

Conservatism and Aggregation: The Effect on Cost of Equity Capital and the Efficiency of Debt Contracts*

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Abstract

This paper studies the joint effect of conservatism and aggregation on the cost of equity capital and the efficiency of debt contracts. In the model, a firm's two assets are valued at either the lower-of-cost-or-market or fair value and the accounting report aggregates the value of the two assets. While the process of aggregation leads inevitably to a loss of information, what information is lost depends on the accounting regime. In the conservative regime, gains are ignored while in the fair value regime gains are off-set against losses.

The paper studies the joint effect of conservatism and aggregation in two settings. First, it considers an all-equity firm and compares the valuation of equity when investors observe an accounting report under either the conservative or the fair-value regime. The model predicts that cost of equity capital is lower in the conservative regime than in the fair value regime if the firm's production function exhibits sufficiently decreasing marginal returns. Second, the paper takes a contractual perspective and considers the efficiency of debt contracts when debt covenants must be written in terms of the accounting report. The paper shows the maximum capital that can be raised by a debt contract which implements efficient post-contractual decisions is higher in the conservative than in the fair value regime.

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1 Introduction

This paper studies the effect of conservatism and aggregation on the cost of equity capital and the efficiency of debt contracts. It considers a specific form of conservative accounting, namely the lower-of-cost-or-market rule for asset values. The lower-of-cost-or-market rule, i.e., recognition of unrealized losses but not unrealized gains, is widely considered the archetype of conservative accounting and has a natural benchmark in fair value accounting where gains and losses are treated symmetrically. By relying on specific accounting rules such as the lower-of-cost-or-market and fair value accounting for long-lived assets, informational properties of financial reports follow directly from the accounting rules instead of being assumed exogenously.

Conservatism winsorizes data on the upper tail. In empirical studies, data is winsorized if “outliers” are likely to contain errors and as a result cause summary statistics of the empirical analysis to be less informative. In accounting, a similar notion exists that supports conservatism based on the conjecture that positive news such as gains are more likely to be erroneous due to manipulation by management than losses (e.g., Watts 2003) and that financial data is therefore more informative if positive news are excluded. Moreover, excluding positive news may deter management from manipulating financial data in the first place (Chen, Hemmer and Zhang 2007, Gao 2012). This paper puts forward a different explanation for winsorizing accounting values. It argues that the financial report can be more informative or useful when financial information such as asset values is winsorized even if manipulation by management is not a concern. In particular, the model shows that conservative accounting is sometimes preferred when considered in conjunction with aggregation, another innate characteristic of accounting.

Aggregation refers to the practice of summarizing raw underlying data into summary measures. Aggregation is a key feature of accounting: countless transactions that occur during a reporting period are aggregated into a set of financial statements at the end of the period which contains only relatively few line items. For example, even though the company likely owns many different machinery, buildings and properties, it reports just a single number on its balance sheet which reflects the aggregate value of all its property, plant and equipment. While the process of aggregation leads inevitably to a loss of information, what information is lost in the process of aggregation or, conversely, what information is reflected in aggregate measures, depends on whether the data has been winsorized.

To illustrate this observation consider the example of two assets, each of which can either maintain its value μ_x , lose Δ in value or gain Δ in value. First, consider the aggregate report of the fair value for the two assets. Since both gains and losses are recognized, the aggregate report simply states the sum of the value of the two assets. As a result, in the fair value regime, a report $r_f = 2\mu_x$ indicates that either both asset have not changed in value or, alternatively, that the gain in value of one asset offset the loss in value of the other asset. Next, consider the aggregate report in the case of conservative accounting. Since gains are not recognized, the aggregate report equals the prior value of the two assets, $2\mu_x$, net of any losses. Hence, the financial report in the conservative regime does not offset gains against losses and in that sense does not conceal losses in individual asset values. Of course, conservative accounting leads to another form of information loss. For instance, in the conservative regime, the report $r_c = 2\mu_x$ merely reveals that neither asset lost value but does not indicate whether none, one or both assets gained in value.

The above example illustrates the importance of jointly considering conservatism and aggregation. If asset values were not aggregated or if there were only one asset, fair value accounting would always produce the more informative report (in a Blackwell sense). Jointly considering measurement rules and aggregation as characteristic features of accounting systems therefore emphasizes that fair value accounting does not always provide more informative reports than conservative accounting even in the absence of manipulation.

The paper studies the joint effect of conservatism and aggregation in two settings. Section 2 considers an all-equity firm and compares the valuation of equity when investors observe a financial report that measures asset values either conservatively or at their fair value. Section 3 takes a contractual perspective and considers the efficiency of debt contracts when debt covenants must be written in terms of the financial report.

In Section 2, risk-averse investors value an all-equity firm upon observing a financial report that aggregates the value of the firm's two assets. It shows that, on average, financial reports in the conservative regime are more informative and result in higher equity valuations than fair value accounting if the firm's production function (with the two assets as input factors) exhibits sufficiently decreasing marginal returns. This is intuitive. If the production function is sufficiently concave in each of the asset values the effect of a gain on firm value is small relative to the effect of a loss. As a result, investors do not forego much valuable information due to the fact that conservative accounting does not recognize gains. In contrast, the possibility that gains may offset

losses in the fair value regime imposes significant uncertainty on investors resulting in lower equity valuations on average. Of course, when the firm's production function features constant marginal returns, fair value accounting provides more informative financial reports. The fact that concavity of the production function and the accounting regime jointly determine the informativeness of financial reports suggests that, depending on the firm's industry and the asset class, different accounting regimes might be best suited to minimize firms' future cost of equity capital. To the extent that standard setters in practice are concerned about the effect of accounting rules on firms' cost of equity capital, this prediction lends support to mixed-attribute accounting. In particular, it might explain why, on the one hand, we observe fair value accounting such as mark-to-market as the measurement rule for financial assets that are less likely to have synergies with the firm's other assets aggregated into the same financial statement item while, on the other hand, we observe conservative accounting such as lower-of-cost-or-market as the preferred measurement rule for productive assets on the balance sheet with significant synergies among them.

Section 2 also illustrates that, in the model, the effect of the accounting regimes on firms' equity valuation is not diversifiable. In particular, it derives a multi-firm capital asset pricing model in which firms' cost of equity capital are measured as the expected return. It shows that if a firm's production function again exhibits sufficiently decreasing marginal returns, the firm's cost of equity capital is lower on average when the firm's accounting system is conservative than when the firm's assets are accounted for at their fair value.

Section 3 considers the effect of accounting regimes on the efficiency of debt contracts and hence provides a contractual rather than an informational perspective of the joint effects of aggregation and conservatism. The literature has considered several incentive problems between equityholders, management and debtholders. For instance, managers might pay higher than optimal dividends (e.g., Kalay 1982, Leftwich 1983, Healy and Palepu 1990), continue projects even though it is optimal to terminate them (e.g., Holmström and Tirole 1998 and 2000, Tirole 2006), or dilute debtholders' claims by substituting more risky assets (e.g., Gavish and Kalay 1983). Some of these shareholder-debtholder conflicts may be addressed through covenants (e.g., Jensen and Meckling 1976, Aghion and Bolton 1992, Gârleanu and Zwiebel 2009). Accounting regimes may therefore affect the efficiency of debt contracts if there is an incentive problem that can be addressed through covenants which are based on financial reports. Section 3 outlines such a setting in which debt covenants may alleviate suboptimal liquidation decisions at an interim date. Following Tirole

(2006), I assume that the manager may engage in ‘bad’ behavior such as shirking or taking private benefits and hence requires a certain amount of pay-for-performance in order to exhibit optimal behavior. As a result of this moral hazard problem, the firm cannot pledge its entire cash flows to the lender and it is optimal for the firm to raise long-term funds. Due to potential incentive conflicts between the manager and the lender, contingent control allocation may allow the firm to implement a more efficient liquidation decision at the intermediate date (Aghion and Bolton 1992). The contingent control allocation or debt covenants must be written in terms of contractible information about the value of the firm’s assets. While the individual asset values may be publicly observable, I assume that only the aggregate asset value as reflected in the financial report is contractible. This provides a channel for accounting regimes to affect the efficiency of debt contracts.¹

Section 3 shows that the efficient liquidation decision can be implemented in either accounting regime as long as the capital to be raised in form of debt is not too high. This is intuitive. If the manager is always allocated control, renegotiation at the interim stage ensures that the efficient liquidation decision is achieved. However, the manager’s bargaining power in the renegotiation stage limits the amount of cash the lender expects to obtain if he is not allocated control. As a result, the amount of capital that can be raised in the form of debt is limited when the manager always maintains control of the firm. If the required amount of capital is higher the lender must obtain control rights in some states of the world. Allocating the decision rights to the lender guarantees the lender higher cash flows than if the manager is allocated control. However, when the lender obtains the control rights, he tends to liquidate the firm’s assets inefficiently. In particular, he may prefer to liquidate an asset that did not lose any value in the first period even though it is efficient not to. Hence, the lender is most likely to implement the efficient liquidation decision if he is only allocated control over the firm’s assets when a loss occurred. Implementing the efficient liquidation decision may therefore be supported by a debt covenant that allocates control rights to the lender if and only if at least one of the firm’s assets lost value. The conservative regime always identifies whether one of the firm’s assets incurred a loss in value. In contrast, the fair value regime may offset gains against losses. In particular, in the fair value regime, a report $r_f = 2\mu_x$ does not distinguish whether neither asset changed in value or whether the gain of one asset offset the loss of the other asset. Avoiding inefficient liquidation decisions, which would occur if the lender

¹In order to highlight that the effects studied in Section 3 are specific to debt contracting and cannot be attributed to decreasing marginal returns of firms’ production technology or investors’ risk-aversion, I assume in this section that the firm’s production technology exhibits constant marginal returns and potential lenders are risk-neutral.

were to be allocated control when neither asset changed in value, therefore requires that, in the fair value regime (but not in the conservative regime), the lender is also not allocated control when one asset lost value while the other gained value. As a result, the maximum capital that can be raised by a debt contract which implements the efficient liquidation decision is weakly higher in the conservative than in the fair value regime.

More generally, the paper shows that debt contracts are more efficient when covenants can be written in terms of conservative rather than fair value reports as long as the capital to be raised in the form of debt is below a certain threshold. For higher amounts, fair value accounting may result in more efficient debt contracts. This is intuitive. If the required amount of capital is so high that allocating control to the lender only when assets lost value does not allow the lender to break even the lender must be allocated control also when no asset lost value. In that case, the lender must always be allocated control in the conservative regime because conservative accounting does not distinguish between an asset maintaining its value and an asset gaining in value. In contrast, fair value allows for a more differentiated allocation of control rights because, e.g., the lender may be allocated control when both assets maintained their value while the manager may retain control if one asset maintained its value while the other asset gained in value.

The paper contributes to the literature in four ways. First, it provides a potential explanation for why conservative accounting is pervasive in practice and provides a framework for empirical evidence consistent with contracting efficiency being enhanced by conservatism. For instance, Basu (1997) and Ball, Kothari and Robin (2000) as well as subsequent papers provide evidence that conservatism is observed in the U.S. and internationally. Bushman and Piotroski (2006) finds that earnings reflect bad news in a more timely manner than good news in countries in which firms rely extensively on private bonds. Related, Beatty, Weber and Yu (2008) find evidence consistent with financial reporting conservatism reflecting lenders' demand for conservatism while Zhang (2008) documents that lenders offer lower interest rates to borrowers with more conservative financial reporting.²

Second, it starts with a generally accepted specification of conservatism, the lower-of-cost-or-market rule, and derives the properties of the financial reports based on this definition. In this way, the model avoids the controversy of having to define the informational properties of conservative reports in an abstract manner. This is helpful because, even though the concept of conservatism

²However, interest rates are not a comprehensive measure of contracting efficiency in the model studied in this paper. Also, see Gigler et al. (2009).

seems intuitive, the informational properties of conservative accounting numbers are controversial. In particular, models differ in whether they consider an information system in which financial reports that contain good news are more informative than financial reports that contain bad news as conservative or aggressive. In Guay and Verrecchia (2006), Gow (2009), and Göx and Wagenhofer (2010), more conservative accounting refers to low reports being more informative while in Chen et al. (2007), Gigler et al. (2009), Gao (2011), and Lu et al. (2011) more conservative accounting refers to high reports being more informative. Of course, whether information systems that generate high reports that are more informative than low reports are considered to be conservative or aggressive has implications for the interpretation of models' predictions. Relying on intuitive representations of conservative measurement rules, such as the lower-of-cost-or-market in this paper, avoids this controversy (see, e.g., Beyer et al., 2010; and Berger, 2011).

Third, the paper identifies conditions when conservatism improves the efficiency of debt contracts. It therefore adds to the theoretical literature that studies the effect of information structures on the design of debt covenants (e.g., Sridhar and Magee 1997, Gârleanu and Zwiebel 2009, Gigler et al. 2009, Caskey and Hughes 2012). Moreover, the paper shows that – contrary to popular beliefs – conservatism can be beneficial not only in the context of debt contracts but also in the context of equity valuation. As a result, the perceived tension between financial information useful for contracting and financial information useful for valuation might be less severe than previously thought.

Forth, it relates conservatism to aggregation, another key characteristic of accounting. As far as I am aware, this is the first paper that considers the properties of conservative financial reports when conservatism (i.e., lower-of-cost-or-market) is applied as a measurement rule to two individual assets whose values are then aggregated.³ It identifies conditions under which ignoring gains is beneficial and therefore directly addresses the concern expressed in Guay and Verrecchia (2006) that

“[...] the crux of the debate about the merits of conservatism should revolve around

³This paper takes aggregation exogenous. Few papers in the literature study the benefits of aggregating multiple signals into a single disclosed report, including Gigler and Hemmer (2002) and Dye and Sridhar (2004). Gigler and Hemmer (2002) study the benefits of aggregation in a contracting setting in which aggregation has positive incentive effects because it limits the agent's strategy space. Dye and Sridhar (2004) derive conditions under which aggregate reports yield more efficient investment decisions than disaggregate reports in a capital market setting because aggregation limits managers' incentives to misreport earnings. Aggregation of information in the sense of transforming relatively fine into relatively coarse information has been studied in, e.g., Kwon, Newman and Suh (2001) and Fan and Zhang (2012).

the question of why, conditional on the decision to incorporate some difficult-to-verify information into accounting statements, economic gains are recognized in a less timely manner than economic losses. We perceive that much of the existing literature on conservatism is focused exclusively on why information about losses should be incorporated in financial statements in a timely manner, with little if any, research on why information about gains should be excluded from timely recognition in financial statements.”

The remainder of the paper is organized as follows. Section 2 examines the role of conservatism in equity valuation. Section 3 studies the effect of conservatism on the efficiency of debt contract and Section 4 concludes. All proofs are relegated to the Appendix.

2 Role of conservatism in equity valuation

This section emphasizes the role of conservatism in the valuation of equity capital and assumes that the firm is financed entirely by equity. In a capital market setting in which risk-averse investors price the firm’s equity, this section compares financial reports in terms of their informativeness about future cash flows and their effect on firms’ cost of capital when the financial reports are prepared according to conservative accounting (lower-of-cost-or-market) or according to fair value accounting.

2.1 Model setup

The simplest case that allows us to study the joint effect of measurement rules and aggregation is a firm with two assets for which the firm issues a single financial report that reflects the sum of the value of its two assets. The model therefore assumes that the firm has two assets and that the value of each asset is measured according to either a fair value measurement rule or a conservative measurement rule. The sequence of events is the following.

At the beginning of period 1 ($t = 0$), the firm purchases two assets. It uses the assets in its production process which generates cash flows at the end of period 2 ($t = 2$). The firm’s accounting system generates a report about the value of the firm’s assets upon purchase and at the end of period 1 ($t = 1$). Investors trade the firm’s shares upon observing the financial report.

At the beginning of the first period, the firm purchases two identical assets, $i = 1, 2$, at a price of μ_x per asset and reports their aggregate value. This initial report equals $2\mu_x$ independent of the accounting system since both fair value accounting and historical cost accounting value

assets initially at their purchase price. In the following analysis, μ_x , is therefore considered public knowledge and we focus on the financial report about the values of the firm’s assets at $t = 1$. The values of the firm’s two assets change over time and equal $x_{i,t}$ at time $t = 1, 2$. While asset values change over time, the changes are zero on average such that μ_x denotes common prior beliefs about asset i ’s value, i.e., $\mu_x = E[\tilde{x}_{i,1}] = E[\tilde{x}_{i,2}]$, and the asset value at time $t = 1$ equals the expected value at time 2, $x_{i,1} = E[\tilde{x}_{i,2}|x_{i,1}]$.

