the return on the market $r_{d,t+1}$ is utilized instead. In Table IA.5 we repeat the analysis above, except that $r_{d,t+1}$ replaces $r_{c,t+1}$ in the stochastic discount factor, and hence in the construction of N_R, N_{DR}, and N_V. The fact the market return is a levered asset relative to the consumption/wealth return exacerbate the inference problems shown earlier. In particular, Table IA.5 shows that when the IES is equal to two, the volatility of the implied consumption shocks is about 15%, relative to the true volatility of only 2.5%. Moreover, the correlation structure with various shocks is distorted in a significant manner. It is interesting to note that now even when IES is equal to one the consumption innovation shocks are misspecified. The small difference between the case of ignoring volatility altogether and the case in which volatility is included indicates that much of the misspecification arise in the construction of the return and discount rate innovations, N_R, and N_{DR} respectively. The market return, being a levered return relative to the consumption return, yields much too volatile implied consumption innovations. Further, the distinction between $r_{d,t+1}$ and $r_{c,t+1}$ leads to a distorted innovation structure even when the underlying economy has constant volatility (see Panel B of Table IA.5). Campbell (1996) (Table 9) reports the implied consumption innovations based on equation (IA.12) when volatility is ignored and the return and discount rate shocks are read off a VAR using observed financial data. The volatility of the consumption innovations when the IES is assumed to be 2 is about 22%, not far from the quantity displayed in our simulated model in Table IA.5.[2] As in our case, lower IES values lead to somewhat smoother implied consumption innovations. While Campbell (1996) concludes that this evidence is more consistent with a low IES, the analysis here suggests that in fact this evidence is consistent with an environment in which the IES is greater than one and the innovation structure contains a volatility component.

Our mis-specification results are based on the benchmark model specification which features a prominent role for the volatility risk. Indeed, the persistence of volatility shocks is 0.999, and its contribution to the risk premium is about 50%. We evaluate the role of the volatility risk using alternative model calibrations, and report our findings in Table IA.3-IA.4. For the first model M1, we reduce the persistence of volatility to 0.95 and simultaneously increase the scale of volatility shocks σ_w to 4.3e-05 to keep the benchmark market risk premium of 6.71%. As can be seen from the Tables, the volatility risk plays an important role to correctly identify consumption and SDF dynamics, though, its contribution is quantitatively somewhat smaller than in the benchmark case. Indeed, for IES equal to 2, the volatility of consumption shock is 4.8% when volatility is ignored relative to the calibrated 2.5%, and the correlation of the two consumption shocks is only 0.5. For the second robustness check, we lower the volatility persistence to $\nu = 0.95$ and increase the persistence of expected growth risks ρ to 0.984 again to target the benchmark value for the market risk premium. In this case, while qualitatively biases from ignoring volatility are the same as before, quantitatively they are much smaller now as the role of the volatility risks is substantially reduced relative to the cash-flow risks. Indeed, in this case the volatility risk premium on the market declines to 0.05%, relative to 2.68% in the benchmark case. Finally, our last robustness specification uses the volatility parameters based on the estimates of the Macro-DCAPM-SV model. In the data, the estimated persistence of volatility shocks is 0.63 and the standard deviation is 0.07% on the annual frequency. To convert these numbers to monthly frequency, we set $\nu = 0.63^{1/12} = 0.962$, and let the unconditional variance of volatility at monthly frequency to be equal to 1/12th of the variance of the annual volatility which leads to the estimates of σ_w of 3.65e-05. Thus, the volatility calibration in this setup is consistent with our empirical evidence in the Macro-DCAMP-SV model. The implications for the bias in consumption and SDF dynamics are similar to the benchmark case. When IES is equal to 2, the volatility of consumption shock is 5.1% when volatility is ignored relative to the calibrated 2.5%, the correlation of the two consumption shocks is 0.5, and the volatility of the SDF is biased downward by about one-third.

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Notes

[1]See Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2011) for a discussion of the long-run risks channels for the asset markets and specifically the role of volatility risks, Bansal, Khatchatrian, and Yaron (2005) for an early extensive empirical evidence on the role of volatility risks, Eraker and Shaliastovich (2008), Bansal and Shaliastovich (2010), and Drechsler and Yaron (2011) for the importance of volatility risks for derivative markets, and Bansal and Shaliastovich (2013) for the importance of volatility risks for the bond and currency markets.

[2]The data used in Campbell (1996) is from 1890-1990 which leads to slightly higher volatility numbers than the calibrated model produces.

Table IA.1

Benchmark Configuration of Model Parameters

The Table reports the baseline calibration of the parameters of the preferences and consumption and dividend dynamics in the long-run risks model. The model is calibrated on monthly frequency.

Preferences	δ	γ	ψ	
	0.9987	10	2	
Consumption	μ	ρ	φ_e	
	0.0015	0.975	0.038	
Volatility	σ_g	ν	σ_w	
	0.0072	0.999	2.8e-06	
Dividend	μ_d	ϕ	φ_d	π
	0.0015	2.5	5.96	2.6

Table IA.2

Asset Market Calibration

The Table reports the model-implied risk premia for wealth and market returns, the decomposition of the risk premia into the risk compensation for cash-flow, discount rate and volatility risks, and the correlations of the volatility shock with the discount rate shock for the market and wealth portfolios. Population values based on the long-run risks model with stochastic volatility.