More specifically, the simplest distributional assumptions that illustrate the trade-offs between conservative and fair-value accounting from an informational perspective, and the ones adopted here, are as follows. At time 1, the change in each asset’s value can be either zero, positive or negative compared to the price μ_x at which the asset was purchased. To allow the asset value to remain the same or to change in either direction while at the same time keeping calculations tractable, I assume that $x_{i,1} \in \{\mu_x - \Delta, \mu_x, \mu_x + \Delta\}$ where $\Delta \in [0, \mu_x]$ with positive and negative changes to the asset value occurring with probability $\Pr(\tilde{x}_{i,1} = \mu_x + \Delta) = \Pr(\tilde{x}_{i,1} = \mu_x - \Delta) = \frac{p}{2}$ and the asset value remaining the same with probability $\Pr(\tilde{x}_{i,1} = \mu_x) = 1 - p$. The changes in asset values are independent across assets.⁴ For simplicity, I assume $x_{i,1} = x_{i,2}$, i.e., that the asset values do not change between time 1 and 2.

The company uses the assets as inputs in its production process. The firm’s production technology allows the firm to produce goods worth $V(x_{1,2}, x_{2,2})$ where the production or value function is given by $V(x_{1,2}, x_{2,2}) = k(x_{1,2}^\gamma + x_{2,2}^\gamma)$ for $\gamma \in (0, 1]$ and $k > 0$.⁵ The production function is homogenous of degree γ and exhibits weakly decreasing marginal returns. The production function is assumed to be symmetric with respect to the firm’s two assets. This captures the notion that, on balance sheets, similar assets are aggregated into a single line-item and similar assets are likely to contribute to firm value in similar ways.⁶

The value of the firm’s assets at $t = 1$ are not directly observable to outside investors. Instead,

⁴While verifiability is not the focus of this paper, the definition of conservatism as lower-of-cost-or-market is consistent with Basu (1997)’s definition of conservatism as a higher degree of verification for gains than for losses if we presume that, following “good news” ($x_{i,1} = \mu_x + \Delta$), the uncertainty in $x_{i,2}$ is too high to warrant recognition while, following “bad news” ($x_{i,1} = \mu_x - \Delta$), it is not.

⁵The parameter k does not play a central role in the following analysis. However, by assuming that k is sufficiently high, we can ensure that selling the asset at its market value $x_{i,t}$ is only a second-best option compared to using the asset for its intended production purpose within the firm, i.e., $E[k\tilde{x}_{i,2}^\gamma|x_{i,t}] > x_{i,t}$.

⁶Of course, it is straight-forward to construct non-symmetric production functions that yield predictions similar to the ones derived below. Also, the function assumes that firm’s output depends on the value of firm’s assets at $t = 2$. For instance, a machinery might be damaged in period 2 and as a result, its value and the firm’s output decrease. However, the model’s main trade-off is independent of whether asset values at $t = 2$ or $t = 1$ determine firm output. In fact, for simplicity, as described above, asset values are assumed to remain the same between date 1 and date 2.

investors observe a financial report r generated by the firm’s accounting system. For exogenous reasons, the firm can generate only an aggregate report of the values of its two assets. We consider two accounting regimes. In the fair value accounting regime, the report generated simply equals the sum of the values of the firm’s two assets at date 1, i.e.,

$$r_f = x_{1,1} + x_{2,1}. \quad (1)$$

In contrast, the conservative accounting regime applies the lower-of-cost-or-market rule to each asset. In particular, it compares the asset’s carrying value (equal to its historical acquisition costs), μ_x , to the asset’s fair value at date 1 and recognizes the lower of the two in the financial report at date 1, i.e.,

$$r_c = \min \{ \mu_x, x_{1,1} \} + \min \{ \mu_x, x_{2,1} \}. \quad (2)$$

Depending on the accounting regime implemented by the firm, investors observe either the fair value report r_f or the conservative report r_c but not both. Whether the firm’s report is prepared according to the fair value or conservative accounting regime is publicly observable and not a choice variable of the firm. This reflects conservatism as prescribed by accounting standards and implemented in firms’ accounting information systems.⁷

It is evident that, in the model, the asset value $x_{i,t}$ refers to the exit or market value of the firm’s assets at date t rather than to the asset’s incremental contribution to firm value. The fact that book values are based on assets’ exit/market values rather than on the entity specific value in use is due to the difficulties associated with calculating and verifying the latter and broadly consistent with accounting practices.⁸ For instance, consider a firm whose output is described by a Cobb-Douglas production function $kx_1^\gamma x_2^\gamma$ or a production function with fixed proportions $k \min \{ x_1, x_2 \}$.⁹ In both

⁷Firms’ financial statements may differ in the degree of their conservatism due to the reporting flexibility inherent in many financial statement items and management’s incentives to manipulate financial reports (e.g., Dye, 2002; Chen et al., 2007). To the extent that investors correctly infer the degree of conservatism in the financial report, similar properties of cost of capital and stock prices continue to hold.

⁸Consider, for instance, accounting for property, plant and equipment under IFRS and US GAAP, a prominent example of the fair value vs. historical cost debate. US GAAP applies the lower-of-cost-or-market rule when accounting for property, plant and equipment. According to FAS 144, property, plant and equipment (long-lived assets) are impaired if the asset’s carrying value exceeds its fair value where the fair value is determined using quoted market prices or, if those are unavailable, market prices of similar assets. In contrast, IFRS allows the use of the revaluation model under which assets are carried on the balance sheet at their fair value where fair value is also defined as “the amount for which the asset could be exchanged between knowledgeable, willing parties in an arm’s length transaction” (IFRS 2, IAS 16).

⁹For these alternative production functions, qualitatively similar results as the ones described in the following sections, apply. Generally, for production functions $V(x_1, x_2)$ that are symmetric, i.e., $V(a, b) = V(b, a)$, and have decreasing marginal returns, the results hold because $V(\mu_x - \Delta, \mu_x + \Delta) < V(\mu_x, \mu_x)$. However, note that if the production function is such that the sum of asset values $x_1 + x_2$ is a sufficient statistic for (x_1, x_2) with respect to firm

cases, the value in use, i.e., the incremental contribution of an asset to firm value, depends on the other asset's value and requires detailed knowledge of the firm's production function. Hence, the value in use might not be readily determinable and verifiable by, e.g., auditors. As a result, financial reports based on exit/market values are significantly easier to implement and verify.

2.2 Effect of conservatism on the informativeness of financial reports

Before analyzing the effect of accounting regimes on firms' cost of capital, this section studies the informativeness of financial reports about future cash flows as a preliminary step. In particular, this section derives investors' ability to predict firm value based on the financial report, r_j , $j \in \{f, c\}$, as measured by the average residual variance of firm value at $t = 1$, $E[Var(V(\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_j)]$, and compares it across the two accounting regimes.

In the following analysis, it is often desirable to characterize a change in asset value in terms of its impact on firm value relative to the case when the asset value remains at μ_x , as defined in the following definition.¹⁰

Definition 1 *The “upside potential” of an asset on firm value is measured by $u \equiv \left(1 + \frac{\Delta}{\mu_x}\right)^\gamma$; the “downside risk” of an asset on firm value is measured by $d \equiv \left(1 - \frac{\Delta}{\mu_x}\right)^\gamma$.*

From $\Delta \in (0, \mu_x)$ and $\gamma \in (0, 1]$, it follows that $u \in (1, 2)$ and $d \in (0, 2 - u)$. Note that the upside potential is greater if u is higher while the downside risk is greater if d is lower.

In the fair value accounting regime, the report can take on the values $2\mu_x - 2\Delta$, $2\mu_x - \Delta$, $2\mu_x$, $2\mu_x + \Delta$ and $2\mu_x + 2\Delta$. For each report generated by the fair value accounting regime, Table 1 shows the distribution of possible firm values and the resulting conditional variance of firm value.¹¹

If they were able to observe the individual asset values $x_{1,1}$ and $x_{2,1}$, investors would be able to perfectly predict firm value (due to the simplifying assumption that asset values do not change between $t = 1$ and $t = 2$). As a result, investors' residual uncertainty upon observing the financial report as calculated in the right most column of Table 1 reflects the loss of information due to aggregation. Table 1 shows that, in the fair value accounting regime, information is lost when the firm issues the intermediate report $r_f = 2\mu_x$. Such a report implies that either the value of both

value V , such as $V(x_1, x_2) = k(x_1 + x_2)^\gamma$, the fair value accounting system will always generate more informative financial reports and result in lower cost of capital.

¹⁰For similar assumptions regarding the distribution as well as the notion of “upside potential” and “downside risk” see Bertomeu, Beyer and Dye (2011).

¹¹Supporting calculations for Table 1 and Table 2 are included in the Appendix.

r_f	$\Pr(\tilde{r}_f = r_f)$	$\Pr(V(\tilde{x}_{1,2}, \tilde{x}_{2,2}) r_f)$	$Var(V(\tilde{x}_{1,2}, \tilde{x}_{2,2}) r_f)$
$2\mu_x - 2\Delta$	$\frac{p^2}{4}$	$V(\mu_x - \Delta, \mu_x - \Delta)$ with prob. 1	0
$2\mu_x - \Delta$	$p(1-p)$	$\begin{cases} V(\mu_x - \Delta, \mu_x) & \text{with prob. } \frac{1}{2} \\ V(\mu_x, \mu_x - \Delta) & \text{with prob. } \frac{1}{2} \end{cases}$	0
$2\mu_x$	$(1-p)^2 + \frac{p^2}{2}$	$\begin{cases} V(\mu_x - \Delta, \mu_x + \Delta) & \text{with prob. } \frac{\frac{p^2}{4}}{(1-p)^2 + \frac{p^2}{2}} \\ V(\mu_x, \mu_x) & \text{with prob. } \frac{(1-p)^2}{(1-p)^2 + \frac{p^2}{2}} \\ V(\mu_x + \Delta, \mu_x - \Delta) & \text{with prob. } \frac{\frac{p^2}{4}}{(1-p)^2 + \frac{p^2}{2}} \end{cases}$	$(k\mu_x\gamma)^2 \frac{\frac{p^2}{2}(1-p)^2}{\left((1-p)^2 + \frac{p^2}{2}\right)^2} * (u+d-1)^2$
$2\mu_x + \Delta$	$p(1-p)$	$\begin{cases} V(\mu_x, \mu_x + \Delta) & \text{with prob. } \frac{1}{2} \\ V(\mu_x + \Delta, \mu_x) & \text{with prob. } \frac{1}{2} \end{cases}$	0
$2\mu_x + 2\Delta$	$\frac{p^2}{4}$	$V(\mu_x + \Delta, \mu_x + \Delta)$ with prob. 1	0

Table 1: Fair Value Accounting Regime

assets has remained unchanged, i.e., $x_{1,1} = x_{2,1} = \mu_x$, or that one asset lost value while the other asset gained in value, i.e., $x_{i,1} = \mu_x - \Delta$ and $x_{j,1} = \mu_x + \Delta$, $i \neq j$. The fact that investors cannot distinguish between these two scenarios is a key feature of fair value accounting: bad news are offset against good news.

The fact that investors are uncertain about asset values when the firm reports $r_f = 2\mu_x$ is sensitive to the model's assumption that gains and losses, if they occur, are of the same magnitude. While this might seem like a restrictive assumption at first, it is an artefact of the discrete distribution and would not have to be imposed if asset values were continuously distributed at time $t = 1$ or if the financial report aggregates the value of many assets. Assuming that gains and losses are of the same magnitude therefore allows us to capture the offsetting of good news against bad news in the fair value accounting regime while keeping the model tractable. By the same token, it is important that the support of the asset values at time 1 is, at a minimum, ternary. Distributions with binary support, as they have been used in models of conservatism in the prior literature, cannot capture the feature of bad news being offset against good news.¹²

¹²The fact that aggregation does not lead to a loss in information under the fair value reporting regime if asset

In the conservative accounting regime, good news is ignored and hence the report r_c can take on only values below the prior value of the two assets of $2\mu_x$, specifically $r_c \in \{2\mu_x - 2\Delta, 2\mu_x - \Delta, 2\mu_x\}$. Similar to the information presented for the fair value accounting regime in Table 1, Table 2 summarizes the distribution of possible firm values and the resulting conditional variance of firm value for each possible financial report, r_c , in the conservative accounting regime.

r_c	$\Pr(\tilde{r}_c = r_c)$	$\Pr(V(\tilde{x}_{1,2}, \tilde{x}_{2,2}) r_c)$	$Var(V(\tilde{x}_{1,2}, \tilde{x}_{2,2}) r_c)$
$2\mu_x - 2\Delta$	$\frac{p^2}{4}$	$V(\mu_x - \Delta, \mu_x - \Delta)$ with prob. 1	0
$2\mu_x - \Delta$	$p(1 - \frac{p}{2})$	$\left\{ \begin{array}{ll} V(\mu_x - \Delta, \mu_x) & \text{with prob. } \frac{1-p}{2-p} \\ V(\mu_x, \mu_x - \Delta) & \text{with prob. } \frac{1-p}{2-p} \\ V(\mu_x - \Delta, \mu_x + \Delta) & \text{with prob. } \frac{1}{2} \frac{p}{2-p} \\ V(\mu_x + \Delta, \mu_x - \Delta) & \text{with prob. } \frac{1}{2} \frac{p}{2-p} \end{array} \right.$	$2(k\mu_x^\gamma)^2 \frac{p(1-p)}{(2-p)^2} (u-1)^2$
$2\mu_x$	$\frac{1}{4}(2-p)^2$	$\left\{ \begin{array}{ll} V(\mu_x, \mu_x) & \text{with prob. } \frac{4(1-p)^2}{(2-p)^2} \\ V(\mu_x, \mu_x + \Delta) & \text{with prob. } \frac{2p(1-p)}{(2-p)^2} \\ V(\mu_x + \Delta, \mu_x) & \text{with prob. } \frac{2p(1-p)}{(2-p)^2} \\ V(\mu_x + \Delta, \mu_x + \Delta) & \text{with prob. } \frac{p^2}{(2-p)^2} \end{array} \right.$	$4(k\mu_x^\gamma)^2 \frac{p(1-p)}{(2-p)^2} (u-1)^2$

Table 2: Conservative Accounting Regime

Table 2 illustrates that conservative accounting provides investors with precise information in the worst state of the world. When $r_c = 2\mu_x - 2\Delta$ investors know that both assets are valued at $x_{i,1} = \mu_x - \Delta$, $i = 1, 2$. Following more positive reports, i.e., $r_c = 2\mu_x - \Delta$ and $r_c = 2\mu_x$, investors

values are binary holds true even if we consider more than two assets. To see this consider the following scenario. Suppose there are n identical assets for which $x_{i,1} \in \{\mu_x - \Delta, \mu_x + \Delta\}$ and the firm issues report $r_f = \sum_i x_{i,1}$. The report can range from $n(\mu_x - \Delta)$ when all assets loose value to $n(\mu_x + \Delta)$. Specifically, the possible values for the report are $n\mu_x + k\Delta$ for $k \in \{-n, -(n-1), -(n-2), \dots, -1, 0, 1, \dots, n-2, n-1, n\}$. We know that

$$r_f = n_+(\mu_x + \Delta) + (n - n_+)(\mu_x - \Delta)$$

where n_+ denotes the number of assets that gained in value and $n - n_+$ the number of assets that lost in values. All values in the equation except n_+ are known. Hence, we can solve for n_+ as

$$n_+ = \frac{r_f - n(\mu_x - \Delta)}{2\Delta}.$$

The fact that we can solve for the number of assets that gained in value, and, equivalently, the number of assets that lost in value, implies that aggregation does not lead to a loss of information if the support of the asset values is $\{\mu_x - \Delta, \mu_x + \Delta\}$ rather than $\{\mu_x - \Delta, \mu_x, \mu_x + \Delta\}$.

cannot perfectly infer asset values. However, by ignoring gains, conservative financial reports allow investors to unequivocally identify the losses in asset values that occurred in between date 0 and date 1. In particular, investors know that if $r_c = 2\mu_x - 2\Delta$ both assets must have lost value, if $r_c = 2\mu_x - \Delta$ exactly one asset must have lost value, and if $r_c = 2\mu_x$ investors know that neither asset has lost value. The information that is lost in the conservative financial report is whether any asset that did not lose value is valued at μ_x or $\mu_x + \Delta$.

In contrast to the fair value accounting regime, aggregation is not the cause of information loss in the conservative accounting regime: whether the firm reports r_c as described above or the (conservative) valuation $\min\{\mu_x, x_{i,1}\}$ for each asset separately does not make a difference in terms of the information conveyed to investors. In both cases, i.e., whether they observe r_c or $\min\{\mu_x, x_{1,1}\}$ and $\min\{\mu_x, x_{2,1}\}$, investors can infer exactly the same information, namely the number of assets that lost value.¹³ This suggests that aggregation has less bearing in conservative accounting regimes than in fair value accounting regimes where all loss of information is due to aggregation. This observation also suggests that by limiting the model to just two assets rather than considering the more realistic scenario of financial statement items aggregating many assets might actually favor the fair value accounting regime.

Building on the informational properties of the fair value and conservative accounting regimes in Tables 1 and 2, we next compare the two accounting regimes. Figure 1 illustrates the information sets in fair value and conservative accounting regimes. Consistent with the earlier discussion, the figure illustrates that they cannot be ranked in a Blackwell sense, i.e., neither regime provides information sets that are a partition of the information sets of the other regime. For instance, on the one hand, the fair value accounting regime allows investors to discriminate between the states $(x_{1,1}, x_{2,1}) = (\mu_x - 1, \mu_x)$ and $(\mu_x - 1, \mu_x + 1)$ because the report is $r_f = 2\mu_x - 1$ and $r_f = 2\mu_x$, respectively, while the conservative accounting regime provides the same report, $r_c = 2\mu_x - 1$, in both states. On the other hand, the conservative accounting regime allows investors to discriminate between the states $(x_{1,1}, x_{2,1}) = (\mu_x - 1, \mu_x + 1)$ and (μ_x, μ_x) because the report is $r_c = 2\mu_x - 1$ and $r_c = 2\mu_x$, respectively, while the fair value accounting regime provides the same report, $r_f = 2\mu_x$, in both states.

¹³The fact that aggregation does not lead to a loss in information is not an artefact of limiting the model to two assets. It can easily be seen that extending the model to n asset would not change this observation. If losses can be of different magnitude (which is precluded in the model due to the restriction to two assets as discussed above), then investors learn the total magnitude of losses but not the number of assets across which these losses occurred.

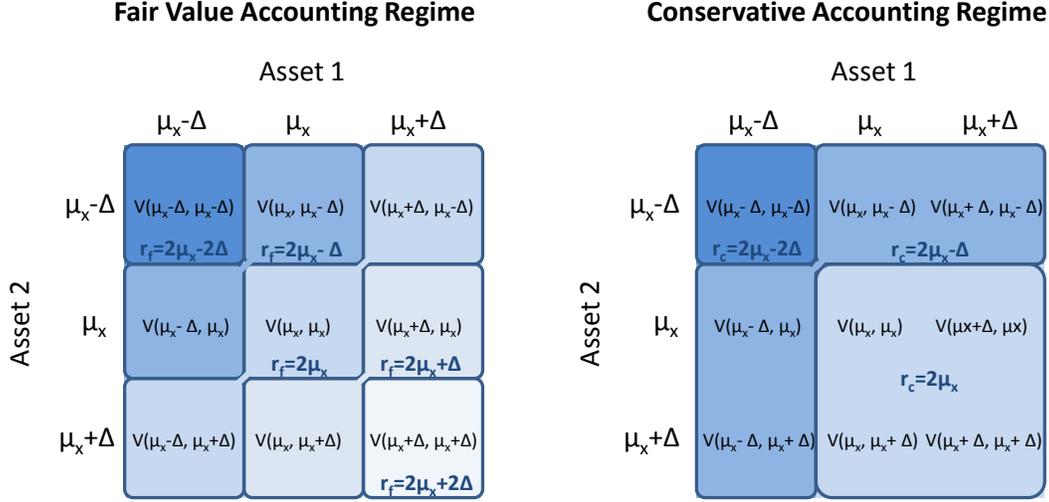


Figure 1: Information sets under fair value and conservative accounting regimes

Since the accounting regimes cannot be ranked in a Blackwell sense, which regime is more informative depends on the decision problem at hand (Blackwell 1951). Intuitively, if it is “more important” to know the average asset value, the fair value accounting regime will dominate. If, however, it is “more important” to know whether some assets incurred a loss, the conservative accounting regime will dominate.

Providing financial information that is useful to investors is a stated objective of standard setters.¹⁴ The standard setter does not specify how capital providers’ ability to predict future cash flows should best be measured. Since (the inverse of) variance is a widely accepted measure of predictability or, conversely, of risk, the extent to which investors are able to predict future cash flows based on financial reports as measured by (the inverse of) cash flow variance provides some indication of the circumstances under which fair value accounting is preferred to conservative accounting, and vice versa. In the context of the model, the conditional variance serves as a building block for deriving the effect of conservatism on the firm’s cost of capital studied in the following section. The following Lemma characterizes whether the fair value or the conservative accounting regime on average result in less uncertainty as measured by the variance of firm value conditional on the financial report issued at time 1. If the ratio is greater than 1, the fair value accounting regime on average provides more precise information than the conservative accounting regime. If

¹⁴For instance, FASB emphasizes the importance of providing information that is useful for capital providers in reaching their investment decisions and therefore aims at providing information that allows capital providers to assess “the entity’s ability to generate net cash flows” and, specifically, to assess the amount, timing, and uncertainty of future cash flows” (FASB 2008, p. 3).

the ratio is less than 1, the reverse is true.

Lemma 1 *The ratio of the average conditional variance in the conservative accounting regime to the average conditional variance in the fair value accounting regime,*

(a) *is given by*

$$\frac{E[\text{Var}(V(\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_c)]}{E[\text{Var}(V(\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_f)]} = \frac{(18 - 7p) \left((1 - p)^2 + \frac{p^2}{2} \right)}{p(1 - p)(2 - p)} \left(\frac{u - 1}{u + d - 2} \right)^2;$$

(b) *increases in u and d and is U-shaped in p ;*

(c) *converges to 0 for $u \rightarrow 1$ and $d < 1$;*

(d) *converges to ∞ for $d \rightarrow 2 - u$ and $u > 1$.*

Lemma 1 establishes that, despite the fact that conservative accounting ignores information, i.e., despite the fact that conservative accounting does not recognize gains, the reports of conservative accounting regimes can be more informative about future firm value than financial reports of fair value accounting regimes which recognize both gains and losses. Lemma 1 part (c) shows that this is the case if the upside potential u is sufficiently low as long as there is still some downside risk, i.e., $d < 1$. Conversely, Lemma 1 part (d) shows that reports of fair value accounting regimes are more informative if downside risk is sufficiently low, i.e., if d is sufficiently close to its upper bound $2 - u$, as long as there is still some upside potential, i.e., $u > 1$. In particular, if $d = 2 - u$, the production function has constant marginal returns, i.e., $V(x_{1,2}, x_{2,2}) = k(x_{1,2} + x_{2,2})$. In that case, knowing the sum of asset values as revealed by the fair value financial report, $r_f = x_{1,2} + x_{2,2}$ is sufficient for investors to perfectly infer firm value. As a result, fair value accounting is more informative than conservative accounting.¹⁵

Production exhibits non-zero downside risk ($0 < d < 1$) while having minimal upside potential ($u \rightarrow 1$) when asset values are sufficiently uncertain and the production function is sufficiently concave. Asset values exhibit more uncertainty if potential losses or gains as measured by Δ are larger in magnitude. Similarly, the production function is more concave if γ is sufficiently low. It follows that conservative accounting provides more informative reports if Δ is sufficiently high and γ is sufficiently low. This is established in the following proposition.

¹⁵Let π denote the first factor of the ratio of the average conditional variance in the conservative accounting regime to the average conditional variance in the fair value accounting regime. Then, the ratio is less than 1 if and only if $u(\sqrt{\pi} + 1) \leq 2 + \sqrt{\pi} - d$.

Proposition 1 *There exist $\Delta^* < \mu_x$ and $\gamma^* > 0$ such that for all $(\Delta, \gamma) \in \{(\Delta, \gamma) \mid \Delta > \Delta^* \text{ and } \gamma < \gamma^*\}$ the conservative accounting regime is more informative in the sense that $\frac{E[\text{Var}(V(\tilde{x}_{1,2}, \tilde{x}_{2,2})|r_c)]}{E[\text{Var}(V(\tilde{x}_{1,2}, \tilde{x}_{2,2})|r_f)]} < 1$.*

Conservative financial reports are more informative than fair value financial reports if the production function is sufficiently concave because decreasing marginal returns have different effects on the residual uncertainty investors face in the conservative versus the fair value accounting regime. In the fair value accounting regime, decreasing marginal returns imply that a loss in value of one of the assets is not fully offset by a gain in value of the other asset. As a result, investors who learn that the average asset value has not changed (as they do in the fair value accounting regime when $r_f = 2\mu_x$) still face uncertainty with respect to firm value because they cannot discern whether both assets are valued at μ_x and firm value is $V(\mu_x, \mu_x) = 2k\mu_x^\gamma$ or whether one asset has gained while the other asset has lost in value and firm value is $V(\mu_x - \Delta, \mu_x + \Delta) = 2k((\mu_x - \Delta)^\gamma + (\mu_x + \Delta)^\gamma)$ which is lower than $2k\mu_x^\gamma$. The higher the uncertainty about the firm's asset values, Δ , and the more concave the production function, the larger is the difference between these two scenarios and offsetting bad news against good news significantly limits investors' ability to predict firm value.

In contrast, in the conservative accounting regime, investors always learn about decreases in asset values. What investors don't know is whether an asset has gained in value or whether its value just remained unchanged. However, the more concave the production function is, i.e., the lower γ and the more decreasing the marginal returns are, the smaller is the effect of a gain in asset value on firm value. As a result, investors do not forego much valuable information due to the fact that conservative accounting does not recognize gains. This illustrates the importance of jointly considering the informational effects of value measurement and aggregation.

2.3 Effect of conservatism on cost of capital

This section derives explicit equilibrium asset prices that allows us to compare the cost of capital of a firm under the two accounting regimes. Consistent with the asset pricing literature, cost of capital is defined to be the expected return on the firm's equity capital that equates the firm's stock price at the beginning of the period to the expected cash flow investors receive at the end of the period.

In order to differentiate between systematic and idiosyncratic risk, I extend the setting to multiple firms and allow liquidating dividends V_j to be correlated across firms. Specifically, assume that there are $j = 1..N_F$ identical firms which are owned by the N_I identical investors.

In order to introduce correlation among firms' cash flows, suppose there is a "market state" $\omega_M \in \{2d, 1+d, u+d, 2, u+1, 2u\}$ with probabilities $\left\{ \frac{p^2}{4}, p(1-p), \frac{p^2}{2}, (1-p)^2, p(1-p), \frac{p^2}{4} \right\}$. For each firm, the indicator variable \tilde{I}_j^M equals 1 if the firm value V_j is determined by the market state, i.e., $V_j = k\mu_x\omega_M$, and it equals 0 if the firm value V_j is distributed independently of the market state and the value of all other firms. The probability of a firm value being determined by the market state is denoted by $q_j \equiv \Pr(\tilde{I}_j^M = 1)$ where \tilde{I}_j^M are distributed independently across firms and independent of all other random variables in the model.

This setup has the following two desirable features. First, it preserves the ex ante distribution of a firm's cash flows described in Section 2.1. Second, it provides a parsimonious model of idiosyncratic and systematic risk. When firm j 's cash flows are determined by the market state, i.e., when $\tilde{I}_j^M = 1$, firm j 's cash flows are correlated with the cash flows of all other firms whose cash flows are also determined by the market state exposing investors to systematic risk. In contrast, when $\tilde{I}_j^M = 0$ then firm j 's cash flows are independent of the cash flows of all other firms and firm j 's cash flows exhibit only idiosyncratic risk. The probability $q_j = \Pr(\tilde{I}_j^M = 1)$ is therefore a measure of a firm's exposure to systematic risk.

Based on the report issued by the firm at date 1, risk-averse investors price the firm at $P_1(r)$. In the economy, there are N_I identical investors who invest their initial wealth W_0 into the firm's equity and a risk-free asset. Investors' utility is quadratic over their consumption in period 1 and 2. In particular,

$$U(c_1, c_2) = c_1(2c^* - c_1) + bc_2(2c^* - c_2)$$

where b is the personal discount factor that measures investors' impatience and c^* is the satiation point, $c_t < c^*$. The satiation point c^* determines investors' absolute and relative risk-aversion with both measures of risk-aversion being higher if the satiation point is lower. Similar to utility functions with constant relative or constant absolute risk aversion, the quadratic utility exhibits hyperbolic absolute risk aversion (HARA), i.e., the absolute risk aversion is a hyperbolic function of consumption. We can consider the quadratic utility as a second-order Taylor series approximation to investors' actual utility and it is used here for tractability reasons. Since the support of the firm value, and therefore the support of investors' consumption, is bounded from above, the main criticism of quadratic utility – that marginal utility is negative for sufficiently high consumption levels – can be circumvented by assuming that the satiation point c^* is sufficiently high.

Investors act as price takers and allocate their budget W_0 among first period consumption c_{1i}

and investments in the risk-free asset and the (risky) firm equity. Let ω_{ij} denote the fraction of firm j that investor i purchases at the price P_{1j} at date 1 and let ω_{if} denote the amount investor i invests into the risk-free asset whose price is normalized to 1. Then, investor i consumes

$$c_{1i} = W_0 - \omega_{if} - \sum_{j=1}^{N_F} \omega_{ij} P_{1j} \quad (3)$$

in period 1. In period 2, the investor consumes the entire payoffs of his investments because he lives for only two periods, i.e.,

$$\tilde{c}_{2i} = \omega_{if} R_f + \sum_{j=1}^{N_F} \omega_{ij} \tilde{V}_j \quad (4)$$

where the risk-free rate is given exogenously. Investor i chooses the investment into the risk-free asset and the risky assets to maximize his expected utility, i.e.,

$$\max_{\omega_{if}, \omega_{ij}, j=1..N_F} E [U (c_{1i}, \tilde{c}_{2i}) | \Omega]$$

where Ω denotes investors' information set at the time they make their investment and consumption decision in period 1. Since investors' utility is quadratic, the first two moments of the distribution of \tilde{V}_j are sufficient to characterize investors' optimization problem and, as a result, the CAPM holds.

In the following, we assume that investors observe firm j 's financial report r_j prior to making their investment and consumption decision, i.e., $\Omega = \{r_j\}$. Firm j prepares its financial report either according to the fair value or the conservative accounting regime. For simplicity, I assume that firms $k \neq j$ are identical with respect to their exposure to systematic risk, i.e., $q_k = q$ for $k \neq j$. The following lemma describes the capital market equilibrium for the case of a large economy in which the number of investors and firms approach infinity.¹⁶

Lemma 2 *Let*

$$\kappa = \frac{1}{2} \left(\sqrt{(c^* + R_f (c^* - W_0))^2 + 4 (1 + bR_f^2) Cov [\tilde{V}_{k \neq j}, \tilde{V}_{l \neq k, j} | r_j]} - (c^* + R_f (c^* - W_0)) \right). \quad (5)$$

In a large economy, in which investors observe firm j 's financial report, r_j ,

(a) *firm j 's equilibrium price is given by*

$$P_j (r_j) = \frac{E [\tilde{V}_j | r_j]}{R_f} + \frac{Cov [\tilde{V}_j, \tilde{V}_{k \neq j} | r_j]}{Cov [\tilde{V}_{k \neq j}, \tilde{V}_{l \neq k, j} | r_j]} \left(P_{k \neq j} (r_j) - \frac{E [\tilde{V}_{k \neq j} | r_j]}{R_f} \right). \quad (6)$$

¹⁶The lemma assumes $N_F = N_I$. $N_F \neq N_I$ is equivalent to scaling the investor specific parameters c^* and W_0 , in particular if $N_F = \eta N_I$ is equivalent to $c^{*'} = \eta c^*$ and $W_0' = \eta W_0$. This assumption implies that the expected wealth per investor remains constant as the economy grows. Similar assumptions have been made, e.g., by Hughes et al. (2007) and Lambert et al. (2007).

where the average price of a firm in the economy is given by $P_{k \neq j}(r_j) = R_f^{-1} \left(E \left[\tilde{V}_{k \neq j} | r_j \right] - \kappa \right)$;
and

(b) firm j 's required rate of return in period 2 is given by

$$E \left[\tilde{R}_j | r_j \right] = R_f + \beta_j(r_j) \times (R_M(r_j) - R_f) \quad (7)$$

where its beta is given by

$$\beta_j(r_j) = \frac{q_j \text{Var} \left[\tilde{V}_j | r_j \right]}{(1 - q_j) \text{Var} \left[\tilde{V}_{k \neq j} | r_j \right] + q_j \text{Var} \left[\tilde{V}_j | r_j \right] + q_j (1 - q_j) \left(E \left[\tilde{V}_{k \neq j} \right] - E \left[\tilde{V}_j | r_j \right] \right)^2} \frac{P_{k \neq j}}{q P_j} \quad (8)$$

and the market premium is given by

$$R_M(r_j) - R_f = \frac{\kappa}{E \left[\tilde{V}_{k \neq j} | r_j \right] - \kappa} R_f. \quad (9)$$

Lemma 2 derives the equilibrium prices and a firm's cost of capital in period 2 if investors observe its financial report. It provides both the firm's stock price (part *a*) and its required rate of return or cost of capital (part *b*). As expected, the CAPM holds due to investors' quadratic utility. In addition to deriving the CAPM, specifying investors' utility and initial endowment allows us to explicitly derive the market capitalization (as measured by $P_{k \neq j}(r_j)$) and the market premium, $R_M - R_f$. Both the market capitalization and the market premium are determined by κ which measures the risk-premium in dollars for an average firm $k \neq j$, i.e., $\kappa = E[\tilde{V}_k | r_j] - R_f P_k(r_j)$. As one would expect, the risk-premium κ depends on both investor and firm characteristics. In particular, the risk-premium is higher if investors are more risk-averse (corresponding to a lower satiation point, c^*), less wealthy (lower initial endowment, W_0), or more patient (higher personal patience, b), and the risk-premium increases in the economy's systematic risk (higher covariance among firms' cash flows, $Cov[\tilde{V}_{k \neq j}, \tilde{V}_{l \neq k, j} | r_1]$).