	Wealth	Market
Total Risk Premium	2.63	6.71
Cash-flow Risk Premium	1.26	4.00
Discount Rate Risk Premium	0.04	0.03
Vol. Risk Premium	1.33	2.68
Corr. of discount rate with vol shock	0.83	0.96

Table IA.3

Mis-Specification of Consumption Dynamics

The Table shows the volatility of the consumption shock and its correlation with the true consumption shock in the calibrated long-run risks model (column "True"), and in the specification in which the volatility risks are ignored in construction of the innovations (column "Ignore Vol"). Benchmark Model output is based on the benchmark calibration of the model shown in Table IA.1. $M1$ Model lowers volatility persistence to $\nu = 0.95$ and increases the volatility of volatility σ_w to 4.3e-05 to match the risk premium on the market. In $M2$, the volatility persistence is $\nu = 0.95$ and the persistence of expected consumption is increased to $\rho = 0.984$ to match the market risk premium. For $M3$, the volatility calibration is based on the estimates from the Macro-DCAPM-SV model at a monthly frequency, and involves setting $\nu = 0.962$ and $\sigma_w = 3.65e-05$. Population values based on the long-run risks model with stochastic volatility for various levels of the inter-temporal elasticity of substitution (IES). Volatility is annualized, in percent.

	IES = 2		IES= 1		IES = 0.75	
	Ignore Vol	True	Ignore Vol	True	Ignore Vol	True
Benchmark Model:						
Vol of cons. shock	5.98	2.49	2.49	2.49	2.08	2.49
Corr. with true cons. shock	0.42	1.00	1.00	1.00	0.81	1.00
M1: ν=*0.95,* σ_w= *4.3e-05*						
Vol of cons. shock	4.81	2.49	2.49	2.49	2.69	2.49
Corr. with true cons. shock	0.52	1.00	1.00	1.00	0.93	1.00
M2: ν=*0.95,* ρ= *0.984*						
Vol of cons. shock	2.54	2.49	2.49	2.49	2.50	2.49
Corr. with true cons. shock	0.98	1.00	1.00	1.00	0.99	1.00
M3: ν=*0.962,* σ_w= *3.65e-05*						
Vol of cons. shock	5.14	2.49	2.49	2.49	2.73	2.49
Corr. with true cons. shock	0.49	1.00	1.00	1.00	0.91	1.00

Table IA.4

Mis-Specification of SDF Dynamics

The Table shows the volatility of the stochastic discount factor (SDF) shock, its correlation with the true SDF shock, and the level of the market risk premium in the calibrated long-run risks model (column "True"), and in the specification in which the volatility risks are ignored in construction of the innovations (column "Ignore Vol"). Benchmark Model output is based on the benchmark calibration of the model shown in Table IA.1. $M1$ Model lowers volatility persistence to $\nu = 0.95$ and increases the volatility of volatility σ_w to 4.3e-05 to match the risk premium on the market. In $M2$, the volatility persistence is $\nu = 0.95$ and the persistence of expected consumption is increased to $\rho = 0.984$ to match the market risk premium. For $M3$, the volatility calibration is based on the estimates from the Macro-DCAPM-SV model at a monthly frequency, and involves setting $\nu = 0.962$ and $\sigma_w = 3.65e-05$. Population values based on the long-run risks model with stochastic volatility for various levels of the inter-temporal elasticity of substitution (IES). Volatility is annualized, in percent.

	IES = 2		IES= 1		IES = 0.75	
	Ignore Vol	True	Ignore Vol	True	Ignore Vol	True
Benchmark Model:						
Vol of SDF shock	42.28	66.64	40.91	66.71	40.41	74.47
Corr. with true SDF shock	0.64	1.00	0.61	1.00	0.49	1.00
Market Risk Premium	4.03	6.71	3.21	5.20	2.74	4.44
M1: ν=0.95, σ_w= 4.3e-05						
Vol of SDF shock	42.48	57.72	40.91	55.08	39.94	53.34
Corr. with true SDF shock	0.77	1.00	0.74	1.00	0.73	1.00
Market Risk Premium	4.03	6.71	3.22	5.20	2.75	4.28
M2: ν=0.95, ρ= 0.984						
Vol of SDF shock	57.88	58.07	55.25	55.43	53.52	53.69
Corr. with true SDF shock	0.99	1.00	0.99	1.00	0.99	1.00
Market Risk Premium	6.65	6.71	5.15	5.19	4.24	4.27
M3: ν=0.962, σ_w= 3.65e-05						
Vol of SDF shock	42.43	60.14	40.91	57.50	39.99	55.75
Corr. with true SDF shock	0.74	1.00	0.71	1.00	0.69	1.00
Market Risk Premium	4.03	7.12	3.20	5.51	2.73	4.53

Table IA.5

Consumption Innovation Ignoring Volatility and Consumption Return

The Table shows the volatility of the consumption shock and its correlation with the true consumption shock in the calibrated long-run risks model (column "True"); in the specification in which dividend return is substituted for consumption return (column "Mkt Vol"); and in the specification in which the volatility risks are ignored in construction of the innovations (column "Ignore Vol"). Population values based on the long-run risks model with stochastic volatility (Panel A) and constant volatility (Panel B) for various levels of the inter-temporal elasticity of substitution (IES). Volatility is annualized, in percent.

	IES = 2			IES= 1			IES = 0.75		
	Ignore Vol	With Vol	True	Ignore Vol	With Vol	True	Ignore Vol	With Vol	True
Panel A: Model with Time-varying Volatility									
Vol of cons. shock	14.97	11.60	2.49	10.84	10.84	2.49	10.30	10.79	2.49
Corr. with True cons. shock	0.43	0.56	1.00	0.60	0.60	1.00	0.63	0.60	1.00
Panel B: Model with Constant Volatility									
Volatility of cons. shock	10.34	10.34	2.49	10.30	10.30	2.49	10.28	10.28	2.49
Corr. with True cons shock	0.63	0.63	1.00	0.63	0.63	1.00	0.63	0.63	1.00