Part (b) of Lemma 2 also derives the return beta of firm j which is equal to the ratio of the covariance of firm j 's return with the market return to the variance of the market return. Beta increases in the residual variance of firm j 's cash flows, $\text{Var}[\tilde{V}_j | r_j]$. This is intuitive since with probability q_j the variance constitutes systematic risk.

In the equilibrium described in Lemma 2, a firm's required rate of return will vary with the

realization of its financial report.¹⁷ To effectively compare accounting regimes, we must therefore compare the average required rate of return where the average is taken over the possible realizations of the financial report each regime generates, i.e., $E[E[\tilde{R}_j|r_j]] = \sum_{r_j} E[\tilde{R}_j|r_j] \Pr(\tilde{r}_j = r_j)$ where both $E[\tilde{R}_j|r_j]$ and $\Pr(\tilde{r}_j = r_j)$ depend on the accounting regime. The following proposition establishes conditions under which a firm's average equity cost of capital is lower in the conservative accounting regime than in the fair value accounting regime.

Proposition 2 *In a large economy, in which investors observe firm j 's financial report, r_j , there exist $\Delta^{**} < \mu_x$ and $\gamma^{**} > 0$ such that for all $(\Delta, \gamma) \in \{(\Delta, \gamma) | \Delta > \Delta^{**} \text{ and } \gamma < \gamma^{**}\}$ firm j 's average equity cost of capital $E[E[\tilde{R}_j|r_j]]$ is lower if its financial report is generated in the conservative accounting regime than in the fair value accounting regime.*

Proposition 2 establishes that a firm's average equity costs of capital are lower in the conservative accounting regime than in the fair value accounting regime if the downside risk of its operation is significantly higher than its upside potential. The downside risk of a firm's operation is significantly higher than its upside potential if the uncertainty as measured by the potential variation in asset values, Δ , is sufficiently large and the production function is sufficiently concave. This is intuitive. As mentioned in the discussion of Lemma 2, a firm's cost of capital are in part determined by the residual variance of its cash flows. Hence, if investors are better able to predict the firm's cash flows based on its financial report, the residual uncertainty and its cost of capital are lower. Proposition 1 established that conservative accounting allows investors to better predict firm value if the downside risk of a firm's operation is significantly higher than its upside potential because conservative accounting does not offset gains against losses.

If either costs of capital or the predictability of future cash flows are adequate measures of the desirability of accounting standards, Proposition 1 and 2 suggest that, depending on the firm's industry and the asset class, different accounting regimes might be preferable, lending support to mixed-attribute accounting. In particular, the propositions might support a preference for the mark-to-market rule for assets that exhibit near constant marginal returns and a preference for the lower-of-cost-or-market rule for productive assets that exhibit decreasing marginal returns. Presumably, financial assets are unlikely to exhibit significantly decreasing marginal returns lending support for

¹⁷Not all papers that study the effect of information on cost of capital integrate over the realization of signals. The reason is that when both cash flows and the noisy signal of cash flows are normally distributed the conditional variance of cash flows is independent of the signal realization and, as a result, there is no need to integrate over signal realization if cost of capital are entirely determined by cash flow variance (e.g., Lambert et al. 2007).

the fair value option for financial assets and liabilities introduced in SFAS 159. In contrast, the predictions of the model suggest that caution should be exercised with respect to applying fair value accounting to productive assets such as the revaluation model for property, plant and equipment under IFRS (IAS 16) if the productive assets may exhibit significantly decreasing marginal returns. Alternatively, the analysis emphasizes the importance of disclosing gains and losses separately when assets are valued at fair values.

3 Role of conservatism in debt financing

The previous section considered the joint effect of aggregation and conservatism from an informational perspective in the context of equity valuation. This section takes a contractual perspective and considers the efficiency of debt contracts. In order to examine the role of accounting regimes in debt contracting, there must exist an incentive problem that cannot be addressed by interest rates alone. The literature has considered several incentive problems between equityholders, management and debtholders. For instance, managers might pay higher than optimal dividends (e.g., Kalay 1982, Leftwich 1983, Healy and Palepu 1990), continue projects even though it is optimal to terminate them (e.g., Tirole 2006), or dilute debtholders' claims by substituting more risky assets (e.g., Gavish and Kalay 1983). Some of these shareholder-debtholder conflicts may be addressed through covenants (e.g., Jensen and Meckling 1976, Aghion and Bolton 1992, Sridhar and Magee 1997).

The following section outlines a setting in which debt covenants may alleviate suboptimal liquidation decisions at an interim date. Since the covenants must be written in terms of contractible information such as financial reports, the accounting regime that produces these reports may affect the efficiency of debt contracts.

3.1 Model of debt covenants

The model of debt covenants builds upon the single-firm setup introduced in Section 2.1. From the analysis in Sections 2.2 and 2.3, we know that for constant marginal returns, i.e., $\gamma = 1$, the fair value accounting system is always more informative about the firm's cash flows and results in lower cost of equity capital for an all-equity firm. In order to highlight that the effects studied in this section are specific to debt contracting and cannot be attributed to decreasing marginal returns of firms' production technology or investors' risk-aversion, I assume $\gamma = 1$ and that both the manager

and potential lenders are risk-neutral. For simplicity, I also assume that the risk-free rate is zero, i.e., $R_f = 1$, and that $k = 1$.

The firm requires a fixed investment of I at date 0. The manager finances the investment with a standard debt contract at date $t = 0$. The lender provides capital I at $t = 0$ and in return receives a prioritized payment of the debt's face value F at $t = 2$. The owner/manager is assumed to have limited liability such that the lender receives the minimum of the face value F and the firm's pledgeable cash flows at $t = 2$. In addition to the interest rate, the loan contract may specify a debt covenant that assigns control rights to the lender based on the realization of the financial report r . Lenders behave competitively such that the loan yields a zero profit. As a result, the surplus to the manager also measures the efficiency of the debt contract in equilibrium.

In order to introduce an incentive conflict between debtholders and shareholders, I introduce a moral hazard problem in period 2 (see, e.g., Holmström and Tirole 1998, 2000; Tirole 2006 Chapter 5). After signing a debt contract at $t = 0$ and acquiring the two assets, the manager has the option to liquidate any of the two assets at $t = 1$. If it is liquidated, an asset yields a deterministic liquidation value of L at $t = 2$. If it is not liquidated, the asset produces uncertain cash flows $\tilde{x}_{i,2}$ at $t = 2$. As in Section 2.1, we assume for simplicity that asset values do not change in period 2, i.e., that $\tilde{x}_{i,2} = x_{i,1}$. However, for an asset to realize its value $x_{i,2}$ at the end of period 2, the manager must "behave" in the sense that he must not take any private benefits in period 2. "Behaving" yields cash flows of $x_{i,2}$ while "misbehaving" yields a private benefit c to the manager and zero cash flows at the end of period 2. Hence, the manager must be compensated c in order to induce him to behave resulting in pledgeable cash flows of $x_{i,2} - c$ if the asset is not liquidated. This moral hazard problem exists even if the lender is allocated control. In order to ensure that "behaving" is efficient even if the asset lost value, suppose that $\mu_x - \Delta > c$.¹⁸ Moreover, I assume that the post-financing decision, i.e., whether to liquidate any of the two assets, is non-contractible, consistent with the literature on debt covenants.

Suppose that liquidating the asset is only efficient if the asset lost value in the first period, i.e., $\mu_x > L > \mu_x - \Delta$. In order to introduce a need for a debt covenant, the parameter values must be such that the manager's and the lender's incentives are misaligned and that neither the manager

¹⁸In the absence of the *ex post* moral hazard problem, the Coase Theorem would ensure that the initial allocation of decision rights does not affect the efficiency of the outcome (Coase 1960). Alternatively, costly renegotiation (e.g., Sridhar and Magee 1997, Caskey and Hughes 2012) or information asymmetry at the intermediate stage (e.g., Gârleanu and Zwiebel 2009) may prevent full efficiency to be attainable independent of the initial allocation of decision rights.

nor the lender will always implement the first best liquidation decision if allocated control at $t = 1$. In particular, suppose the lender may prefer to liquidate the asset even if it's value didn't change in period 1, i.e., $L > \mu_x - c$.¹⁹ In contrast, the manager may prefer not to liquidate the asset even if the asset lost value. This is the case if the face value of the debt is sufficiently high such that the manager's compensation c when the asset is not liquidated exceeds his residual claim against the firm's cash flows if the asset is liquidated. However, if the value of the asset increased in period 1, both the manager and the lender prefer not to liquidate the asset, i.e., $\mu_x + \Delta - c > L$.

While only the financial report r is contractible, both the manager and the lender observe the realizations of the individual asset values at $t = 1$, $x_{1,1}$ and $x_{2,1}$. This non-contractible information provides potential to improve the efficiency of the liquidation decisions by renegotiating the debt contract at $t = 1$. Any renegotiation is assumed to be costless and any surplus from renegotiation is split such that the manager retains fraction α of the surplus and the lender receives fraction $1 - \alpha$.²⁰

3.2 Need for debt covenants

A debt covenant is necessary only if control rights are optimally allocated to the manager in some states of the world and to the lender in others. In order to determine when debt covenants are necessary, I first examine the outcome when the control rights are either always allocated to the manager or always allocated to the lender.

If the manager is always allocated control, he may choose not to liquidate an asset even though it lost value and hence it would therefore be efficient to liquidate the asset. There is then scope for renegotiation. In fact, renegotiation guarantees that the efficient liquidation decision is always implemented when the manager is allocated control. However, allocating control rights to the manager limits the returns the lender can capture and hence limits the capital the lender is willing to provide ex ante.

Lemma 3 *Suppose the manager is always allocated control. Then, renegotiation guarantees that*

¹⁹The lender only prefers to liquidate the asset when $x_{i,1} = \mu_x$ if the face value of his debt is sufficiently high. Otherwise, he is indifferent.

²⁰The assumption that renegotiation is feasible is consistent with empirical evidence. Robert and Sufi (2009) document that about 90 percent of long-term private credit arrangements of publicly traded U.S. firms with financial institutions are renegotiated before maturity. Consistent with a high frequency of renegotiation, Chava and Roberts (2008) find that at inception of a debt contract, the average covenant threshold differs from the current value of the relevant accounting ratio by about one standard deviation only.

the efficient liquidation decisions is implemented and capital I can be raised as long as $I \leq \underline{I}$ where

$$\underline{I} = 2(\mu_x - c) + p(1 - \alpha)(L - (\mu_x - \Delta)).$$

If the required capital exceeds \underline{I} , the lender needs to be allocated control rights at least in some states of the world in order for the lender's individual rationality constraint to be satisfied. However, when the lender is allocated control, renegotiation does not guarantee that the efficient liquidation decision is implemented. The reason is that the lender may prefer to liquidate an asset even though it did not lose any value. If that is the case, the lender can obtain a (maximum) payoff from the asset of L . In contrast, foregoing the liquidation option allows the lender to only obtain a (maximum) payoff of $\mu_x - c$ because the moral hazard problem in period 2 prevents the lender from extracting the entire value of the asset. Since $\mu_x - c$ is less than L , the lender does not agree to renegotiation when he is allocated control. Always allocating control to the lender therefore implements the efficient liquidation decision only if the face value of debt is sufficiently low such that the lender does not prefer to liquidate an asset even though it did not lose value.

Lemma 4 *Suppose the lender is always allocated control. Then, the efficient liquidation decisions is implemented and capital I can be raised as long as $I \leq 2(\mu_x - c)$. If $I > 2(\mu_x - c)$, the lender sometimes liquidates an asset that did not lose value, i.e., $x_{i,1} \geq \mu_x$. If $I > \bar{I}$ always allocating control to the lender is necessary to raise capital I where*

$$\bar{I} = 2(\mu_x - c) + p\Delta + 2p\left(1 - \frac{p}{2}\right)(L - (\mu_x - c)) + (1 - p)(L - (\mu_x - c)) \max\{2(1 - p), p\}.$$

Lemma 4 establishes that the efficient liquidation decision can be obtained without the need for a debt covenant by always allocating the control rights to the lender as long as the required debt capital is less than $2(\mu_x - c)$. Since $2(\mu_x - c)$ is lower than the threshold \underline{I} in Lemma 3, the option to always allocate control to the lender does not extend the set of parameter values for which the efficient liquidation decision can be implemented without the need for a debt covenant. Hence, debt covenants may be valuable when $I > \underline{I}$. However, if the capital requirement I is so high that the lender is only willing to provide the loan if he is always allocated control, then there is again no need for an accounting based debt covenant. This is the case for $I > \bar{I}$.²¹

²¹The case of $I > \bar{I}$ seems less relevant in practice since standard debt contracts do not allocate control rights to the lender unconditionally.

3.3 Efficiency of debt contracts with covenants

When (i) allocating the decision rights always to the manager is not feasible, but (ii) at the same time the required amount of capital does not make it necessary to always allocate control to the lender, unilateral control allocations are not optimal. In other words, when the required amount of capital is in between the upper and lower bounds established in the previous section, $I \in (\underline{I}, \bar{I})$, then contingent control allocation dominates unilateral control allocations. Since the allocation of control rights must be contingent on the realization of the financial report, the accounting regime may affect the efficiency of the debt contract.

Under either accounting regime, the efficient liquidation decision can be implemented as long as the required amount of capital I that has to be raised in the form of debt capital is not too high. This is intuitive. As established in Lemma 3, the efficient liquidation decision can be achieved when $I \leq \underline{I}$ by always allocating control to the manager. If I exceeds \underline{I} , the lender must obtain decision rights in some states of the world. When the lender is allocated control, he will implement the efficient liquidation decision as long as the face value of the debt issued is not too high. This implies that there is a maximum amount of capital that can be raised in form of a debt contract with a covenant such that the efficient liquidation decision is implemented. Proposition 3 establishes that the maximum amount of capital that can be raised is higher in the conservative than in the fair value accounting regime.

Proposition 3 *The efficient liquidation decision can be implemented for $I \in [0, I_c]$ in the conservative regime and for $I \in [0, I_f]$ in the fair value accounting regime. The maximum capital that can be raised by a debt contract which implements the efficient liquidation decision is weakly higher in the conservative than in the fair value accounting regime, i.e., $I_c \geq I_f$. The difference $I_c - I_f$ is greater if α , L , p and c are higher and Δ and μ_x are lower.*

If the required amount of capital exceeds the amount \underline{I} that can be raised when the manager always maintains control of the firm, the lender must obtain decision rights in some states of the world. Allocating the decision rights to the lender guarantees the lender higher cash flows than if the manager is allocated control and renegotiation takes place. When the lender has the right to decide whether an asset is liquidated, he tends to liquidate more often than is efficient. In particular, he may prefer to liquidate an asset that did not lose any value in the first period even though it is efficient not to. The reason is that the lender can extract the liquidation value L if the

asset is liquidated but only $\mu_x - c$ if the asset is not liquidated. However, there are two states of the world in which the lender always, i.e., independent of the debt's face value, takes the efficient liquidation decision. First, when both assets lost value in the first period, i.e., $x_{i,1} = \mu_x - \Delta$ for $i = 1, 2$, the lender will always choose to liquidate both assets. Second, when one asset lost value and the other gained value in the first period, i.e., when $x_1 = (\mu_x - \Delta, \mu_x + \Delta)$, the lender will always liquidate the asset that lost value but not the one that gained value.²²

Under either accounting regime it is optimal to allocate the decision rights to the lender when both assets lost value. The reason is that in either accounting regime the state when both assets lost value is unambiguously identified by the report $r = 2\mu_x - 2\Delta$. In contrast, allocating the decision rights to the lender when one asset lost value and the other asset gained value in period 1 requires that the decision rights are also allocated to the lender in another state of the world. The reason is that the report $r = r(\mu_x - \Delta, \mu_x + \Delta)$ does not unambiguously identify the state in which one asset lost value and the other gained value. Under the conservative accounting regime, the financial report $r_c(\mu_x - \Delta, \mu_x + \Delta) = 2\mu_x - \Delta$ is also issued when one asset lost value while the value of the other asset did not change, i.e., when $x_1 = (\mu_x - \Delta, \mu_x)$. Hence, when allocating the decision rights to the lender when $(\mu_x - \Delta, \mu_x + \Delta)$ requires that the decision rights must also be allocated to the lender when $x_1 = (\mu_x - \Delta, \mu_x)$. For the lender to implement the efficient liquidation decision when $x_1 = (\mu_x - \Delta, \mu_x)$, the debt's face value must be no higher than $L + \mu_x - c$.

In contrast, in the fair value regime, allocating the decision rights to the lender when $x_1 = (\mu_x - \Delta, \mu_x + \Delta)$ requires that the decision rights must also be allocated to the lender when $x_1 = (\mu_x, \mu_x)$ because the report equals $r_f = 2\mu_x$ in both states. For the lender to implement the efficient liquidation decision when $x_1 = (\mu_x, \mu_x)$, the debt's face value must be less than $2(\mu_x - c)$. However, when the face value is $F = 2(\mu_x - c)$ the amount of capital the lender is willing to provide is $2(\mu_x - c)$ which is less than \underline{I} . Hence, allocating the decision rights to the lender when $r_f = 2\mu_x$ does not increase the maximum capital that can be raised in the fair value accounting regime while guaranteeing efficient liquidation decisions. This implies that the debt contracts that guarantee efficient liquidation decisions either (i) allocate the control to the manager except in the case when both assets lost value; or (ii) limit the face value to $L + \mu_x - c$ and allocate control

²²In fact, there is a third state of the world in which the lender always takes the efficient liquidation decision. When both assets gained value in period 1, i.e., $x_1 = (\mu_x + \Delta, \mu_x + \Delta)$, the lender never chooses to liquidate an asset. However, allocating the decision rights to the lender does not increase the amount of cash that can be paid to the lender, and hence, it is irrelevant whether the lender or manager has the decision rights in this state.

to the manager when at least one asset lost value in the conservative regime (i.e., when $x_1 \in \{(\mu_x - \Delta, \mu_x - \Delta), (\mu_x - \Delta, \mu_x), (\mu_x - \Delta, \mu_x + \Delta)\}$) or when at least one asset lost value and the other did not gain value in the fair value regime (i.e., when $x_1 \in \{(\mu_x - \Delta, \mu_x - \Delta), (\mu_x - \Delta, \mu_x)\}$). As a result, the maximum capital that can be raised by a debt contract which implements the efficient liquidation decision is weakly higher in the conservative than in the fair value accounting regime.

Proposition 3 also establishes that the difference between the maximum capital that can be raised by a debt contract which implements the efficient liquidation decision in the conservative accounting regime versus the fair value accounting regime is greater if α, L and c are higher and p, Δ and μ_x are lower. The intuition is as follows. The conservative accounting regime's dominance stems from the fact that the lender can be allocated decision rights when one asset lost value and one asset gained value, i.e., when $x_1 = (\mu_x - \Delta, \mu_x + \Delta)$. Then, in the conservative regime his maximum payoff is $L + \mu_x - c$. Under the fair value accounting regime, his maximum payoff is $L + \mu_x - c$ only if $\Delta \geq c$ otherwise the manager prefers not to liquidate any asset and renegotiation occurs. This provides the lender with a payoff of $2\mu_x - 2c + (1 - \alpha)(L + \mu_x + \Delta - 2\mu_x)$. As a result, the capital the lender is willing to provide in the conservative regime is greater relative to the fair value regime if α, L , and c are higher and Δ and μ_x are lower. Since $x_1 = (\mu_x - \Delta, \mu_x + \Delta)$ occurs with probability $\frac{p^2}{2}$, the capital the lender is willing to provide in the conservative regime is also greater relative to the fair value regime if p is higher.

Conversely, for any $I \in (I_f, I_c]$, the market value of the firm's equity is higher in the conservative accounting regime than in the fair value accounting regime at $t = 0$ because equityholders are the residual claimants and benefit from the efficient liquidation decision. The market value of the firm's debt is the same in either accounting regime and equals I because lenders are risk-neutral and the debt market is competitive. However, the stated interest rate, $F/I - 1$, is higher in the fair value accounting regime than in the conservative accounting regime. This follows immediately from the fact that the lender is not allocated control when $x_1 = (\mu_x - \Delta, \mu_x + \Delta)$ in the fair value accounting regime, and hence, expects a lower payoff in that state of the world. Corollary 1 formalizes this finding.

Corollary 1 *For $I \in (I_f, I_c]$, the interest rate is higher in the fair value regime than in conservative accounting regime.*

The benefits of conservative accounting may also extend to cases in which the required amount of capital is higher than the maximum capital that can be raised by a debt contract that implements the efficient liquidation decision. However, Proposition 4 shows that this is the case only up to a threshold of required capital and that for exceedingly high amounts of required capital, the fair value accounting regime dominates. This is intuitive. Consider the case of the required capital being so high that allocating control to the lender when at least one asset lost value (i.e., when $r_c < 2\mu_x$) isn't sufficient. Then, under conservative accounting the only option is to always allocate control to the lender. In contrast, under fair value accounting, control allocation can be contingent as long as $I < \bar{I}$.

Proposition 4 *There exists $I^* \in (\underline{I}, \bar{I})$ such that for $I \leq I^*$ the conservative accounting regime weakly dominates and for $I > I^*$ the fair value regime weakly dominates in terms of the efficiency of the debt contract where*

$$I^* = L + \mu_x - c + \max \left\{ p\Delta - (1-p)^2 (L - (\mu_x - c)), 0 \right\}.$$

4 Conclusion

This paper studies the interdependencies of conservatism and aggregation both from an informational perspective and from a contractual perspective. It therefore adds to the relatively small literature that provides models supporting the widely held beliefs that conservatism is a desirable feature of accounting information systems.

In addition to the two settings studied in the paper, conservative accounting may be preferred over fair value accounting due to the effect of aggregation in other circumstances as well. For instance:

- Suppose the firm's ability to borrow is constrained such that the firm may be illiquid even though it is solvent. In particular, there may be uncertainty related to when the firm's assets produce cash inflows. For simplicity assume that with some probability q asset 1 produces cash inflows in the near term while asset 2 won't produce cash inflows until later and with probability $1-q$ the reverse is true. If financial reports are used to forecast the firm's illiquidity then conservative accounting may be preferred to fair value accounting because conservative accounting unambiguously identifies losses.

- Empirical evidence suggests that future cash flow uncertainty is higher if the firm incurred a loss rather than a gain. Assuming a similar relation hold on an asset-by-asset basis, conservative accounting may be preferred to fair value accounting for predicting future cash flow uncertainty or expected payoffs of concave contracts such as debt contracts.
- Alternatively, effectively incentivizing the manager or making informed termination decisions may require identifying losses in asset values. Since conservative accounting unambiguously identifies losses, conservative accounting may support more effective compensation contracts and corporate governance than fair value accounting.

Moreover, for the same reasons that conservative accounting may be preferred to fair value accounting, accounting standards that specify a higher threshold for assets than for liabilities with respect to the accuracy necessary for recognition on the balance sheet can result in financial reports that are more useful for valuation and contracting.

Appendix

Tables 1 and 2

For Table 1, we need to compute the conditional variance when $r_f = 2\mu_x$.

$$\begin{aligned}
& \text{Var}(V(\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_f = 2\mu_x) \\
&= (k\mu_x^\gamma)^2 \left(\frac{\frac{p^2}{2}}{(1-p)^2 + \frac{p^2}{2}} (u+d)^2 + \frac{(1-p)^2}{(1-p)^2 + \frac{p^2}{2}} 2^2 - \left(\frac{\frac{p^2}{2}}{(1-p)^2 + \frac{p^2}{2}} (u+d) + \frac{(1-p)^2}{(1-p)^2 + \frac{p^2}{2}} 2 \right)^2 \right) \\
&= (k\mu_x^\gamma)^2 \frac{\frac{p^2}{2} (1-p)^2}{\left((1-p)^2 + \frac{p^2}{2} \right)^2} (u+d-2)^2
\end{aligned}$$

Note that $u+d \in [1, 2]$, or equivalently, $\left(1 + \frac{\Delta}{\mu_x}\right)^\gamma + \left(1 - \frac{\Delta}{\mu_x}\right)^\gamma \in [2^\gamma, 2] \subseteq [1, 2]$ because

$$\frac{\partial}{\partial \Delta'} \left((1 + \Delta')^\gamma + (1 - \Delta')^\gamma \right) \Big|_{\Delta' = \frac{\Delta}{\mu_x}} = \gamma \left(1 + \frac{\Delta}{\mu_x} \right)^{\gamma-1} - \gamma \left(1 - \frac{\Delta}{\mu_x} \right)^{\gamma-1} \leq 0$$

and $\left(1 + \frac{\Delta}{\mu_x}\right)^\gamma + \left(1 - \frac{\Delta}{\mu_x}\right)^\gamma$ equals 1 for $\Delta = \mu_x$ the firm value equals $2^\gamma \in [1, 2]$ and for $\Delta = 0$ the firm value equals 2.

For Table 1, we need to compute the conditional variance when $r_c = 2\mu_x - \Delta$ and $r_c = 2\mu_x$.

$$\begin{aligned}
& \text{Var}(V(\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_c = 2\mu_x - \Delta) \\
&= (k\mu_x^\gamma)^2 \left(2 \frac{1-p}{2-p} (1+d)^2 + \frac{p}{2-p} (u+d)^2 - \left(2 \frac{1-p}{2-p} (1+d) + \frac{p}{2-p} (u+d) \right)^2 \right) \\
&= 2 (k\mu_x^\gamma)^2 \frac{p(1-p)}{(2-p)^2} (u-1)^2
\end{aligned}$$

$$\begin{aligned}
& \text{Var}(V(\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_c = 2\mu_x) \\
&= (k\mu_x^\gamma)^2 \left(\frac{4(1-p)^2}{(2-p)^2} 2^2 + \frac{4p(1-p)}{(2-p)^2} (u+1)^2 + \frac{p^2}{(2-p)^2} (2u)^2 \right) \\
&\quad - (k\mu_x^\gamma)^2 \left(\frac{4(1-p)^2}{(2-p)^2} 2 + \frac{4p(1-p)}{(2-p)^2} (u+1) + \frac{p^2}{(2-p)^2} 2u \right)^2 \\
&= 4 (k\mu_x^\gamma)^2 \frac{p(1-p)}{(2-p)^2} (u-1)^2
\end{aligned}$$

Proof of Lemma 1

$$\begin{aligned}
E[\text{Var}(V(\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_f)] &= \left((1-p)^2 + \frac{p^2}{2} \right) \text{Var}(V(\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_f = 2\mu_x) \\
&= \frac{1}{2} (k\mu_x^\gamma)^2 \frac{p^2 (1-p)^2}{(1-p)^2 + \frac{p^2}{2}} (u+d-2)^2
\end{aligned}$$

$$\begin{aligned}
& E [\text{Var} (V (\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_c)] \\
&= p \left(1 - \frac{p}{2}\right) \text{Var} (V (\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_c = 2\mu_x - \Delta) + \frac{1}{4} (2-p)^2 \text{Var} (V (\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_c = 2\mu_x) \\
&= 2 \left(p \left(1 - \frac{p}{2}\right) + 2\frac{1}{4} (2-p)^2 \right) (k\mu_x^\gamma)^2 \frac{p(1-p)}{(2-p)^2} (u-1)^2 \\
&= \frac{1}{2} (k\mu_x^\gamma)^2 \frac{p(1-p)(18-7p)}{2-p} (u-1)^2
\end{aligned}$$

For notational convenience, let A denote the ratio of expected variance in the conservative reporting regime to expected variance in the fair value reporting regime.

$$\begin{aligned}
A &= \frac{E [\text{Var} (V (\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_c)]}{E [\text{Var} (V (\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_f)]} \\
&= \frac{\frac{1}{2} (k\mu_x^\gamma)^2 \frac{p(1-p)(18-7p)}{2-p} (u-1)^2}{\frac{1}{2} (k\mu_x^\gamma)^2 \frac{p^2(1-p)^2}{(1-p)^2 + \frac{p^2}{2}} (u+d-1)^2} = \frac{\frac{p(1-p)(18-7p)}{2-p}}{\frac{p^2(1-p)^2}{(1-p)^2 + \frac{p^2}{2}}} \left(\frac{u-1}{u+d-2} \right)^2 \\
&= \frac{(18-7p) \left((1-p)^2 + \frac{p^2}{2} \right)}{p(1-p)(2-p)} \left(\frac{u-1}{2-u-d} \right)^2
\end{aligned}$$

We first consider how the second factor, $F_2 = \left(\frac{u-1}{2-u-d} \right)^2$ of A depends on u and d .

$$\frac{\partial F_2}{\partial u} = 2 \left(\frac{u-1}{2-u-d} \right) \frac{-(2-u-d) - (u-1)}{(2-u-d)^2} = 2 \frac{(u-1)(1-d)}{(2-u-d)^3} \geq 0$$

where the inequality follows from $u \geq 1$, $d \leq 2-u \leq 1$ and $2-u-d \geq 2-u-(2-u) = 0$.

$$\frac{\partial F_2}{\partial d} = 2 \frac{(u-1)^2}{(2-u-d)^3} > 0$$

Recall that $d \in [0, 2-u]$ and $u \in [1, 2]$. This suggests that A reaches its minimum for $u = d = 1$ and that A reaches its maximum for $u = 2$ and $d = 0$.

$$F_2|_{u=d=1} = 0$$

$$F_2|_{u=2,d=0} \rightarrow \infty$$

Clearly, $A|_{u=d=1} = 0$ is a global minimum and $A|_{u=2,d=0}$ is a global maximum.

Next, we consider how the first factor $F_1 = \frac{(18-7p) \left((1-p)^2 + \frac{p^2}{2} \right)}{p(1-p)(2-p)}$ of A depends on p . We want to show that F_1 is U-shaped on $[0, 1]$. Note that F_1 is strictly positive on $[0, 1]$. Hence, we have to show that F_1 is convex.

$$\begin{aligned}
\frac{\partial}{\partial p} F_1 &= -\frac{172 - 216p + 202p^2 - 88p^3 + 19p^4}{2 p^2 (1-p)^2 (2-p)^2} \\
\frac{\partial^2}{\partial p^2} F_1 &= \frac{144 - 648p + 1188p^2 - 1058p^3 + 498p^4 - 132p^5 + 19p^6}{p^3 (1-p)^3 (2-p)^3}
\end{aligned}$$

We want to show that $\frac{\partial^2}{\partial p^2} F_1 > 0$. This is equivalent to showing that the numerator is positive. Plotting the numerator yields

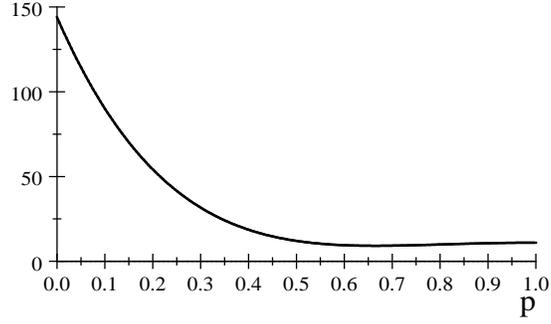


Figure 2: Numerator of $\frac{\partial^2}{\partial p^2} F_1 > 0$

The plot clearly illustrates that the numerator of $\frac{\partial^2}{\partial p^2} F_1$ is positive and hence F_1 is convex on $[0, 1]$.²³

For $p \rightarrow 0$ and $p \rightarrow 1$, $F_1 \rightarrow \infty$.

Consider the case of $\gamma = 1$. This is equivalent to $d = 2 - u$.

$$\begin{aligned} \frac{E [Var (V (\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_c)]}{E [Var (V (\tilde{x}_{1,2}, \tilde{x}_{2,2}) | r_f)]} \Big|_{d=2-u} &= \frac{(18 - 7p) \left((1-p)^2 + \frac{p^2}{2} \right)}{p(1-p)(2-p)} \left(\frac{u-1}{u+d-2} \right)^2; \\ &= \frac{(18 - 7p) \left((1-p)^2 + \frac{p^2}{2} \right)}{p(1-p)(2-p)} \left(\frac{2-d-1}{2-d+d-1} \right)^2 \end{aligned}$$

Proof of Proposition 1

In the proof of Lemma 1, we established that $A = 0$ for $u = 1$ and $d < 1$ which is satisfied for $\Delta = \mu_x$ and $\gamma \rightarrow 0$. The proposition follows from continuity.

Proof of Lemma 2

Part (a). First and second period consumption are given in equation (3) and (4), respectively. The expected value and variance of \tilde{c}_{2i} are given by

$$\begin{aligned} E [\tilde{c}_{2i}] &= \omega_{if} R_f + \sum_{j=1}^{N_F} \omega_{ij} E [\tilde{V}_j] \\ Var [\tilde{c}_{2i}] &= \sum_{j=1}^{N_F} \sum_{k=1}^{N_F} \omega_{ij} \omega_{ik} Cov [\tilde{V}_j, \tilde{V}_k] \end{aligned}$$

²³A formal proof of $\frac{\partial^2}{\partial p^2} F_1 > 0$ is available from the author. It is omitted for brevity.

Investor i 's expected utility is given by

$$\begin{aligned} E[U(c_{1i}, c_{2i})] &= E[c_{1i}(2c^* - c_{1i}) + b\tilde{c}_{2i}(2c^* - \tilde{c}_{2i})] \\ &= c_{1i}(2c^* - c_{1i}) + 2bc^*E[\tilde{c}_{2i}] - bVar[\tilde{c}_{2i}] - bE[\tilde{c}_{2i}]^2 \end{aligned}$$

Substituting first period consumption as given in (3) yields

$$\begin{aligned} E[U(c_{1i}, c_{2i})] &= \left(W_0 - \omega_{if} - \sum_{j=1}^{N_F} \omega_{ij} P_j\right) \left(2c^* - \left(W_0 - \omega_{if} - \sum_{j=1}^{N_F} \omega_{ij} P_j\right)\right) \\ &\quad + b \left(2c^* - \left(\omega_{if} R_f + \sum_{j=1}^{N_F} \omega_{ij} E[\tilde{V}_j]\right)\right) \left(\omega_{if} R_f + \sum_{j=1}^{N_F} \omega_{ij} E[\tilde{V}_j]\right) \\ &\quad - b \sum_{j=1}^{N_F} \sum_{k=1}^{N_F} \omega_{ij} \omega_{ik} Cov[\tilde{V}_j, \tilde{V}_k] \end{aligned}$$

The first order condition with respect to ω_{if} is given by

$$-c^* + \left(W_0 - \omega_{if} - \sum_{j=1}^{N_F} \omega_{ij} P_j\right) + bR_f \left(c^* - \left(\omega_{if} R_f + \sum_{j=1}^{N_F} \omega_{ij} E[\tilde{V}_j]\right)\right) = 0 \quad (10)$$

and the first order condition with respect to ω_{i1} (exemplary for ω_{ij}) is given by

$$\left(c^* - \left(\omega_{if} R_f + \sum_{k=1}^{N_F} \omega_{ik} E[\tilde{V}_k]\right)\right) \left(E[\tilde{V}_1] - P_1 R_f\right) - \sum_{k=1}^{N_F} \omega_{ik} Cov[\tilde{V}_1, \tilde{V}_k] = 0 \quad (11)$$

For notational convenience, let the market capitalization and aggregated cash flows be denoted by $P_M = \sum_{j=1}^{N_F} P_j$ and $\tilde{V}_M = \sum_{j=1}^{N_F} \tilde{V}_j$ respectively. Summing up the FOC wrt ω_{i1} over investors $i = 1..N_I$ and imposing the market clearing condition for risky assets $\sum_{i=1}^{N_I} \omega_{ij} = 1$ for $j = 1..N^I$ yields

$$\begin{aligned} \left(\sum_{i=1}^{N_I} c^* - \left(\sum_{i=1}^{N_I} \omega_{if} R_f + \sum_{k=1}^{N_F} \sum_{i=1}^{N_I} \omega_{ik} E[\tilde{V}_k]\right)\right) \left(E[\tilde{V}_1] - P_1 R_f\right) - \sum_{k=1}^{N_F} \sum_{i=1}^{N_I} \omega_{ik} Cov[\tilde{V}_1, \tilde{V}_k] &= 0 \\ \left(N_I c^* - \left(\sum_{i=1}^{N_I} \omega_{if} R_f + E[\tilde{V}_M]\right)\right) \left(E[\tilde{V}_1] - P_1 R_f\right) - Cov[\tilde{V}_1, \tilde{V}_M] &= 0 \end{aligned}$$

Aggregating across firms $j = 1..N_F$ using the notation introduced above yields

$$\left(N_I c^* - \left(\sum_{i=1}^{N_I} \omega_{if} R_f + E[\tilde{V}_M]\right)\right) \left(E[\tilde{V}_M] - P_M R_f\right) - Var[\tilde{V}_M] = 0 \quad (12)$$

Substituting into the pricing equation of firm 1 and solving for P_1 (and substituting j for 1) and taking the limit for $N_F = N_I \rightarrow \infty$ yields the price in (6). To obtain the market premium in terms of dollars, we need to solve for $\sum_{k=1}^{N_F} \omega_{if}$. From the first order condition for ω_{if} in (10) we obtain $\sum_{k=1}^{N_F} \omega_{if} = \frac{-N_I(c^* - W_0) - P_M + bR_f(N_I c^* - E[\tilde{V}_M])}{1 + bR_f^2}$. Substituting into the aggregated first order condition for risky assets in (12) yields

$$\left(E \left[\tilde{V}_M\right] - P_M R_f\right)^2 + (N_I c^* + N_I R_f (c^* - W_0)) \left(E \left[\tilde{V}_M\right] - P_M R_f\right) - (1 + b R_f^2) \text{Var} \left[\tilde{V}_M\right] = 0$$

Solving the quadratic equation for the market risk-premium $E \left[\tilde{V}_M\right] - P_M R_f$ yields

$$E \left[\tilde{V}_M\right] - P_M R_f = \frac{1}{2} \sqrt{(N_I c^* + N_I R_f (c^* - W_0))^2 + 4 \left(1 + b R_f^2\right) \text{Var} \left[\tilde{V}_M\right]} - \frac{1}{2} (N_I c^* + N_I R_f (c^* - W_0))$$

Scaling by N_F and taking the limit for $N_F = N_I \rightarrow \infty$ yields κ in (5).

Part (b). Let $\tilde{R}_j = \frac{\tilde{V}_j}{P_j}$ denote the return of firm j and $\tilde{R}_M = \sum_{j=1}^{N_F} \frac{P_j}{P_M} \tilde{R}_j$ denote the value-weighted return on the market portfolio. Then,

$$\begin{aligned} \text{Cov} \left[\tilde{R}_M, \tilde{R}_j\right] &= \sum_{k=1}^{N_F} \frac{P_k}{P_M} \text{Cov} \left[\tilde{R}_j, \tilde{R}_k\right] \\ \text{Var} \left[\tilde{R}_M\right] &= \sum_{j=1}^{N_F} \sum_{k=1}^{N_F} \frac{P_j P_k}{P_M^2} \text{Cov} \left[\tilde{R}_j, \tilde{R}_k\right] \end{aligned}$$

Substituting into the pricing equation in $\sum_{k=1}^{N_F} \omega_{if}$ yields the familiar CAPM relation in (7). The market premium equals

$$R_M - R_f = \sum_{j=1}^{N_F} \frac{P_j}{P_M} \frac{E \left[\tilde{V}_j\right]}{P_j} - R_f = \frac{E \left[\tilde{V}_M\right] - P_M R_f}{P_M}.$$

Scaling both the numerator and denominator by N_F and taking the limit for $N_F = N_I \rightarrow \infty$ and substituting $\lim_{N_F = N_I \rightarrow \infty} \frac{E \left[\tilde{V}_M\right] - P_M R_f}{N_F} = \kappa$ yields the market premium in (9).

WLOG assume that firm 1 issues the report and assume $j, k \neq 1$. To obtain the firm 1's beta, note that

$$\lim_{N_F \rightarrow \infty} \frac{N_F \text{Cov} \left[\tilde{R}_M, \tilde{R}_1\right]}{\text{Var} \left[\tilde{R}_M\right]} = \frac{\text{Cov} \left[\tilde{V}_j, \tilde{V}_1\right]}{\text{Cov} \left[\tilde{V}_j, \tilde{V}_{k \neq j}\right]} \frac{P_j}{P_1}$$

where

$$\text{Cov} \left[\tilde{V}_1, \tilde{V}_j | r_1\right] = \Pr \left(\tilde{I}_{M1} = \tilde{I}_{Mk} = 1 | r_1\right) \text{Var} \left[\tilde{V}_1 | r_1\right] + \left(1 - \Pr \left(\tilde{I}_{M1} = \tilde{I}_{Mk} = 1 | r_1\right)\right) * 0 = q_1 q \text{Var} \left[\tilde{V}_1 | r_1\right]$$

and

$$\begin{aligned} \text{Cov} \left[\tilde{V}_j, \tilde{V}_{k \neq j} | r_1\right] &= q^2 \text{Cov} \left[\tilde{V}_j, \tilde{V}_{k \neq j} | r_1, \tilde{I}_{Mk} = \tilde{I}_{Mj} = 1\right] \\ &= q^2 \left(\sum_{s=1}^6 \Pr \left(\tilde{V}_j = V_s | r_1, \tilde{I}_{Mj} = 1\right) V_s^2 - \left(\sum_{s=1}^6 \Pr \left(\tilde{V}_j = V_s | r_1, \tilde{I}_{Mj} = 1\right) V_s \right)^2 \right) \\ &= q^2 \left((1 - q_1) \text{Var} \left[\tilde{V}_j\right] + q_1 \text{Var} \left[\tilde{V}_1 | r_1\right] + q_1 (1 - q_1) \left(E \left[\tilde{V}_j\right] - E \left[\tilde{V}_1 | r_1\right]\right)^2 \right) \end{aligned}$$

where the expected value and variance of \tilde{V}_j are given by

$$\begin{aligned} E[\tilde{V}_j] &= 2k\mu_x^\gamma \left(1 - \frac{p}{2}(2 - u - d)\right) \\ Var[\tilde{V}_j] &= Var[\tilde{x}_{1j} + \tilde{x}_{2j}] = Var[\tilde{x}_{1j}] + Var[\tilde{x}_{2j}] = 2(k\mu_x^\gamma)^2 \left(1 - \frac{p}{2}(2 - u^2 - d^2) - \left(1 - \frac{p}{2}(2 - u - d)\right)^2\right) \end{aligned}$$

Proof of Proposition 2

As in Lemma 2, WLOG we assume that firm 1 issues the report. Let $j, k \neq 1$.

First, we consider the fair value accounting regime. $Cov[\tilde{V}_1, \tilde{V}_j | r_1] = 0$ for $r_1 \neq 2\mu_x$. For $r_1 = 2\mu_x$

$$Cov[\tilde{V}_1, \tilde{V}_j | r_1 = 2\mu_x] = q_1 q (k\mu_x^\gamma)^2 \frac{\frac{p^2}{2}(1-p)^2}{\left((1-p)^2 + \frac{p^2}{2}\right)^2} (2 - u - d)^2.$$

The systematic risk conditional on the financial report is given by

$$\begin{aligned} Cov[\tilde{V}_j, \tilde{V}_k | r_1 = 2\mu_x - 2\Delta] &= 2(k\mu_x^\gamma)^2 q^2 (1 - q_1) \left(\begin{aligned} &\left(1 - \frac{p}{2}(2 - u^2 - d^2) - \left(1 - \frac{p}{2}(2 - u - d)\right)^2\right) \\ &+ 2q_1 \left(1 - \frac{p}{2}(2 - u - d) - d\right)^2 \end{aligned} \right) \\ Cov[\tilde{V}_j, \tilde{V}_{k \neq j} | r_1 = 2\mu_x - \Delta] &= (k\mu_x^\gamma)^2 q^2 (1 - q_1) \left(\begin{aligned} &2 \left(1 - \frac{p}{2}(2 - u^2 - d^2) - \left(1 - \frac{p}{2}(2 - u - d)\right)^2\right) \\ &+ q_1 \left(2 \left(1 - \frac{p}{2}(2 - u - d)\right) - (1 + d)\right)^2 \end{aligned} \right) \\ Cov[\tilde{V}_j, \tilde{V}_{k \neq j} | r_1 = 2\mu_x + \Delta] &= (k\mu_x^\gamma)^2 q^2 (1 - q_1) \left(\begin{aligned} &2 \left(1 - \frac{p}{2}(2 - u^2 - d^2) - \left(1 - \frac{p}{2}(2 - u - d)\right)^2\right) \\ &+ q_1 \left(2 \left(1 - \frac{p}{2}(2 - u - d)\right) - (1 + u)\right)^2 \end{aligned} \right) \\ Cov[\tilde{V}_j, \tilde{V}_{k \neq j} | r_1 = 2\mu_x + 2\Delta] &= 2(k\mu_x^\gamma)^2 q^2 (1 - q_1) \left(\begin{aligned} &\left(1 - \frac{p}{2}(2 - u^2 - d^2) - \left(1 - \frac{p}{2}(2 - u - d)\right)^2\right) \\ &+ 2q_1 \left(1 - \frac{p}{2}(2 - u - d) - u\right)^2 \end{aligned} \right) \\ Cov[\tilde{V}_j, \tilde{V}_{k \neq j} | r_1 = 2\mu_x] &= q^2 (k\mu_x^\gamma)^2 \left(\begin{aligned} &2(1 - q_1) \left(1 - \frac{p}{2}(2 - u^2 - d^2) - \left(1 - \frac{p}{2}(2 - u - d)\right)^2\right) \\ &+ q_1 \frac{\frac{p^2}{2}(1-p)^2}{\left((1-p)^2 + \frac{p^2}{2}\right)^2} (2 - u - d)^2 \left(1 + \frac{1}{2}(1 - q_1)(2 - 3p)^2\right) \end{aligned} \right) \end{aligned}$$

It is easy to see that for $u \rightarrow 1$ and $d < 1$, $Cov[\tilde{V}_1, \tilde{V}_j | r_1 = 2\mu_x] > 0$ and $Cov[\tilde{V}_j, \tilde{V}_k | r_1]$ is finite.

Hence the average cost of capital exceeds the risk-free rate.

Next, consider the conservative accounting regime. The covariance of firm 1's cash flow with the market conditional on the financial report r_1 is given by

$$\begin{aligned} Cov[\tilde{V}_1, \tilde{V}_j | r_1 = 2\mu_x - 2\Delta] &= 0 \\ Cov[\tilde{V}_1, \tilde{V}_j | r_1 = 2\mu_x - \Delta] &= 2q_1 q (k\mu_x^\gamma)^2 \frac{p(1-p)}{(2-p)^2} (u-1)^2 \\ Cov[\tilde{V}_1, \tilde{V}_j | r_1 = 2\mu_x] &= 4q_1 q (k\mu_x^\gamma)^2 \frac{p(1-p)}{(2-p)^2} (u-1)^2 \end{aligned}$$

and the systematic risk conditional on the financial report is given by

$$\begin{aligned}
Cov \left[\tilde{V}_j, \tilde{V}_k | r_1 = 2\mu_x - 2\Delta \right] &= 2(k\mu_x^\gamma)^2 q^2 (1 - q_1) \left(\begin{aligned} &\left(1 - \frac{p}{2} (2 - u^2 - d^2) - \left(1 - \frac{p}{2} (2 - u - d) \right)^2 \right) \\ &+ 2q_1 \left(1 - \frac{p}{2} (2 - u - d) - d \right)^2 \end{aligned} \right) \\
Cov \left[\tilde{V}_j, \tilde{V}_k | r_1 = 2\mu_x - \Delta \right] &= q^2 (k\mu_x^\gamma)^2 \left(\begin{aligned} &2(1 - q_1) \left(1 - \frac{p}{2} (2 - u^2 - d^2) - \left(1 - \frac{p}{2} (2 - u - d) \right)^2 \right) \\ &+ 2q_1 \frac{p(1-p)}{(2-p)^2} (u - 1)^2 \\ &+ q_1 (1 - q_1) \left(2 \left(1 - \frac{p}{2} (2 - u - d) \right) - \frac{2(1-p)(1+d)+p(u+d)}{2-p} \right)^2 \end{aligned} \right) \\
Cov \left[\tilde{V}_{k \neq 1}, \tilde{V}_{j \neq 1, k} | r_1 = 2\mu_x \right] &= q^2 \left(\begin{aligned} &(1 - q_1) Var \left[\tilde{V}_j \right] + 4q_1 (k\mu_x^\gamma)^2 \frac{p(1-p)}{(2-p)^2} (u - 1)^2 \\ &+ q_1 (1 - q_1) (k\mu_x^\gamma)^2 \frac{p^2}{(2-p)^2} \left(2(1 - p) - 2d + p(d + u) \right)^2 \end{aligned} \right)
\end{aligned}$$

For $u \rightarrow 1$ and $d < 1$, we obtain $Cov \left[\tilde{V}_1, \tilde{V}_j | r_1 \right] = 0$ for all r_1 and $Cov \left[\tilde{V}_j, \tilde{V}_k | r_1 \right] > 0$ finite for all r_1 . Hence, the expected cost of capital equals the risk-free rate.

The statement of the proposition follows from continuity.

Proof of Lemma 3

Suppose $F = 2(\mu_x + \Delta - c)$. In any state $(x_{1,1}, x_{2,1})$, the manager will choose not to liquidate any of the assets. Without renegotiation, the manager's payoff is $2c$ and the lender's payoff is $x_{1,1} + x_{2,1} - 2c$. With renegotiation, the lender's expected payoff is

$$\begin{aligned}
\underline{I} &= 2 \left(\frac{p}{2} (\mu_x - \Delta - c + (1 - \alpha)(L - (\mu_x - \Delta))) + (1 - p)(\mu_x - c) + \frac{p}{2} (\mu_x + \Delta - c) \right) \\
&= 2(\mu_x - c) + p(1 - \alpha)(L - (\mu_x - \Delta))
\end{aligned}$$

Proof of Lemma 4

Part a. If $(\mu_x - \Delta, \mu_x - \Delta)$, the lender always chooses to liquidate both assets. If $(\mu_x - \Delta, \mu_x)$, the lender chooses to liquidate one asset if $F \leq L + \mu_x - c$ and the lender chooses to liquidate both assets if $F > L + \mu_x - c$. If (μ_x, μ_x) , the lender chooses to liquidate no asset if $F \leq 2(\mu_x - c)$, to liquidate one asset if $F \in (2(\mu_x - c), L + \mu_x - c]$ and to liquidate both assets if $F > L + \mu_x - c$. Hence, the lender always implements the efficient liquidation decision as long as $F \leq 2(\mu_x - c)$. Suppose $F = 2(\mu_x - c)$. Then, the debt is risk-free (because the lowest payoff is $2L > 2(\mu_x - c)$) and lender's expected payoff equals also $2(\mu_x - c)$.

Part b. Suppose we allocate control rights to the manager in the state $(\mu_x, \mu_x + \Delta)$ and $F =$

$2(\mu_x + \Delta)$. Then, the lender's expected payoff is

$$\begin{aligned}
I &= \frac{p^2}{4}(2L) + p(1-p)(2L) + \frac{p^2}{2}(L + \mu_x + \Delta - c) + (1-p)^2(2L) \\
&\quad + p(1-p)(2\mu_x + \Delta - 2c) + \frac{p^2}{4}(2\mu_x + 2\Delta - 2c) \\
&= \frac{p^2}{4}(2L) + p(1-p)(2L) + \frac{p^2}{2}(L + \mu_x + \Delta - c) + (1-p)^2(2\mu_x - 2c) \\
&\quad + p(1-p)(2\mu_x + \Delta - 2c) + \frac{p^2}{4}(2\mu_x + 2\Delta - 2c) + 2(1-p)^2(L - (\mu_x - c)) \\
&= 2(\mu_x - c) + p\Delta + 2p\left(1 - \frac{p}{2}\right)(L - (\mu_x - c)) + 2(1-p)^2(L - (\mu_x - c)) \tag{13}
\end{aligned}$$

Suppose we allocate control rights to the manager in the state (μ_x, μ_x) and $F = 2(\mu_x + \Delta)$. Then, the lender's expected payoff is

$$\begin{aligned}
I &= \frac{p^2}{4}(2L) + p(1-p)(2L) + \frac{p^2}{2}(L + \mu_x + \Delta - c) + (1-p)^2(2\mu_x - 2c) \\
&\quad + p(1-p)(L + \mu_x + \Delta - c) + \frac{p^2}{4}(2\mu_x + 2\Delta - 2c) \\
&= 2(\mu_x - c) + p\Delta + 2p\left(1 - \frac{p}{2}\right)(L - (\mu_x - c)) + p(1-p)(L - (\mu_x - c)) \tag{14}
\end{aligned}$$

where (13) is greater than (14) if

$$\begin{aligned}
(1-p)^2(2L) + p(1-p)(2\mu_x + \Delta - 2c) &> (1-p)^2(2\mu_x - 2c) + p(1-p)(L + \mu_x + \Delta - c) \\
2(1-p)^2(L - (\mu_x - c)) &> p(1-p)(L - (\mu_x - c)) \\
2(1-p) &> p \\
\frac{2}{3} &> p
\end{aligned}$$

Suppose we allocated control rights to the manager in the state $(\mu_x - \Delta, \mu_x)$ and $F = 2(\mu_x + \Delta)$. Then, the lender's expected payoff is

$$\begin{aligned}
I &= \frac{p^2}{4}(2L) + p(1-p)(2\mu_x - \Delta - 2c) + \frac{p^2}{2}(L + \mu_x + \Delta - c) \\
&\quad + (1-p)^2(2L) + p(1-p)(L + \mu_x + \Delta - c) + \frac{p^2}{4}(2\mu_x + 2\Delta - 2c) \tag{15}
\end{aligned}$$

This is greater than (14) only if

$$\begin{aligned}
p(1-p)(2\mu_x - \Delta - 2c) + (1-p)^2(2L) &> p(1-p)(2L) + (1-p)^2(2\mu_x - 2c) \\
2(1-p)^2(L - (\mu_x - c)) - p(1-p)\Delta &> 2p(1-p)(L - (\mu_x - c)) \\
(1-2p)(L - (\mu_x - c)) &> \frac{p}{2}\Delta
\end{aligned}$$

However, this is never satisfied when $p > \frac{2}{3}$ which is necessary for (14) to be greater than (13). (15) is never greater than (13) because

$$\begin{aligned} p(1-p)(2\mu_x - \Delta - 2c) + p(1-p)(L + \mu_x + \Delta - c) &> p(1-p)(2L) + p(1-p)(2\mu_x + \Delta - 2c) \\ p(1-p)(L - (\mu_x - c)) - p(1-p)\Delta &> 2p(1-p)(L - (\mu_x - c)) \\ -p(1-p)\Delta &> p(1-p)(L - (\mu_x - c)) \end{aligned}$$

Proof of Proposition 3

We derive the maximum capital that can be raised while still implementing efficient liquidation in four scenarios: (i) if the manager is allocated control except when $(\mu_x - \Delta, \mu_x - \Delta)$, (ii) if the manager is allocated control except when $(\mu_x - \Delta, \mu_x - \Delta)$ and $(\mu_x - \Delta, \mu_x)$, (iii) if the manager is allocated control except when $(\mu_x - \Delta, \mu_x - \Delta)$, $(\mu_x - \Delta, \mu_x)$, and $(\mu_x - \Delta, \mu_x + \Delta)$; and (iv) if the manager is allocated control except when $(\mu_x - \Delta, \mu_x - \Delta)$, $(\mu_x - \Delta, \mu_x)$, $(\mu_x - \Delta, \mu_x + \Delta)$, and (μ_x, μ_x) . Allocating control to the lender when $(\mu_x, \mu_x + \Delta)$ or when $(\mu_x + \Delta, \mu_x + \Delta)$ does not increase the payoff to the lender and hence does not increase the maximum capital that can be raised.

Lemma 5 *If $I \leq I_1$ we can implement first best by allocating decision rights to the manager except when $(\mu_x - \Delta, \mu_x - \Delta)$ where*

$$I_1 = \underline{I} + \frac{p^2}{2}\alpha(L - (\mu_x - \Delta)) + \frac{p^2}{2}c$$

(I_1 is the old I_0)

Proof. Suppose $F = 2(\mu_x + \Delta - c)$ and we allocate the decision rights to the manager except when $(\mu_x - \Delta, \mu_x - \Delta)$. Without renegotiation, when the manager is allocated control, the manager's payoff is $2c$ and the lender's payoff is $x_{1,1} + x_{2,1} - 2c$. With renegotiation, the lender's expected payoff is

$$\begin{aligned} I_1 &= \frac{p^2}{4}(2L) + p(1-p)(2\mu_x - \Delta - 2c + (1-\alpha)(L + \mu_x - (2\mu_x - \Delta))) \\ &\quad + \frac{p^2}{2}(2\mu_x - 2c + (1-\alpha)(L + \mu_x + \Delta - 2\mu_x)) \\ &\quad + (1-p)^2(2\mu_x - 2c) + p(1-p)(2\mu_x + \Delta - 2c) + \frac{p^2}{4}(2\mu_x + 2\Delta - 2c) \\ &= \underline{I} + \frac{p^2}{2}\alpha(L - (\mu_x - \Delta)) + \frac{p^2}{2}c \end{aligned}$$

■

Lemma 6 *Under the conservative regime, the maximum capital that can be raised while implementing the efficient liquidation decision is*

$$\bar{I}_c^{FB} = \max \{I_1, I_2\}$$

where $I_2 = L + \mu_x - c - (1 - p)^2 (L - (\mu_x - c))$

Proof. From Lemma 5 we know that the maximum capital that can be raised when allocating decision rights to the manager except when $(\mu_x - \Delta, \mu_x - \Delta)$ is I_1 and that debt contract implements first best.

Suppose the decision rights are allocated to the manager except when $(\mu_x - \Delta, \mu_x - \Delta)$, $(\mu_x - \Delta, \mu_x)$ and $(\mu_x - \Delta, \mu_x + \Delta)$. First best can be implemented only if the lender liquidates just one asset when $(\mu_x - \Delta, \mu_x)$, i.e., if $F \leq L + \mu_x - c$. The reason is that no renegotiation can take place when the lender is allocated control. With this allocation of control, the lender will liquidate two assets when $(\mu_x - \Delta, \mu_x - \Delta)$ and one asset when $(\mu_x - \Delta, \mu_x)$ or $(\mu_x - \Delta, \mu_x + \Delta)$ and the manager will never liquidate an asset when allocated control.

$$\begin{aligned} I(F = L + \mu_x - c) &= \frac{p^2}{4}F + p(1-p)F + \frac{p^2}{2}F + (1-p)^2(2\mu_x - 2c) + p(1-p)F + \frac{p^2}{4}F \\ &= F - (1-p)^2(F - 2(\mu_x - c)) \\ &= L + \mu_x - c - (1-p)^2(L - (\mu_x - c)) \end{aligned}$$

This is the same as in Lemma 7.

Suppose the lender is always allocated control. Then, the efficient liquidation decisions is implemented and capital I can be raised as long as $I \leq 2(\mu_x - c)$ which is less than I_1 . ■

Lemma 7 *Under the fair value regime, the maximum capital that can be raised while implementing the efficient liquidation decision is*

$$\bar{I}_c^{FB} = \begin{cases} \max \{I_1, I_2\} & \text{if } c \geq \Delta \\ \max \{I_1, I'_2\} & \text{if } c < \Delta \end{cases} .$$

where $I_2 = L + \mu_x - c - (1 - p)^2 (L - (\mu_x - c))$ and $I'_2 = I_2 - \frac{p^2}{2} (\alpha (L - (\mu_x - c)) + (1 - \alpha) (c - \Delta))$

Proof. First best can be implemented only if the lender liquidates just one asset when $(\mu_x - \Delta, \mu_x)$, i.e., if $F \leq L + \mu_x - c$. If $F = L + \mu_x - c$, then the manager liquidates one asset when $(\mu_x - \Delta, \mu_x + \Delta)$

if $\Delta > c$ and liquidates no asset if $\Delta < c$. (because the manager liquidates one asset when $(\mu_x - \Delta, \mu_x + \Delta)$ if $F \leq L + \mu_x + \Delta - 2c$). Suppose $\Delta \geq c$.

$$\begin{aligned}
I_2 &= \frac{p^2}{4}F + p(1-p)F + \frac{p^2}{2}F + (1-p)^2(2\mu_x - 2c) + p(1-p)F + \frac{p^2}{4}F \\
&= F - (1-p)^2(F - 2(\mu_x - c)) \\
&= L + \mu_x - c - (1-p)^2(L - (\mu_x - c))
\end{aligned}$$

Suppose $\Delta < c$. Note that this implies that renegotiation will take place when $(\mu_x - \Delta, \mu_x + \Delta)$. Without renegotiation, the lender's payoff is $2\mu_x - 2c$. With renegotiation, the lender's payoff is $2\mu_x - 2c + (1-\alpha)(L + \mu_x + \Delta - 2\mu_x)$. Note that the face value is greater than the lender's payoff, i.e.,

$$\begin{aligned}
F = L + \mu_x - c &> 2\mu_x - 2c + (1-\alpha)(L + \mu_x + \Delta - 2\mu_x) \\
L - (\mu_x - c) &> (1-\alpha)(L - (\mu_x - \Delta))
\end{aligned}$$

$$\begin{aligned}
\text{sufficient to show that } L - (\mu_x - c) &> L - (\mu_x - \Delta) \\
c &> \Delta
\end{aligned}$$

which is true by assumption. Hence, the maximum capital that the lender provides is

$$\begin{aligned}
&I'_2 \\
&= \frac{p^2}{4}F + p(1-p)F + \frac{p^2}{2}(2\mu_x - 2c + (1-\alpha)(L + \mu_x + \Delta - 2\mu_x)) + (1-p)^2(2\mu_x - 2c) + p(1-p)F + \frac{p^2}{4}F \\
&= F - (1-p)^2(F - 2(\mu_x - c)) - \frac{p^2}{2}(F - (2\mu_x - 2c + (1-\alpha)(L + \mu_x + \Delta - 2\mu_x))) \\
&= L + \mu_x - c - (1-p)^2(L - (\mu_x - c)) - \frac{p^2}{2}((L - (\mu_x - c)) - (1-\alpha)(L - (\mu_x - \Delta))) \\
&= L + \mu_x - c - (1-p)^2(L - (\mu_x - c)) - \frac{p^2}{2}(\alpha(L - (\mu_x - c)) + (1-\alpha)(c - \Delta))
\end{aligned}$$

Suppose the lender is allocated control also when $(\mu_x - \Delta, \mu_x + \Delta)$ and when (μ_x, μ_x) . Then, the efficient liquidation decisions is implemented and capital I can be raised as long as $I \leq 2(\mu_x - c)$ which is less than I_1 . ■

Lemma 8 \bar{I}_f^{FB} is strictly less than \bar{I}_c^{FB} if $c > \Delta$ and

$$\left(1 - p + \alpha \left(1 - \frac{p}{2}\right)\right)(L - (\mu_x - c)) + \left(1 - \alpha \left(1 - \frac{p}{2}\right)\right)(c - \Delta) - \frac{p}{2}c > 0.$$

This is the case if α, L and c are high and p, Δ and μ_x are low.

Proof. $\bar{I}_f^{FB} < \bar{I}_c^{FB}$ only if $c > \Delta$ and $\bar{I}_c^{FB} > I_1$, i.e., if $I_2 - I_1 > 0$.

$$\begin{aligned}
& I_2 - I_1 \\
&= L + \mu_x - c - (1-p)^2(L - (\mu_x - c)) - 2(\mu_x - c) - p(1-\alpha)(L - (\mu_x - \Delta)) - \frac{p^2}{2}\alpha(L - (\mu_x - \Delta)) - \frac{p^2}{2}c \\
&= p(2-p)(L - (\mu_x - c)) - p\left((1-\alpha) + \frac{p}{2}\alpha\right)(L - (\mu_x - c)) + p\left((1-\alpha) + \frac{p}{2}\alpha\right)(c - \Delta) - \frac{p^2}{2}c \\
&= p\left(\left(2-p - (1-\alpha) - \frac{p}{2}\alpha\right)(L - (\mu_x - c)) + \left((1-\alpha) + \frac{p}{2}\alpha\right)(c - \Delta) - \frac{p}{2}c\right) \\
&= p\left(\left(1-p + \left(1 - \frac{p}{2}\right)\alpha\right)(L - (\mu_x - c)) + \left(1 - \left(1 - \frac{p}{2}\right)\alpha\right)(c - \Delta) - \frac{p}{2}c\right)
\end{aligned}$$

We want to determine when $I_2 - I_1$ is strictly positive. Let the second factor be denoted by A .

Then,

$$\begin{aligned}
\frac{\partial A}{\partial \alpha} &= \left(1 - \frac{p}{2}\right)(L - (\mu_x - \Delta)) > 0 \\
\frac{\partial A}{\partial L} &= 2\left(1 - \frac{p}{2}\right) - \left(1 - \alpha\left(1 - \frac{p}{2}\right)\right) = 1 - p + \alpha\left(1 - \frac{p}{2}\right) > 0 \\
\frac{\partial A}{\partial c} &= 2\left(1 - \frac{p}{2}\right) - \frac{p}{2} = 2 - \frac{3}{2}p > 0 \\
\frac{\partial A}{\partial p} &= -(L - (\mu_x - c)) - \frac{\alpha}{2}(L - (\mu_x - \Delta)) - \frac{1}{2}c < 0 \\
\frac{\partial A}{\partial \Delta} &= -\left(1 - \alpha\left(1 - \frac{p}{2}\right)\right) < 0 \\
\frac{\partial A}{\partial \mu_x} &= -\frac{\partial A}{\partial L} < 0
\end{aligned}$$

■

Lemma 9 *The difference between $\bar{I}_c^{FB} - \bar{I}_f^{FB}$ is either 0 or equals $I_2 - I'_2 = \frac{p^2}{2}(\alpha(L - (\mu_x - c)) + (1 - \alpha)(c - \Delta))$.*

Proof. Follows immediately from the previous lemmas.

$$\begin{aligned}
\frac{\partial (I_2 - I'_2)}{\partial \alpha} &= \frac{p^2}{2}(L - (\mu_x - c) - (c - \Delta)) = \frac{p^2}{2}(L - (\mu_x - \Delta)) > 0 \\
\frac{\partial (I_2 - I'_2)}{\partial L} &= \frac{p^2}{2}\alpha > 0 \\
\frac{\partial (I_2 - I'_2)}{\partial c} &= \frac{p^2}{2} > 0 \\
\frac{\partial (I_2 - I'_2)}{\partial p} &= p(\alpha(L - (\mu_x - c)) + (1 - \alpha)(c - \Delta)) > 0 \\
\frac{\partial (I_2 - I'_2)}{\partial \Delta} &= \frac{p^2}{2}(1 - \alpha) < 0 \\
\frac{\partial (I_2 - I'_2)}{\partial \mu_x} &= -\frac{\partial A}{\partial L} < 0
\end{aligned}$$

■

4.1 Proof of Proposition 4

We want to show that for $I \leq \max\{I_4, L + \mu_x - c\}$, the conservative regime weakly dominates and for $I > \max\{I_4, L + \mu_x - c\}$, the fair value regime weakly dominates where

$$\begin{aligned} \max\{I_4, L + \mu_x - c\} &= \max\left\{L + \mu_x - c + p\Delta - (1-p)^2(L - (\mu_x - c)), L + \mu_x - c\right\} \\ &= L + \mu_x - c + \max\left\{p\Delta - (1-p)^2(L - (\mu_x - c)), 0\right\} \end{aligned}$$

Lemma 10 *Suppose $I > \bar{I}_c^{FB}$ such that first best cannot be implemented in the conservative accounting regime. Then, if $p \leq \frac{1}{2}$ then for $(\bar{I}_c^{FB}, I_4]$ it is optimal to increase the face value of debt allocating decision rights to the lender only when $(\mu_x - \Delta, \mu_x - \Delta)$, $(\mu_x - \Delta, \mu_x)$, and $(\mu_x - \Delta, \mu_x + \Delta)$. For $(I_4, L + \mu_x - c]$, it is optimal to extend the lender's decision rights to all states of the world while setting the face value to $F = L + \mu_x - c$ ($(I_4, L + \mu_x - c] \neq \emptyset$ iff $\frac{p}{(1-p)^2} < \frac{L - (\mu_x - c)}{\Delta}$). If $p > \frac{1}{2}$, then for $(\bar{I}_c^{FB}, L + \mu_x - c]$ it is optimal to extend the lender's decision rights to all states of the world while setting the face value to $F = L + \mu_x - c$. For $(L + \mu_x - c, I_4]$, it is optimal to increase the face value of debt allocating decision rights to the lender only when $(\mu_x - \Delta, \mu_x - \Delta)$, $(\mu_x - \Delta, \mu_x)$, and $(\mu_x - \Delta, \mu_x + \Delta)$ ($(L + \mu_x - c, I_4] \neq \emptyset$ iff $\frac{p}{(1-p)^2} > \frac{L - (\mu_x - c)}{\Delta}$). For $I > \max\{I_4, L + \mu_x - c\}$, the lender must always be allocated the decision rights and the face value must exceed $L + \mu_x - c$.*

Proof. (Recall that $\bar{I}_c^{FB} = \max\{I_1, I_2\}$ where I_1 is obtained if the lender is only allocated decision rights when $(\mu_x - \Delta, \mu_x - \Delta)$ but the face value is $F = 2(\mu_x + \Delta - c)$ while I_2 is obtained if the lender is allocated decision rights when $(\mu_x - \Delta, \mu_x - \Delta)$, $(\mu_x - \Delta, \mu_x)$, and $(\mu_x - \Delta, \mu_x + \Delta)$ but the face value is limited to $F = L + \mu_x - c$).

Suppose $I > \bar{I}_c^{FB} = \max\{I_1, I_2\}$ such that first best cannot be implemented. Then, we have two options: either to increase the face value F or to increase the lender's decision rights. First, suppose, we keep $F = L + \mu_x - c$ and extend the lender's decision rights, i.e., we allocate the decision rights always to the lender. Then, the lender will only deviate from the efficient liquidation decision when (μ_x, μ_x) in which case he will liquidate one asset. Maximum capital we can raise if we offer $F = L + \mu_x - c$, is $I = F = L + \mu_x - c$ (risk-free). The asset value is

$$\begin{aligned} V_3 &= \frac{p^2}{4}(2L) + p(1-p)(L + \mu_x) + \frac{p^2}{2}(L + \mu_x + \Delta) + (1-p)^2(L + \mu_x) \\ &\quad + p(1-p)(2\mu_x + \Delta) + \frac{p^2}{4}(2\mu_x + 2\Delta) \\ &= 2\mu_x + p\Delta - (1-p(1-p))(\mu_x - L) \end{aligned}$$

Suppose, instead, we increase the face value but the lender is allocated control only when $(\mu_x - \Delta, \mu_x - \Delta)$, $(\mu_x - \Delta, \mu_x)$, and $(\mu_x - \Delta, \mu_x + \Delta)$. The lender will choose to liquidate both assets when $(\mu_x - \Delta, \mu_x - \Delta)$ and $(\mu_x - \Delta, \mu_x)$ and liquidate one asset when $(\mu_x - \Delta, \mu_x + \Delta)$.

$$\begin{aligned} V_4 &= \frac{p^2}{4}(2L) + p(1-p)(2L) + \frac{p^2}{2}(L + \mu_x + \Delta) + (1-p)^2(2\mu_x) + p(1-p)(2\mu_x + \Delta) + \frac{p^2}{4}(2\mu_x + 2\Delta) \\ &= 2\mu_x + p\Delta - 2p\left(1 - \frac{p}{2}\right)(\mu_x - L) \end{aligned}$$

Since

$$\begin{aligned} V_4 &> V_3 \\ 2\mu_x + p\Delta - 2p\left(1 - \frac{p}{2}\right)(\mu_x - L) &> 2\mu_x + p\Delta - (1-p(1-p))(\mu_x - L) \\ 0 &< (1-2p)(1-p) \\ p &< \frac{1}{2} \end{aligned}$$

increasing the face value is preferred when $p < \frac{1}{2}$ while extending the lender's decision rights is preferred when $p > \frac{1}{2}$. The intuition is that extending the decision rights has negative implications when (μ_x, μ_x) in which case he will liquidate one asset even though it is efficient to not liquidate any asset. This occurs with probability $(1-p)^2$. In contrast, increasing the face value has negative implications when $(\mu_x - \Delta, \mu_x)$ in which case the lender will liquidate both assets even though it is efficient to liquidate just one asset. This occurs with probability $p(1-p)$. Hence, if p low increasing the face value is preferred and when p high extending the decision rights is preferred.

The maximum investment that can be supported by increasing the face value is (for $F = 2(\mu_x + \Delta - c)$)

$$\begin{aligned} I_4 &= \frac{p^2}{4}(2L) + p(1-p)(2L) + \frac{p^2}{2}(L + \mu_x + \Delta - c) + (1-p)^2(2\mu_x - 2c) \\ &\quad + p(1-p)(2\mu_x + \Delta - 2c) + \frac{p^2}{4}(2\mu_x + 2\Delta - 2c) \\ &= 2\mu_x - 2c + p\Delta - p^2(L - (\mu_x - c)) + 2p(L - (\mu_x - c)) \\ &= L + \mu_x - c + p\Delta - (1-p)^2(L - (\mu_x - c)) \end{aligned}$$

and the maximum investment that can be supported by extending the lender's decision rights is $I = F = L + \mu_x - c$.

Hence, if $p < \frac{1}{2}$ then for $(\bar{I}_c^{FB}, I_4]$ we increase the face value of the debt. If $\frac{p}{(1-p)^2} \geq \frac{L - (\mu_x - c)}{\Delta}$ then $I_4 \geq L + \mu_x - c$ and hence extending the decision rights instead of increasing the face value

does not increase the amount of capital that can be raised. If $\frac{p}{(1-p)^2} < \frac{L-(\mu_x-c)}{\Delta}$ then $I_4 < L + \mu_x - c$ and hence extending the decision rights instead of increasing the face value is the preferred option in the conservative accounting regime when $I \in (I_4, L + \mu_x - c]$.

If $p > \frac{1}{2}$ then for $(\bar{I}_c^{FB}, L + \mu_x - c]$ we extend the decision rights of the lender. If $\frac{p}{(1-p)^2} \leq \frac{L-(\mu_x-c)}{\Delta}$ then $I_4 \leq L + \mu_x - c$ and hence increasing the face value instead of extending the decision rights does not increase the amount of the capital that can be raised. If $\frac{p}{(1-p)^2} > \frac{L-(\mu_x-c)}{\Delta}$ then $I_4 > L + \mu_x - c$ and hence increasing the face value instead of extending the decision rights is the preferred option in the conservative accounting regime when $I \in (L + \mu_x - c, I_4]$.

For $I > \max\{I_4, L + \mu_x - c\}$, the lender must always be allocated the decision rights and the face value must exceed $L + \mu_x - c$. ■

Lemma 11 *Suppose $I > \bar{I}_f^{FB}$ such that first best cannot be implemented in the conservative accounting regime. Then, if $p \leq \frac{1}{2}$ then for $(\bar{I}_f^{FB}, I_4^*]$ it is optimal to increase the face value of debt allocating decision rights to the lender only when $(\mu_x - \Delta, \mu_x - \Delta)$ and $(\mu_x - \Delta, \mu_x)$. For $(I_4^*, L + \mu_x - c]$, it is optimal to extend the lender's decision rights to $(\mu_x - \Delta, \mu_x + \Delta)$ and (μ_x, μ_x) while setting the face value to $F = L + \mu_x - c$. If $p > \frac{1}{2}$, then for $(\bar{I}_c^{FB}, L + \mu_x - c]$ it is optimal to extend the lender's decision rights to $(\mu_x - \Delta, \mu_x + \Delta)$, and (μ_x, μ_x) . while setting the face value to $F = L + \mu_x - c$. For $(L + \mu_x - c, I_4]$, it is optimal to increase the face value of debt allocating decision rights to the lender only when $(\mu_x - \Delta, \mu_x - \Delta)$ and $(\mu_x - \Delta, \mu_x)$.*

Proof. Suppose $I > \bar{I}_f^{FB}$ such that first best cannot be implemented. Then, we have two options: either to increase the face value F or to increase the lender's decision rights. First, suppose, we keep $F = L + \mu_x - c$ and extend the lender's decision rights, i.e., we allocate the decision rights to the lender when $r_f \leq 2\mu_x$. Then, the lender will only deviate from the efficient liquidation decision when (μ_x, μ_x) in which case he will liquidate one asset. Maximum capital we can raise if we offer $F = L + \mu_x - c$, is $I = F = L + \mu_x - c$ (risk-free). The asset value is

$$V_5 = 2\mu_x + p\Delta - (1 - p(1 - p))(\mu_x - L)$$

Suppose, instead, we increase the face value but the lender is allocated control only when $(\mu_x - \Delta, \mu_x - \Delta)$ and $(\mu_x - \Delta, \mu_x)$. The lender will choose to liquidate both assets when $(\mu_x - \Delta, \mu_x - \Delta)$ and $(\mu_x - \Delta, \mu_x)$ and liquidate one asset when $(\mu_x - \Delta, \mu_x + \Delta)$.

$$V_4 = 2\mu_x + p\Delta - 2p\left(1 - \frac{p}{2}\right)(\mu_x - L)$$

Since $V_4 > V_5$ iff $p < \frac{1}{2}$, increasing the face value is preferred when $p < \frac{1}{2}$ while extending the lender's decision rights is preferred when $p > \frac{1}{2}$.

The maximum investment that can be supported by increasing the face value is (for $F = 2(\mu_x + \Delta - c)$) depends on how Δ and c compare. If $F = L + \mu_x - c$, then the manager liquidates one asset when $(\mu_x - \Delta, \mu_x + \Delta)$ if $\Delta > c$ and liquidates no asset if $\Delta < c$. (because the manager liquidates one asset when $(\mu_x - \Delta, \mu_x + \Delta)$ if $F \leq L + \mu_x + \Delta - 2c$). Suppose $\Delta \geq c$. Then, the maximum investment that can be supported by increasing the face value is (for $F = 2(\mu_x + \Delta - c)$) is I_4 . Suppose $\Delta < c$. Then, the maximum investment that can be supported by increasing the face value is (for $F = 2(\mu_x + \Delta - c)$) is

$$\begin{aligned} I'_4 &= \frac{p^2}{4}(2L) + p(1-p)(2L) + \frac{p^2}{2}(2\mu_x - 2c + (1-\alpha)(L + \mu_x + \Delta - 2\mu_x)) + (1-p)^2(2\mu_x - 2c) \\ &\quad + p(1-p)(2\mu_x + \Delta - 2c) + \frac{p^2}{4}(2\mu_x + 2\Delta - 2c) \\ &= I_4 - \frac{1}{2}p^2(c + \alpha(L - (\mu_x - \Delta))) \end{aligned}$$

Let

$$I_4^* = \begin{cases} I_4 & \text{if } c \geq \Delta \\ I_4 - \frac{1}{2}p^2(c + \alpha(L - (\mu_x - \Delta))) & \text{if } c < \Delta \end{cases}$$

The maximum investment that can be supported by extending the decision rights (for $F = L + \mu_x - c$) is $I = F = 2(\mu_x + \Delta - c)$. Note that extending the decision rights even further does not increase the amount of capital that can be raised. Hence, for $I \leq \max\{I_4, L + \mu_x - c\}$, the conservative regime weakly dominates. For $I > \max\{I_4, L + \mu_x - c\}$, the fair value regime weakly dominates because it has more options in terms of choosing between extending the lender's control rights and increasing the face value. ■

Lemma 12 $I^* \in (\underline{I}, \bar{I})$.

Proof. We want to show that $I^* > \underline{I}$. First suppose $\max\{p\Delta - (1-p)^2(L - (\mu_x - c)), 0\} = 0$ which implies that $L - (\mu_x - c) > \frac{p}{(1-p)^2}\Delta$.

$$\begin{aligned} I^* &> \underline{I} \\ L + \mu_x - c + \max\{p\Delta - (1-p)^2(L - (\mu_x - c)), 0\} &\geq 2(\mu_x - c) + p(1-\alpha)(L - (\mu_x - \Delta)) \\ L + \mu_x - c &\geq 2(\mu_x - c) + p(1-\alpha)(L - (\mu_x - \Delta)) \\ \left(1 - (1-p)^2(1-\alpha)\right)\Delta &\geq (1-p)^2(1-\alpha)(L - \mu_x) \\ \left(1 - (1-p)^2(1-\alpha)\right)\Delta &\geq 0 > (1-p)^2(1-\alpha)(L - \mu_x) \end{aligned}$$

Next, suppose $\max \left\{ p\Delta - (1-p)^2(L - (\mu_x - c)), 0 \right\} > 0$

$$I^* > \underline{I}$$

$$L + \mu_x - c + \max \left\{ p\Delta - (1-p)^2(L - (\mu_x - c)), 0 \right\} \geq 2(\mu_x - c) + p(1-\alpha)(L - (\mu_x - \Delta))$$

$$L + \mu_x - c + p\Delta - (1-p)^2(L - (\mu_x - c)) \geq 2(\mu_x - c) + p(1-\alpha)(L - (\mu_x - \Delta))$$

$$\left(1 - (1-p)^2\right)(L - (\mu_x - c)) + p\alpha\Delta \geq p(1-\alpha)(L - \mu_x)$$

$$\left(1 - (1-p)^2\right)(L - (\mu_x - c)) + p\alpha\Delta \geq 0 > p(1-\alpha)(L - \mu_x)$$

Next, we want to show that $I^* < \bar{I}$. First suppose $\max \left\{ p\Delta - (1-p)^2(L - (\mu_x - c)), 0 \right\} = 0$

$$I^* < \bar{I}$$

$$L + \mu_x - c < 2(\mu_x - c) + p\Delta + 2p\left(1 - \frac{p}{2}\right)(L - (\mu_x - c))$$

$$+ (1-p)(L - (\mu_x - c)) \max \{2(1-p), p\}$$

$$(1-p)^2(L - (\mu_x - c)) < p\Delta + (1-p)(L - (\mu_x - c)) \max \{2(1-p), p\}$$

$$0 < p\Delta + (1-p)(L - (\mu_x - c)) \max \{1-p, -1\}$$

Next, suppose $\max \left\{ p\Delta - (1-p)^2(L - (\mu_x - c)), 0 \right\} > 0$

$$I^* < \bar{I}$$

$$L + \mu_x - c + p\Delta - (1-p)^2(L - (\mu_x - c)) < 2(\mu_x - c) + p\Delta + 2p\left(1 - \frac{p}{2}\right)(L - (\mu_x - c))$$

$$+ (1-p)(L - (\mu_x - c)) \max \{2(1-p), p\}$$

$$\left(1 - (1-p)^2\right)(L - (\mu_x - c)) < 2p\left(1 - \frac{p}{2}\right)(L - (\mu_x - c))$$

$$+ (1-p)(L - (\mu_x - c)) \max \{2(1-p), p\}$$

$$0 < (1-p) \max \{2(1-p), p\}$$

■

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